Constraints on muon-specific dark forces

Savely G. Karshenboim,[1,2] David McKeen,[3] and Maxim Pospelov[4,5]

[1] *Max-Planck-Institut für Quantenoptik, Garching 85748, Germany*
[2] *Pulkovo Observatory, St. Petersburg, 196140, Russia*
[3] *Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[4] *Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia V8P 5C2, Canada*
[5] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada*

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The recent measurement of the Lamb shift in muonic hydrogen allows for the most precise extraction of the charge radius of the proton which is currently in conflict with other determinations based on $e-p$ scattering and hydrogen spectroscopy. This discrepancy could be the result of some new muon-specific force with O(1–100) MeV force carrier—in this paper we concentrate on vector mediators. Such an explanation faces challenges from the constraints imposed by the $g-2$ of the muon and electron as well as precision spectroscopy of muonic atoms. In this work we complement the family of constraints by calculating the contribution of hypothetical forces to the muonium hyperfine structure. We also compute the two-loop contribution to the electron parity-violating amplitude due to a muon loop, which is sensitive to the muon axial-vector coupling. Overall, we find that the combination of low-energy constraints favors the mass of the mediator to be below 10 MeV and that a certain degree of tuning is required between vector and axial-vector couplings of new vector particles to muons in order to satisfy constraints from muon $g-2$. However, we also observe that in the absence of a consistent standard model embedding high-energy weak-charged processes accompanied by the emission of new vector particles are strongly enhanced by $(E/m_V)^2$, with E a characteristic energy scale and m_V the mass of the mediator. In particular, leptonic W decays impose the strongest constraints on such models completely disfavoring the remainder of the parameter space.

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I. INTRODUCTION

The persistent discrepancy of the measured muon $g-2$ and the standard model (SM) prediction at the level of $\sim 3\sigma$ [1] has generated a lot of experimental and theoretical activity in search of a possible explanation. Among the new physics explanations for this discrepancy are weak scale solutions [2] *and* possible new contributions from light and very weakly coupled new particles (see, e.g., Ref. [3]). For the latter case, there must be additional observable effects that involve muons and new forces mediated by light particles.

Recently, a new intriguing discrepancy has emerged since the Lamb shift in muonic hydrogen has been measured at PSI. The 2010–2012 results [4,5] combined with the QED calculations of the same quantity allow for a very accurate extraction of the charge radius of the proton. The result stands in sharp contradiction with the determination of the proton charge radius in electron-proton scattering experiments and from high-precision spectroscopy of "normal" hydrogen and deuterium, as summarized in the CODATA review [6]. The combined discrepancy stands now at more than 7σ, with 5σ discrepancies with H spectroscopy and scattering separately, and therefore should be taken very seriously. Unlike the case of the $g-2$ discrepancy, this latest contradiction *cannot* be a result of new physics at the weak scale.

Broadly speaking, there are several logical pathways toward resolving the present contradiction:

(1) The muonic atom results are obtained by only one group and could contain an unaccounted source of error. However, so far, no credible candidates for a systematic shift on the order of 0.3 meV have been found. Moreover, the measurement of two lines in muonic hydrogen exhibit full self-consistency [4,5]. At the level of accuracy set by the current size of the discrepancy, $\delta E \sim 0.3$ meV, the QED part of the muonic hydrogen Lamb shift calculation is comparatively simple and has been checked by many groups. For a compilation of the related theoretical issues, see Ref. [7].

(2) Strong interactions could affect the Lamb shift in μH via a two-photon polarization diagram. Standard calculations based on a dispersive approach (see, e.g., Ref. [8] for the latest evaluations) show no room for a contribution that could account for the discrepancy. Still, some of the input to these calculations has model dependence built in [9], and exaggerating this dependence to the extreme [10] could hypothetically provide a large frequency shift.

In this case, however, one should expect drastic deviations for the hadronic two-photon effects elsewhere [11] which are not observed. In addition, approaches based on chiral perturbation theory also indicate that the polarizability contribution is not large enough to explain the discrepancy [12]. Therefore, this is also an unlikely proposition.

(3) The problem could lie with the determination of r_p in standard hydrogen. Notice that in order to be consistent with the muonic hydrogen Lamb shift, determinations based on *both* methods, $e-p$ scattering and hydrogen spectroscopy, would have to be incorrect or have overstated precision.

(4) Finally, it is also possible that some "intermediate range" force is responsible for the discrepancy. Should such a new force carrier exist in the MeV–100 MeV mass range, it could potentially affect the μH Lamb shift directly. Constructing a model that would be not immediately ruled out by the existing constraints on dark forces in this range is a difficult challenge [13,14].

Further background information and discussion can be found in the recent review [15].

The search for a resolution to the r_p discrepancy is important because it carries strong implications for the precision of theoretical evaluation of the muon $g-2$. Suppose, for example, that either "unexpected" effects of strong interactions (solution 2 above) or some new physics (solution 4) is responsible for inducing, e.g., a large proton-muon interaction term,

$$\Delta\mathcal{L} \simeq C(\bar{\psi}_\mu\psi_\mu)(\bar{\psi}_p\psi_p), \tag{1}$$

where coefficient the C needs to be $\sim(4\pi\alpha)\times 0.01\ \mathrm{fm}^2$ in order to explain the discrepancy in r_p measurements. This effective interaction is shown on the left of Fig. 1. One can then estimate the typical shift to the muon $g-2$ that this interaction would imply by integrating out the proton, leading to the two-loop effect on the right of Fig. 1. (Other charged hadrons presumably would contribute as well.) Using (1) as a starting point, we perform a simple estimate by rescaling the well-known perturbative formula for the two-loop Higgs/heavy quark contributions to the muon $g-2$ found in, e.g., Ref. [16]. Since we are converting a dimension-6 operator in (1) into the dimension-5 $g-2$ operator, the result is linearly divergent and presumably is stabilized by some hadronic scale Λ_{had}, where neither the coefficient C nor the proton-photon vertex can be considered local. Taking a wide range for Λ_{had}, from a proton mass scale m_p to a very light dynamical scale $\sim m_\pi$, one arrives at the following estimates of a typical expected shift for the muon anomalous magnetic moment,

