Gauge fields as Goldstone bosons triggered by spontaneously broken supersymmetry

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The emergent gauge theories are reconsidered in light of supersymmetry and an appropriate emergence conjecture is formulated. Accordingly, it might be expected that only global symmetries are fundamental symmetries of nature, whereas local symmetries and associated massless gauge fields could solely emerge due to spontaneous breaking of the underlying spacetime symmetries involved, such as relativistic invariance and supersymmetry. We further argue that this breaking, taken in the form of the nonlinear σ-model-type pattern for vector fields or superfields, puts essential restrictions on geometrical degrees of freedom of a physical field system that makes it adjust itself in such a way that its global internal symmetry G turns into the local symmetry $G_{\rm loc}$. Otherwise, a given field system could lose too many degrees of freedom, thus getting unphysical, which would make it impossible to set the required initial conditions in an appropriate Cauchy problem, or to choose self-consistent equal-time commutation relations in quantum theory. Remarkably, this emergence process may naturally be triggered by supersymmetry, as is illustrated in detail by an example of a general supersymmetric QED model which is then extended to the Standard Model and GUTs. The requirement of vacuum stability in such a class of models makes both Lorentz invariance and supersymmetry become spontaneously broken in the visible sector. As a consequence, the massless photon and other gauge bosons appear as the corresponding Goldstone and pseudo-Goldstone zero modes and special local invariance is simultaneously generated. Due to this invariance, all possible Lorentz violations turn out to be completely canceled out among themselves. However, broken supersymmetry effects related to the existence of a light pseudo-Goldstino (being essentially a photino) are still left in the theory. It typically appears in the low-energy particle spectrum as the eV-scale stable lightest supersymmetric particle or the electroweak-scale long-lived next-to-lightest supersymmetric particle, and in both cases it is accompanied by a very light gravitino that could be considered as some observational signature in favor of emergent supersymmetric theories.

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I. INTRODUCTION

It is now conventional wisdom that internal gauge symmetries form the basis of modern particle physics, being most successfully realized within the celebrated Standard Model (SM) of quarks and leptons and their fundamental strong, weak, and electromagnetic interactions. At the same time, local gauge invariance, contrary to the global symmetry case, may look like a cumbersome geometrical input rather than a "true" physical principle, especially in the framework of an effective quantum field theory (QFT) becoming, presumably, irrelevant at very high energies. In this connection, one could wonder whether there is any basic dynamical reason that necessitates gauge invariance and the associated masslessness of gauge fields as some emergent phenomenon arising from a more profound level of dynamics. By analogy with a dynamical origin of massless scalar particle excitations, which is very well understood in terms of spontaneously broken global internal symmetries [1], one could think that the origin of massless gauge fields as vector Nambu-Goldstone (NG) bosons is related to the spontaneous violation of Lorentz invariance which is in fact the minimal spacetime global symmetry underlying particle physics. This well-known approach, providing a viable alternative to quantum electrodynamics [2], gravity [3], and Yang-Mills theories [4], has a long history that started over 50 years ago.

However, the role of Lorentz invariance may change, and its spontaneous violation may not be the only reason why massless photons could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest is related to supersymmetry which has made a serious impact on particle physics in the last few decades (though it has not been yet discovered). Actually, as we will see, the situation is changed dramatically in the SUSY inspired emergent gauge theories. In sharp contrast to non-SUSY analogs, it appears that the spontaneous Lorentz invariance violation (SLIV) caused by an arbitrary potential of vector superfield $V(x,\theta,\bar{\theta})$ never goes any further than some nonlinear gauge constraint put on its vector field component $A_\mu(x)$ associated with a photon. This allows us to think that physical Lorentz invariance is somewhat protected by SUSY, thus only requiring the "condensation" of the gauge degree of freedom in the vector field A_μ. The point is, however, that even in the case when SLIV is not physical it inevitably leads to the generation of massless photons as vector NG bosons, provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance.

The paper is organized in the following way. In the next section we give a brief sketch of existing emergent gauge theories in light of supersymmetry. This help us to see more clearly the significant changes which appear to be necessary in a supersymmetric context, and to properly formulate an emergence conjecture, in Sec. III. We give a detailed presentation of emergent gauge invariant Abelian and non-Abelian theories and show the somewhat fundamental relationship between spontaneous Lorentz violation and emergent gauge invariance due to which all SLIV contributions to physical processes completely cancel out among themselves. In essence, the only way for SLIV to manifest itself observationally may appear to be if gauge invariance in these theories turns out to be broken in an explicit rather than spontaneous way. As a result, the SLIV cancellation mechanism does not work longer and one inevitably comes to physical Lorentz violation, as is explicitly demonstrated in Sec. III E. In the next section we consider the supersymmetric QED model extended by an arbitrary polynomial potential of a massive vector superfield that breaks gauge invariance in the SUSY invariant phase. However, the requirement of vacuum stability in such a class of models makes both supersymmetry and Lorentz invariance become spontaneously broken. As a consequence, the massless photino and photon appear as the corresponding Nambu-Goldstone zero modes in an emergent SUSY QED, and also a special gauge invariance is simultaneously generated. Due to this invariance, all observable relativistically noninvariant effects appear to be completely canceled out and physical Lorentz invariance is recovered. Further in Sec. V, all basic arguments developed in SUSY QED are generalized successively to the Standard Model and grand unified theories (GUTs). For definiteness, we focus on the $U(1)\times SU(N)$ symmetrical theories. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case, one only has the SUSY invariant phase in the theory that makes it inappropriate for an outgrowth of an emergence process. As possible realistic realizations, the Standard Model case with the electroweak $U(1)\times SU(2)$ symmetry and flipped $SU(5)$ GUT are briefly discussed. Phenomenological implications are largely given in Sec. VI. The most interesting part of them is related to the massless photino mentioned above. This photino being mixed with another Goldstino appearing from a spontaneous SUSY violation in the hidden sector essentially turns into the light pseudo-Goldstino whose physics is considered in significant detail. This physics is unambiguously related to the class of models where SUSY breaks, at least partially, in the visible sector as well. This is the only class of models where emergent supersymmetric QED or the Standard Model can be self-consistently realized. And finally in Sec. VII, we summarize the main results and conclude.

II. PHOTONS AS NAMBU-GOLDSTONE ZERO MODES: A BRIEF SKETCH

Below, we briefly comment on some known models where an idea of emergent gauge theory according to which photons and other gauge fields may appear as Nambu-Goldstone zero modes is realized in one way or the other. They include the composite models, where this idea was considered for the first time [2,5–7], and three other ones: the vector field potential-based models [8,9], the vector field constraint-based models [10–12], and models with external vector backgrounds [13,14], together with their supersymmetric extensions [15–17]. Some quick summary on them may be useful before we finally turn to emergent SUSY models introduced recently [18], which we consider in significant detail in subsequent sections.

A. Composite models

The first models [2] realizing the SLIV conjecture were based on the four-Fermi interaction where the photon appears as a fermion-antifermion pair composite state in complete analogy with massless composite scalar fields (identified with pions) in the original Nambu–Jona-Lasinio model [1]. This old idea is better expressed nowadays in terms of an effective field theory where the standard QED Lagrangian is readily obtained through the corresponding loop radiative effects due to the N fermion species involved [5–7]. Also, instead of the old four-Fermi model, one can start with the generalized effective action with all possible multi-Fermi interactions [6]:

$$L(\psi,\bar{\psi})=\bar{\psi}_i(i\gamma\partial-\mathrm{m})\psi_i+N\sum_{n=1}^{\infty}\mathrm{G}_{2n}\left[\frac{\bar{\psi}_i\gamma_\mu\psi_i}{N}\right]^{2n}. \quad (1)$$

Here summation over flavor indices i (and spacetime indices μ) is implied so that the Lagrangian $L(\psi,\bar{\psi})$ possesses a $U(N)$ global flavor symmetry. This model is evidently nonrenormalizable and can be only considered as an effective theory valid at sufficiently low energies. The dimensionful couplings G_{2n} are proportional to appropriate powers of some UV cutoff Λ and are ultimately related to some energy scale up to which this effective theory is valid, $\mathrm{G}_{2n}\sim\Lambda^{4-6n}$. Factors of N in (1) are chosen in such a way as to provide a well-defined large N limit so that the correlators for the properly normalized fermion bilinears $(\bar{\psi}_i\gamma_\mu\psi_i)/N$ will scale as N^0.

