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No pair production of open strings in a plane-wave background

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We consider whether an external electric field may cause the pair production of open strings in a type IIA plane-wave background. The boundary states of D-branes with condensates are constructed in the Green-Schwarz formulation of superstring theory with the light-cone gauge. The cylinder diagrams are computed with massive theta functions. Although the value of the electric field is bounded by the upper value, there is no pole in the amplitudes and it indicates that no pair production occurs in the plane-wave background. This result would be universal for a class of plane-wave backgrounds.

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I. INTRODUCTION

The AdS/CFT correspondence [1–3] gives a nice laboratory to argue new aspects of string theory and gauge theories. The classical dynamics of strings on the AdS space is related to nonperturbative phenomena in strongly coupled field theories. An example of this fascinating subject is the pair production of pairs of particle and antiparticle in the presence of an external electric field. It is originally studied in quantum electrodynamics (QED) [4] and often called the Schwinger effect [5] (for further developments, see [6,7]).

Recently, a holographic scenario to study the Schwinger pair production was proposed by Semenoff and Zarembo [8], where the production rate of W bosons in the Coulomb phase (for various generalizations, see [9–11]). The potential analysis is done in the holographic way [12]. The holographic analysis is applicable to confining gauge theories [13–16].

In relation to this progress, an interesting issue is to consider the pair production of strings on the AdS space and argue its holographic dual. However, it is technically difficult to analyze the dynamics of strings on the AdS space; hence, it would be reasonable to consider a plane-wave background as an approximation of the target space geometry. It is known that the pair production of strings occurs in flat space [17,18] (for the analysis based on the Green-Schwarz formulation, see [19]), but it is not obvious whether it can occur on curved backgrounds.

Here we will focus on a type IIA pp-wave background as a concrete example. We consider if an external electric field can induce the pair production of open strings. The boundary states of D-branes with condensates are considered in the Green-Schwarz formulation of superstring theory with the light-cone gauge. The cylinder diagrams are computed with massive theta functions. The value of the electric field is found to be bounded by the upper value. On the other hand, there is no pole in the amplitudes and it indicates that no pair production occurs in the plane-wave background.[1] Although just one example of plane-wave backgrounds is studied here, it is expected that this result would be universal for general plane-wave backgrounds.

This paper is organized as follows. Section II gives a brief review of the Green-Schwarz formulation of type IIA superstring theory on a plane-wave background preserving 24 supersymmetries. In Sec. III, boundary states with condensates are constructed. In Sec. IV, we compute cylinder amplitudes between parallel D-branes with condensates. Section V argues the pole structure of the amplitudes after moving to the open string picture. It is shown that there is no pole in the amplitudes, and therefore the pair production does not occur. This result indicates that there is no pair production on general plane-wave backgrounds. Section VI is devoted to the conclusion and discussion. Appendix A explains in detail that there is no pole in the D2-brane amplitude.

II. SETUP

This section gives a brief review of the construction of a light-cone Hamiltonian for a closed superstring in a type IIA plane-wave background, the details of which can be found in [21].

A. A type IIA plane-wave background

The type IIA plane-wave background is given by [22–24]

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[1]In fact, it has been reported that there is no vacuum polarization in plane waves in the context of quantum field theory [20]. Our result may be considered as its stringy version.

$$ds^2 = -2dx^+dx^- - A(x^I)(dx^+)^2 + \sum_{I=1}^{8}(dx^I)^2,$$
$$F_{+123} = \mu, \qquad F_{+4} = -\frac{\mu}{3}, \tag{2.1}$$

where $x^I = (x^i, x^{i'})$ and the scalar function $A(x^I)$ is defined as

$$A(x^I) \equiv \sum_{i=1}^{4}\frac{\mu^2}{9}(x^i)^2 + \sum_{i'=5}^{8}\frac{\mu^2}{36}(x^{i'})^2 \qquad (\mu\colon \text{const}). \tag{2.2}$$

The relative coefficients are fixed so that the background preserves 24 supersymmetries.

B. The Green-Schwarz action of type IIA plane-wave string

The Green-Schwarz action of a type IIA superstring on this background is simplified by fixing the kappa symmetry with the light-cone gauge condition,

$$\Gamma^+\theta = 0, \qquad X^+ = p^+\tau. \tag{2.3}$$

Here p^+ is the total momentum conjugate to X^- and τ is time on the world sheet. Then the light-cone gauge fixed action is given by[2]

$$S_{LC} = -\frac{1}{2}\int d^2\sigma\left[\eta^{mn}\partial_m X^I\partial_n X^I + \frac{m^2}{9}(X^i)^2 + \frac{m^2}{36}(X^{i'})^2 + \bar\theta\Gamma^-\partial_\tau\theta + \bar\theta\Gamma^{-9}\partial_\sigma\theta - \frac{m}{4}\bar\theta\Gamma^-\left(\Gamma^{123} + \frac{1}{3}\Gamma^{49}\right)\theta\right], \tag{2.4}$$

where a new parameter m is defined as

$$m \equiv \mu p^+. \tag{2.5}$$

This is the mass parameter characterizing the masses of the fields on the world sheet. Then the Majorana fermion θ is the combination of Majorana-Weyl fermions θ^1 and θ^2 with opposite ten-dimensional $SO(1,9)$ chiralities; that is, $\theta = \theta^1 + \theta^2$, where $\theta^1(\theta^2)$ has positive (negative) chirality. The Dirac conjugate of θ is defined as $\bar\theta \equiv i\theta^T\Gamma^0$.

The fermionic part in the action (2.4) is written in the 32-component notation. It is now convenient to rewrite the action in the 16-component spinor notation. Let us first introduce a representation of $SO(1,9)$ gamma matrices as follows:

[2]We set $2\pi\alpha' = 1$ for our convention. Here η^{mn} is the flat world-sheet metric with the world-sheet coordinates $\sigma^m = (\tau, \sigma)$.

$$\Gamma^0 = -i\sigma^2 \otimes \mathbf{1}_{16}, \qquad \Gamma^{11} = \sigma^1 \otimes \mathbf{1}_{16}, \qquad \Gamma^I = \sigma^3 \otimes \gamma^I,$$
$$\Gamma^9 = -\sigma^3 \otimes \gamma^9, \qquad \Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^{11}). \tag{2.6}$$

Here σ's are Pauli matrices and $\mathbf{1}_{16}$ the 16×16 unit matrix. Then γ^I are the 16×16 symmetric real gamma matrices satisfying the spin(8) Clifford algebra $\{\gamma^I, \gamma^J\} = 2\delta^{IJ}$, which are reducible to the $\mathbf{8_s} + \mathbf{8_c}$ representation of spin (8). Note that, in our representation, Γ^9 is taken to be the $SO(1,9)$ chirality operator and γ^9 becomes the $SO(8)$ chirality operator,

$$\gamma^9 = \gamma^1\gamma^2\cdots\gamma^8. \tag{2.7}$$

It is convenient to introduce the spinor notation

$$\theta^A = \frac{1}{2^{1/4}}\begin{pmatrix}0\\ \psi^A\end{pmatrix},$$

so as to satisfy the kappa symmetry fixing condition of (2.3), where the superscript A denotes the $SO(1,9)$ chirality. Then the action S_{LC} is rewritten as

$$S_{\mathrm{LC}} = -\frac{1}{2}\int d^2\sigma\left[\eta^{mn}\partial_m X^I\partial_n X^I + \frac{m^2}{9}(X^i)^2 + \frac{m^2}{36}(X^{i'})^2 - i\psi^1_+\partial_+\psi^1_+ - i\psi^1_-\partial_+\psi^1_- - i\psi^2_+\partial_-\psi^2_+ - i\psi^2_-\partial_-\psi^2_- + 2i\frac{m}{3}\psi^2_+\gamma^4\psi^1_- - 2i\frac{m}{6}\psi^2_-\gamma^4\psi^1_+\right], \tag{2.8}$$

where we have introduced the light-cone coordinates on the world sheet and the associated derivatives are defined as $\partial_\pm \equiv \partial_\tau \pm \partial_\sigma$. Here the subscripts $\pm$ in $\psi^A_\pm$ represent the eigenvalues ±1 of γ^{1234}. In our convention, the fermion has the same $SO(1,9)$ and $SO(8)$ chiralities measured by Γ^9 and γ^9, respectively.

C. Mode expansions (bosons)

The light-cone action (2.8) is quadratic in fields; hence, the quantization of closed string is carried out exactly.

