PHYSICAL REVIEW D **90,** 067301 (2014)

Closed string thermodynamics and a blue tensor spectrum

Robert H. Brandenberger,[1,*] Ali Nayeri,[2,†] and Subodh P. Patil[3,‡]
[1]*Department of Physics, McGill University, Montréal, Quebec H3A 2T8, Canada*
[2]*Institute for Quantum Studies and Department of Physics, Chapman University, Orange, California 92866, USA*
[3]*Theory Group, CERN, Case C01600, Geneva CH-1211, Switzerland*
(Received 26 March 2014; published 22 September 2014)

The BICEP-2 team has recently reported the positive detection of cosmic microwave background B-mode polarization. Although uncertainties due to galactic dust foregrounds remain, it is a constructive exercise to work out the implications of presuming some part of the detected B-mode signal to be due to primordial gravitational waves. Were a positive detection of a tensor-to-scalar ratio larger than $r \gtrsim \mathcal{O}(10^{-2})$ confirmed, detecting a tilt in the tensor spectrum comparable to that observed for the scalar power spectrum becomes in principle possible. We wish to explore in this brief paper the possibility of there being a blue tilt to the primordial gravitational-wave spectrum. Such a tilt would be incompatible with standard inflationary models, although it was predicted some years ago in the context of a mechanism that thermally generates the primordial perturbations through a Hagedorn phase of string cosmology. By scrutinizing the data with priors informed by a model that is immediately falsifiable, but which *predicts* features that may be favored by the data—namely a blue tensor tilt with an induced and complementary red tilt to the scalar spectrum, with a naturally large tensor-to-scalar ratio that relates to both—we offer a useful straw model against which to test the predictions of single-field inflation.

DOI: 10.1103/PhysRevD.90.067301 PACS numbers: 98.80.Cq

I. INTRODUCTION

The BICEP-2 team recently announced the possible detection of primordial cosmic microwave background (CMB) B-mode polarization, seemingly implying a tensor-to-scalar ratio of $r = 0.2 \pm 0.05$ [1] were dust foregrounds to contribute negligibly. The positive detection of primordial gravitational waves would constitute a major advance for early Universe cosmology, giving us a new diagnostic tool with which to scrutinize models of the very early Universe against observational data. Conventional adiabatic cosmological fluctuations do not predict any B-mode polarization at the linear level in cosmological perturbation theory. Hence in the context of the simplest models, primordial B-mode polarization must be due to gravitational waves.[1]

Although the BICEP-2 Collaboration's analysis took $n_T = 0$ as a prior in its simulated data, we wish to ask whether a suppression of power in the BB angular power spectrum at large angular scales relative to smaller scales might be present in the data, in particular in the B2 × Keck cross correlation function at long wavelengths (which is less sensitive to systematic noise[2]) and in the B2 × B2 correlation function (see e.g. Figs. 2 and 9 in Ref. [1]), although the latter is more susceptible to contamination from foregrounds (see Refs. [5–7] for some concerns recently raised on the latter point). Whether this suppression is statistically significant, and more importantly, present after the proper subtraction of the dust foreground remains to be seen. If it is, it could be interpreted as indicative of a positive tilt of the primordial tensor spectrum at the largest angular scales [8], which would be very hard to interpret in the context of the standard inflationary paradigm of early Universe cosmology (see also Ref. [9] for an analysis of the additional tension between measuring a large r with the small-k scalar power spectrum).

String gas cosmology naturally provides a theoretical basis for considering models with a blue spectrum of gravitational waves, and this paper is concerned with working out the details of the detection of a large tensor-to-scalar ratio for this scenario. Further justification to carefully study these models comes from an observational issue: when interpreted in the context of a simple inflationary model, the large tensor-to-scalar ratio $r = 0.2$ that BICEP-2 favors is in tension with the Planck results [10] which favors a value of $r < 0.11$. This is because gravitational waves lead to a contribution to the angular power spectrum of CMB temperature anisotropies which boosts the small l values relative to the values in the region of the Doppler peak. A blue spectrum of gravitational waves mitigates this problem since it suppresses the small-l

*rhb@physics.mcgill.ca
†nayeri@chapman.edu
‡subodh.patil@cern.ch

[1]Note, however, that beyond the simplest single-field scalar models, there are other sources of B-mode polarization e.g. from cosmic strings [2]. B-mode polarization will also be produced by lensing of E-mode polarization, which in turn is directly generated from cosmological fluctuations, and this B-mode lensing signal has in fact recently been discovered by the South Pole [3] and the Polarbear telescopes [4].