The action (1) can be rewritten using the standard trick of introducing an auxiliary field A_μ:

$$L(\psi, \bar{\psi}, A_\mu) = \bar{\psi}_i(i\gamma\partial - \gamma A - \mathrm{m})\psi_i - N\mathrm{V}(A_\mu A^\mu). \quad (2)$$

The potential V is a power series in $A_\mu A^\mu$,

$$\mathrm{V}(A_\mu A^\mu) = \frac{\mu^2}{2} A_\mu A^\mu - \frac{\lambda_c^{(4)}}{4}(A_\mu A^\mu)^2 + \cdots, \quad (3)$$

with coefficients chosen as

$$\mu^2 = \frac{1}{2\mathrm{G}_2}, \qquad \lambda_c^{(4)} = \frac{1}{4}\frac{\mathrm{G}_4}{\mathrm{G}_2^4}, \ldots \quad (4)$$

and by solving the algebraic equations of motion for A_μ and substituting back into (2), one recovers the starting Lagrangian (1). If instead one integrates out the fermions ψ_i, an effective action emerges in terms of the composite A^μ field alone, which acquires its own dynamics:

$$S_{\mathrm{eff}} = N \int d^4x \left[\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + \mathrm{V}(A_\mu A^\mu) + A_\mu J^\mu + \cdots\right], \quad (5)$$

where the coupling constant e^2 is given by

$$\mathrm{e}^2 = \frac{12\pi^2}{\ln(\Lambda^2/\mathrm{m}^2)}, \quad (6)$$

with Λ standing for the UV cutoff mentioned above.[1] Since the fermions ψ_i are minimally coupled to the vector field A_μ in (2), its kinetic term generated in this way appears gauge invariant provided that a gauge invariant cutoff is chosen. Furthermore, since there are N species of fermions, the effective action (5) has an overall factor of N. And lastly, introducing in the basic Lagrangian the minimal couplings of some extra matter fields Ψ_I $(I = 1, 2, \ldots)$ to the basic fermions ψ_i via conserved currents, $J^\mu(\Psi)\bar{\psi}_i\gamma_\mu\psi_i$, one generates the minimal matter couplings given in (5), which are also gauge invariant.

Let us turn now to the Lorentz violation in the model. As is readily seen from Eqs. (3)–(4), the quartic term in the effective action S_{eff} may only appear when higher-order terms, beyond the four-Fermi interaction, are activated in the basic Lagrangian (1). This quartic term is enough to generate the familiar Mexican hat structure of the potential V (3) and induce spontaneous Lorentz violation through the nonvanishing vacuum expectation value (VEV) of the vector field A_μ (for more detail, see the next section). Thereby, three Lorentz generators become broken for both timelike and spacelike Lorentz violation. As a result, the three massless NG modes associated with this symmetry breaking emerge. They might be interpreted as the photon components. However, owing to the lack of gauge invariance in the starting fermion Lagrangian (1) the effective theory for the composite vector field (5) is not entirely gauge invariant either. Apart from the vector field polynomial terms, it also contains many vector field-derivative terms [represented by ellipses in (5)]. These terms may badly break gauge invariance unless they are properly suppressed by taking the number of fermion species N to be large enough. The absence of well-defined approximate gauge invariance could make it hard to explicitly demonstrate that these NG modes emerging due to spontaneous Lorentz violation really form together a massless photon as a gauge field candidate. Rather, there would be in general three separate massless Goldstone modes, two of which may mimic the transverse photon polarizations, while the third one must be appropriately suppressed.

Nevertheless, as was argued in [6], it appears possible to arrange an effective theory the way that gauge invariance is violated at leading order in N only by potential terms in (5). At this order the gauge invariant form of the kinetic terms in (5) implies that only the transverse NG bosons propagate, exactly as in the conventional Lorentz invariant electrodynamics. As a consequence, interactions between conserved matter currents $J^\mu(\Psi)$ give the standard QED results at leading order plus Lorentz noninvariant corrections occurring at order $1/N$. The third NG boson effects are also suppressed by $1/N$. Altogether, one comes to the emergent effective QED where the spontaneously broken Lorentz invariance may appear as a controllable approximate symmetry in low-energy physics.

B. Potential-based models

One could think that composite models contain too many prerequisites and complications related to the large number of basic fermion species involved, their proper arrangement, nonrenormalizability of the fundamental multi-Fermi Lagrangian, stability under radiative corrections, and so on indefinitely. This approach contains in fact a cumbersome invisible sector which induces the effective emergent theory. A natural question arises whether one could start from the effective vector field theory (5) instead, thus having spontaneous Lorentz violation from the outset.

Actually, by making a proper redefinition of the vector field $A_\mu \to ieA_\mu$ in (5), one comes to a conventional QED-type Lagrangian extended by arbitrary vector field potential energy terms which explicitly break gauge invariance. For a minimal potential containing bilinear and quartic vector field terms, one comes to the Lagrangian

$$L_V = L_{\mathrm{QED}} - \frac{\lambda_c}{4}(A_\mu A^\mu - n^2\mathrm{M}^2)^2, \quad (7)$$

[1]This value (6) simply follows from the usual vacuum polarization integral. Although quadratic divergence does not appear in the loop diagrams thanks to the global current conservation, logarithmic divergences do. Note that all couplings and masses [see also (4)] appearing in the emergent effective theory are evaluated at zero four-momenta.

where the coupling constant λ_c is determined as in (5), the mass term $n^2\mathrm{M}^2$ (properly expressed through parameters of the effective theory) stands for the proposed SLIV scale, while n_μ is a properly oriented unit Lorentz vector, $n^2 = n_\mu n^\mu = \pm 1$. This partially gauge invariant model, sometimes referred to as the "bumblebee" model [8] (see also [9] and references therein), means in fact that the vector field A_μ develops a constant background value

$$\langle A_\mu \rangle = n_\mu \mathrm{M}, \tag{8}$$

and Lorentz symmetry $SO(1,3)$ breaks down to $SO(3)$ or $SO(1,2)$ depending on whether n_μ is timelike ($n^2 = +1$) or spacelike ($n^2 = -1$).[2] By expanding the vector field around this vacuum configuration,

$$A_\mu(x) = a_\mu(x) + n_\mu(\mathrm{M} + \mathcal{A}), \qquad n_\mu a_\mu = 0, \tag{9}$$

one finds that the a_μ field components, which are orthogonal to the Lorentz-violating direction n_μ, describe a massless vector Nambu-Goldstone boson, while the $\mathcal{A}$ field corresponds to a massive Higgs mode away from the potential minimum. Due to the presence of this mode, the model may lead to some physical Lorentz violation in terms of the properly deformed dispersion relations for the photon and matter fields involved that appear from the corresponding radiative corrections to their kinetic terms [6].