Let us first consider the bosonic sector of the theory. The equations of motion for the bosonic coordinates X^I are obtained from the action (2.8) like

$$\eta^{mn}\partial_m\partial_n X^i - \left(\frac{m}{3}\right)^2 X^i = 0,$$
$$\eta^{mn}\partial_m\partial_n X^{i'} - \left(\frac{m}{6}\right)^2 X^{i'} = 0. \tag{2.9}$$

Here the fields are subject to the periodic boundary condition

$$X^I(\tau, \sigma + 2\pi) = X^I(\tau, \sigma).$$

The solutions are given in the form of mode expansion,

$$\begin{aligned} X^i(\tau,\sigma) &= x^i\cos\left(\frac{m}{3}\tau\right)+\frac{1}{2\pi}p^i\frac{3}{m}\sin\left(\frac{m}{3}\tau\right)\\ &\quad+i\sqrt{\frac{1}{4\pi}}\sum_{n\neq 0}\frac{1}{\omega_n}(\alpha_n^i\phi_n(\tau,\sigma)+\tilde{\alpha}_n^i\tilde{\phi}_n(\tau,\sigma)),\\ X^{i'}(\tau,\sigma) &= x^{i'}\cos\left(\frac{m}{6}\tau\right)+\frac{1}{2\pi}p^{i'}\frac{6}{m}\sin\left(\frac{m}{6}\tau\right)\\ &\quad+i\sqrt{\frac{1}{4\pi}}\sum_{n\neq 0}\frac{1}{\omega'_n}(\alpha_n^{i'}\phi'_n(\tau,\sigma)+\tilde{\alpha}_n^{i'}\tilde{\phi}'_n(\tau,\sigma)),\end{aligned} \tag{2.10}$$

where x^I and p^I are center-of-mass variables, coefficients for zero modes, and α_n^I and $\tilde{\alpha}_n^I$ are the expansion coefficients for the nonzero modes. The basis functions for nonzero modes are given by

$$\phi_n(\tau,\sigma)=\mathrm{e}^{-i\omega_n\tau-in\sigma},\qquad \tilde{\phi}_n(\tau,\sigma)=\mathrm{e}^{-i\omega_n\tau+in\sigma},\tag{2.11}$$

$$\phi'_n(\tau,\sigma)=\mathrm{e}^{-i\omega'_n\tau-in\sigma},\qquad \tilde{\phi}'_n(\tau,\sigma)=\mathrm{e}^{-i\omega'_n\tau+in\sigma},\tag{2.12}$$

with the wave frequencies

$$\begin{aligned}\omega_n &= \mathrm{sign}(n)\sqrt{\left(\frac{m}{3}\right)^2+n^2},\\ \omega'_n &= \mathrm{sign}(n)\sqrt{\left(\frac{m}{6}\right)^2+n^2}.\end{aligned}\tag{2.13}$$

Note that the reality of X^I requires that $\alpha_n^{I\dagger}=\alpha_{-n}^I$ and $\tilde{\alpha}_n^{I\dagger}=\tilde{\alpha}_{-n}^I$.

The next is to promote the expansion coefficients in (2.10) to operators with the canonical quantization. The canonical commutation relations (at equal time) for the bosonic fields are given by

$$[X^I(\tau,\sigma),\mathcal{P}^J(\tau,\sigma')]=i\delta^{IJ}\delta(\sigma-\sigma'),\tag{2.14}$$

where $\mathcal{P}^J=\partial_\tau X^J$ is the canonical conjugate momentum of X^J. Then one can read off the following commutation relations between the mode operators,

$$\begin{aligned} &[x^I,p^J]=i\delta^{IJ},\qquad [\alpha_n^i,\alpha_m^j]=\omega_n\delta^{ij}\delta_{n+m,0},\\ &[\alpha_n^{i'},\alpha_m^{j'}]=\omega'_n\delta^{i'j'}\delta_{n+m,0},\qquad [\tilde{\alpha}_n^i,\tilde{\alpha}_m^j]=\omega_n\delta^{ij}\delta_{n+m,0},\\ &[\tilde{\alpha}_n^{i'},\tilde{\alpha}_m^{j'}]=\omega'_n\delta^{i'j'}\delta_{n+m,0}.\end{aligned}\tag{2.15}$$

These relations will be used in considering boundary states in the next section.

D. Mode expansions (fermions)

For the fermionic sector of the theory, the fermionic fields are split into the two parts like (ψ_-^1,ψ_+^2) and (ψ_+^1,ψ_-^2). The equations of motion for the former part are obtained as

$$\partial_+\psi_-^1+\frac{m}{3}\gamma^4\psi_+^2=0,\qquad \partial_-\psi_+^2-\frac{m}{3}\gamma^4\psi_-^1=0.\tag{2.16}$$

The nonzero mode solutions of these equations are given by using the modes, (2.11). For the zero-mode part of the solution, we impose a condition that, at $\tau=0$, the solution behaves just as that of the massless case. The mode expansions for the fermionic coordinates are then

$$\begin{aligned}\psi_-^1(\tau,\sigma)&=c_0\tilde{\psi}_0\cos\left(\frac{m}{3}\tau\right)-c_0\gamma^4\psi_0\sin\left(\frac{m}{3}\tau\right)+\sum_{n\neq 0}c_n\left(\tilde{\psi}_n\tilde{\phi}_n(\tau,\sigma)-i\frac{3}{m}(\omega_n-n)\gamma^4\psi_n\phi_n(\tau,\sigma)\right),\\ \psi_+^2(\tau,\sigma)&=c_0\psi_0\cos\left(\frac{m}{3}\tau\right)+c_0\gamma^4\tilde{\psi}_0\sin\left(\frac{m}{3}\tau\right)+\sum_{n\neq 0}c_n\left(\psi_n\phi_n(\tau,\sigma)+i\frac{3}{m}(\omega_n-n)\gamma^4\tilde{\psi}_n\tilde{\phi}_n(\tau,\sigma)\right),\end{aligned}\tag{2.17}$$

where the chirality condition is that $\gamma^{1234}\psi_n=\psi_n$ and $\gamma^{1234}\tilde{\psi}_n=-\tilde{\psi}_n$ for all n. The normalization constants c_0 and c_n are given by

$$c_0=\frac{1}{\sqrt{2\pi}},\qquad c_n=\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{1+(\frac{3}{m})^2(\omega_n-n)^2}}.$$

Then let us consider the other part (ψ_+^1,ψ_-^2). The equations of motion are given by, respectively,

$$\partial_+\psi_+^1-\frac{m}{6}\gamma^4\psi_-^2=0,\qquad \partial_-\psi_-^2+\frac{m}{6}\gamma^4\psi_+^1=0.\tag{2.18}$$

The solutions are found to be

$$\psi^1_+(\tau,\sigma) = c'_0\tilde{\psi}'_0\cos\left(\frac{m}{6}\tau\right) + c'_0\gamma^4\psi'_0\sin\left(\frac{m}{6}\tau\right) + \sum_{n\neq 0} c'_n\left(\tilde{\psi}'_n\tilde{\phi}'_n(\tau,\sigma) + i\frac{6}{m}(\omega'_n - n)\gamma^4\psi'_n\phi'_n(\tau,\sigma)\right),$$
$$\psi^2_-(\tau,\sigma) = c'_0\psi'_0\cos\left(\frac{m}{6}\tau\right) - c'_0\gamma^4\tilde{\psi}'_0\sin\left(\frac{m}{6}\tau\right) + \sum_{n\neq 0} c'_n\left(\psi'_n\phi'_n(\tau,\sigma) - i\frac{6}{m}(\omega'_n - n)\gamma^4\tilde{\psi}'_n\tilde{\phi}'_n(\tau,\sigma)\right), \tag{2.19}$$

where the chirality conditions are described by $\gamma^{1234}\psi'_n = -\psi'_n$ and $\gamma^{1234}\tilde{\psi}'_n = \tilde{\psi}'_n$. The normalization constants c_0 and c_n are given by

$$c'_0 = \frac{1}{\sqrt{2\pi}}, \qquad c'_n = \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{1+(\frac{6}{m})^2(\omega'_n - n)^2}}.$$

Promoting the expansion coefficients to operators with the canonical quantization again, the canonical anticommutation relations (at equal time) are obtained as

$$\{\psi^A_\pm(\tau,\sigma), \psi^B_\pm(\tau,\sigma')\} = \delta^{AB}\delta(\sigma-\sigma'). \tag{2.20}$$

Then the anticommutation relations between the mode operators are determined as

$$\{\psi_n, \psi_m\} = \delta_{n+m,0}, \qquad \{\tilde{\psi}_n, \tilde{\psi}_m\} = \delta_{n+m,0}, \tag{2.21}$$

$$\{\psi'_n, \psi'_m\} = \delta_{n+m,0}, \qquad \{\tilde{\psi}'_n, \tilde{\psi}'_m\} = \delta_{n+m,0}. \tag{2.22}$$