$$\Delta(a_\mu) \sim -C\times\frac{\alpha m_\mu m_p}{8\pi^3}\times\begin{cases}1.7; & \Lambda_{\mathrm{had}}\sim m_p\\ 0.08; & \Lambda_{\mathrm{had}}\sim m_\pi\end{cases}, \tag{2}$$

which, after inputting the value of C implied by the r_p discrepancy results in

$$5\times 10^{-9} \lesssim |\Delta(a_\mu)| \lesssim 10^{-7}. \tag{3}$$

Clearly, the upper range of this possible shift is enormous while the lower range is still large, on the order of the existing discrepancy in muon $g-2$. It is three times the size of the current estimates for the hadronic light-by-light contributions and 1 order of magnitude larger than the uncertainty claimed for that contribution. These estimates show that if indeed large muon-proton interactions are responsible for the r_p discrepancy one can no longer insist that theoretical calculations of the muon $g-2$ are under control. Thus, a resolution of the r_p problem is urgently needed in light of the new significant investments made in the continuation of the experimental $g-2$ program.

In this paper, we entertain the possibility (solution 4) that a new vector force is responsible for the discrepancy. Our goal is to investigate the status of this vector force in light of the $g-2$ results for the electron and muon and to derive additional constraints from the hyperfine structure of muonium. As we will show, the presence of a parity-violating coupling to the muon is a very likely consequence of such models, and in light of that, we calculate the two-loop constraint on the parity-violating muon-nucleon forces imposed by ultraprecise tests of parity in the electron sector. We believe that our analysis is timely, given the new experimental information that will soon emerge from the measurement of the Lamb shift in muonic deuterium and helium and the new efforts at making the ordinary hydrogen measurements more precise.

Our approach to the new force is purely phenomenological. At the same time, it is important to realize that the embedding of such a new force into the structure of the SM is very difficult, and so far no fully consistent models of such a new interaction have been proposed. (The closest attempt, the gauged μ_R model of Ref. [14], suffers from a gauge anomaly and thus must be regarded as an effective model up to some ultraviolet scale, close to the weak scale.) Therefore, even a phenomenologically successful model that would explain the r_p discrepancy and pass through all

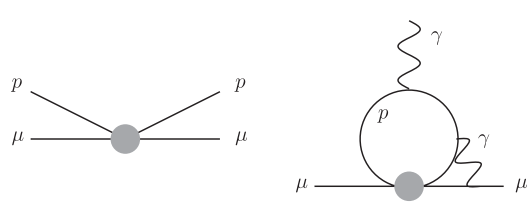

FIG. 1. Left: the effective proton-muon interaction resulting from unexpectedly large QCD effects or new physics that is responsible for the r_p discrepancy. Right: the two-loop contribution to the muon $g-2$ that results from the interaction on the left after integrating out the proton.

additional constraints should be viewed at this point as an exercise which can be taken more seriously only if a credible SM embedding is found, or if the new force hypothesis finds further experimental support.

We illustrate the need for the consistent SM embedding explicitly, by considering the high-energy constraints on the muon-specific vector force. We show that normally not-so-precise observables such as W-boson decay branching fractions become extremely constraining, since they are affected by the muon-specific force because of the breaking of the full SM gauge invariance. We observe that $\sim(E/m_\mu)^2$ enhancement of all charged current effects is a generic price for the absence of a consistent SM embedding, which strongly disfavors such models.

This paper is organized as follows. In the next section, we introduce a model for an intermediate-range force and determine the parameter values suggested by the r_p anomaly. In Sec. III we calculate the one-loop contribution to the muonium hyperfine structure. Section IV contains the calculation of the two-loop transfer of the parity violation in the muon sector to electrons. Section V has a the discussion of the high-energy constraints. Section VI combines all the constraints on the model, and we reach our conclusions in Sec. VII.

II. INTERMEDIATE-RANGE FORCE

We will choose an entirely phenomenological approach and allow for one new particle to mediate the new force between muons and protons. Motivated by dark photon models [17], we assume that the new particle mostly interacts with the electromagnetic current and, in addition, has further vector and axial-vector coupling to muons. The interaction Lagrangian for this choice is given by

$$\begin{aligned}\mathcal{L}_{\rm int} &= -V_\nu[\kappa J_\nu^{\rm em} - \bar\psi_\mu(g_V\gamma_\nu + g_A\gamma_\nu\gamma_5)\psi_\mu]\\ &= -V_\nu[e\kappa\bar\psi_p\gamma_\nu\psi_p - e\kappa\bar\psi_e\gamma_\nu\psi_e\\ &\quad - \bar\psi_\mu((e\kappa + g_V)\gamma_\nu + g_A\gamma_\nu\gamma_5)\psi_\mu + \cdots],\end{aligned} \tag{4}$$

where the last two lines describe interaction of the vector, V, with the relevant fields: electron, muon, and proton. We use positive $e = (4\pi\alpha)^{1/2}$. The constant κ is the mixing angle between the photon and V. It is a safe assumption that this mixing must be small. g_V and g_A are the new phenomenological muon-specific couplings that are introduced in this paper by hand.