[2]Which we note can only boost the autocorrelation function.

angular power spectrum relative to the spectrum of larger values of l.

Assuming that space-time is described by general relativity and that matter obeys the "weak energy condition," inflation generically predicts a red spectrum of gravitational waves, i.e. $n_T < 0$. This is because the amplitude of the gravitational waves on a scale k is set by the Hubble expansion rate H at the time when that scale exits the Hubble radius, and during inflation $\dot{H} < 0$. For single-field slow-roll models this relation is precisely

$$n_T = -2\epsilon, \tag{1}$$

with $\epsilon \coloneqq -\dot{H}/H^2$. A challenge for standard inflationary cosmology in light of the BICEP-2 data is that the tensor-to-scalar ratio $r = 0.2$ implies a large field excursion over the duration in which the observed modes in the CMB were produced [11]:

$$\Delta\phi \gtrsim M_{\rm pl}. \tag{2}$$

Constructing a model that safely accomplishes this is challenging from the perspective of effective field theory, as field excursions comparable to the cutoff of the theory typically generate large anomalous dimensions for operators that were initially suppressed (by appropriate powers of the cutoff), potentially spoiling the requisite conditions for inflation to occur as it progresses.[3] However, this is not to say that this might not be accomplished in the context of some fundamental theory construction (see Ref. [13] for an interesting claim and Ref. [14] for a counter-claim); within the context of string theory, large field excursions certainly appear to be problematic [15].

In this paper we wish to work through the implications of the BICEP-2 results for a mechanism to generate the primordial perturbations from the thermodynamics of *closed* strings in a quasistatic background, which

(i) naturally generates a large tensor-to-scalar ratio, and
(ii) *predicts* a blue tilt to the tensor spectrum with
(iii) a complimentary red tilt to the scalar spectrum, both of which relate to r.

This construction relies upon a background that consists of a quasistatic initial state in the Einstein frame, whose specific realization can be addressed in the context of particular string constructions (see Refs. [16,17] for some recent attempts), but whose existence we will take for granted in the following as far as the study of fluctuations is concerned, just as one typically does in the context of inflationary cosmology.[4] In fact, this is the very premise of the effective theory of the adiabatic mode [19]—the so-called effective theory of inflation. In what follows, we will first address plausible constructions that could give rise to the requisite background as motivation for the subsequent section—the main focus of this paper—where we argue that the thermodynamics of closed strings in the early Universe can naturally generate a large, blue-titled tensor-mode background.

Our goal is to provide observations with a novel, predictive, and falsifiable model which can inform the formulation of priors when analyzing the data in a manner that is easily contrasted against the predictions of inflationary cosmology. Whether there are hints for a blue tensor tilt in the data is to be viewed as secondary to the goal of providing a "straw model" against which to contrast the predictions of inflation, the scientific utility of which needs no further elaboration.

II. CLOSED STRING THERMODYNAMICS AND A QUASISTATIC INITIAL UNIVERSE

The geometry of string theory is a very rich and complex subject. There exist very distinct geometries, sometimes with very distinct topologies that are indistinguishable from one another as far as physical processes involving strings are concerned. Known as "dualities" [20], the connections between these geometries are one of the most striking features of string theory that persist at low energies, a pervasive manifestation of which is the T-duality symmetry that relates strings in a very large universe (relative to the string scale) to strings in a very small universe. In the absence of any background fluxes, in the context of heterotic string theory for example, this implies the duality

$$G_{ab} \leftrightarrow G^{-1}_{ab} \tag{3}$$

where G_{ab} is the (target space) metric of space-time. The implications of this duality for early Universe cosmology have been studied extensively in various constructions [21,22]. The particular context we are concerned with, "string gas cosmology," is a paradigm of early Universe cosmology initially proposed in Ref. [23] to explain why only three of the nine spatial dimensions of string theory can be macroscopic. Within a particular realization of this framework, given certain assumptions, one can naturally generate a large tensor background with a spectrum that is blue tilted [24], with a red-tilted scalar spectrum [25].[5]

The cosmological model we consider [24,25] is based on the thermodynamics of *closed* heterotic strings. Due to the existence of an exponential tower of oscillatory string modes, there is a maximal temperature T_H which a thermal gas of strings can attain [27]. The existence of winding modes in addition to the center-of-mass momentum modes

[3] The so-called sensitivity of large field models to "Planck slop" [12].