E. The light-cone Hamiltonian

The light-cone Hamiltonian of the theory is given by

$$H = \frac{1}{p^+}\int_0^{2\pi} d\sigma \mathcal{H}, \tag{2.23}$$

with the Hamiltonian density $\mathcal{H}$ derived from $S_{\rm LC}$ given in (2.8),

$$\mathcal{H} = \frac{1}{2}(\mathcal{P}^I)^2 + \frac{1}{2}(\partial_\sigma X^I)^2 + \frac{1}{2}\left(\frac{m}{3}\right)^2(X^i)^2 + \frac{1}{2}\left(\frac{m}{6}\right)^2(X^{i'})^2$$
$$-\frac{i}{2}\psi^1_-\partial_\sigma\psi^1_- + \frac{i}{2}\psi^2_+\partial_\sigma\psi^2_+ + i\frac{m}{3}\psi^2_+\gamma^4\psi^1_-$$
$$-\frac{i}{2}\psi^1_+\partial_\sigma\psi^1_+ + \frac{i}{2}\psi^2_-\partial_\sigma\psi^2_- - i\frac{m}{6}\psi^2_-\gamma^4\psi^1_+. \tag{2.24}$$

By plugging the mode expansions for the fields, Eqs. (2.10), (2.17), and (2.19), into Eq. (2.23), the light-cone Hamiltonian becomes

$$H = E_0 + E + \tilde{E}, \tag{2.25}$$

where E_0, E, and $\tilde{E}$ are given by

$$E_0 = \frac{\pi}{p^+}\left[\left(\frac{p^I}{2\pi}\right)^2 + \left(\frac{m}{3}\right)^2(x^i)^2 + \left(\frac{m}{6}\right)^2(x^{i'})^2 - \frac{i}{\pi}\frac{m}{3}\tilde{\psi}_0\gamma^4\psi_0 + \frac{i}{\pi}\frac{m}{6}\tilde{\psi}'_0\gamma^4\psi'_0\right],$$
$$E = \frac{1}{p^+}\sum_{n=1}^{\infty}(\alpha^I_{-n}\alpha^I_n + \omega_n\psi_{-n}\psi_n + \omega'_n\psi'_{-n}\psi'_n),$$
$$\tilde{E} = \frac{1}{p^+}\sum_{n=1}^{\infty}(\tilde{\alpha}^I_{-n}\tilde{\alpha}^I_n + \omega_n\tilde{\psi}_{-n}\tilde{\psi}_n + \omega'_n\tilde{\psi}'_{-n}\tilde{\psi}'_n). \tag{2.26}$$

The first part E_0 is the zero-mode contribution and the remaining two parts, E and $\tilde{E}$, are the contributions of the nonzero modes. Here E and $\tilde{E}$ are the normal-ordered expressions, which do not have zero-point energy in total because the zero-point energy coming from the bosonic fields is exactly canceled by that of the fermionic ones.

The zero-mode contribution E_0 has the form of simple harmonic oscillators. Hence, it can be conveniently rewritten in terms of the creation and annihilation operators. For the bosonic part, the creation and annihilation operators are defined as

$$a^{i\dagger} \equiv \sqrt{\frac{3\pi}{m}}\left(\frac{p^i}{2\pi} + i\frac{m}{3}x^i\right), \qquad a^i \equiv \sqrt{\frac{3\pi}{m}}\left(\frac{p^i}{2\pi} - i\frac{m}{3}x^i\right),$$
$$a^{i'\dagger} \equiv \sqrt{\frac{6\pi}{m}}\left(\frac{p^{i'}}{2\pi} + i\frac{m}{6}x^{i'}\right), \qquad a^{i'} \equiv \sqrt{\frac{6\pi}{m}}\left(\frac{p^{i'}}{2\pi} - i\frac{m}{6}x^{i'}\right), \tag{2.27}$$

and for the fermionic part, those are given by

$$\chi^\dagger \equiv \frac{1}{\sqrt{2}}(\psi_0 - i\gamma^4\tilde{\psi}_0), \qquad \chi \equiv \frac{1}{\sqrt{2}}(\psi_0 + i\gamma^4\tilde{\psi}_0),$$
$$\chi'^\dagger \equiv \frac{1}{\sqrt{2}}(\psi'_0 + i\gamma^4\tilde{\psi}'_0), \qquad \chi' \equiv \frac{1}{\sqrt{2}}(\psi'_0 - i\gamma^4\tilde{\psi}'_0). \tag{2.28}$$

Note that the chirality conditions are now rewritten as $\gamma^{12349}\chi = -\chi$ and $\gamma^{12349}\chi' = \chi'$. Equations (2.15), (2.21), and (2.22) lead to the nonvanishing (anti)commutation relations,

$$[a^I, a^{J\dagger}] = \delta^{IJ}, \qquad \{\chi, \chi^\dagger\} = 1, \qquad \{\chi', \chi'^\dagger\} = 1. \tag{2.29}$$

Then, after taking the normal ordering, the zero-mode contribution to H is given by

$$E_0 = \frac{m}{6p^+}(2a^{i\dagger}a^i + a^{i'\dagger}a^{i'} + 2\chi^\dagger\chi + \chi'^\dagger\chi'). \qquad (2.30)$$

Here the zero-point energy vanishes as in the case of E and $\tilde{E}$.

The light-cone Hamiltonian H of (2.25) with (2.26) and (2.30) will be used to describe time evolution of closed string states.

III. BOUNDARY STATES WITH CONDENSATES

In this section, boundary states for D-branes with condensates are constructed in the Green-Schwarz formulation of type IIA superstring theory on the plane-wave background. The boundary states constructed here will be utilized to compute the cylinder diagrams between parallel D-branes in the closed-string description.

A. The bosonic part

It is well known that, in the light-cone gauge (2.3), the light-cone closed-string coordinates $X^\pm$ satisfy the Dirichlet boundary condition on the boundary state for D-brane basically due to the open/closed string channel duality [25,26]. That is, letting the boundary state $|B\rangle$, it follows that

$$\partial_\sigma X^\pm |B\rangle = 0.$$

These conditions imply that D-branes are instantonic objects and restrict the dimensionality of a Dp-brane to the range $0 \le p+1 \le 8$.

For the spatial coordinates, the boundary condition can be taken as

$$(\partial_+ X^I - M_{IJ}\partial_- X^J)|B\rangle = 0, \qquad (3.1)$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$ as defined in (2.8). The matrix M_{IJ} eventually describes a rotation. In the absence of boundary condensates, its explicit form is given by

$$M_{IJ} = \pm\delta_{IJ}\begin{cases} + & \text{for } I \in D \\ - & \text{for } I \in N \end{cases}, \qquad (3.2)$$

where $I \in D(N)$ means that I denotes the Dirichlet (Neumann) direction. Plugging this matrix into (3.1), the Dirichlet or Neumann boundary condition is imposed for X^I like

$$\partial_\sigma X^I |B\rangle = 0, \quad \text{or} \quad \partial_\tau X^I |B\rangle = 0.$$

In the presence of boundary condensates, M_{IJ} is no longer the form of (3.2) as it should be.

Note that, even without knowing the explicit form of M_{IJ} in the presence of boundary condensates, boundary states can be constructed at least at the formal level. In terms of the bosonic modes in the mode expansion (2.10) with (2.27), the boundary condition (3.1) is rewritten as

$$\begin{aligned} &(a^I - M_{IJ}a^{J\dagger})|B\rangle = 0, \\ &(\alpha_n^I - M_{IJ}\tilde{\alpha}_{-n}^J)|B\rangle = 0 \quad (n \ge 1). \end{aligned} \qquad (3.3)$$

Then one can solve these conditions with the method of constructing the coherent state. The resulting state is given by

$$|B\rangle_{\mathrm{B}} = \mathrm{e}^{\sum_{n>0}(\frac{1}{\omega_n}M_{ij}\alpha_{-n}^i\tilde{\alpha}_{-n}^j + \frac{1}{\omega_n'}M_{i'j'}\alpha_{-n}^{i'}\tilde{\alpha}_{-n}^{j'})}\,\mathrm{e}^{\frac{1}{2}M_{ij}a^{i\dagger}a^{j\dagger} + \frac{1}{2}M_{i'j'}a^{i'\dagger}a^{j'\dagger}}|0\rangle, \qquad (3.4)$$

where the subscript B in $|B\rangle_{\mathrm{B}}$ means the bosonic part of the boundary state. This expression shows that the problem of constructing a boundary state reduces to the problem of finding an appropriate matrix M_{IJ} under a given setup.

In the following, we are concerned with the nonvanishing boundary condensates. Here the condensates are supposed to be constant electromagnetic fields $\mathcal{F}_{IJ}$ on the D-brane world volume, and hence all the indices of $\mathcal{F}_{IJ}$ are in Neumann directions.