The interaction via a conserved current, $\kappa J_\nu^{\rm em}$, allows for a UV completion via kinetic mixing and is totally innocuous. The muon-specific couplings g_V and g_A are much more problematic from the point of view of UV completion and full SM gauge invariance. Notice that in parallel to the kinetic mixing type coupling $V_\nu\kappa J_\nu^{\rm em}$ there exists another "safe" coupling via the baryonic current, $V_\nu(\bar\psi_p\gamma_\nu\psi_p + \bar\psi_n\gamma_\nu\psi_n)$. The reason we suppress it in this paper is because of the extra phenomenological problems it creates, chiefly the additional O(10–100 fm) range force for neutrons—a possibility that is very constrained by neutron scattering experiments. It may look strange that the new force introduced in (4) includes parity violation for muons. In fact, as we will see shortly, the g_A coupling is necessary to cancel the excessive one-loop contribution to the muon $g-2$ generated by the g_V coupling.

Having formulated our starting point with the Lagrangian in Eq. (4), it is easy to present a combination of couplings that alleviates the current r_p discrepancy. Choosing the same sign for κ and g_V/e will create an additional attractive force between protons and muons. It will be interpreted as the difference between charge radii observed in regular and muonic hydrogen:

$$\begin{aligned}\Delta r^2|_{\mu\rm H} - \Delta r^2|_{\rm H} &= -\frac{6\kappa(\kappa + g_V/e)}{m_V^2} + \frac{6\kappa^2}{m_V^2} = -\frac{6\kappa(g_V/e)}{m_V^2}\\ &\simeq -0.06\ {\rm fm}^2 \times \frac{(20\ {\rm MeV})^2}{m_V^2}\\ &\quad \times \frac{\kappa}{(3\times10^{-6})^{1/2}} \times \frac{g_V/e}{0.06}.\end{aligned} \tag{5}$$

Here we explicitly assume that the momentum transfer in the μH system, αm_μ, is smaller than the mass of the mediator, m_V. In the second line, we have normalized the coupling in such a way as to factor out the size of the suggested correction for r_p, which corresponds to a relative shift of the squared radius of 0.06 fm^2. At the same time, we have normalized m_V and κ on their values that correspond to the borderline of the constraint that comes from combining the electron $g-2$ measurement with QED theory and the independent atomic physics determination of α.

Equation (5) makes clear the fact that, given the strong constraints on κ and m_V, only relatively large values for the muon-specific coupling g_V are capable of correcting the r_p anomaly. At the same time, it is clear that the muon $g-2$ value will be in conflict with $g_V \sim 0.06$ *unless* there is a significant degree of cancellation between g_V^2- and g_A^2-proportional contributions. Fortunately, such contributions are of the opposite sign, and the possibility of cancellation does exist. Moreover, since in the limit of $m_V \ll m_\mu$ the contribution of the axial-vector coupling to anomalous magnetic moment a_μ is parametrically enhanced compared to the vector coupling,

$$\begin{aligned}&\frac{\Delta a_\mu(g_A)}{\Delta a_\mu(g_V)} \simeq -\frac{2g_A^2}{g_V^2} \times \frac{m_\mu^2}{m_V^2},\\ &\Rightarrow g_A^{\rm tuned} = \pm g_V \times \frac{m_V}{\sqrt{2}m_\mu},\end{aligned} \tag{6}$$

such a cancellation can be achieved with a relatively small value of $g_A \sim {\rm few}\times10^{-4}$. Such small values of g_A still induce a parity-violating amplitude for muons well above the level suggested by the weak interactions. However, the direct tests of neutral current parity violation for muons at low energy have not been carried out directly [18], and the existence of enhanced parity-violating effects involving muons should be regarded as an opportunity to test these models in the future [19]. We also note that the similar tuning of vector against axial-vector contribution is *not* possible for the electron $g-2$, mainly because of the lack of corresponding enhancement for the axial-vector contribution and excessively strong constraints on new axial couplings for electrons.

Finally, we comment on the possibility that a scalar particle mediates a long-range force. On one hand, the constraints from $g-2$ of the electron are milder because it is reasonable to expect that the coupling would scale proportional to mass, $g_S^e/g_S^\mu \sim m_e/m_\mu$. On the other hand, the coupling to neutrons that would also be a generic consequence of such a model would limit $g_S^{n,p}$ to below the 10^{-3}–10^{-4} level, requiring the coupling to muons be $\sim 10^{-2}$ and larger. As in the vector case, the correction to $g-2$ of the muon is too large. Unlike the vector case, one cannot use the opposite parity coupling to cancel this contribution. This is because the cancellation can be achieved only when the pseudoscalar coupling is approximately the same as the scalar one, $g_P^\mu \simeq g_S^\mu$. This maximally CP-violating case leads to unacceptably large electric dipole moments of neutrons and heavy atoms, even after making a generous allowance for the suppression coming from the two-loop mediation mechanism. Therefore, one needs *extra* light states beyond a single scalar. We therefore abandon this possibility and concentrate on the vector force (4), where only one new particle is introduced.

III. MUONIUM hfs AND NEW PHYSICS

The best experimental result on the muonium 1s hyperfine structure (hfs) interval is [20]

$$\nu(1s, \text{hfs}) = 4463302.776(51)\ \text{kHz}. \tag{7}$$

To compare it with theory, one has to find the leading term, the so-called Fermi energy,

$$\frac{E_\text{F}}{h} = \frac{16\alpha^2}{3\pi}\frac{\mu_\mu}{\mu_\text{B}} cR_\infty \left(1+\frac{m_e}{m_\mu}\right)^{-3} = \frac{16\alpha^2}{3\pi}\frac{(1+a_\mu)m_e}{m_\mu} cR_\infty \left(1+\frac{m_e}{m_\mu}\right)^{-3}, \tag{8}$$

and QED corrections to it [21,22]. The Fermi energy can be presented in terms of fundamental constants in many different ways, but unavoidably, when describing the HFS interaction of a muon (and electron), one has to input either the muon magnetic moment or the muon mass in appropriate units.