However, as was argued in [19], a bumblebeelike model appears generally unstable; its Hamiltonian is not bounded from below beyond the constrained phase space determined by the nonlinear condition $A_\mu A^\mu = n^2\mathrm{M}^2$. With this condition imposed, the massive Higgs mode never appears, the Hamiltonian is positive, and the model is physically equivalent to the nonlinear constraint-based QED, which we consider in the next section. Apart from the instability, the potential-based models were shown [20] to be obstructed from having a consistent ultraviolet completion, whereas the most of viable effective theories possess such a completion. The problems mentioned certainly appear in the effective theories emerging from the composite models as well. Nevertheless, since a natural mass scale associated with spontaneous Lorentz violation is presumably of the Planck-scale order, only quantum-gravity theory might make the ultimate conclusion on physical viability of such models at all energy scales.

C. Constraint-based models

This class of models starts directly with the nonlinearly realized Lorentz symmetry for underlying vector field $A_\mu(x)$ through the "length-fixing" constraint

$$A_\mu A^\mu = n^2\mathrm{M}^2 \tag{10}$$

implemented into a conventional QED. The constraint-based models were first studied by Nambu a long ago [10] (see also [21]), and in more detail in recent years [11,12,22–25]. The constraint (10) is in fact very similar to the constraint appearing in the nonlinear σ model for pions [26], $\sigma^2 + \pi^2 = f_\pi^2$, where f_π is the pion decay constant. Rather than impose by postulate, the constraint (10) may be implemented into the standard QED Lagrangian L_{QED} through the invariant Lagrange multiplier term,

$$L = L_{\mathrm{QED}} - \frac{\lambda}{2}(A_\mu A^\mu - n^2\mathrm{M}^2), \qquad n^2 = n_\mu n^\mu = \pm 1, \tag{11}$$

provided that initial values for all fields (and their momenta) involved are chosen so as to restrict the phase space to values with a vanishing multiplier function $\lambda(x)$, $\lambda = 0$. Actually, due to an automatic conservation of the matter current in QED, an initial value $\lambda = 0$ will remain for all time.[3] In a general case, when nonzero values of λ are also allowed, it appears problematic to have a stable theory with a positive Hamiltonian (for a detailed discussion, see [19]). It is worth noting that, though the Lagrange multiplier term formally breaks gauge invariance in the Lagrangian (11), this breaking is in fact reduced to the nonlinear gauge choice (10). On the other hand, since gauge invariance is no longer generically assumed, it seems that the vector field constraint (10) might be implemented into the general vector field theory (7) rather than the gauge invariant QED in (11). The point is, however, that both theories are equivalent once the constraint (10) holds. Indeed, due to the simple structure of the vector field polynomial in (7), they lead to practically the same equations of motion in both cases.

One way or another, the constraint (10) means in essence that the vector field A_μ develops the VEV (8), causing again an appropriate (timelike or spacelike) Lorentz violation at a scale M. The point is, however, that in sharp contrast to the nonlinear σ model for pions, the nonlinear QED theory, due to gauge invariance in the starting Lagrangian L_{QED} in (11) or in (7), ensures that all the physical Lorentz-violating effects turn out to be nonobservable. Actually, as was shown in the tree [10] and one-loop approximations [11], the nonlinear constraint (10) implemented as a supplementary condition appears in essence as a possible gauge choice for the vector field A_μ, while the S matrix remains unaltered under such a gauge convention. So, as generally expected,

[2] Note that such freedom in the choice of either n^2 value exists in fact for the minimal vector field potential in (7). For the higher-order terms included, the potential may have a minimum for only positive or only negative n^2.

[3] Interestingly, this solution with the Lagrange multiplier field $\lambda(x)$ being vanished can technically be realized by introducing in the Lagrangian (11) an additional Lagrange multiplier term of the type $\xi\lambda^2$, where $\xi(x)$ is a new multiplier field. One can now easily confirm that a variation of the modified Lagrangian $L + \xi\lambda^2$ with respect to the ξ field leads to the condition $\lambda = 0$, whereas a variation with respect to the basic multiplier field λ preserves the vector field constraint (10).

the SLIV inspired by the nonlinear constraint (10), while producing an ordinary photon as a true Goldstone vector boson (a_μ),

$$A_\mu = a_\mu + n_\mu \sqrt{\mathrm{M}^2 - n^2 a^2}, \quad n_\mu a_\mu = 0\,(a^2 \equiv a_\mu a^\mu), \quad (12)$$

leaves physical Lorentz invariance intact.[4] Similar results were also confirmed for spontaneously broken massive QED [22], non-Abelian theories [23], and tensor field gravity [25], which will be discussed from a supersymmetry point of view in later sections.

To conclude, the constraint-based emergent gauge theories are in fact indistinguishable from conventional gauge theories. Their emergent nature could only be seen when taking the nonlinear gauge condition (10). Any other gauge, e.g., Coulomb gauge, is not in line with the emergent picture, since it breaks Lorentz invariance in an explicit rather than spontaneous way. As to observational evidence in favor of emergent theories, the only way for SLIV to cause physical Lorentz violation would appear only if gauge invariance in these theories were really broken [27], rather than merely constrained by some gauge condition. This leads us to some general observation that, in contrast to the spontaneous violation of internal symmetries, SLIV seems not to necessarily imply a physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, this may ultimately result in a nonlinear gauge choice in an otherwise gauge invariant and Lorentz invariant theory. In substance, the SLIV ansatz, due to which the vector field $A_\mu(x)$ develops the VEV (8), may itself be treated as a pure gauge transformation with a gauge function linear in coordinates, $\omega(x) = n_\mu x^\mu \mathrm{M}$. From this viewpoint, gauge invariance in QED leads to the conversion of SLIV into gauge degrees of freedom of the massless photon that emerges. This is what one could refer to as the generic nonobservability of SLIV in QED. Moreover, as was shown some time ago [28], gauge theories, both Abelian and non-Abelian, can be obtained by themselves from the requirement that the physical nonobservability of SLIV be induced by vector fields rather than from the standard gauge principle.

We will hereafter refer to this case of the constraint-based models as an “inactive” SLIV, as opposed to an “active” SLIV leading to physical Lorentz violation which appears if gauge invariance is explicitly broken, as is presented later in Sec. III E.

D. Models with external vector backgrounds

Although we are mainly focused here on spontaneous Lorentz violation, it must not be ruled out that Lorentz invariance might be explicitly, rather than spontaneously, broken at high energies. This has attracted considerable attention in recent years as an interesting phenomenological possibility appearing in direct Lorentz noninvariant extensions of QED and the Standard Model [13,14,29,30]. They are generically regarded as effective theories originated from a more fundamental theory at some large scale probably related to the Planck scale M_P. These extensions may be in a certain measure motivated [8] by a string theory according to which an explicit (from a QFT point of view) Lorentz violation might be in essence a spontaneous Lorentz violation related to hypothetical tensor-valued fields acquiring nonzero VEVs in some nonperturbative vacuum. These VEVs appear effectively as a set of external background constants so that interactions with these coefficients have preferred spacetime directions in an effective QFT framework. The full SM extension (SME) [14] is then defined as an effective gauge invariant field theory obtained when all such Lorentz-violating vector and tensor field backgrounds are contracted term by term with SM (and gravitational) fields. However, without a completely viable string theory, it is not possible to assign definite numerical values to these coefficients. Moreover, not to have disastrous consequences (especially when these coefficients are contracted with nonconserved currents) one also has to additionally propose that observable Lorentz-violating effects are properly suppressed [13,14,29,30], which in many cases is a serious theoretical problem. Therefore, one has in essence a purely phenomenological approach, treating the above arbitrary coefficients as quantities to be bounded in experiments as if they would simply appear due to explicit Lorentz violation. Actually, in sharp contrast to the above formulated SLIV in a pure QFT framework, there is nothing in the SME itself that requires that these Lorentz-violation coefficients emerge due to a process of a spontaneous Lorentz violation. Indeed, the corresponding massless vector (tensor) NG bosons are not required to be generated, nor do these bosons have to be associated with photons or any other gauge fields of SM.