In the presence of $\mathcal{F}_{IJ}$, the boundary condition for Neumann directions is given by

$$(\partial_\tau X^I + \mathcal{F}_{IJ}\partial_\sigma X^J)|B\rangle = 0 \quad (I, J \in N). \qquad (3.5)$$

By rewriting this condition in terms of $\partial_\pm$, the relation between M_{IJ} and $\mathcal{F}_{IJ}$ is obtained as

$$M_{IJ} = -\left(\frac{1-\mathcal{F}}{1+\mathcal{F}}\right)_{IJ} \quad (I, J \in N), \qquad (3.6)$$

while $M_{IJ} = \delta_{IJ}$ when $I, J \in D$. It is convenient here to take a frame so that $\mathcal{F}_{IJ}$ becomes the block-diagonal form,

$$\mathcal{F} = \mathrm{diag}(\mathcal{F}_{(1)}, \mathcal{F}_{(2)}, \ldots, \mathcal{F}_{(p/2)}), \qquad (3.7)$$

where each block $\mathcal{F}_{(a)}$ is given by

$$\mathcal{F}_{(a)} = \begin{pmatrix} 0 & f_a \\ -f_a & 0 \end{pmatrix}. \qquad (3.8)$$

From (3.6), we see that M_{IJ} has also the block-diagonal form. Recalling that M_{IJ} is a rotation matrix, the 2×2 block $M_{(a)}$ of M associated with $\mathcal{F}_{(a)}$ represents a rotation in a certain two-dimensional plane labeled by a:

$$M_{(a)} = -\frac{1}{1+f_a^2}\begin{pmatrix} 1-f_a^2 & -2f_a \\ 2f_a & 1-f_a^2 \end{pmatrix} = e^{\varphi_a T_{(a)}} = \begin{pmatrix} \cos\varphi_a & \sin\varphi_a \\ -\sin\varphi_a & \cos\varphi_a \end{pmatrix}. \quad (3.9)$$

Here the first line is derived from (3.6) with (3.7) and (3.8), $T_{(a)}$ is the rotation generator in the two-dimensional plane labeled by a,

$$T_{(a)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (3.10)$$

and φ_a is the rotation angle related to the constant background f_a as follows:

$$\cos\varphi_a = -\frac{1-f_a^2}{1+f_a^2}, \qquad \sin\varphi_a = \frac{2f_a}{1+f_a^2}. \quad (3.11)$$

B. The fermionic part

Next we consider the fermionic part. The fermionic boundary state in the absence of boundary condensates has been constructed in [27]. Like the bosonic part, the basic structure of the fermionic boundary state is not changed even in the presence of boundary condensates. Thus we will just quote some essential results obtained in [27] without repeating the detailed analysis and focus upon the fermionic counterpart of the matrix M_{IJ}.

The fermionic boundary state is constructed by requiring that the boundary state preserves some amount of supersymmetries possessed by the IIA plane-wave background. Here, we demand that half of the supersymmetries are preserved (i.e., 1/2-BPS condition) and that the fermionic modes satisfy the following *Ansätze* as the boundary conditions,

$$(\chi - \gamma^{123}\hat{M}\chi^\dagger)|B\rangle = 0, \qquad (\chi' - \gamma^{123}\hat{M}\chi'^\dagger)|B\rangle = 0,$$
$$(\tilde{\psi}_n + i\hat{M}\psi_{-n})|B\rangle = 0, \qquad (\tilde{\psi}'_n + i\hat{M}\psi'_{-n})|B\rangle = 0 \quad (n>0). \quad (3.12)$$

Here $\hat{M}$ is an orthogonal matrix and the fermionic counterpart of the matrix M_{IJ}. The consistency of the 1/2 supersymmetry preserving condition with these *Ansätze* leads to the conditions which determine the matrix $\hat{M}$,

$$\gamma^J M_{JI} + \hat{M}\gamma^I\hat{M}^T = 0,$$
$$M_{JI}\gamma^J\gamma^{123} + \hat{M}\gamma^I\gamma^{123}\hat{M} = 0. \quad (3.13)$$

In the absence of boundary condensates, by using M_{IJ} of (3.2), the solution that satisfies the conditions (3.13) is simply given by the product of gamma matrices with indices in a particular set of Neumann directions,

$$\hat{M} = \gamma^{I_1 I_2 \cdots I_{p+1}} \quad (I_m \in N, I_1 < I_2 < \cdots < I_{p+1}), \quad (3.14)$$

TABLE I. Spanning directions of Dp-brane instantons in the type IIA plane-wave background. The branes are 1/2-BPS when sitting at the origin of the transverse space. $\#_N$ denotes the number of Neumann directions. The indices are defined as $\hat{i}, \hat{j} = 1, 2, 3$ and $i', j' = 5, 6, 7, 8$.

$\#_N (= p+1)$	Spanning directions
1	$(\hat{i})$
3	$(\hat{i}, \hat{j}, 4)$, $(4, i', j')$
5	$(1, 2, 3, i', j')$, $(\hat{i}, 5, 6, 7, 8)$
7	$(\hat{i}, \hat{j}, 4, 5, 6, 7, 8)$

for each 1/2-BPS Dp-brane boundary state. In general, possible configurations of Dp-branes are restricted on plane-wave backgrounds. The possible configurations in the present case [27] are summarized in Table I.

Like in the bosonic case, the fermionic boundary state can be constructed even without knowing the explicit form of $\hat{M}$ in the presence of boundary condensates. Namely, by solving the *Ansätze* (3.12) with the method of constructing the coherent state, the fermionic boundary state is obtained as

$$|B\rangle_{\rm F} = e^{-i\sum_{n>0}(\psi_{-n}\hat{M}\tilde{\psi}_{-n} + \psi'_{-n}\hat{M}\tilde{\psi}'_{-n})} e^{\frac{1}{2}\chi^\dagger\gamma^{123}\hat{M}\chi^\dagger + \frac{1}{2}\chi'^\dagger\gamma^{123}\hat{M}\chi'^\dagger}|0\rangle, \quad (3.15)$$

where the subscript F in $|B\rangle_{\rm F}$ means the fermionic part of the boundary state and the redefined fermionic zero modes χ and χ' of (2.28) have been used.

Note that $\hat{M}$ is related to M_{IJ} under the conditions given in (3.13). Hence, when boundary condensates in the form of (3.7) are turned on, $\hat{M}$ is determined from M_{IJ} of (3.6) with the 2×2 blocks given in (3.9). It should be remarked that (3.6) is only for the Neumann directions, but M_{IJ} for the Dirichlet directions is still given by (3.2). Though one may directly solve (3.13) for $\hat{M}$, it is easier to determine $\hat{M}$ by taking the group theoretical viewpoint that $\hat{M}$ and M are the rotation matrices in the spinor and vector representations, respectively.

This can be illustrated by an explicit example. Let us consider a D2-brane spanning along the 1, 2, and 4 directions (see Table I) and turn on a boundary condensate on the 1-2 plane. Then, M_{IJ} is given by $(e^{\varphi T})_{IJ}$ for $I, J = 1, 2$, and otherwise by (3.2). Here, T is the rotation generator in the 1-2 plane given by (3.10), and φ is related to the boundary condensate through (3.11).[3] Now, the spinor representation of the rotation in the 1-2 plane by an angle φ is simply given by $e^{\frac{\varphi}{2}\gamma^{12}}$, which is the part of $\hat{M}$ that realizes the rotation. The full form of $\hat{M}$ is given by

$$\hat{M} = e^{\frac{\varphi}{2}\gamma^{12}}\gamma^4 = \left(\cos\frac{\varphi}{2} + \gamma^{12}\sin\frac{\varphi}{2}\right)\gamma^4, \quad (3.16)$$

[3] The subscript in (3.11), which distinguishes two-dimensional planes, is not necessary in the present case.

which satisfies (3.13), as it should be. Note that the boundary condensate is absent when $\varphi = \pi$, as can be seen from (3.11). Then $\hat{M}$ is reduced to just γ^{124}, and this is nothing but the expression implied by (3.14).

Also for the other branes listed in Table I, various boundary condensates can be turned on and the corresponding $\hat{M}$'s are determined by following the same manner. Some examples will be given in the next section.