Presently, it is the determination of the muon mass (or muon magnetic moment) [20] that dominates the uncertainty of the theoretical prediction [6,21],

$$\nu(1s, \text{hfs}) = 4463302.89(27)\ \text{kHz}, \tag{9}$$

leading to the following comparison of theory and experiment:

$$\frac{\nu^\text{exp} - \nu^\text{th}}{\nu^\text{exp}} = (-2.5 \pm 1.2 \pm 6.1) \times 10^{-8}. \tag{10}$$

The concordance determines room for possible exotic corrections, which we limit at 2σ,

$$\left|\frac{\Delta E_\text{hfs}}{E_\text{hfs}}\right| < 1.24 \times 10^{-7}. \tag{11}$$

One has to remember that calculation of the Fermi energy involves fundamental constants, and any effect of new physics would affect their determination as well (see, e.g., Ref. [23]). Here, we are most interested in the mediator mass range that is higher than the typical momenta of atomic constituents, and much higher than that of macroscopic physics. Therefore, the atomic determination of α and m_e/m_μ are unaffected by new physics. Indeed, the value for m_e/m_μ comes from measurements of the hyperfine structure of muonium in a magnetic field. The magnetic field dependence and the determination of the field through free proton precession produce a value for m_e/m_μ, which corresponds to very low momentum transfers and is not sensitive to short-range effects. Therefore, we can safely proceed by calculating the contributions from the box diagram in Fig. 2.

In the limit $m_\mu \gg m_V$, the calculation is simple, and we adjust the known formula for the Zemach correction [24] in the hydrogen atom to calculate the contribution of the V-mediated force in muonium,

$$\frac{\Delta E_\text{hfs}}{E_\text{hfs}} \simeq \frac{2\alpha m_{e\mu}}{\pi^2}\int\frac{d^3p}{p^4}\left[\frac{G_E(-p^2)G_M(-p^2)}{\mu_\mu} - 1\right], \tag{12}$$

where p^2 is the square of the spacelike momentum, $m_{e\mu}$ is the reduced mass of the muon and electron, and $G_{E(M)}$ are the electric and magnetic form factors. The V-mediated Yukawa contribution can be interpreted as effective $G_{E(M)}$ form factors given by

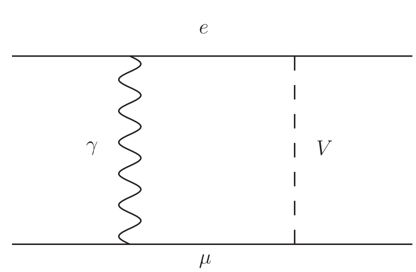

FIG. 2. One-loop diagram (plus all possible crossings) contributing at $1/m_V$ order to the muonium HFS.

$$G_{E(M)} - 1 = \frac{\alpha'}{\alpha} \times \frac{1}{p^2 + m_V^2} = \frac{\kappa(\kappa + g_V/e)}{p^2 + m_V^2}, \quad (13)$$

which defines the exotic coupling α' in our model. Performing the remaining integral, and taking $m_{e\mu} = m_e$, one arrives at a rather simple result:

$$\frac{\Delta E_{\rm hfs}}{E_{\rm hfs}} = \frac{8\alpha' m_e}{m_V} = \frac{8\alpha\kappa(\kappa + g_V/e)m_e}{m_V}. \quad (14)$$

The full result, without taking $m_V \ll m_\mu$, is derived in the Appendix, which exploits the existing more precise calculations of the two-photon contribution due to hadronic vacuum polarization [25]. Either way, comparing Eq. (10) with the r_p-suggested choice of couplings, Eq. (5), one can see that the muonium HFS provides a nontrivial constraint on the model.

IV. TWO-LOOP INDUCED PNC AMPLITUDE

The necessity of introducing an axial-vector coupling results in stronger-than-weak amplitudes for parity nonconservation (PNC) effects involving muons. However, since there are no direct constraints on neutral current PNC with muons at low energy, we are led to consider the two-loop mediation mechanism shown in Fig. 3 that transfers parity violation from the muon to the electron sector. Typically, two-loop corrections to PNC amplitudes are not expected to be large. However, because in our model we start with an effective four-fermion $\bar\psi_p\gamma^\nu\psi_p\bar\psi_\mu\gamma_\nu\psi_\mu$ interaction with a coupling (of mass dimension -2) that is much larger than G_F while the precision in measuring the weak charge is better than 1%, one can expect a reasonably strong constraint despite the two-loop suppression.

Currently, the most precise experimental determination of the PNC amplitude for ^{133}Cs [26] is supplemented by high-accuracy atomic calculations [27,28] that give a very

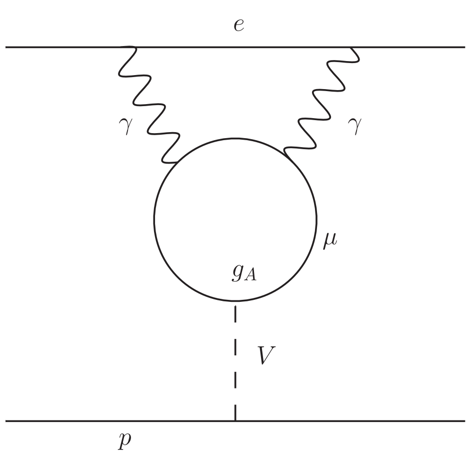

FIG. 3. Two-loop diagram with the closed muon loop contributing to the atomic PNC amplitude. This diagram does not decouple in the large m_μ limit.

