Probing *CP* violation with the first three years of ultrahigh energy neutrinos from IceCube

Animesh Chatterjee,^{1,*} Moon Moon Devi,^{2,†} Monojit Ghosh,^{3,‡} Reetanjali Moharana,^{1,§} and Sushant K. Raut^{3,||}

¹Harish-Chandra Research Institute, Jhunsi, Allahabad 211 019, India

²Tata Institute of Fundamental Research, Mumbai 400 005, India

³Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

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The IceCube Collaboration has recently announced the discovery of ultrahigh energy neutrino events. These neutrinos can be used to probe their production source, as well as leptonic mixing parameters. In this work, we have used the first IceCube data to constrain the leptonic *CP* violating phase δ_{CP} . For this, we have analyzed the data in the form of flux ratios. We find that the fit to δ_{CP} depends on the assumptions made on the production mechanism of these astrophysical neutrinos. Consequently, we also use this data to impose constraints on the sources of the neutrinos.

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I. INTRODUCTION

The study of cosmic particles, and through them the study of astrophysical phenomena has gradually moved up the energy scale over the last few decades. GeV scale cosmic rays have already been observed in atmospheric neutrino experiments for many years. With the recent detection of ultrahigh energy (UHE) neutrinos at the IceCube detector [1,2], we have emphatically entered the TeV regime. In fact, the PeV energy events seen by IceCube underscore the prospects of neutrino astrophysics with large telescopes.

The first data set announced by the IceCube Collaboration consists of 28 events above 25 TeV, detected over a period of 662 days of live time (May 2011–May 2012 with 79 strings, and May 2012-May 2013 with 86 strings). Seven out of these 28 events are tracks signifying $(\nu_{\mu} + \bar{\nu}_{\mu})$ charged-current (CC) events; while the other 21 are showers indicating either $(\nu_e + \bar{\nu}_e)$ or $(\nu_\tau + \bar{\nu}_\tau)$, or $(\nu_{\mu} + \bar{\nu}_{\mu})$ neutral-current (NC) events [3]. This 4σ detection marked the first discovery of UHE neutrinos. Further data were collected for next one year. For the full 988 days, IceCube collected 37 events, adding one track and seven shower events, with one produced by a coincident pair of background muons from unrelated air showers that cannot be reconstructed with a single direction and energy, to the previously detected 28 events including the neutrino with the highest energy ever detected, 2000 TeV [2].

UHE cosmic rays secondaries like photons and neutrinos carry information about their production (source) and propagation. UHE neutrinos can be produced by a wide array of astrophysical and cosmological processes. It may be possible to probe the mechanism of their production by observing them at neutrino telescopes. Moreover, as these neutrinos travel from their source to the earth, they oscillate. Therefore, one can use this information to constrain mixing in the leptonic sector. In this paper, we have used the first data set from IceCube to address questions about astrophysical neutrino production and neutrino oscillations.

Data from various neutrino oscillation experiments have constrained the mixing angles and mass-squared differences at the $\leq 10\%$ level [4]. However, the value of the *CP* violating phase δ_{CP} is not constrained by the data. The measurement of δ_{CP} is one of the outstanding problems in particle physics today, since *CP* violation in the leptonic sector can be linked to leptogenesis and the matterantimatter asymmetry of the Universe [5]. Measuring the value of δ_{CP} can also provide valuable insights into new physics beyond the Standard Model, since *CP* violation can arise in various models of neutrino mass generation through complex couplings of neutrinos to other particles, or through complex vacuum expectation values [6].