C. The full boundary state

In summary, by collecting the bosonic and fermionic parts of the boundary state, (3.4) and (3.15), the full boundary state for a Dp-brane instanton $|Dp\rangle$ is given by

$$|\mathrm{D}p\rangle = \mathcal{N}\mathrm{e}^{\sum_{n>0}(\frac{1}{\omega_n}M_{ij}\alpha^i_{-n}\tilde{\alpha}^j_{-n}+\frac{1}{\omega'_n}M_{i'j'}\alpha^{i'}_{-n}\tilde{\alpha}^{j'}_{-n}-i\psi_{-n}\hat{M}\tilde{\psi}_{-n}+i\psi'_{-n}\hat{M}\tilde{\psi}'_{-n})}|\mathrm{D}p\rangle_0, \tag{3.17}$$

where $\mathcal{N}$ is the normalization constant and $|\mathrm{D}p\rangle_0$ is the part composed of zero modes,

$$|\mathrm{D}p\rangle_0 = \mathrm{e}^{\frac{1}{2}M_{ij}a^{i\dagger}a^{j\dagger}+\frac{1}{2}M_{i'j'}a^{i'\dagger}a^{j'\dagger}}\mathrm{e}^{\frac{1}{2}\chi^\dagger\gamma^{123}\hat{M}\chi^\dagger+\frac{1}{2}\chi'^\dagger\gamma^{123}\hat{M}\chi'^\dagger}|0\rangle. \tag{3.18}$$

In the next section we will compute cylinder diagrams with the constructed boundary states.

IV. PARALLEL BRANES WITH BOUNDARY CONDENSATES

In this section, we compute cylinder diagrams that describe the interaction between two parallel D-branes. The typical expression of the amplitude is given by

$$\begin{aligned}\mathcal{A}_{\mathrm{D}p_1;\mathrm{D}p_2}(X^+,X^-,\mathbf{q}_1,\mathbf{q}_2) &= \int\frac{dp^+dp^-}{2\pi i}\mathrm{e}^{-ip^+X^- - ip^-X^+}\langle\mathrm{D}p_1,-p^+,-p^-,\mathbf{q}_1|\left(\frac{1}{p^+(p^- - H)}\right)|\mathrm{D}p_2,p^+,p^-,\mathbf{q}_2\rangle \\ &= \int_{-\infty}^{\infty}\frac{dp^+}{p^+}\mathrm{e}^{-ip^+X^-}\theta(p^+)\langle\mathrm{D}p_1,-p^+,\mathbf{q}_1|\mathrm{e}^{-iHX^+}|\mathrm{D}p_2,p^+,\mathbf{q}_2\rangle,\end{aligned} \tag{4.1}$$

where H is the closed-string light-cone Hamiltonian (2.25) and $X^\pm = (x_2^\pm - x_1^\pm)$ are the separation of two branes in the light-cone directions. Then $\mathbf{q}_1$ and $\mathbf{q}_2$ describe the transverse positions. The $i\epsilon$ prescription, which induces the step function $\theta(p^+)$ [28], is used in going to the last line.

Let us perform a usual Wick rotation on the world sheet, $t = i\tau/\pi$ (with π for later convenience), or in terms of the string coordinate,

$$t = i\frac{X^+}{\pi p^+}. \tag{4.2}$$

Then the resulting amplitude has the form

$$\begin{aligned}&\mathcal{A}_{\mathrm{D}p_1;\mathrm{D}p_2}(X^+,X^-,\mathbf{q}_1,\mathbf{q}_2) \\ &\quad = \int_0^\infty\frac{dt}{t}\mathrm{e}^{\frac{X^+X^-}{\pi t}}\tilde{\mathcal{A}}_{\mathrm{D}p_1;\mathrm{D}p_2}(t,\mathbf{q}_1,\mathbf{q}_2),\end{aligned} \tag{4.3}$$

where we have introduced the following quantity,

$$\begin{aligned}&\tilde{\mathcal{A}}_{\mathrm{D}p_1;\mathrm{D}p_2}(t,\mathbf{q}_1,\mathbf{q}_2) \\ &\quad = \langle\mathrm{D}p_1,-p^+,\mathbf{q}_1|\mathrm{e}^{-2\pi t(Hp^+/2)}|\mathrm{D}p_2,p^+,\mathbf{q}_2\rangle.\end{aligned} \tag{4.4}$$

In the following, we will consider the amplitude with identical D-branes (i.e., $p_1 = p_2$) sitting at the origin of the transverse space, that is, $\mathbf{q}_1 = \mathbf{q}_2 = 0$.

A. A general prescription to compute the amplitudes

The amplitude is evaluated in the standard way, and all the following building blocks for the amplitude calculation are obtained by following [25,28,29]. Note that the zero-point energy is not taken into account because it is canceled out between the bosonic and fermionic contributions in the final expression.

1. The bosonic part (for nonzero modes)

Let us first see the bosonic oscillator part. For each of the Neumann directions without boundary condensate and each of the Dirichlet directions, when the direction is in the 1234 space (or in the 5678 space), the contribution is given by

$$\prod_{n>0}(1-q^{\omega_n})^{-1}\quad\left(\text{or}\quad\prod_{n>0}(1-q^{\omega'_n})^{-1}\right),\qquad q\equiv\mathrm{e}^{-2\pi t}. \tag{4.5}$$

For the presence of boundary condensates in a two-dimensional plane labeled by a, when the two-dimensional plane is in the 1234 space, the contribution is given by

$$\prod_{n>0}(1-q^{\omega_n}\mathrm{e}^{i\phi_a})^{-1}(1-q^{\omega_n}\mathrm{e}^{-i\phi_a})^{-1}, \tag{4.6}$$

where ϕ_a is the difference between two boundary condensates on the two parallel branes represented by $\varphi_a^{(1)}$ and $\varphi_a^{(2)}$, respectively,

$$\phi_a = \varphi_a^{(1)} - \varphi_a^{(2)}. \tag{4.7}$$

From (3.11), the angles that describe the boundary condensates are represented by

$$\cos\varphi_a^{(1)} = -\frac{1-f_a^{(1)2}}{1+f_a^{(1)2}}, \qquad \cos\varphi_a^{(2)} = -\frac{1-f_a^{(2)2}}{1+f_a^{(2)2}}. \tag{4.8}$$

On the other hand, when the two-dimensional plane is in the 5678 space, the contribution is given by

$$\prod_{n>0}(1-q^{\omega'_n}\mathrm{e}^{i\phi_a})^{-1}(1-q^{\omega'_n}\mathrm{e}^{-i\phi_a})^{-1}. \tag{4.9}$$

2. The fermionic part (for nonzero modes)

For the fermionic oscillator part, the contribution from the modes ψ_n and $\tilde{\psi}_n$ is given by

$$\prod_{n>1}\prod_{s_1,s_2,\ldots=\pm1}(1-q^{\omega_n}\mathrm{e}^{\frac{i}{2}(s_1\phi_1+s_2\phi_2+\ldots)})^{d(s_1,s_2,\ldots)}, \tag{4.10}$$

where s_a's are the eigenvalues of the spinors for the rotation generator given by the product of two γ matrices in the corresponding two-dimensional plane labeled by a.[4] The difference ϕ_a is given in (4.7). Then $d(s_1, s_2, \ldots)$ is the multiplicity for a given sequence of $(s_1, s_2, \ldots)$. Though it is not explicitly denoted, there is a constraint on the values of s_a such that the product of all s_a's is consistent with the chiralities of ψ_n and $\tilde{\psi}_n$ (which are listed in Table II).

Similarly, the contribution from the modes ψ'_n and $\tilde{\psi}'_n$ is given by

$$\prod_{n>1}\prod_{s'_1,s'_2,\ldots=\pm1}(1-q^{\omega'_n}\mathrm{e}^{\frac{i}{2}(s'_1\phi_1+s'_2\phi_2+\ldots)})^{d(s'_1,s'_2,\ldots)}, \tag{4.11}$$

where the product of all s'_a's should be consistent with the chiralities of ψ'_n and $\tilde{\psi}'_n$ listed in Table II. Although the expressions given in (4.10) and (4.11) are rather symbolic, the meaning of them will be clarified in later explicit evaluations of various amplitudes.

3. The zero modes

The next is to consider the contributions from the bosonic zero modes. For each of the directions without boundary condensates, if it is in the 1234 space (or in the 5678 space), the contribution is given by

[4] The eigenvalues of the antisymmetric product of two γ matrices are $\pm i$. The symbol s_a for these eigenvalues represents only the sign.

TABLE II. The chiralities of the fermionic modes for γ^9, γ^{1234}, and γ^{5678}.