[4] Requiring that inflation exists in the context of a consistent quantum theory requires considerable tuning at the level of the low-energy effective description [18] (its so-called UV sensitivity).

[5] As has been remarked since this model was proposed [26], the detection of a blue spectrum of tensor modes can be viewed as a prediction for cosmological observations, first made in the context of string theory, that would falsify the inflationary paradigm if observed.

is the representation of the T duality (3) on the matter content of the universe, wherein physics on a torus of radius R is equivalent to that on a torus of radius l_s^2/R, where l_s is the string length. This duality leads to the temperature/radius relation for a weakly coupled gas of strings that plateaus around $R = l_s$ (cf. Fig. 1 of Ref. [28]). It thus seems reasonable to conjecture that the cosmological singularity might be dynamically resolved by the energetics of the so-called Hagedorn phase.[6]

The model of Refs. [24,25] is based on the premise that the Universe starts in the quasistatic Hagedorn phase when the temperature is only very slightly lower than the Hagedorn temperature.[7] The decay of string winding modes will eventually enable three spatial dimensions to become large, while the others are forever confined by string winding modes [23].[8] The decay of the string winding modes leads to a smooth transition to the radiation-dominated phase of standard cosmology. The transition time between the quasistatic Hagedorn phase with constant scale factor $a(t)$ and the radiation phase with $a(t) \sim t^{1/2}$ is denoted by t_R, since it plays a role similar to the reheating time in inflationary cosmology.

In Fig. 1 we show the evolution of various scales in string gas cosmology. In this sketch, the vertical axis is time, and the horizontal axis represents physical distance. The two light red curves which are vertical in the Hagedorn phase indicate the physical wavelengths of two different fluctuation modes. The solid blue curve which grows linearly in the radiation phase and is at infinity early in the Hagedorn phase is the Hubble radius $l_H(t) = H^{-1}(t)$, where the inverse expansion rate $H(t) = \dot{a}/a$ (where the dot indicates the derivative with respect to time t). The Hubble radius separates scales on which fluctuations oscillate (sub-Hubble modes) from those where the oscillations are frozen out and the amplitude of the modes is squeezed (see Ref. [31] for discussions of how cosmological perturbations evolve), namely the super-Hubble modes.

The first remark to make is that the horizon is much larger than the Hubble radius (in fact it is infinite if time extends to $-\infty$). Hence, string gas cosmology addresses the horizon problem of standard cosmology in a complimentary way to inflation. Secondly, it is clear that cosmological fluctuations begin on sub-Hubble scales and evolve after t_R for a long time at super-Hubble lengths. The sub-Hubble origin of the scales makes it possible to have a causal generation mechanism of fluctuations; the super-Hubble period of evolution will lead to acoustic oscillations at late times in both the angular power spectrum of CMB anisotropies and in the matter power spectrum [32].

[6]As has been explicitly demonstrated in the context of type II strings in Refs. [17,29].

[7]As explained in Ref. [23], the temperature difference depends inversely on the entropy.

[8]The role played by T duality in ensuring moduli stabilization was discussed in detail in Ref. [30].

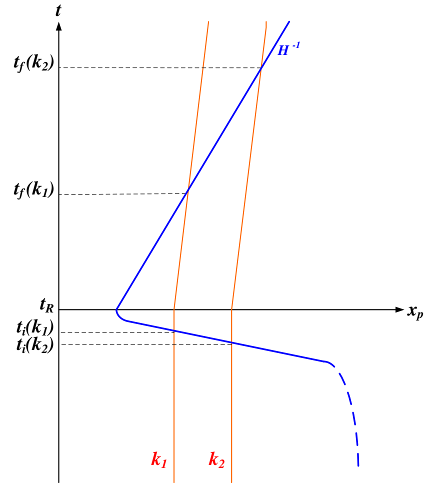

FIG. 1 (color online). Space-time sketch of the evolution in string gas cosmology. The vertical axis is time, and the horizontal axis is physical distance. The time t_R corresponds to the transition between the Hagedorn phase and the radiation phase. The thick blue curve labeled by H^{-1} indicates the Hubble radius, and the two thin red curves which are vertical during the Hagedorn phase correspond to the physical wavelengths of fluctuation modes labeled by k_1 and k_2.

Having set the scene for the background, we now turn to reviewing why this string cosmological background can lead to a spectrum of cosmological perturbations with a red tilt and gravitational waves with a blue tilt, generated by the thermodynamics of strings.