The measurement of δ_{CP} through oscillations of atmospheric/artificially produced neutrinos is difficult using existing technology, and therefore new strategies have to be devised. Many interesting proposals exist in the literature for getting an evidence of *CP* violation and/or measuring δ_{CP} (for a nonexhaustive list, see Refs. [7–9]). In this paper, for the first time, we analyze actual UHE neutrino data to measure δ_{CP} and determine the source of astrophysical neutrinos. In Ref. [7], the author discussed in detail the complementary nature of astrophysical and terrestrial neutrino experiments in *CP* studies. In that paper (and more recently in Ref. [8]), data in the form of flavor ratios of observed neutrinos were used. In this paper, we have analyzed data from IceCube using a similar approach to get a hint about the value of δ_{CP} .

animesh@hri.res.in

moonmoon4u@tifr.res.in

monojit@prl.res.in

reetanjalim@uj.ac.za

sushant@prl.res.in

II. ASTROPHYSICAL SOURCES

The data recorded by the IceCube telescope were the first evidence of extraterrestrial events in the UHE range. These neutrinos can have their origin in extragalactic astrophysical sources like low power gamma-ray burst (GRB) jets in stars [10] or active galactic nuclei (AGN) cores [11]. (Note, however, that based on the data collected by the photon detectors Fermi, MAGIC, HESS, etc. in the 100 GeV-TeV range, one can predict galactic sources of TeV neutrinos [12].) The energy of the 28 detected neutrino events are in the range 25–2000 TeV. By tracing the hadronic origin [13] of these events, one can estimate the proton energies at their sources to be within 0.5-40 PeV. Supernova remnants (SNRs), AGNs, GRBs and other astrophysical sources can accelerate protons to such energies (and above) by the Fermi acceleration mechanism. The interactions of these protons with soft photons or matter from the source can give UHE neutrinos through the following process: $p\gamma$, $pp \rightarrow \pi^{\pm}X, \ \pi^{\pm} \rightarrow \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu}), \ \mu^{\pm} \rightarrow e^{\pm}\bar{\nu}_{\mu}(\bar{\nu}_{\mu})\nu_{e}(\bar{\nu}_{e})$ [14] with a flux ratio of ϕ_{ν_e} : ϕ_{ν_u} : $\phi_{\nu_\tau} = 1:2:0$ (known as πS process). Some of the muons, due to their light mass, can get cooled in the magnetic field quickly resulting in a neutrino flux ratio of 0:1:0 (µDS process). K mesons, produced from $p\gamma$ interactions with a cross section two orders of magnitude less than pions, will cool in the magnetic field of the source at higher energies compared to the pions. $K^+ \rightarrow \mu^+ \bar{\nu^{\mu}}$ is the dominant channel of neutrino production from cooled pions, with a branching fraction of 63%, and with the same flux ratio as the pion decay [15]. The $p\gamma$ interaction also produces high energy neutrons which would decay as $n \rightarrow p + e^- + \bar{\nu}_e$ to antineutrinos [16] with the flux ratio of 1:0:0 (nS process). The relative contribution of each channel depends on different parameters of the astrophysical source like the magnetic field, the strength of the shock wave and density of photon background [17]. Apart from neutrinos these processes also produce high energy photons inside the source. Correlation of high energy photons with the UHE neutrinos can be considered as a signature of hadronic production inside the source. For example, a TeV neutrino can have an accompanying TeV photon at the source. However due to attenuation in the background radiation during propagation, PeV photons will have typical mean free path ~ 10 kpc [18]. Thus, the associated photons of TeV neutrinos from extragalactic sources cannot reach earth.

III. ANALYSIS

The main sources of astrophysical neutrinos in the energy range 10 to 1200 TeV are the π S, μ DS and *n*S channels. However, the exact fraction of events in the detector from each of these sources is not known. Therefore, we have introduced relative fractions k_1 , k_2 and k_3 for these three sources, respectively, which are

treated as free parameters in the problem subject to the normalization constraint $\sum k_i = 1$. In this paper, we have not considered any other subdominant mode of neutrino production.