	γ^9	γ^{1234}	γ^{5678}
ψ_n, χ	$-$	$+$	$-$
$\tilde{\psi}_n$	$+$	$-$	$-$
ψ'_n, χ'	$-$	$-$	$+$
$\tilde{\psi}'_n$	$+$	$+$	$+$

$$(1-q^{m/3})^{-1/2} \quad (\text{or} \quad (1-q^{m/6})^{-1/2}). \tag{4.12}$$

For a two-dimensional plane labeled by a with a boundary condensate, if it is in the 1234 space, the contribution is

$$(1-\cos\varphi_a^{(1)}\cos\varphi_a^{(2)}q^{m/3})^{-1}, \tag{4.13}$$

while if it is in the 5678 space, the contribution is

$$(1-\cos\varphi_a^{(1)}\cos\varphi_a^{(2)}q^{m/6})^{-1}. \tag{4.14}$$

Then let us see the fermionic zero modes. The contribution from the mode χ is given by

$$\prod_{s_1,s_2,\ldots=\pm1}(1-q^{m/3}\mathrm{e}^{\frac{i}{2}(s_1\phi_1+s_2\phi_2+\ldots)})^{d(s_1,s_2,\ldots)/2}, \tag{4.15}$$

where the product of s_a's is under the same constraint given to (4.10). Similarly, the contribution from the mode χ' is

$$\prod_{s'_1,s'_2,\ldots=\pm1}(1-q^{m/6}\mathrm{e}^{\frac{i}{2}(s'_1\phi_1+s'_2\phi_2+\ldots)})^{d(s'_1,s'_2,\ldots)/2}, \tag{4.16}$$

where the product of s'_a's is under the same constraint given to (4.11).

By following the general prescription, we will consider concrete examples in the next subsection.

B. Examples

In the following, let us consider three types of D-branes concretely and compute the corresponding amplitudes.

1. Parallel D6-branes

First of all, let us consider parallel D6-branes which extend along the (1,2,4,5,6,7,8) directions (see Table I). We turn on boundary condensates in the 12, 56, and 78 planes. Then the matrix M is expressed as

$$M = \begin{pmatrix} M_{(1)} & & & & \\ & +1 & & & \\ & & -1 & & \\ & & & M_{(2)} & \\ & & & & M_{(3)} \end{pmatrix}. \tag{4.17}$$

Here $M_{(a)} = \mathrm{e}^{\varphi_a T_{(a)}}$ $(a = 1, 2, 3)$ is the 2×2 block given in (3.9), where $T_{(1)}$, $T_{(2)}$, and $T_{(3)}$ are the rotation generators in the 12, 56, and 78 planes, respectively.

On the other hand, $\hat{M}$ is given by

$$\hat{M} = \mathrm{e}^{\frac{\varphi_1}{2}\gamma^{12}}\gamma^4 \mathrm{e}^{\frac{\varphi_2}{2}\gamma^{56}}\mathrm{e}^{\frac{\varphi_3}{2}\gamma^{78}} = \mathrm{e}^{\frac{\varphi_1}{2}\gamma^{12}}\mathrm{e}^{\frac{\varphi_2}{2}\gamma^{56}}\mathrm{e}^{\frac{\varphi_3}{2}\gamma^{78}}\gamma^4. \tag{4.18}$$

For the contributions of ψ_n, $\tilde{\psi}_n$, and χ to the amplitude, the sequences of eigenvalues (s_1, s_2, s_3) in (4.10) and (4.15) are obtained from Table II as

$$(s_1, s_2, s_3) = (\pm, +, +), (\pm, -, -), \tag{4.19}$$

for each of which the multiplicity is 1, $d(s_1, s_2, s_3) = 1$. For ψ'_n, $\tilde{\psi}'_n$, and χ', the sequences of eigenvalues (s'_1, s'_2, s'_3) are

$$(s'_1, s'_2, s'_3) = (\pm, +, -), (\pm, -, +), \tag{4.20}$$

for each of which the multiplicity is 1.

The basic building blocks from (4.5) to (4.16) lead to the following factorized form,

$$\tilde{\mathcal{A}}_{\mathrm{D6;D6}}(t) = \tilde{\mathcal{A}}^{(0)}_{\mathrm{D6;D6}}(t)\tilde{\mathcal{A}}^{(\mathrm{osc})}_{\mathrm{D6;D6}}(t). \tag{4.21}$$

Here $\tilde{\mathcal{A}}^{(0)}_{\mathrm{D6;D6}}(t)$ comes from the zero mode,

$$\begin{aligned}\tilde{\mathcal{A}}^{(0)}_{\mathrm{D6;D6}}(t) = {}& (1 - q^{m/3})^{-1}(1 - \cos\varphi_1^{(1)}\cos\varphi_1^{(2)} q^{m/3})^{-1} \\ & \times (1 - \cos\varphi_2^{(1)}\cos\varphi_2^{(2)} q^{m/6})^{-1}(1 - \cos\varphi_3^{(1)}\cos\varphi_3^{(2)} q^{m/6})^{-1} \\ & \times (1 - q^{m/3}\mathrm{e}^{\frac{i}{2}(\phi_1+\phi_2+\phi_3)})^{1/2}(1 - q^{m/3}\mathrm{e}^{\frac{i}{2}(-\phi_1+\phi_2+\phi_3)})^{1/2} \\ & \times (1 - q^{m/3}\mathrm{e}^{\frac{i}{2}(\phi_1-\phi_2-\phi_3)})^{1/2}(1 - q^{m/3}\mathrm{e}^{\frac{i}{2}(-\phi_1-\phi_2-\phi_3)})^{1/2} \\ & \times (1 - q^{m/6}\mathrm{e}^{\frac{i}{2}(\phi_1+\phi_2-\phi_3)})^{1/2}(1 - q^{m/6}\mathrm{e}^{\frac{i}{2}(-\phi_1+\phi_2-\phi_3)})^{1/2} \\ & \times (1 - q^{m/6}\mathrm{e}^{\frac{i}{2}(\phi_1-\phi_2+\phi_3)})^{1/2}(1 - q^{m/6}\mathrm{e}^{\frac{i}{2}(-\phi_1-\phi_2+\phi_3)})^{1/2},\end{aligned} \tag{4.22}$$

and $\tilde{\mathcal{A}}^{(\mathrm{osc})}_{D6;D6}(t)$ is from the nonzero mode,

$$\begin{aligned}\tilde{\mathcal{A}}^{(\mathrm{osc})}_{\mathrm{D6;D6}}(t) = \prod_{n>0} & (1 - q^{\omega_n})^{-2}(1 - q^{\omega_n}\mathrm{e}^{i\phi_1})^{-1}(1 - q^{\omega_n}\mathrm{e}^{-i\phi_1})^{-1} \\ & \times (1 - q^{\omega'_n}\mathrm{e}^{i\phi_2})^{-1}(1 - q^{\omega'_n}\mathrm{e}^{-i\phi_2})^{-1}(1 - q^{\omega'_n}\mathrm{e}^{i\phi_3})^{-1}(1 - q^{\omega'_n}\mathrm{e}^{-i\phi_3})^{-1} \\ & \times (1 - q^{\omega_n}\mathrm{e}^{\frac{i}{2}(\phi_1+\phi_2+\phi_3)})(1 - q^{\omega_n}\mathrm{e}^{\frac{i}{2}(-\phi_1+\phi_2+\phi_3)}) \\ & \times (1 - q^{\omega_n}\mathrm{e}^{\frac{i}{2}(\phi_1-\phi_2-\phi_3)})(1 - q^{\omega_n}\mathrm{e}^{\frac{i}{2}(-\phi_1-\phi_2-\phi_3)}) \\ & \times (1 - q^{\omega'_n}\mathrm{e}^{\frac{i}{2}(\phi_1+\phi_2-\phi_3)})(1 - q^{\omega'_n}\mathrm{e}^{\frac{i}{2}(-\phi_1+\phi_2-\phi_3)}) \\ & \times (1 - q^{\omega'_n}\mathrm{e}^{\frac{i}{2}(\phi_1-\phi_2+\phi_3)})(1 - q^{\omega'_n}\mathrm{e}^{\frac{i}{2}(-\phi_1-\phi_2+\phi_3)}).\end{aligned} \tag{4.23}$$

It is now convenient to introduce the "massive" thetalike function defined in [30,31]

$$\Theta_{(a,b)}(\tau;\nu) \equiv e^{4\pi\tau_2\Delta(\nu;a)}\prod_{n\in\mathbf{Z}}\left|1 - \mathrm{e}^{-2\pi\tau_2\sqrt{\nu^2+(n+a)^2}+2\pi\tau_1(n+a)+2\pi ib}\right|^2, \tag{4.24}$$

where $\tau = \tau_1 + i\tau_2$ and $\Delta(\nu;a)$ is the zero-point energy. Actually, we are not concerned with $\Delta(\nu;a)$ because it disappears in the final expression.