Apart from Lorentz violation in the Standard Model, one can generally think that the vacuum in quantum gravity may also determine a preferred rest frame at the microscopic level. If such a frame exists, it must be very much hidden in low-energy physics since, as was mentioned above, numerous observations severely limit the possibility of Lorentz-violating effects for the SM fields [13,14,29,30]. However, the constraints on Lorentz violation in the gravitational sector are generally far weaker. This allows us to introduce a pure gravitational Lorentz violation having no significant impact on the SM physics. An elegant way that is close in spirit to our SLIV model (11), (12) seems to appear in the so-called Einstein-aether theory [31]. This is in essence a general covariant theory in which local Lorentz invariance is broken by some vector “aether” field u_μ defining the preferred frame. This field is similar to our

[4] Indeed, the nonlinear QED contains a plethora of Lorentz- and CPT-violating couplings when it is expressed in terms of the zero photon modes a_μ. However, the contributions of all these couplings to physical processes completely cancel out among themselves.

constrained vector field A_μ, apart from that this field is taken to be unit $u_\mu u^\mu = 1$. It spontaneously breaks Lorentz symmetry down to a rotation subgroup, just like our constrained vector field A_μ does for a timelike Lorentz violation. So, they both give a nonlinear realization of Lorentz symmetry, thus leading to its spontaneous violation and inducing the corresponding Goldstone-like modes. The crucial difference is that, while modes related to the vector field A_μ are collected into the physical photon, modes associated with the unit vector field u_μ (one helicity-0 and two helicity-1 modes) exist on their own, appearing in some effective SM and gravitational couplings. Some of them might disappear, being absorbed by the corresponding spin-connection fields related to local Lorentz symmetry in the Einstein-aether theory. In any case, while aether field u_μ can significantly change dispersion relations of the fields involved, thus leading to many gravitational and cosmological consequences of preferred frame effects, it certainly cannot be a physical gauge field candidate (say, the photon in QED).

E. Supersymmetric models

While there are many papers in the literature on Lorentz noninvariant extensions of supersymmetric models (for some interesting ideas, see [15–17,32] and references therein), an emergent gauge theory in a SUSY context has only recently been introduced [18]. Actually, the situation was shown to be seriously changed in a SUSY context which certainly disfavors some emergent models considered above. It appears that, while the constraint-based models of an inactive SLIV successfully matches supersymmetry, the composite and potential-based models of an active SLIV leading to physical Lorentz violation cannot be conceptually realized in the SUSY context. The reason is that, in contrast to an ordinary vector field theory where all kinds of polynomial terms $(A_\mu A^\mu)^n$ $(n = 1, 2, \ldots)$ can be included into the Lagrangian in a Lorentz invariant way, SUSY theories only admit the bilinear mass term $A_\mu A^\mu$ in the vector field potential energy. As a result, without stabilizing high-linear [at least quartic, as in (7)] vector field terms, the potential-based SLIV never occurs in SUSY theories. The same could be said about composite models as well: the fundamental Lagrangian with multi-Fermi current-current interactions (1) cannot be constructed from any matter chiral superfields. So, all the models considered above, except for the constraint-based models, are ruled out in the SUSY framework and, therefore, between the two basic SLIV versions, active and inactive, SUSY unambiguously chooses the inactive SLIV case.

Meanwhile, some efforts have been made [15–17] over the last decade to construct Lorentz-violating operators for matter and gauge fields interacting with external vector field backgrounds in the supersymmetric QED and Standard Model. These backgrounds, according to the SME approach [14] discussed above, are generated by some Lorentz-violating dynamics at an ultraviolet scale of order the Planck scale. An advantage over the ordinary SME was shown to be that in the supersymmetric Standard Model, the lowest possible dimension for such operators is 5. Therefore, they are suppressed by at least one power of an ultraviolet energy scale, providing a possible explanation for the smallness of Lorentz violation and its stability against radiative corrections. All possible dimension-5 and dimension-6 Lorentz-violating operators in the SUSY QED [17] were classified, their properties were analyzed at the quantum level, and their observational consequences in this theory were described. These operators, as was confirmed, do not induce destabilizing D terms, nor gauge anomaly and the Chern-Simons term for photons. Dimension-5 Lorentz-violating operators were shown to be constrained by low-energy precision measurements at 10^{-10}–10^{-5} level in units of the inverse Planck scale, while the Planck-scale suppressed dimension-6 operators are allowed by observational data.

Also, the supersymmetric extension of the Einstein-aether theory [33] discussed above has been constructed. It has been found that the dynamics of the superaether is somewhat richer than that of its non-SUSY counterpart. In particular, the model possesses a family of inequivalent vacua exhibiting different symmetry breaking patterns while remaining stable and ghost free. Interestingly enough, as long as the aether VEV preserves spatial supersymmetry (SUSY algebra without boosts), the Lorentz breaking does not propagate into the SM sector at the renormalizable level. The eventual breaking of SUSY, that must be incorporated in any realistic model, is unrelated to the dynamics of the aether. It is assumed to come from a different source characterized by a lower energy scale. However, in spite of the superaether model's own merits, an important final step which would lead to natural accommodation of this model into the supergravity framework has not yet been done.

III. GAUGE THEORIES EMERGING FROM CONSTRAINTS

A. An emergence conjecture revised

Given the current status of models considered above, it may seem that an "emergence level" of an effective theory is decreased when going from the original composite models to the vector field theoretical ones. At first glance, the latter models look less fundamental if it is granted that emergent degrees of freedom (gauge bosons) are necessarily built of more fundamental degrees of freedom (fermions). However, the compositeness itself hardly is important for emergent theories and, in essence, one can equally specify the emergent gauge bosons simply as the NG modes associated with spontaneous Lorentz violation, no matter they are composite or elementary.

Another seemingly depreciating point might be that the vector field theoretical models are taken to possess gauge invariance from the outset (being partial in the potential-based models and full in the constraint-based ones), whereas in the composite models [2] one tries to derive it, though this has not yet been really achieved. However, the most important side of the nonlinear vector field constraint (10) was shown [34] (see also [35,36]) to be that one does not need to especially postulate the starting gauge invariance. Normally, one can start in the framework of an arbitrary relativistically invariant Lagrangian which is proposed only to possess some global internal symmetry. Nonetheless, looking for the theories which are compatible with the vector field constraint (10), one inevitably comes to gauge invariance. Namely, gauge invariance in such theories has to appear in essence as a response of an interacting field system to putting the covariant constraint (10) on its dynamics, provided that we allow parameters in the corresponding Lagrangian density to be adjusted so as to ensure self-consistency without losing too many degrees of freedom. Otherwise, a given field system could get unphysical in the sense that a superfluous reduction in the number of degrees of freedom would make it impossible to set the required initial conditions in the appropriate Cauchy problem. Namely, it would be impossible to specify arbitrarily the initial values of the vector and other field components involved, as well as the initial values of the momenta conjugated to them. Furthermore, in quantum theory, to choose self-consistent equal-time commutation relations would also become impossible [37].