Neutrinos oscillate during propagation, and our aim is to observe these oscillations. Given that the value of E/L for such neutrinos is very large compared to the mass-squared differences between the neutrino mass states, we can only observe the average oscillation probability. Therefore, the probabilities take the simple form:

$$P(\nu_{\alpha} \to \nu_{\beta}) \equiv P_{\alpha\beta} = \sum_{i} |U_{\alpha_{i}}|^{2} |U_{\beta i}|^{2}.$$
 (1)

It is worth emphasizing that this oscillation probability depends only on the mixing angles and *CP* phase, but not on the mass-squared differences. Therefore, unlike in beam-based experiments where knowledge of the mass hierarchy is essential for *CP* sensitivity [19], in this case we can (at least in principle) detect *CP* violation without suffering from the hierarchy degeneracy. Also note that $P_{\alpha\beta} = P_{\beta\alpha}$, therefore the probability can only be an even function of δ_{CP} . As a consequence, we can treat neutrino and antineutrino oscillations on an equal footing. Another consequence of this is that every value of δ_{CP} allowed by the data will be accompanied by a degenerate solution $-\delta_{CP}$.

The distinction between tracks (which we assume to be ν_{μ} CC events) and showers (which we assume to be ν_{e} or ν_{τ} , or ν_{μ} NC events) is quite clear in the IceCube detector. We have folded the relative initial fluxes with the oscillation probabilities to get the relative number of events at the detector. Separation of muon events into CC and NC has been done using the ratio of the cross sections at the relevant energy [20]. We have done a simple analysis using the total events, instead of binning the data in energy and angle. Since the probability is almost independent of energy, this simplification is not expected to affect the analysis. This also allows us to neglect the effect of energy resolution. In Ref. [1], the number of background events in the IceCube data set is estimated to be $10.6^{+5.0}_{-3.6}$. Of these, 6.0 ± 3.4 are expected to be veto penetrating atmospheric muons and $4.6^{+3.7}_{-1.2}$ are from the atmospheric neutrino background above energy 10 TeV. The background assumed by IceCube could be an overestimation [21], since (a) it has been estimated by extrapolating data and (b) for atmospheric neutrinos the background has been calculated from 10 TeV while the events have been detected with lowest energy, nearly 28 TeV. Therefore, we have used an estimate of 3 background atmospheric muon tracks and 3.4 (the lower limit) background atmospheric neutrinos. IceCube has predicted a total of 8.4 ± 4.2 muon events and $6.6^{+5.9}_{-1.6}$ atmospheric neutrinos [2] including the next set of neutrino events detected for the period of 988 days. Using the same analysis method we have taken the lowest limit of

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the backgrounds for our calculation. We have separated the background atmospheric neutrinos into tracks and showers using the same cross sections as mentioned earlier. These background events are subtracted from the data set in our analysis. The neutrino background flux ratio has been included as 0:1:0, (which is close to 0.05:1:0 estimated in Ref. [22]) since TeV range muons will penetrate all the way through the atmosphere.

In Refs. [7,8,23,24], the authors have proposed the use of the variable $R = N_{\mu}/(N_e + N_{\tau})$ for the study of *CP* violation with astrophysical sources, where N_{α} is the flux of $\nu_{\alpha} + \bar{\nu}_{\alpha}$ at the detector. This variable helps by eliminating the overall source and detector-dependent normalization. Moreover, as studies of the up/down ratio as well as data/MC ratio in atmospheric neutrinos have shown, taking ratios of event rates can also reduce the effect of systematics [25]. For our study, we have constructed a similar quantity $\rho = N_{\text{track}}/N_{\text{shower}}$, with the flavor compositions of the track and shower events as mentioned before.