III. TENSOR MODES FROM AN EARLY HAGEDORN PHASE

Since in the initial Hagedorn phase of string cosmology matter is a thermal gas of strings, the initial conditions for scalar and tensor metric fluctuations are thermal rather than vacuum and the energy-momentum tensor correlation functions are determined by closed string thermodynamics rather than by open or point-particle thermodynamics [33].

The calculation of the spectrum of scalar [25] and tensor [24] fluctuations in string gas cosmology proceeds in three steps. In the first, the matter correlation functions are evaluated using the results of the closed string thermal partition function given in Ref. [34]. The second step is to use the Einstein constraint equations, presuming our quasistatic background to be a given *in the Einstein frame*[9], to determine the cosmological fluctuations and gravitational waves from the matter correlation functions mode by mode when the modes k exit the Hubble radius at the times $t_i(k)$. The third step is to evolve the gravitational fluctuations until the present time using the usual theory of cosmological fluctuations.

[9]From Ref. [28] we know this is a nontrivial assumption. However, see Ref. [17] for suggestions as to how one could accomplish this in the context of type II superstrings.

The metric including cosmological fluctuations $\Phi(\mathbf{x},\eta)$ and gravitational waves $h_{ij}(\mathbf{x},\eta)$ can be written in the form [31]

$$ds^2 = a^2(\eta)\{(1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\}, \tag{4}$$

where η is the conformal time related to the physical time via $dt = a(t)d\eta$. The scalar metric fluctuations are determined via the energy density perturbations via

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G_N^2 k^{-4}\langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle, \tag{5}$$

where the pointed brackets indicate *thermal* expectation values, T^μ_ν is the energy-momentum tensor, and G_N is Newton's gravitational constant. The gravitational waves are given by the off-diagonal (i.e. $i \neq j$) pressure fluctuations:

$$\langle|h(k)|^2\rangle = 16\pi^2 G_N^2 k^{-4}\langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle. \tag{6}$$

To determine the energy density fluctuations, we use the fact that in thermal equilibrium the position-space perturbations are given by the specific heat capacity C_V at fixed volume $V = R^3$,

$$\langle\delta\rho^2\rangle = \frac{T^2}{R^6}C_V. \tag{7}$$

For a thermal gas of heterotic strings C_V is given by

$$C_V \approx 2\frac{R^2/l_s^3}{T(1-T/T_H)}, \tag{8}$$

and hence the power spectrum of Φ, defined by

$$P_\Phi(k) \equiv \frac{1}{2\pi^2}k^3|\Phi(k)|^2 \tag{9}$$

is determined to be [25]

$$P_\Phi(k) = \left(\frac{l_{\mathrm{Pl}}}{l_s}\right)^4 \frac{T(k)}{T_H}\frac{1}{1-T(k)/T_H}, \tag{10}$$

where $T(k)$ is the temperature at the time $t_i(k)$ when mode k exits the Hubble radius. As inferred from Fig. 1, the temperature $T(k)$ decreases as k increases, since large-k modes exit the Hubble radius later. Since $T(k)$ is close to the Hagedorn temperature, it is the denominator of the right-hand side of Eq. (10) which dominates the final amplitude. Hence, the spectrum of scalar metric fluctuations has a red tilt (larger amplitude at larger wavelengths). Neglecting running, the tilt can be computed as [defining $\hat{T}(k) \coloneqq T(k)/T_H$]

$$n_s - 1 = (1-\hat{T}(k))^{-1}k\frac{d\hat{T}(k)}{dk}, \tag{11}$$

which is negative since $d\hat{T}/dk < 0$, and arbitrarily small in the limit of a sudden transition (in which case $d\hat{T}/dk \equiv 0$). The power spectrum of the tensor modes, which is produced by fluctuations of the wound strings around a compact space, is given by Eq. (6) and the correlation function $C^i{}_j{}^i{}_j(R)$ $(i \neq j)$, namely the mean square fluctuation of $T^i{}_j$ $(i \neq j)$ in a region of radius $R = k^{-1}$,

$$P_h(k) = 16\pi^2 G_N^2 k^{-4} C^i{}_j{}^i{}_j(R). \tag{12}$$

The correlation function $C^i{}_j{}^i{}_j$ on the right-hand side of the above equation follows from the thermal closed string partition function and was computed in Refs. [33,35] (see also Ref. [36] for a more general treatment), with the result that for temperatures close to the Hagedorn value

$$P_h(k) \sim \left(\frac{l_{\mathrm{Pl}}}{l_s}\right)^4 \frac{T(k)}{T_H}(1-T(k)/T_H)\ln^2\left[\frac{1}{l_s^2k^2}(1-T(k)/T_H)\right]. \tag{13}$$