By using the massive theta functions, the amplitude can be rewritten into a simpler form,

$$\tilde{\mathcal{A}}_{\mathrm{D6;D6}}(t) = \frac{(1-q^{m/3}\mathrm{e}^{i\phi_1})^{1/2}(1-q^{m/3}\mathrm{e}^{-i\phi_1})^{1/2}}{(1-\cos\varphi_1^{(1)}\cos\varphi_1^{(2)}q^{m/3})} \times \frac{(1-q^{m/6}\mathrm{e}^{i\phi_2})^{1/2}(1-q^{m/6}\mathrm{e}^{-i\phi_2})^{1/2}}{(1-\cos\varphi_2^{(1)}\cos\varphi_2^{(2)}q^{m/6})}$$
$$\times \frac{(1-q^{m/6}\mathrm{e}^{i\phi_3})^{1/2}(1-q^{m/6}\mathrm{e}^{-i\phi_3})^{1/2}}{(1-\cos\varphi_3^{(1)}\cos\varphi_3^{(2)}q^{m/6})} \frac{\Theta^{1/2}_{(0,(\phi_1+\phi_2+\phi_3)/4\pi)}(it;m/3)\Theta^{1/2}_{(0,(\phi_1-\phi_2-\phi_3)/4\pi)}(it;m/3)}{\Theta^{1/2}_{(0,0)}(it;m/3)\Theta^{1/2}_{(0,\phi_1/2\pi)}(it;m/3)}$$
$$\times \frac{\Theta^{1/2}_{(0,(\phi_1+\phi_2-\phi_3)/4\pi)}(it;m/6)\Theta^{1/2}_{(0,(\phi_1-\phi_2+\phi_3)/4\pi)}(it;m/6)}{\Theta^{1/2}_{(0,\phi_2/2\pi)}(it;m/6)\Theta^{1/2}_{(0,\phi_3/2\pi)}(it;m/6)}. \tag{4.25}$$

We would like to note that, if the boundary condensates are absent, the above amplitude simply becomes one. This is consistent with the previous result [27] of the amplitude calculation without boundary condensates.

2. Parallel D4-branes

Let us consider parallel D4-branes. The world volume of the D4-branes extends along the (1,2,3,5,6) directions. Then we turn on boundary condensates in the 1-2 and 5-6 planes.

It is possible to compute the amplitude in the same way as in the case of D6-branes. But it can also be obtained simply by taking the magnetic background in the 78 plane to be infinite from the result on D6-branes. Note that x^3 (x^4) should be understood as the Neumann (Dirichlet) direction at that time.

Let us consider the limit $f_3^{(1)}, f_3^{(2)} \to \infty$ (equivalently $\varphi_3^{(1)}, \varphi_3^{(2)} \to 0$, and thus $\phi_3 \to 0$) in (4.25). Then the x^7 and x^8 directions are turned into the Dirichlet ones. The resulting amplitude is given by

$$\tilde{\mathcal{A}}_{\mathrm{D4;D4}}(t) = \frac{(1-q^{m/3}\mathrm{e}^{i\phi_1})^{1/2}(1-q^{m/3}\mathrm{e}^{-i\phi_1})^{1/2}}{(1-\cos\varphi_1^{(1)}\cos\varphi_1^{(2)}q^{m/3})} \times \frac{(1-q^{m/6}\mathrm{e}^{i\phi_2})^{1/2}(1-q^{m/6}\mathrm{e}^{-i\phi_2})^{1/2}}{(1-\cos\varphi_2^{(1)}\cos\varphi_2^{(2)}q^{m/6})}$$
$$\times \frac{\Theta^{1/2}_{(0,(\phi_1+\phi_2)/4\pi)}(it;m/3)\Theta^{1/2}_{(0,(\phi_1-\phi_2)/4\pi)}(it;m/3)}{\Theta^{1/2}_{(0,0)}(it;m/3)\Theta^{1/2}_{(0,\phi_1/2\pi)}(it;m/3)}$$
$$\times \frac{\Theta^{1/2}_{(0,(\phi_1+\phi_2)/4\pi)}(it;m/6)\Theta^{1/2}_{(0,(\phi_1-\phi_2)/4\pi)}(it;m/6)}{\Theta^{1/2}_{(0,0)}(it;m/6)\Theta^{1/2}_{(0,\phi_2/2\pi)}(it;m/6)}. \tag{4.26}$$

3. Parallel D2-branes

Finally, we shall consider parallel D2-branes. The world-volume of the D2 branes expands along the (1,2,4) directions. We turn on the boundary condensate in the 1-2 plane. Then it is straightforward to obtain the following amplitude,

$$\tilde{\mathcal{A}}_{\mathrm{D2;D2}}(t) = \frac{(1-2\cos(\phi/2)q^{m/3}+q^{2m/3})(1-2\cos(\phi/2)q^{m/6}+q^{2m/6})}{(1-q^{m/3})(1-q^{m/6})^2(1-\cos\varphi^{(1)}\cos\varphi^{(2)}q^{m/3})}$$
$$\times \prod_{n>0} \frac{(1-2\cos(\phi/2)q^{\omega_n}+q^{2\omega_n})^2(1-2\cos(\phi/2)q^{\omega'_n}+q^{2\omega'_n})^2}{(1-q^{\omega_n})^2(1-q^{\omega'_n})^4(1-2\cos(\phi)q^{\omega_n}+q^{2\omega_n})}. \tag{4.27}$$

This is also obtained from the amplitude for the D4-branes (4.26) by taking $f_2^{(1)}, f_2^{(2)} \to \infty$ (equivalently $\varphi_2^{(1)}, \varphi_2^{(2)} \to 0$, and thus $\phi_2 \to 0$).

In terms of the massive theta-like function, the amplitude can be rewritten as

$$\tilde{\mathcal{A}}_{\mathrm{D2;D2}}(t) = \frac{(1-q^{m/3}\mathrm{e}^{i\phi})^{1/2}(1-q^{m/3}\mathrm{e}^{-i\phi})^{1/2}}{(1-\cos\varphi^{(1)}\cos\varphi^{(2)}q^{m/3})} \frac{\Theta_{(0,\phi/4\pi)}(it;m/3)\Theta_{(0,\phi/4\pi)}(it;m/6)}{\Theta^{1/2}_{(0,0)}(it;m/3)\Theta^{1/2}_{(0,\phi/2\pi)}(it;m/3)\Theta_{(0,0)}(it;m/6)}, \tag{4.28}$$

where the subscript 1 has been omitted from all of the angles because the boundary condensate is turned on only in the 1-2 plane.

V. NO PAIR PRODUCTION OF OPEN STRINGS

This section considers the possibility of the pair production of open strings from the viewpoint of the pole structure of the amplitudes. As an example, we concentrate on the case of two parallel D2-branes.

First of all, it is necessary to move from the cylinder diagram (closed string channel) to an annulus one (open string channel). It is carried out by performing the transformation $t \to t' = 1/t$. By the way, the resulting amplitude is the one containing the magnetic condensate. In order to investigate the issue of open string pair production, it is necessary to have an electric condensate. It is difficult to accomplish it directly because we are now working with the light-cone gauge. Still, the amplitude with an electric condensate can be anticipated from the magnetic one through the replacement $f \to if$ (equivalently $\phi \to i\phi$).[5] This anticipation will be supported later from agreement with the result in the flat limit.

Then the pole structure on the real t' axis leads to an imaginary part in the expression of the energy, which is given by the sum over the residues of the poles. This is interpreted as the sign of the pair creation of open strings.

To see the pole structure, recall the transformation law of massive thetalike functions under the S transformation $\tau \to -1/\tau$ [30,31],

$$\Theta_{(a,b)}\left(-\frac{1}{\tau};|\tau|\nu\right) = \Theta_{(b,-a)}(\tau;\nu). \tag{5.1}$$

Then the D2 amplitude in (4.28) is rewritten as

$$\begin{aligned}\tilde{\mathcal{A}}_{\mathrm{D2;D2}}(t) &= \frac{\Theta_{(0,\phi/4\pi)}(it;m/3)\Theta_{(0,\phi/4\pi)}(it;m/6)}{\Theta^{1/2}_{(0,\phi/2\pi)}(it;m/3)}\times\cdots\\ &\xrightarrow{t\to t'=1/t}\frac{\Theta_{(-\phi/4\pi,0)}(it';m/3t')\Theta_{(-\phi/4\pi,0)}(it';m/6t')}{\Theta^{1/2}_{(-\phi/2\pi,0)}(it';m/3t')}\times\cdots\\ &\xrightarrow{\phi\to i\phi}\frac{\Theta_{(-i\phi/4\pi,0)}(it';m/3t')\Theta_{(-i\phi/4\pi,0)}(it';m/6t')}{\Theta^{1/2}_{(-i\phi/2\pi,0)}(it';m/3t')}\times\cdots,\end{aligned} \tag{5.2}$$

where "…" denote the factors irrelevant to the pole structure. By the definition of the massive thetalike function (4.24), the last line does not lead to any pole in the real t' axis (For details of the proof, see Appendix A). Thus it has been shown that there is no pair creation of open strings.