We have constructed the quantity ρ^{data} using the IceCube data, and calculated ρ^{theory} for a certain value of δ_{CP} as described above. Background events are subtracted from the data, as mentioned above. The statistical χ^2 is then computed using the Gaussian definition

$$\chi^2(\delta_{CP}) = \left(\frac{\rho^{\text{data}} - \rho^{\text{theory}}}{\sigma_{\rho}}\right)^2,\tag{2}$$

where $\sigma_{\rho} = \sqrt{\rho^{\text{data}}(1-\rho^{\text{data}})/N}$, with *N* being the number of data points [26]. We have incorporated systematic effects using the method of pulls, with a systematic error of 5%. Note that we have marginalized the $\Delta \chi^2$ over the mixing angles $(\theta_{23}, \theta_{13}, \theta_{12})$ within the ranges $\theta_{23} = 35^\circ$ to 55° , $\sin^2 2\theta_{13} =$ 0.085 to 0.115 and $\theta_{12} = 30^\circ$ to 36° , respectively. The priors added are $\sigma(\sin^2 2\theta_{13}) = 0.01$, $\sigma(\sin^2 2\theta_{23}) = 0.1$ and $\sigma(\sin^2 \theta_{12}) = 0.0155$.

IV. RESULTS

To demonstrate the impact of the origin of these astrophysical neutrinos on the precision of δ_{CP} , we start with various possibilities, like single, double or a combination of three sources as the origin. First, in Fig. 1 we show the fit to the data as a function of δ_{CP} for the single source assumption. The upper row shows the results of our analysis of the full three-year data set. We have also included the results from analyzing data from only the first two years (lower row) to show the improvement in results from additional data.

In the left panels we assume that all the events seen at IceCube are purely of astrophysical origin whereas in the right panels we include the effect of backgrounds. The latter is the realistic assumption. From these figures we can see that, in case of no background the π S source is favored by the data as compared to the *n*S and μ DS source(though

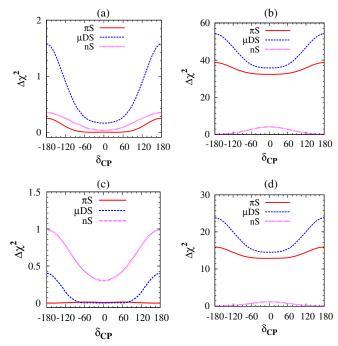


FIG. 1 (color online). Fit to δ_{CP} considering a single source, i.e., $\pi S (k_1 = 1, k_2 = 0, k_3 = 0, \text{ solid red}), \mu DS (k_1 = 0, k_2 = 1, k_3 = 0, \text{ dashed blue}), nS (k_1 = 0, k_2 = 0, k_3 = 1, \text{ dotted magenta}), considering all the events being from astrophysical environment. Panel [A]: Three-year data, without background. Panel [B]: Three-year data, with background. Panel [C]: Two-year data, without background. Panel [D]: Two-year data, with background.$

the sensitivity is quite small, as $\Delta \chi^2$ is always <1.5). However, when we include the background, the scenario changes completely. Pure πS and pure μDS sources are ruled out by the data at > 3σ , while the pure nS source is favored by data, though it is not sufficient to put any significant constraint on the value of δ_{CP} . This has also been pointed out recently in Ref. [21]. This result can be understood qualitatively in the following way. In the second column of Table I we have listed the theoretically calculated values for track by shower ratio for all the three sources keeping the oscillation parameters fixed at their tribimaximal (TBM) values ($\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$, $\sin^2\theta_{12} = \frac{1}{3}$,¹ whereas the third column contains the experimental values of the track by shower ratio without and with backgrounds. We can clearly see that for a pure signal, the track to shower ratio for πS is closest to the data. But the difference becomes quite high when backgrounds are taken under consideration, resulting in a very high $\Delta \chi^2$. A comparison of the upper and lower panels shows a marked increase in $\Delta \chi^2$. This shows the importance of

¹Due to the present nonzero value of θ_{13} , there will be deviations from the TBM values but as shown in Ref. [8], this deviation is quite small.

TABLE I. Theoretical values of track by shower ratio for all the three sources along with experimental values with and without background.