The key factor $(1-T(k)/T_H)$ now appears in the numerator and hence leads to a blue spectrum. Neglecting running (and thus the logarithmic factor as well), the tilt can be computed as

$$\begin{aligned} n_T &= \frac{1-2\hat{T}(k)}{1-\hat{T}(k)}k\frac{d\hat{T}(k)}{dk} \\ &= -(n_s-1)(2\hat{T}(k)-1), \end{aligned} \tag{14}$$

where we see the complimentarity between the tilt of the scalar and tensor spectra. The fact that we obtain a blue spectrum of gravitational waves is readily understood. The spectrum of gravitational waves is determined by the anisotropic pressure perturbations. Since deeper in the Hagedorn phase, i.e. at higher $T(k)$, the pressure is smaller, the anisotropic pressure fluctuations should be smaller, as well. Hence, the amplitude of the gravitational-wave spectrum will increase towards the ultraviolet, corresponding to a blue spectrum. Furthermore, we can also compute the tensor-to-scalar ratio as

$$r = (1-\hat{T})^2\ln^2\left[\frac{1}{l_s^2k^2}(1-\hat{T}(k))\right]. \tag{15}$$

Requiring COBE normalization for the power spectrum for the comoving curvature perturbation [25], in addition to requiring a tensor-to-scalar ratio of 0.2 [1] fixes the string length to be given by $l_{Pl} = 0.0016 l_s$, and that the modes we observe exited when the temperature of the Universe was $T \sim 0.99T_H$. The latter implies that the tensor tilt is essentially equal and opposite to the scalar tilt,

$$n_T \approx -(n_s - 1), \tag{16}$$

the precise value of which depends on the manner in which the background exited the Hagedorn phase.

It may seem that one would be hard pressed to be able to detect a tilt as small as $n_T \sim -(n_s-1) \sim 0.03$. However, as observed in Ref. [37],[10] any detection of a large tensor background $[r \gtrsim \mathcal{O}(10^{-2})]$ would bring down the cosmic variance limitation on the two-sigma detectability of

$n_T + r/4.8$ to be comparable to $|n_s - 1| \sim 0.03$. Whether any future (all-sky) surveys of the B-mode background can reach an angular resolution and sensitivity that comes close to cosmic variance limits remains to be seen (see Ref. [38] for a recently tabled proposal). There is clear motivation to aim for such sensitivity from within the inflationary paradigm as well; deviations from the tensor-to-scalar consistency relation are powerful probes of a variety of new physics [37].

ACKNOWLEDGMENTS

We wish to thank Matt Dobbs, Gil Holder and Cumrun Vafa for valuable discussions and correspondence. S. P. is supported by a Marie Curie Intra-European Fellowship of the European Community's 7th Framework Program under contract number PIEF-GA-2011-302817. R. B. is supported by an NSERC Discovery Grant, and by funds from the Canada Research Chair program.