Let us dwell upon this point in more detail. Conventionally, while a standard variation principle requires the equations of motion to be satisfied, for a general four-vector field A_μ it is still possible (in contrast to scalar and fermion fields) to eliminate one extra component in order to describe a pure spin-1 particle by imposing a supplementary condition. Typically, this is covariantly achieved by taking the divergence from a general vector field equation of motion. In the massive vector field case there are three physical spin-1 states to be described by the A_μ field. Similarly in the massless vector field case, although there are only two physical (transverse) photon spin states, one cannot construct a massless four-vector field A_μ as a linear combination of creation and annihilation operators for helicity ± 1 states in a relativistically covariant way, unless one fictitious state is added [38]. So, in both the massive and massless vector field cases, only one component of the A_μ field may be eliminated and still preserve physical Lorentz invariance. Now, once the SLIV constraint (10) is imposed, it is therefore not possible to satisfy another supplementary condition since this would superfluously restrict the number of degrees of freedom for the vector field. To avoid this, its equation of motion should be automatically divergenceless, that is, only possible in the gauge invariant theory. Thus, due to spontaneous Lorentz violation determined by the constraint (10), being the only possible covariant and holonomic vector field constraint, the theory has to acquire on its own gauge-type invariance, which gauges the starting global symmetry of the interacting vector and matter fields involved. In such a way, one comes to the gauge symmetry emergence (GSE) conjecture:

Let there be given an interacting field system containing some vector field (or vector field multiplet) A_μ together with fermion (ψ), scalar (ϕ) and other matter fields in an arbitrary relativistically invariant Lagrangian $L(A_\mu, \psi, \phi, \ldots)$ which possesses only global Abelian or non-Abelian internal symmetry G. Suppose that an underlying relativistic invariance of this field system is spontaneously broken in terms of the length-fixing covariant constraint put on vector fields, $A_\mu A^\mu = n^2 \mathrm{M}^2$. If this constraint is preserved under the time development given by the field equations of motion, then in order to be protected from further reduction in degrees of freedom, this system will modify its global symmetry G into a local symmetry G_{loc}, that will in turn convert the vector field constraint itself into a gauge condition, thus virtually resulting in a gauge invariant and Lorentz invariant theory.

So, the nonlinear SLIV condition (10), due to which true vacuum in the theory is chosen and massless gauge fields are generated, may provide a dynamical setting for all underlying internal symmetries involved through the GSE conjecture. One might think that the length-fixing vector field constraint (10) itself seems not to be especially singled out in the present context. Actually, it looks like that the GSE conjecture might be equally formulated for any type of covariant constraint. However, as we argue later in Sec. III D, the SLIV constraint appears to be the only one whose application leads to a full conversion of an internal global symmetry G into a local symmetry G_{loc} that forces a given field system to remain always physical.

B. Emergent Abelian gauge invariance

To see how technically a global internal symmetry may be converted into a local one, let us consider in detail the question of consistency of a possible constraint for a general four-vector field A_μ with its equation of motion in an Abelian symmetry case, $G = U(1)$. In the presence of the SLIV constraint $C(A) = A_\mu A^\mu - n^2 \mathrm{M}^2 = 0$ (10), it follows that the equations of motion can no longer be independent. The important point is that, in general, the time development would not preserve the constraint. So the parameters in the Lagrangian have to be chosen in such a way that effectively we have one less equation of motion for the vector field. This means that there should be some relationship among all the vector and matter field Eulerians $(E_A, E_\psi, ...)$ involved.[5] Such a relationship can quite generally be formulated as a functional—but by locality just a function—of the Eulerians, $F(E_A, E_\psi, \ldots)$, being put equal

[5]Hereafter, the notation E_A stands for the vector field Eulerian $(E_A)^\mu \equiv \partial L/\partial A_\mu - \partial_\nu[\partial L/\partial(\partial_\nu A_\mu)]$. We use similar notations for other field Eulerians as well.

to zero at each spacetime point with the configuration space restricted by the constraint $C(A) = 0$:

$$F(C = 0; E_A, E_\psi, \ldots) = 0. \tag{13}$$

This relationship must satisfy the same symmetry requirements of Lorentz and translational invariance, as well as all the global internal symmetry requirements, as the general starting Lagrangian does. This Lagrangian is supposed to also include the standard Lagrange multiplier term with the field $\lambda(x)$,

$$L^{\rm tot}(A,\psi,\ldots,\lambda) = L(A,\psi,\ldots) - \frac{\lambda}{2}(A_\mu A^\mu - n^2 \mathrm{M}^2), \tag{14}$$

the variation under which results in the above constraint $C(A) = 0$. In fact, the relationship (13) is used as the basis for an emergence of gauge symmetries in the SLIV constrained vector field theories [35,36]. Note that, while the Lagrange multiplier field is presented in the total Lagrangian $L^{\rm tot}$, it does not appear in the equations of motion of the vector field determined by the Eulerian E_A in the expression (13). This occurs naturally, as we explained in the previous section, if initial values for all fields involved are chosen so as to restrict their phase space to values with a vanishing multiplier function $\lambda(x)$ (see also footnote 4).

Let us now consider a "Taylor expansion" of the function F expressed as a linear combination of terms involving various field combinations multiplying or derivatives acting on the Eulerians. We are taking for simplicity only one matter (say, fermion) field ψ in the model. The constant term in this expansion is of course zero since the relation (13) must be trivially satisfied when all the Eulerians vanish, i.e., when the equations of motion are satisfied. We consider just the terms containing field combinations (and derivatives) with the lowest mass dimension 4, corresponding to the Lorentz invariant expressions

$$\partial_\mu (E_A)^\mu, A_\mu (E_A)^\mu, E_\psi \psi, \bar{\psi} E_{\bar{\psi}}, \tag{15}$$

to eventually have an emergent gauge theory at a renormalizable level. The higher-dimension terms we will discuss later in Sec. III D. Now, under the assumption that the SLIV constraint (10) is preserved under the time development given by the equations of motion, we show how gauge invariance of the physical Lagrangian $L(A,\psi)$ in (14) is established. A conventional variation principle applied to the total Lagrangian $L^{\rm tot}(A,\psi,\lambda)$ requires the following equations of motion for the vector field A_μ and the auxiliary field λ to be satisfied:

$$(E_A)^\mu = 0, \qquad C(A) = A_\mu A^\mu - n^2 \mathrm{M}^2 = 0. \tag{16}$$

However, in accordance with general arguments given above, the existence of five equations for the four-component vector field A^μ (one of which is the constraint) means that not all of the vector field Eulerian components can be independent. Therefore, there must be a relationship of the form given in Eq. (13). When being expressed as a linear combination of the Lorentz invariant terms (15), this equation leads to the identity between the vector and matter field Eulerians of the following type,

$$\partial_\mu (E_A)^\mu = it E_\psi \psi - it \bar{\psi} E_{\bar{\psi}}, \tag{17}$$

where t is some constant.[6] This identity immediately signals the invariance of the basic Lagrangian $L(A,\psi)$ in (14) under vector and fermion field local $U(1)$ transformations whose infinitesimal form is given by

$$\delta A_\mu = \partial_\mu \omega, \qquad \delta\psi = it\omega\psi. \tag{18}$$

Here $\omega(x)$ is an arbitrary function, only restricted by the requirement to conform with the nonlinear constraint (10):

$$(A_\mu + \partial_\mu \omega)(A^\mu + \partial^\mu \omega) = n^2 \mathrm{M}^2. \tag{19}$$

Conversely, the identity (17) follows from the invariance of the physical Lagrangian $L(A,\psi)$ under the transformations (78). Indeed, both direct and converse assertions are particular cases of Noether's second theorem [39].

So, we have shown how the choice of a vacuum conditioned by the SLIV constraint (10) enforces the choice of the parameters in the starting Lagrangian $L^{\rm tot}(A,\psi,\lambda)$, so as to convert the starting global $U(1)$ charge symmetry into a local one, thus demonstrating an emergence of gauge symmetry (18) that allows the emerged Lagrangian to be determined in full. For a theory with renormalizable couplings, it is in fact the conventional QED Lagrangian (11) extended by the Lagrange multiplier term,

$$L^{\rm em}(A,\psi,\lambda) = L_{\rm QED}(A,\psi) - \frac{\lambda}{2}(A_\mu A^\mu - n^2 \mathrm{M}^2), \tag{20}$$

which provides the SLIV constraint (10) imposed on the vector field A_μ.