One interpretation of this result is that strings are trapped in a harmonic potential due to the IIA plane wave background. In other words, the string coordinates describe the set of harmonic oscillators. Hence it is impossible to separate the constituents of the produced pairs in an infinite distance even if a pair is produced at a certain instance.

At this point, one may ask if the production rate becomes finite after removing the harmonic potential, that is, after taking the flat space-time limit ($m \to 0$). If it tends to be finite, then the well-known result in flat space-time is reproduced and our interpretation passes an important check.

Let us consider the flat space-time limit. With the help of the expression of the massive thetalike function in $m \to 0$ limit [31],

$$\lim_{m\to 0}\Theta_{(a,b)}(\tau;m) = \mathrm{e}^{-2\pi\tau_2 a^2}\left|\frac{\theta_1(a\tau+b|\tau)}{\eta(\tau)}\right|^2, \tag{5.3}$$

and the usual product form of Jacobi theta function,

$$\theta_1(z|\tau) = 2q^{\frac{1}{8}}\sin(\pi z)\prod_{n=1}^{\infty}(1-q^n)(1-q^n\mathrm{e}^{2\pi iz})(1-q^n\mathrm{e}^{-2\pi iz}) \quad (q\equiv\mathrm{e}^{2\pi i\tau}), \tag{5.4}$$

the last line of (5.2) can be rewritten as

$$\begin{aligned}&\frac{\Theta_{(-i\phi/4\pi,0)}(it';m/3t')\Theta_{(-i\phi/4\pi,0)}(it';m/6t')}{\Theta^{1/2}_{(-i\phi/2\pi,0)}(it';m/3t')}\\ &\times\cdots\xrightarrow{m\to 0}\frac{\theta_1^4(\frac{\phi t'}{4\pi}|it')}{\theta_1(\frac{\phi t'}{2\pi}|it')}\times\cdots\longrightarrow\frac{\sin^4(\frac{\phi t'}{4})}{\sin(\frac{\phi t'}{2})}\times\cdots.\end{aligned} \tag{5.5}$$

The last line explicitly shows that there are an infinite number of poles on the real t' axis. The locations of the poles are specified by

[5] The replacement is not to be regarded as the result of signature change from the Euclidean space to the Minkowskian one, because the time direction is definitely set by the light-cone time in our computation. Rather, it should be anticipated as a natural generalization of the resulting amplitude following the reasoning of [19]. For the issue of signature change from a rigorous viewpoint, see for example Ref. [32].

$$t' = 2\pi(2k+1)/\phi \quad (k \in \mathbf{Z}).$$

This is nothing but the result in flat spacetime [25].[6]

VI. CONCLUSION AND DISCUSSION

We have considered whether an external electric field may cause the pair production of open strings in a type IIA plane-wave background. The boundary states of D-branes with condensates have been constructed in the Green-Schwarz formulation with the light-cone gauge. The cylinder diagrams have been computed with the boundary states and the resulting amplitudes are shown to be expressed in terms of massive theta functions. This is a characteristic property intrinsic to plane-wave backgrounds. As a consequence, although the value of the electric field is bounded by the upper value,[7] there is no pole in the amplitudes and it indicates that no pair production occurs in the plane-wave background. Our result is based on an analysis in a IIA pp-wave background, but the result would be universal for a class of plane-wave backgrounds.

In order to confirm our conjecture that no pair production occurs, it is indispensable to compute the amplitudes in other plane-wave backgrounds. It would be interesting to classify the gravitational backgrounds which allows the pair production. For example, plane-wave backgrounds with flat directions are good candidates. In this sense, adding angular momenta would be able to support the pair production.

The next important question is whether or not the result of no pair production is intrinsic to plane-wave backgrounds. As was stated in the Introduction, it is interesting to study the possibility of the pair production in AdS backgrounds. A plane-wave background appears as an approximation of the AdS geometry times an internal space, while the geometry of flat space always appears by considering a small and local region and the pair creation seems possible on it. Actually, a Penrose limit [33] of the AdS geometry may lead to flat space, depending on the choice of the null geodesic. Thus, our argument would not be able to exclude the possibility that the pair production of strings occurs in the AdS backgrounds. The phenomenon that no pair production occurs may be an artifact in the plane-wave approximation.

The study of the pair production of open strings on curved backgrounds would reveal a new aspect of the string dynamics.

[6]Although D-strings in type IIB string theory are considered there, the essential point is the same.

[7]We note that the upper bound is not the usual constant critical electric field and comes from the consideration of Eq. (A6), the zero-mode part that is essential in investigating the pair production. From the viewpoint of ϕ, it is easy to see that ϕ has the upper bound from the non-negativity of the inside of the square root, which is given by (A7).

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APPENDIX: THE POLE STRUCTURE OF THE D2-BRANE AMPLITUDE

We will show that there is no pole in the D2-brane amplitude given in (5.2).

Let us begin with the last line of (5.2). The definition of the massive theta function is given in (4.24). The goal is to show that there is no pole on the real t' axis. The amplitude is divided into (1) the $n \neq 0$ contributions and (2) the $n = 0$ contribution. We will consider each of them below.

1. The $n \neq 0$ contributions

In order to study the pole structure, let us consider zero points of the following part,

$$\Theta_{(-i\phi/2\pi,0)}\left(it'; \frac{m}{3t'}\right) = e^{4\pi t' \Delta(m/3t'; -i\phi/2\pi)} \times \prod_{n\in\mathbb{Z}} \left|1 - \exp\left[-\sqrt{M^2 + t'^2(2\pi n - i\phi)^2}\right]\right|^2, \quad \text{(A1)}$$

where we have defined

$$M \equiv \frac{2\pi m}{3}.$$

It is convenient to rewrite the argument of the exponential part as follows:

$$M^2 + t'^2(2\pi n - i\phi)^2 = M^2 + t'^2[(2\pi n)^2 - \phi^2] - i \cdot 4\pi n t'^2 \phi \equiv re^{i\varphi} = r\cos\varphi + ir\sin\varphi. \quad \text{(A2)}$$

Here the parameters are identified as

$$r\sin\varphi = -4\pi n t'^2 \phi, \qquad r\cos\varphi = M^2 + t'^2[(2\pi n)^2 - \phi^2], \quad \text{(A3)}$$

where r and φ are represented by

$$r^2 = (4\pi n t'^2 \phi)^2 + (M^2 + t'^2[(2\pi n)^2 - \phi^2])^2,$$
$$\tan\varphi = \frac{-4\pi n t'^2 \phi}{M^2 + t'^2[(2\pi n)^2 - \phi^2]}. \quad \text{(A4)}$$

Then it is easy to derive the following expression,

$$\left|1 - \exp\left[-\sqrt{M^2 + t'^2(2\pi n - i\phi)^2}\right]\right|^2 = 2 - 2e^{-r^{1/2}\cos(\varphi/2)}\cos\left(r^{1/2}\sin(\varphi/2)\right). \quad \text{(A5)}$$

Because $r \neq 0$ on the real t' axis, the only condition that poles exist is the following,

$$\varphi = \pi, \qquad r^{1/2} = 2\pi N (N \in \mathbb{N}).$$

However, this condition is not satisfied due to the condition (A4).

2. The $n = 0$ contribution

The next is to see the contribution from the $n = 0$ mode.

From the denominator of (5.2), one can read off the $n = 0$ contribution as follows:

$$\Theta^{1/2}_{(-i\phi/2\pi,0)}(it'; m/3t') \longrightarrow 1 - \exp\left[-2\pi t'\sqrt{\left(\frac{m}{3t'}\right)^2 - \left(\frac{\phi}{2\pi}\right)^2}\right]. \quad \text{(A6)}$$

It is easy to see that this factor becomes 0 at

$$t' = \frac{m}{3}\cdot\frac{2\pi}{\phi}. \quad \text{(A7)}$$

At this point, it seems that there should be a pole at this value.

On the other hand, a massive thetalike function on the numerator of (5.2) contains the $n = 0$ contribution given by

$$\Theta_{(-i\phi/4\pi,0)}(it'; m/6t') \longrightarrow \left(1 - \exp\left[-2\pi t'\sqrt{\left(\frac{m}{6t'}\right)^2 - \left(\frac{\phi}{4\pi}\right)^2}\right]\right)^2. \quad \text{(A8)}$$

Interestingly, this factor also becomes 0 at the value of t' in (A7). Noting that the power of (A8) is higher than that of (A6), the value of t' in (A7) does not indicate the existence of a pole but a vanishing point of the amplitude.

In total, we conclude that the D2-brane amplitude given in (5.2) does not have any pole on the real t' axis.

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