[1] BICEP2 Collaboration, Phys. Rev. Lett. **112**, 241101 (2014).
[2] R. J. Danos, R. H. Brandenberger, and G. Holder, Phys. Rev. D **82**, 023513 (2010).
[3] D. Hanson *et al.* (SPTpol Collaboration), Phys. Rev. Lett. **111**, 141301 (2013).
[4] The POLARBEAR Collaboration, arXiv:1403.2369.
[5] H. Liu, P. Mertsch, and S. Sarkar, Astrophys. J. **789**, L29 (2014).
[6] M. J. Mortonson and U. Seljak, arXiv:1405.5857.
[7] R. Flauger, J. C. Hill, and D. N. Spergel, J. Cosmol. Astropart. Phys. 08 (2014) 039.
[8] M. Gerbino, A. Marchini, L. Pagano, L. Salvati, E. Di Valentino, and A. Melchiorri, Phys. Rev. D **90**, 047301 (2014).
[9] C. R. Contaldi, M. Peloso, and L. Sorbo, J. Cosmol. Astropart. Phys. 07 (2014) 014.
[10] Planck Collaboration, arXiv:1303.5076 [Astron. Astrophys. (to be published)].
[11] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
[12] C. P. Burgess, M. Cicoli, and F. Quevedo, J. Cosmol. Astropart. Phys. 11 (2013) 003.
[13] E. Silverstein and A. Westphal, Phys. Rev. D **78**, 106003 (2008).
[14] J. P. Conlon, J. Cosmol. Astropart. Phys. 01 (2012) 033.
[15] C. Vafa, Nucl. Phys. **B766**, 21 (2007).
[16] I. Florakis, C. Kounnas, H. Partouche, and N. Toumbas, Nucl. Phys. **B844**, 89 (2011).
[17] C. Kounnas, H. Partouche, and N. Toumbas, Classical Quantum Gravity **29**, 095014 (2012).
[18] C. P. Burgess and L. McAllister, Classical Quantum Gravity **28**, 204002 (2011); S. Hardeman, J. M. Oberreuter, G. A. Palma, K. Schalm, and T. van der Aalst, J. High Energy Phys. 04 (2011) 009.
[19] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, J. High Energy Phys. 03 (2008) 014.
[20] A. Giveon, M. Porrati, and E. Rabinovici, Phys. Rep. **244**, 77 (1994).
[21] M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993); M. Gasperini and G. Veneziano, Phys. Rep. **373**, 1 (2003).
[22] A. A. Tseytlin and C. Vafa, Nucl. Phys. **B372**, 443 (1992).
[23] R. H. Brandenberger and C. Vafa, Nucl. Phys. **B316**, 391 (1989).
[24] R. H. Brandenberger, A. Nayeri, S. P. Patil, and C. Vafa, Phys. Rev. Lett. **98**, 231302 (2007).
[25] A. Nayeri, R. H. Brandenberger, and C. Vafa, Phys. Rev. Lett. **97**, 021302 (2006).
[26] R. H. Brandenberger, A. Nayeri, S. P. Patil, and C. Vafa, Int. J. Mod. Phys. A **22**, 3621 (2007); R. H. Brandenberger, in *String Cosmology*, edited by J. Erdmenger (Wiley, New York, 2009).
[27] R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965).
[28] R. H. Brandenberger, S. Kanno, J. Soda, D. A. Easson, J. Khoury, P. Martineau, A. Nayeri, and S. P. Patil, J. Cosmol. Astropart. Phys. 11 (2006) 009.
[29] C. Kounnas, H. Partouche, and N. Toumbas, Nucl. Phys. **B855**, 280 (2012); R. H. Brandenberger, C. Kounnas, H. Partouche, S. P. Patil, and N. Toumbas, J. Cosmol. Astropart. Phys. 03 (2014) 015.
[30] S. P. Patil and R. Brandenberger, Phys. Rev. D **71**, 103522 (2005); S. Watson, Phys. Rev. D **70**, 066005 (2004); L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister, and E. Silverstein, J. High Energy Phys. 05 (2004) 030; S. P. Patil and R. H. Brandenberger, J. Cosmol. Astropart. Phys. 01 (2006) 005; R. Brandenberger, Y. K. Cheung, and S. Watson, J. High Energy Phys. 05 (2006) 025; R. J. Danos, A. R. Frey, and R. H. Brandenberger, Phys. Rev. D **77**, 126009 (2008); S. Mishra, W. Xue, R. Brandenberger, and U. Yajnik, J. Cosmol. Astropart. Phys. 09 (2012) 015.
[31] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992); R. H. Brandenberger, Lect. Notes Phys. **646**, 127 (2004).
[32] R. A. Sunyaev and Y. B. Zeldovich, Astrophys. Space Sci. **7**, 3 (1970); P. J. E. Peebles and J. T. Yu, Astrophys. J. **162**, 815 (1970).
[33] A. Nayeri, arXiv:hep-th/0607073.
[34] N. Deo, S. Jain, O. Narayan, and C. I. Tan, Phys. Rev. D **45**, 3641 (1992).
[35] R. H. Brandenberger, A. Nayeri, S. P. Patil, and C. Vafa, Int. J. Mod. Phys. A **22**, 3621 (2007).
[36] T. Biswas, R. Brandenberger, T. Koivisto, and A. Mazumdar, Phys. Rev. D **88**, 023517 (2013).
[37] Y.-S. Song and L. Knox, Phys. Rev. D **68**, 043518 (2003).
[38] PRISM Collaboration, J. Cosmol. Astropart. Phys. 02 (2014) 006.