good agreement of experiment with the SM. For this paper, we shall adopt the bound on new physics contribution to the weak charge of cesium nucleus at the 2σ level following the latest theoretical determination [28],

$$|\Delta Q_W| < 0.86. \quad (15)$$

Crucially, the $V\gamma\gamma$ vertex generated by the muon loop does not decouple in the limit of $m_\mu \to \infty$, because of the properties of the fermionic triangle diagram [29]. Moreover, because of what can be viewed as a gauge anomaly, there is a sensitivity to the ultraviolet cutoff $\Lambda_{\rm UV}$. Performing the direct calculation of the two-loop induced V-electron axial-vector coupling in the limit of small momentum transfer and retaining only the logarithmically enhanced contributions, we arrive at the following result:

$$\mathcal{L}_{\rm eff} = V_\mu\bar\psi_e\gamma_\nu\gamma_5\psi_e \times \frac{3\alpha^2 g_A}{2\pi^2}\log\left(\frac{\Lambda_{\rm UV}^2}{m_\mu^2}\right). \quad (16)$$

Without a UV-complete theory, it is then impossible to make a definitive prediction. We note, however, that the simplest way to cut off the logarithm is to extend the model to τ leptons and take $g_{A\mu} = -g_{A\tau}$. In that case the answer for the new physics contribution to the electron-proton parity-violating interaction and the corresponding effective shift of the weak charge of ^{133}Cs takes the following form:

$$\mathcal{L}_{\rm eff} = \bar\psi_e\gamma^\nu\gamma_5\psi_e\bar\psi_p\gamma_\nu\psi_p \times \frac{e\kappa}{m_V^2} \times \frac{3\alpha^2 g_A}{2\pi^2}\log\left(\frac{m_\tau^2}{m_\mu^2}\right),$$

$$\Delta Q_W = \frac{12\sqrt{2}\alpha^3}{\pi}\log\left(\frac{m_\tau^2}{m_\mu^2}\right)\frac{\kappa(g_A/e)}{G_F m_V^2}. \quad (17)$$

Substituting typical values for the parameters of the model, we arrive at the following shift of the weak charge:

$$|\Delta Q_W| \simeq 1.4 \times \left(\frac{Z}{55}\right)\frac{\kappa(|g_A|/e)}{2.5\times 10^{-6}}\left(\frac{10\ \text{MeV}}{m_V}\right)^2. \quad (18)$$

While the contribution to Q_W can be either positive or negative, we do not keep track of the sign of g_A since the required value for g_A to satisfy $(g-2)_\mu$ can be of either sign, c.f. Eq. (6).

Although atomic parity violation as well as PNC experiments with electron scattering can potentially constrain the size of g_A, there is also a question of how to search for enhanced PNC involving muons directly. References [14,18,19] have pointed out that polarized muon scattering and muonic atoms can be used for these purposes. Here we would like to remark that an alternative way of searching for neutral current PNC with muons is polarized electron scattering with muon pair production, $e_{L(R)} + Z \to e + Z + \mu^+\mu^-$. Parity-violating V-exchange amplitudes pictured in Fig. 4 interfere with the QED diagrams, leading

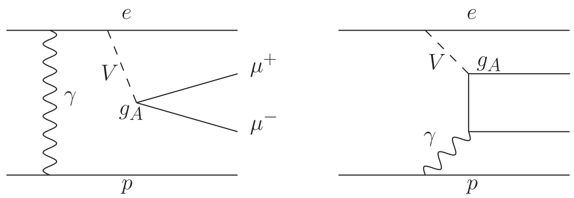

FIG. 4. Typical representatives of muon pair production by electron-proton collision due to a new force. The parity violation will come about due to the presence of the g_A coupling in the interference with the pure QED diagrams.

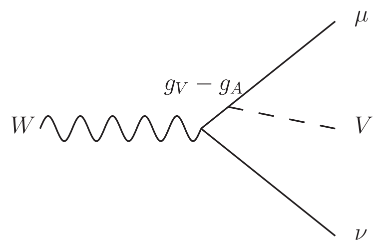

FIG. 5. Diagram that leads to the decay $W \to \mu\nu V$.

to an asymmetry in the muon pair-production cross section by the longitudinally polarized electrons,

$$\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \propto \kappa(g_A/e). \tag{19}$$

While the rate for such a process is rather low, new high-intensity polarized electron beam facilities can conceivably be used to search for such an effect.

V. HIGH-ENERGY CONSTRAINTS

So far, we have dealt with the low-energy observables that are only mildly sensitive to the fact that the effective Lagrangian (4) at a generic point in $\{g_V, g_A\}$ parameter space does not respect the full gauge invariance of the SM. Specifically, we have insisted that the muon neutrinos are uncharged under the new force, due to the fact that their interactions are well known and do not have any room for $O(G_F)$ new physics effects, let alone stronger-than-G_F effects as suggested by the r_p discrepancy. We also do not assume any direct coupling of V to W bosons other than via the kinetic mixing κ. It is then clear that the SM *charged* current processes accompanied by the emission of the light vector boson V from the muon line will be drastically different from a similar process with an emission of a photon. In particular, the interaction of the longitudinal part of the V boson will be enhanced with energy due to the absence of the conservation of the corresponding current. As pointed out in Refs. [30–32], direct production of V from muons in $K \to \mu\nu V$ decays can be enhanced by a factor of m_μ^2/m_V^2 for the $V + A$ current, and even more for the $V - A$ current. In the latter case, it is advantageous to study very high-energy processes (see, e.g., Ref. [33]), where the enhancement can scale as $(\text{Energy})^2/m_V^2$.