[6]Note that the term proportional to the vector field itself, $A_\mu (E_A)^\mu$, which would correspond to the self-interaction of the vector field, is absent in the identity (17). In the presence of this term, the transformations of the vector field given below in (18) would be changed to $\delta A_\mu = \partial_\mu \omega + c\omega A_\mu$. The point is, however, that these transformations cannot in general form a group unless the constant c vanishes, as can be readily confirmed by constructing the corresponding Lie bracket operation for two successive vector field variations. We shall see later that nonzero c-type coefficients necessarily appear in the non-Abelian internal symmetry case, resulting eventually in an emergent gauge invariant Yang-Mills theory.

C. Non-Abelian gauge fields as pseudo-Goldstone modes

We still have only considered a single vector field case with an underlying global $U(1)$ symmetry. However, an extension to a theory possessing from the outset some global non-Abelian symmetry G is quite straightforward [35,36]. Suppose that this theory contains an adjoint vector field multiplet $\boldsymbol{A}_\mu^p$ and some fermion matter field multiplet $\boldsymbol{\psi}$ belonging to one of the irreducible representations of G given by matrices t^p,

$$[t^p, t^q] = if^{pqr}t^r,$$
$$\mathrm{Tr}(t^p t^q) = \delta^{pq}\ (p, q, r = 0, 1, \ldots, \Upsilon - 1), \qquad (21)$$

where f^{pqr} stand for structure constants, while Υ is a dimension of the G group. The corresponding Lagrangian $\mathbb{L}^{\mathrm{tot}}(\boldsymbol{A}_\mu, \boldsymbol{\psi}, \boldsymbol{\lambda})$ is supposed to also include the standard Lagrange multiplier term with the field function $\boldsymbol{\lambda}(x)$:

$$\mathbb{L}^{\mathrm{tot}}(\boldsymbol{A}_\mu, \boldsymbol{\psi}, \boldsymbol{\lambda}) = \mathbb{L}(\boldsymbol{A}_\mu, \boldsymbol{\psi}) - \frac{\boldsymbol{\lambda}}{2}(\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} - \boldsymbol{n}^2\mathbf{M}^2),$$
$$\boldsymbol{n}^2 \equiv \boldsymbol{n}_\mu^p \boldsymbol{n}^{p\mu} = \pm 1, \qquad (22)$$

the variation under which results in the vector field length-fixing constraint

$$C(\boldsymbol{A}) = \boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} - \boldsymbol{n}^2\mathbf{M}^2 = 0 \qquad (23)$$

(where $\boldsymbol{n}_\mu^p$ stands now for some properly oriented "unit" rectangular matrix; see below). The need to preserve the constraint $C(\boldsymbol{A}) = 0$ with time implies that the equations of motion for the vector fields $\boldsymbol{A}_\mu^p$ cannot all be independent. As a result, the so-called "emergence identity," analogous to the identity (17), inevitably occurs:

$$\partial_\mu(\mathbb{E}_{\boldsymbol{A}}^p)^\mu = f^{pqr}\boldsymbol{A}_\mu^q(\mathbb{E}_{\boldsymbol{A}}^r)^\mu + \mathbb{E}_{\boldsymbol{\psi}}(it^p)\boldsymbol{\psi} + \bar{\boldsymbol{\psi}}(-it^p)\mathbb{E}_{\bar{\boldsymbol{\psi}}}. \qquad (24)$$

An identification of the coefficients of the Eulerians on the right-hand side of the identity (24) with the structure constants f^{pqr} and generators t^p (21) of the group G is quite transparent. This simply follows from the fact that the right-hand side of this identity must transform in the same way as the left-hand side, which transforms as the adjoint representation of G. Note that, in contrast to the Abelian case, the term proportional to the vector field multiplet $\boldsymbol{A}_\mu^p$ itself, which corresponds to a self-interaction of non-Abelian vector fields, also appears in the identity (24). Again, Noether's second theorem [39] can be applied directly to this identity in order to derive the gauge invariance of the Lagrangian $\mathbb{L}(\boldsymbol{A}_\mu, \boldsymbol{\psi})$ in (22). Indeed, with the constraint (23) implied, the $\mathbb{L}(\boldsymbol{A}_\mu, \boldsymbol{\psi})$ tends to be invariant under vector and fermion field local transformations having the infinitesimal form

$$\delta\boldsymbol{A}_\mu^p = \partial_\mu\omega^p + f^{pqr}\boldsymbol{A}_\mu^q\omega^r,$$
$$\delta\boldsymbol{\psi} = (it^p)\omega^p\boldsymbol{\psi}, \qquad \delta\bar{\boldsymbol{\psi}} = \bar{\boldsymbol{\psi}}(-it^p)\omega^p. \qquad (25)$$

For a theory with renormalizable coupling constants, this emergent gauge symmetry leads to the conventional Yang-Mills-type Lagrangian,

$$\mathbb{L}^{\mathrm{em}}(\boldsymbol{A}_\mu, \boldsymbol{\psi}, \boldsymbol{\lambda}) = \mathbb{L}_{YM}(\boldsymbol{A}_\mu, \boldsymbol{\psi}) - \frac{\boldsymbol{\lambda}}{2}(\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} - \boldsymbol{n}^2\mathbf{M}^2), \qquad (26)$$

where we also include the corresponding Lagrange multiplier term. As in the above Abelian case, this term does not contribute to the vector field equation of motion in the identity (24).

Now let us turn to the spontaneous Lorentz violation which is caused by the nonlinear vector field constraint (23) determined by the Lagrange multiplier term in (26). Although the Lagrangian $\mathbb{L}^{\mathrm{em}}(\boldsymbol{A}_\mu, \boldsymbol{\psi}, \boldsymbol{\lambda})$ only has an $SO(1,3) \times G$ invariance, the last term in it possesses a much higher accidental symmetry $SO(\Upsilon, 3\Upsilon)$ according to the dimension Υ of the adjoint representation of G to which the vector fields $\boldsymbol{A}_\mu^p$ belong. This symmetry is indeed spontaneously broken at a scale $\mathbf{M}$,

$$\langle \boldsymbol{A}_\mu^p(x) \rangle = \boldsymbol{n}_\mu^p \mathbf{M}, \qquad (27)$$

with the vacuum direction determined now by the unit rectangular matrix $\boldsymbol{n}_\mu^p$ which describes simultaneously both of the non-Abelian SLIV cases, timelike,

$$SO(\Upsilon, 3\Upsilon) \to SO(\Upsilon - 1, 3\Upsilon), \qquad (28)$$

or spacelike,

$$SO(\Upsilon, 3\Upsilon) \to SO(\Upsilon, 3\Upsilon - 1), \qquad (29)$$

depending on the sign of $\boldsymbol{n}^2 \equiv \boldsymbol{n}_\mu^p \boldsymbol{n}^{p\mu} = \pm 1$. In both cases, this matrix has only one nonzero element, subject to the appropriate $SO(1,3)$ and (independently) G rotations. They are, specifically, $\boldsymbol{n}_0^0$ or $\boldsymbol{n}_3^0$ provided that the vacuum expectation value (27) is developed along the $p = 0$ direction in the internal space and along the $\mu = 0$ or $\mu = 3$ direction, respectively, in the ordinary four-dimensional spacetime.