Source	$\frac{N_{\text{track}}}{N_{\text{shower}}}$ (Calculated)	$\frac{N_{\text{track}}}{N_{\text{shower}}}$ (Data)
πS	0.30	
		8/28 = 0.287
		(Without background)
μDS	0.38	-
		0.06 (With background)
nS	0.18	-

additional data in both, excluding certain combinations of sources as well as constraining the value of δ_{CP} .

We have also done an analysis of the events in the energy range 60 TeV < E < 3 PeV considering the three years of IceCube data. This is motivated by the fact that, this energy interval contains the atmospheric muon background less than one. In this energy range there are four track events and 16 shower events with an atmospheric muon background of 0.435 and atmospheric neutrino background of 2.365 [2]. The result is plotted in Fig. 2. In the left panel there is no background and in the right panel background has been considered. From the right panel we can see that we are still getting *n*S as the favored source whereas π S and μ DS sources are excluded at more than 2σ . This is due to the fact that though the atmospheric muon background is less than one in this energy range but due to the presence of atmospheric neutrino background nS is getting preferred over πS source. This can be seen from the left panel where no background is considered. There we can note that the data agree with the final flavor ratio 1:1:1; i.e., it favors the π S source over *n*S source marginally when TBM mixing is assumed. But when we vary the oscillation parameters in their allowed 3σ range then due to the deviation from TBM. *n*S is getting slightly preferred over π S.

In Figs. 3 and 4 we show the fit to the data when neutrinos are coming from two and all three sources, respectively, with equal contributions. These results are for the full data set, and backgrounds have been included in

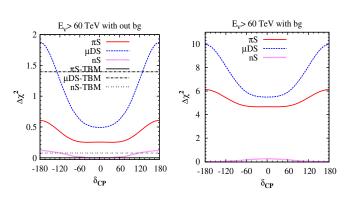


FIG. 2 (color online). Analysis for neutrinos within energy 60 TeV to 3 PeV.

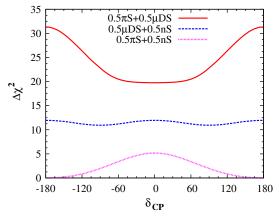


FIG. 3 (color online). Fit to δ_{CP} considering contribution from two sources at a time, in equal proportion, i.e., $k_1 = k_2 = 0.5$, $k_3 = 0$ (solid, red), $k_1 = 0$, $k_2 = k_3 = 0.5$ (dashed, blue), $k_1 = k_3 = 0.5$, $k_2 = 0$ (dotted, magenta) for both old **a** and new **B**.

generating these plots. In Fig. 3, we find only the combination of π S and *n*S neutrinos are allowed at 3σ level. We also see that the *CP* dependence is maximum if the neutrinos come from the combination of π S and μ DS modes. The data may also rule out one-third of δ_{CP} values (approximately -60° to 60°) at $\sim 2\sigma$. The poor sensitivity from *n*S neutrinos is the reason why the combination of π S + μ DS in Fig. 3 has a higher χ^2 than the combinations involving *n*S. When we consider equal contributions from all these channels (Fig. 4), we find that the data favor the first and fourth quadrants of δ_{CP} at 1σ .

We have then performed a check to constrain the astrophysical parameters k_i vs δ_{CP} using the IceCube data, by plotting the allowed countours in the $k_i - \delta_{CP}$ plane. In Fig. 5, we have shown the 2σ (light) and 3σ (dark) contours in the $k_1 - \delta_{CP}$ plane for three fixed values of k_2 . The best-fit point indicated by the data has been marked with a red dot. We see that the data favor a smaller value of k_1 and

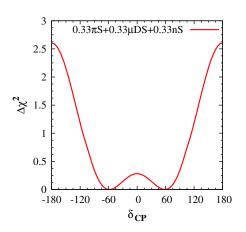


FIG. 4 (color online). Fit to δ_{CP} considering equal contribution from all the sources, i.e., $k_1 = k_2 = k_3 = 0.33$ for both old **a** and new **B**.