One of the best known charged current processes is the leptonic decay of the W boson. When $g_V \neq g_A$ (in other words, when the coupling of the V boson to the left-handed muon is not zero), the decay $W \to \mu\nu V$, shown in Fig. 5, will be enhanced by m_W^2/m_V^2, with the onset of an effectively strong coupling when $(g_V - g_A)m_W/m_V \gtrsim 1$. Since this parameter is indeed larger than 1 for the interesting part of parameter space, one should expect a very strong constraint on the model. Carrying out explicit calculation in the limit of $g_V \gg g_A$, as implied by $(g-2)_\mu$, and to leading order in m_V/m_W and m_μ/m_W, we arrive at

$$\Gamma(W \to \mu\nu V) = \frac{g_V^2}{512\sqrt{2}\pi^3} \frac{G_F m_W^5}{m_V^2} = 1.74\ \text{GeV} \left(\frac{g_V}{10^{-2}}\right)^2 \left(\frac{10\ \text{MeV}}{m_V}\right)^2. \tag{20}$$

Because of the prompt decay of V to an electron-positron pair, and the small value of m_V, this decay will be similar to $W \to \mu\nu\gamma$. In any case, the additional channel leads to the increase of the total W width. The contribution in Eq. (20) should be compared against the current experimental value for the W width, dominated by measurements at the Tevatron [34],

$$\Gamma_W = 2.085 \pm 0.042\ \text{GeV}. \tag{21}$$

Given the agreement of this with SM expectations for $W \to \ell\nu$ and $W \to$ hadrons, we limit the contribution of the $\mu\nu V$ mode to the W width to twice its error, leading to a branching

$$\mathcal{B}(W \to \mu\nu V) < 4.0\% \tag{22}$$

at 2σ. This translates to a limit on the coupling of V to muons of

$$g_V < 2.2 \times 10^{-3} \left(\frac{m_V}{10\ \text{MeV}}\right). \tag{23}$$

It is clear that a large correction to W decay is an example of strong high-energy constraints resulting from the lack of the consistent SM embedding of the starting point in Eq. (4). There are other processes that can be equally problematic for such models. For example, insertion of the virtual V line into the $\mu\nu$ loop in the W self-energy diagram will result in the shift of m_W and will impact the very precisely measured ρ parameter of the electroweak theory. Because of the lack of the full SM gauge invariance, one should expect a powerlike sensitivity to the UV cutoff in such a theory, which is an even stronger enhancement than m_W^2/m_V^2. Thus, indeed, these examples show an utmost need for a consistent SM embedding at the level of the very starting point (4).

VI. COMBINATION OF ALL CONSTRAINTS

Having performed the required calculations of the muonium HFS, atomic PNC, and W decays, we are now ready to compile the constraints on the parameters of our model. We separate all constraints into low-energy and high-energy ones.

Addressing the low-energy constraints first, it is useful to recall that our model has four parameters, $\{m_V, \kappa, g_V, g_A\}$, which enter in the observables in the following combinations:

$$a_e[m_V, \kappa^2]; \quad a_\mu[m_V, (e\kappa + g_V)^2, g_A^2];$$
$$\Delta r_p^2[m_V, \kappa g_V]; \quad \Delta E_{\rm hfs}[m_V, \kappa(e\kappa + g_V)];$$
$$\Delta Q_W[m_V, \kappa g_A]; \quad \Delta E_{\mu{\rm Mg(Si)}}[m_V, \kappa(e\kappa + g_V)]. \tag{24}$$

The last entry here is the constraint imposed by the agreement of the measured $2p - 3d$ transition frequencies in muonic magnesium and silicon with the corresponding QED predictions [35].

Besides the indirect constraints on the model via effects induced by virtual V, there are, of course, direct constraints from the production of V with subsequent decay into e^+e^- pairs, either from e^+e^- colliders or in experiments with fixed targets. Thus, searches for unexpected spikes in the invariant mass of pairs impose additional constraints on κ. The latest compilations [36] show that below m_V of 40 MeV, which is the region of the most interest for us, $g - 2$ of the electron still provides the dominant limits.

To present our results in the most concise form, we choose to saturate the constraint coming from $g - 2$ of the electron combined with atomic determination of α. Taking the 2σ limit on the maximal deviation of a_e (see, e.g., Ref. [37]), we arrive at the maximum allowed κ for a given value of m_V, Currently, this constraint is given by

$$|\Delta a_e| \le 1.64 \times 10^{-12} \Rightarrow |\kappa^{\rm max}| = 1.8 \times 10^{-3} \frac{m_V}{20\ {\rm MeV}}. \tag{25}$$

The latter equation is valid in the scaling regime $m_V \gg m_e$, but we use the full expression in our numerical treatment.

Using this value of $\kappa^{\rm max}$, we determine the required value of g_V that solves the Δr_p^2 discrepancy according to Eq. (5). Specifically, we require that the new physics effect interpreted as $\Delta r^2|_{\mu {\rm H}} - \Delta r^2|_{\rm H}$ is bounded by 2σ of the CODATA value,

$$-0.081\ {\rm fm}^2 \le \Delta r^2|_{\mu{\rm H}} - \Delta r^2|_{\rm H} \le -0.045\ {\rm fm}^2. \tag{26}$$

This creates the preferred value for g_V, pictured as the upper shaded band with solid borders in Fig. 6. For definiteness we take κ to be positive and for our numerical treatment do not assume $\alpha m_\mu \ll m_V$.