As was argued in [23,34], side by side with one true vector Goldstone boson corresponding to the spontaneous violation of the actual $SO(1,3) \otimes G$ symmetry of the Lagrangian $\mathbb{L}^{\mathrm{em}}(\boldsymbol{A}_\mu, \boldsymbol{\psi}, \boldsymbol{\lambda})$, the $\Upsilon - 1$ pseudo-Goldstone vector bosons (PGB) related to the breakings (28), (29) of the accidental symmetry $SO(\Upsilon, 3\Upsilon)$ of the constraint (23) *per se* are also produced.[7] Remarkably, in contrast to

[7]Note that in total there appear $4\Upsilon - 1$ pseudo-Goldstone modes, complying with the number of broken generators of $SO(\Upsilon, 3\Upsilon)$. From these $4\Upsilon - 1$ pseudo-Goldstone modes, 3Υ modes correspond to the Υ three-component vector states, as will be shown below, while the remaining $\Upsilon - 1$ modes are scalar states which will be excluded from the theory.

the familiar scalar PGB case [26], the vector PGBs remain strictly massless, since they are protected by the simultaneously generated non-Abelian gauge invariance. Together with the above true vector Goldstone boson, they also come into play properly completing the whole gauge multiplet of the internal symmetry group G taken.

After the explicit use of this constraint (23), which virtually appears as a single condition put on the vector field multiplet $\boldsymbol{A}_\mu^p$, one can identify the pure Goldstone field modes $\boldsymbol{a}_\mu^p$ as follows:

$$\boldsymbol{A}_\mu^p = \boldsymbol{a}_\mu^p + \boldsymbol{n}_\mu^p \sqrt{\mathbf{M}^2 - \boldsymbol{n}^2 \boldsymbol{a}^2}, \quad \boldsymbol{n}_\mu^p \boldsymbol{a}^{p\mu} = 0 \ (\boldsymbol{a}^2 \equiv \boldsymbol{a}_\mu^p \boldsymbol{a}^{p\mu}). \tag{30}$$

There is also an effective "Higgs" mode $\boldsymbol{n}_\mu^p (\mathbf{M}^2 - \boldsymbol{n}^2 \boldsymbol{a}^2)^{1/2}$ determined by the SLIV constraint. Note that, apart from the pure vector fields, the general zero modes $\boldsymbol{a}_\mu^p$ contain $\Upsilon - 1$ scalar modes, $\boldsymbol{a}_0^{p'}$ or $\boldsymbol{a}_3^{p'}$ $(p' = 1, \ldots, \Upsilon - 1)$, for the timelike $(\boldsymbol{n}_\mu^p = n_0^0 g_{\mu 0} \delta^{p0})$ or spacelike $(\boldsymbol{n}_\mu^p = n_3^0 g_{\mu 3} \delta^{p0})$ SLIV, respectively. They can be eliminated from the theory, if one imposes appropriate supplementary conditions on the $\Upsilon - 1$ fields $\boldsymbol{a}_\mu^p$ which are still free of constraints. Using their overall orthogonality (30) to the physical vacuum direction $\boldsymbol{n}_\mu^p$, one can formulate these supplementary conditions in terms of a general axial gauge for the entire $\boldsymbol{a}_\mu^p$ multiplet:

$$n \cdot \boldsymbol{a}^p \equiv n_\mu \boldsymbol{a}^{p\mu} = 0, \qquad p = 0, 1, \ldots, \Upsilon - 1. \tag{31}$$

Here n_μ is the unit Lorentz vector, analogous to the vector introduced in the Abelian case, which is now oriented in Minkowskian spacetime so as to be "parallel" to the vacuum unit $\boldsymbol{n}_\mu^p$ matrix. This matrix can be taken hereafter in the "two-vector" form,

$$\boldsymbol{n}_\mu^p = n_\mu \boldsymbol{\epsilon}^p, \qquad \boldsymbol{\epsilon}^p \boldsymbol{\epsilon}^p = 1, \tag{32}$$

where $\boldsymbol{\epsilon}^p$ is the unit G group vector belonging to its adjoint representation. As a result, in addition to the Higgs mode excluded earlier by the above orthogonality condition (30), all the other scalar fields are eliminated. Consequently only the pure vector fields, $\boldsymbol{a}_i^p$ $(i = 1, 2, 3)$ or $\boldsymbol{a}_{\mu'}^p$ $(\mu' = 0, 1, 2)$, for timelike or spacelike SLIV, respectively, are left in the theory. Clearly, the components $\boldsymbol{a}_i^{p=0}$ and $\boldsymbol{a}_{\mu'}^{p=0}$ correspond to the true Goldstone boson, for each type of SLIV, respectively, while all the others (for $p = 1, \ldots, \Upsilon - 1$) are vector PGBs. Substituting the parametrization (30) into the emergent Lagrangian (26) and expanding the square root in powers of $\boldsymbol{a}^2/\mathbf{M}^2$, one is led to a highly nonlinear theory in terms of the zero vector modes $\boldsymbol{a}_\mu^p$ which contains a variety of Lorentz- and CPT-violating couplings. However, as in the Abelian symmetry case, they do not lead to physical Lorentz violation effects, which turn out to be strictly canceled among themselves [23], thus giving one more example of an inactive SLIV.

D. Constraints inducing and uninducing gauge invariance

We now turn to a question that naturally arises: whether the length-fixing vector field constraints (10), (23), both for the Abelian and non-Abelian symmetry cases, are of fundamental importance for an emergence conjecture. It seems that the basic relations among all fields' Eulerians, called above the emergence identities (17), (24), might occur for any type of covariant constraints introduced through the corresponding Lagrange multiplier terms. On the other hand, if one keeps in mind the minimal single-field constraints there are only two possible covariant constraints for vector fields in a relativistically invariant theory: the holonomic SLIV constraints (10), (23) and the nonholonomic one, known as the Lorentz condition,

$$C'(A) = \partial_\mu A^\mu = 0, \qquad C'(\boldsymbol{A}) = \partial_\mu \boldsymbol{A}^{p\mu} = 0, \tag{33}$$

for Abelian and non-Abelian vector fields, respectively (the index p enumerates the G group generators). In general, of course, many nonminimal covariant constraints are also possible. However, as we argue below, just the SLIV constraints (10), (23) seem to push the origin of gauge invariance in a theory so as to provide a sufficient number of degrees of freedom for a physical field system evolved over time. Other covariant constraints, when being put on the fields, will lead, at best, to partial gauge invariance.

We consider a general quantum field theory where the vector fields, on their own or together with matter fields, are subject to some covariant constraint(s) whose precise form is yet unknown. Rather than postulate this form in terms of the SLIV constraints (10), (23), as we have done in previous sections, let us try to derive them. We suppose that such constraints could be determined in general by the underlying Lagrangian itself rather than introduced from outside through some Lagrange multiplier terms. Let there be given an interacting field system containing vector field(s) A_μ together with fermion (ψ), scalar (ϕ), and other matter fields in a relativistically invariant Lagrangian $L(A_\mu, \psi, \phi, \ldots)$ which only possesses global Abelian or non-Abelian symmetry G. Suppose that the Lagrangian L is separated into two parts, $L = L_g + \tilde{L}$, which we call the generic and constraint-bearing ones, respectively. We assume that all possible constraint(s) which can be put on the given field system are completely determined by the variation of the Lagrangian $\tilde{L}$ that specifies some extra source current $J_\mu = (\tilde{E}_A)_\mu$ for vector field A_μ. We show below that in order for the given field system to remain physical, this current has to be vanished or conserved, which, in turn, makes the generic Lagrangian L_g become gauge invariant.