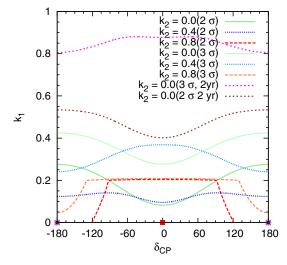


FIG. 5 (color online). Contour plots for allowed region in the $k_1 - \delta_{CP}$ plane, for three representative values of k_2 . The points marked in the respective colors indicate the best-fit point with new IceCube data.

larger values of k_2 and k_3 . Similarly, Fig. 6 shows that for a given value of k_1 , the data disfavor the μ DS process (small value of k_2) but favors the *n*S process (large value of k_3). Likewise, Fig. 7 shows the data favoring the largest possible value of k_3 allowed by the normalization condition. These features can be understood from Fig. 1, where we see that the data prefer the *n*S source. From these contours, we may draw certain contraints on the astrophysical sources most favored. In particular if we obtain a good prior on δ_{CP} from other experiments, then the most favored ratio of k_1 , k_2 and k_3 may be obtained. Alternately, if we obtain a better picture of the sources of the IceCube

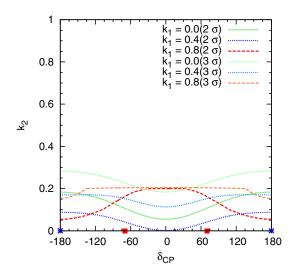


FIG. 6 (color online). Contour plots for allowed region in the $k_2 - \delta_{CP}$ plane, for three representative values of k_1 . The points marked in the respective colors indicate the best-fit point.

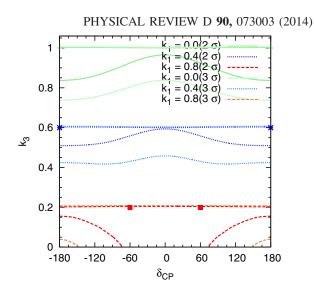


FIG. 7 (color online). Contour plots for allowed region in the $k_3 - \delta_{CP}$ plane, for three representative values of k_1 . The points marked in the respective colors indicate the best-fit point.

events, a refined and constrained range on δ_{CP} would be predicted.

To show the statistical improvement of the three-year data over two-year data, in Fig. 5 we have also plotted the 2σ and 3σ contours for the latter for $k_2 = 0$. Here we can clearly see that for $\delta_{CP} = 0$, three-year data can exclude 73%(91.5%) of k_1 values at $3\sigma(2\sigma)$, whereas the two-year data can only rule out 13%(61%) of K_1 values at $3\sigma(2\sigma)$. For $\delta_{cp} = \pi$ the exclusion percentages are 58%(73%) at $3\sigma(2\sigma)$ for three years and 20%(48%) at $3\sigma(2\sigma)$ for two years. One can understand this qualitatively from the π S curve of Fig. 1 showing a significant improvement in the $\Delta\chi^2$ with three years of data compared to two years.

V. CONCLUSION

In this work, we have analyzed the first IceCube data on TeV-PeV scale neutrinos. We have used the flux ratios of the three neutrino flavors to put constraints on δ_{CP} . We find that the results depend strongly on the source of the neutrinos. After taking into account the effect of backgrounds, we find that the nS source of neutrinos is favored by the data. Depending on the particular combination of sources for these neutrinos, current data can only hint at the allowed region of the δ_{CP} range. However, we have shown that additional data give a remarkable improvement in results, which underlines the importance of future data from IceCube. We have also put constraints on the astrophysical parameters k_1 , k_2 and k_3 that determine which of the modes of neutrino production is more close to the data. In fact if δ_{CP} is measured by other experiments, then IceCube data can be used to determine the production mechanism of these neutrinos. Similar analyses can also be carried out for other parameters related to neutrino physics and astrophysics.

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