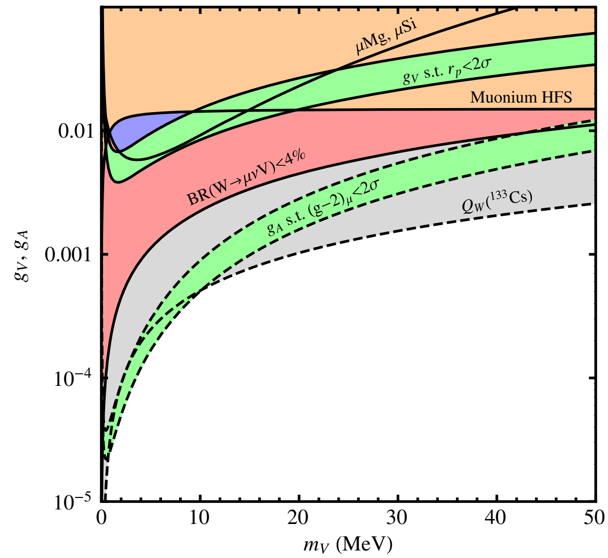

FIG. 6 (color online). Parameter space of the model, when κ is chosen as a function of m_V to saturate the a_e constraint. Solid curves are limits on g_V, while dashed ones are limits on g_A. The upper green shaded band shows the range of values of g_V that alleviate the r_p discrepancy. The lower green shaded band shows values of g_A required so that $(g-2)_\mu$ theory and experiment agree to 2σ, given values of g_V in the upper band. We show constraints on g_V from muonium HFS, muonic Si and Mg, and $W \to \mu\nu V$ decays (solid curves) and on g_A from PNC in ^{133}Cs (dashed curve).

As already stated, such values of g_V are in contradiction with the muon $g - 2$ constraints if $g_A = 0$. Requiring the axial-vector and vector contributions to cancel within the 2σ band around the experimental mean,

$$1.27 \times 10^{-9} \le \Delta a_\mu(g_V + e\kappa) + \Delta a_\mu(g_A) \le 4.47 \times 10^{-9}, \tag{27}$$

we plot the required values of $|g_A|$ as the lower shaded band with dashed borders in Fig. 6.

As expected, rather small values of the axial-vector couplings, $g_A \ll g_V$, are capable of adjusting the muon $g - 2$. However, it must be noted that, despite the possibility of cancellation, the values of g_A are finely tuned to the values of g_V. In other words, to every point in the upper band on Fig. 6, there is exactly one in the lower band in correspondence. The degree of fine-tuning is relatively modest at low values of m_V (e.g., ∼5% at $m_V = 3$ MeV) but quickly becomes rather extreme as m_V is increased (∼1 part in 1000 at $m_V = 30$ MeV).

Besides the g_V and g_A bands, Fig. 6 also shows three low-energy exclusion lines: (1) the atomic PNC constraint on g_A, (2) the muonium HFS constraint on g_V, and (3) the combination of muonic Mg and Si constraints on g_V.

The muonium HFS and the atomic PNC constraints are given by Eqs. (11) and (15), respectively, while for the muonic Si and Mg, we take the weighted mean of the results from Ref. [35] and allow for a 2σ deviation,

$$\left|\frac{\Delta E_{3d-2p}}{E_{3d-2p}}\right| < 6.2\times 10^{-6}. \quad (28)$$

The muonium HFS constraint proves to be rather stringent and disfavors all otherwise acceptable values of m_V above 25 MeV. Notice that, while $\Delta E_{\rm hfs}$ scales inversely proportional to m_V, the constraint line is nearly horizontal because of the $\kappa \propto m_V$ choice from Eq. (25). Muonic Mg and Si are an important constraint, recognized by many groups before [13,14]. It reduces the allowed parameter quite significantly but is unable to close it. (Around $m_V \sim 10$ MeV, for example, the model can still be consistent with all the constraints at $\sim 2\sigma$.) The atomic PNC constraint is also very sensitive to g_A, disfavoring all solutions with $m_V > 10$ MeV. We conclude, though, that the combination of all low-energy constraints cannot decisively exclude the new muonic vector force solution to the r_p puzzle.

However, the addition of the high-energy constraints changes the story: Fig. 6 shows that the $W\to\mu\nu V$ constraint is a factor of a few below the g_V band over the entire parameter range of interest. This is a direct consequence of choosing a zero coupling of V to neutrinos, W bosons, and $g_V \neq g_A$.

VII. DISCUSSION AND CONCLUSIONS

By combining all the constraints, we can assess the phenomenological status of the model with a new "dark photon"-type vector force with additional couplings to the muon. Our main conclusion is that the model designed to "remove" r_p anomaly by adjusting g_V and g_A couplings survives current generation of the low-energy constraints but fails the high-energy tests because of the lack of a consistent SM embedding. The $g-2$ of the muon also requires some fine-tuning. The tuning is minimized in the mass region of very light, $m_V < 10$ MeV, mediators, and it is logical to conclude that this is the preferred mass range for the model.

We provide some further comments below:

(i) The biggest challenge for models of the kind considered here is their embedding into the SM. In particular, the $g_V \gg g_A$ case can be interpreted as mostly $R+L$ coupling. The presence of a significant left-handed component implies large couplings of neutrinos to a new force, which is incompatible with the strength of the force considered here. Therefore, these types of models, unfortunately, remain rather artificial: because of their inability to deal with neutrinos, they are subject to strong high-energy constraints due to $(\mathrm{Energy})^2/m_V^2$ enhancement. The gauged right-handed muon model of Ref. [14] has the least number of pathologies and is not constrained by high-energy processes as $g_V = g_A$, but it requires additional contributions to allow for stringent tunings of the muon $g-2$ and the atomic PNC.

(ii) The fact that muonic HFS turns out to be rather constraining is encouraging, given the fact that a new generation of experiments is being planned.

(iii) The results of the muonic deuterium Lamb shift measurements are about to be released soon by the same group that measured μH. Together with the isotopic shift constraints and accurate theoretical calculations of deuterium polarization, this measurement will be able to shed some additional light on the internal self-consistency of the results. In the speculative world of new physics models, it will provide extra invaluable information on whether an additional coupling of new forces to neutrons is warranted. In particular, it will clarify whether one should revisit constraints on new scalar-mediated forces.

(iv) It is worth emphasizing that the new muon-proton scattering experiment at PSI, MUSE [15], may not detect the presence of the new muon-specific force if the mediator mass is small. Indeed, the experiment will use a momentum transfer of O(100) MeV, which is larger than the preferred mediator mass range. Consequently, the measured charge radius of the proton in the muon-proton scattering may not differ from the $e-p$ result *despite* the possible presence of a new force.

(v) Direct production of new particles with their subsequent decay to electron-positron pairs may be efficiently searched with the muons in the initial state. For example, a careful study of $K\to\mu\nu e^+e^-$ may reveal unexpected peaks at low invariant mass of the pair. A new experiment searching for $\mu\to 3e$ decay [38] will also have capabilities of probing $\mu\to e\bar{\nu}\nu V$ decays.

We believe that future progress in gaining understanding of the r_p problem will come from experiment. Besides the previously mentioned results with the muonic deuterium and the ongoing experiment with muonic helium, one should pay close attention to improvements in experiments with spectroscopy of ordinary hydrogen. If subsequent muonic experiments show no particular anomalies, while the new results with ordinary hydrogen reinforce the r_p problem, it may be worth checking the idea of the electron-specific force, that creates a small amount of *repulsion* between an electron and a proton, and have a mass of the mediator in the range between αm_e and m_e. In this case, one can avoid the constraints from $g-2$ of the electron, as the required scale of couplings will be tiny. But at the same time, the question of consistent embedding of such a new force into the SM will still remain, and such models will face the very same difficulties as "muonic forces" discussed in this work. In addition, sub-MeV mediator masses are also subject to very strong constraints from cosmology and astrophysics.

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APPENDIX: FULL ANSWER FOR MUONIUM HFS

The contribution of an additional vector force to muonium HFS can be extracted from the previous calculations that employed a massive vector in the evaluation due to hadronic vacuum polarization contribution [25]. The contribution of a massive vector reads

$$\frac{\Delta E_{\rm hfs}}{E_{\rm hfs}} = 2\frac{\alpha'}{\pi}\frac{m_e}{m_\mu}K_{\rm Mu}(s), \quad \text{(A1)}$$

where $K_{\rm Mu}(s)$ is discussed below in detail. Here

$$s = m_V^2,$$

and α' is the coupling for the new vector force.

In the limit

$$\frac{m_V}{2m_\mu} \ll 1,$$

one finds

$$K_{\rm Mu}(s\to 0) \to \frac{4\pi m_\mu}{m_V}, \quad \text{(A2)}$$

leading to the result of Eq. (14).

The kernel $K_{\rm Mu}(s)$ was investigated in Ref. [25]. We are interested in m_V around 10 MeV, and thus corrections to the leading term of order m_e/m_V and m_V/m_μ should be under control. The full expression for the kernel is [25]

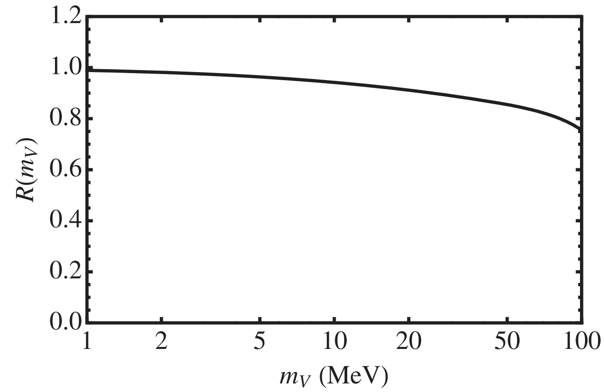

FIG. 7. $R(m_V)$ as defined in Eq. (A5) over the relevant range of m_V.

$$K_{\rm Mu}(s) = -\left(\frac{s}{4m_\mu^2}+2\right)\sqrt{1-\frac{4m_\mu^2}{s}}\log\frac{1+\sqrt{1-\frac{4m_\mu^2}{s}}}{1-\sqrt{1-\frac{4m_\mu^2}{s}}} + \left(\frac{s}{4m_\mu^2}+\frac{3}{2}\right)\ln\frac{s}{m_\mu^2}-\frac{1}{2}. \quad \text{(A3)}$$

Analytically continuing to $s < 4m_\mu^2$, this becomes

$$K_{\rm Mu}(s) = 2\left(\frac{s}{4m_\mu^2}+2\right)\sqrt{\frac{4m_\mu^2}{s}-1}\tan^{-1}\sqrt{\frac{4m_\mu^2}{s}-1} + \left(\frac{s}{4m_\mu^2}+\frac{3}{2}\right)\ln\frac{s}{m_\mu^2}-\frac{1}{2}. \quad \text{(A4)}$$

Normalized on its value at $m_V = 0$, we define the correction factor $R(m_V)$,

$$R(m_V) = \frac{K_{\rm Mu}(s=m_V^2)}{K_{\rm Mu}(0)} = \frac{m_V}{4\pi m_\mu}\times K_{\rm Mu}(s=m_V^2). \quad \text{(A5)}$$

Equation (14) is then generalized to

$$\frac{\Delta E_{\rm hfs}}{E_{\rm hfs}} = \frac{8\alpha' m_e}{m_V}R(m_V), \quad \text{(A6)}$$

at arbitrary values of m_V/m_μ. Over the entire mass range of interest, $R(m_V)$ varies from unity by less than about 20%, as shown in Fig. 7.

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