For the sake of generality, we consider the non-Abelian symmetry case (21), writing the total Lagrangian in an appropriate notation taken above, $\mathbb{L} = \mathbb{L}_g + \tilde{\mathbb{L}}$. We suppose that vector field multiplet $\boldsymbol{A}_\mu^p$ belongs to an adjoint representation of a group G with structure constants f^{pqr}, while matter fields (we leave only fermion fields, for simplicity) transform according to some representation given by matrices $(t^p)^i_j$. Consider first the case when the extra source current for vector fields $\boldsymbol{A}_\mu^p$ vanishes:

$$\mathbb{J}_\mu^p = (\tilde{\mathbb{E}}_{\boldsymbol{A}}^p)_\mu = 0, \qquad p = 0, 1, \ldots, \Upsilon - 1. \tag{34}$$

This allegedly happens due to the appropriately restricted vector field configurations rather than vanishing coupling constants in the Lagrangian $\tilde{\mathbb{L}}$. One can see, however, that such conditions eliminate too many vector field degrees of freedom. Namely, 4Υ degrees appear to be eliminated, whereas only Υ degrees—one for each vector field species—may be excluded. Additional constraints could also appear for matter fields, if they are contained in the Lagrangian $\tilde{\mathbb{L}}$. This means that for these constraints to be admissible, only a special class of the constraint-bearing Lagrangians $\tilde{\mathbb{L}}$ has to be taken. Actually, the only way to proceed, as one may readily confirm, could be the case if the Lagrangian $\tilde{\mathbb{L}}$ would depend on all the fields involved only through the "length squared" invariants $\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu}, \bar{\boldsymbol{\psi}}^i \boldsymbol{\psi}_i$, and so on. This would mean that in the minimal case with the lowest mass dimension couplings, the Lagrangian $\tilde{\mathbb{L}}$ only contains a conventional fourth-order polynomial in vector field $\boldsymbol{A}_\mu^p$,

$$\tilde{\mathbb{L}}_{\text{min}} = -\frac{\boldsymbol{\lambda}_c}{4}(\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} - \boldsymbol{n}^2 \mathbf{M}^2)^2, \tag{35}$$

where $\boldsymbol{\lambda}_c$ and $\boldsymbol{n}^2\mathbf{M}^2$ are the corresponding vector field parameters ($\boldsymbol{n}^2 = \pm 1$). In general, there could be, of course, a variety of high-dimensional vector-vector and vector-fermion couplings of type

$$(\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu})^k, k \geq 3; \qquad (\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu})^l (\bar{\boldsymbol{\psi}}^i \boldsymbol{\psi}_i)^m, \qquad l \geq 0, m \geq 1, \tag{36}$$

and so forth, being properly suppressed by some high-scale mass(es). This structure of the Lagrangian $\tilde{\mathbb{L}}$ provides the following expressions for vector and fermion field Eulerians,

$$\mathbb{J}_\mu^p = (\tilde{\mathbb{E}}_{\boldsymbol{A}}^p)^\mu = 2\boldsymbol{A}^{p\mu} \frac{\partial \tilde{\mathbb{L}}}{\partial(\boldsymbol{A}_\mu^q \boldsymbol{A}^{q\mu})},$$
$$\tilde{\mathbb{E}}_{\boldsymbol{\psi}}(it^p)\boldsymbol{\psi} = \bar{\boldsymbol{\psi}}(it^p)\tilde{\mathbb{E}}_{\bar{\boldsymbol{\psi}}}, \tag{37}$$

the first of which actually reduces all constraints (34) to the single one,

$$\partial\tilde{\mathbb{L}}/\partial(\boldsymbol{A}_\mu^q \boldsymbol{A}^{\mu q}) = 0, \tag{38}$$

while the second one is satisfied automatically. As a result, for the minimal Lagrangian $\tilde{\mathbb{L}}_{\text{min}}$ (35), the condition (38) immediately leads to the SLIV constraint (23). Now, just as in the previous section, assuming that this constraint is preserved under the time development given by the equations of motion, the so-called emergence identity, analogous to identity (24), inevitably occurs:

$$\partial_\mu(\mathbb{E}_{\boldsymbol{A}}^p + \tilde{\mathbb{E}}_{\boldsymbol{A}}^p)^\mu = f^{pqr}\boldsymbol{A}_\mu^q(\mathbb{E}_{\boldsymbol{A}}^r + \tilde{\mathbb{E}}_{\boldsymbol{A}}^r)^\mu + (\mathbb{E}_{\boldsymbol{\psi}} + \tilde{\mathbb{E}}_{\boldsymbol{\psi}})(it^p)\boldsymbol{\psi}$$
$$+ \bar{\boldsymbol{\psi}}(-it^p)(\mathbb{E}_{\bar{\boldsymbol{\psi}}} + \tilde{\mathbb{E}}_{\bar{\boldsymbol{\psi}}}), \tag{39}$$

where the Eulerians for vector and fermion fields are generated by both Lagrangians $\mathbb{L}_g$ and $\tilde{\mathbb{L}}$, respectively. Due to constraints taken (34) and the equation for fermion Eulerians in (37), all the Eulerians generated by the constraint-bearing Lagrangian $\tilde{\mathbb{L}}$ disappear, so that only the generic Lagrangian $\mathbb{L}_g$ contributes to both sides of this identity. This implies according to Noether's second theorem [39] that the generic Lagrangian $\mathbb{L}_g$ is in fact gauge invariant. As to the constraint-bearing Lagrangian $\tilde{\mathbb{L}}$, it may only contain some constant term, and also two-Fermi and multi-Fermi interaction terms. They appear as soon as the constraint equation (38) is solved with respect to $\boldsymbol{A}_\mu^q \boldsymbol{A}^{\mu q}$, which are then substituted back into the $\tilde{\mathbb{L}}$ (35), (36).[8] Actually, the Lagrangian $\tilde{\mathbb{L}}$ also appears to be gauge invariant, and likewise for the generic Lagrangian $\mathbb{L}_g$ [though the constraint (38) itself breaks gauge invariance]. For a minimal Lagrangian $\tilde{\mathbb{L}}_{\text{min}}$ (35) the theory completely coincides with the above SLIV constraint case given by the Lagrangian (26) provided that the constraint (38) in its final form (23) is also included through an appropriate Lagrange multiplier term. Remarkably, symmetry of the constraint (38) uniquely established above from the requirement not to have too many degrees of freedom eliminated is much higher than the symmetry of the whole Lagrangian (26). This, as we could see in the previous section, allows us to treat non-Abelian gauge fields as pseudo-Goldstone bosons.

Let us now turn to the nonzero extra vector field source current $\mathbb{J}^{p\mu}$ which is only required to be conserved,

$$\partial_\mu \mathbb{J}^{p\mu} = \partial_\mu(\tilde{\mathbb{E}}_A^p)^\mu = 0, \tag{40}$$

that gives in principle a sufficient number of constraints (one for each vector field species, $p = 0, 1, \ldots, \Upsilon - 1$). We start deriving the divergenceless conditions for the equations of motion of the vector fields $\boldsymbol{A}_\mu^p$. Indeed, varying the total Lagrangian $\mathbb{L} = \mathbb{L}_g + \tilde{\mathbb{L}}$ and taking four-divergence from the corresponding vector field Eulerians, one has

[8] Such substitution is in principle an allowed procedure, since virtually it does not change the equations of motion of the fields involved.

$$\partial_\mu (\mathbb{E}_{\boldsymbol{A}}^p)^\mu + \partial_\mu (\tilde{\mathbb{E}}_{\boldsymbol{A}}^p)^\mu = 0. \tag{41}$$

Next, since no other constraints than the proposed current conservation (40) are admissible, the first four-divergence term in Eq. (41) has to vanish either identically or as a result of the equations of motion for vector and fermion fields. This implies that in the absence of these equations of motion there must hold the general identity given in (39). However, in contrast to the previous case, the Eulerians generated by the constraint-bearing Lagrangian $\tilde{\mathbb{L}}$, which vanish on the left-hand side of this identity, will give nonzero contributions to its right-hand side. Thus, having different vector field Eulerians in the identity (39), one has to conclude that Noether's second theorem [39] does not hold for this case. This means that gauge invariance fails in general to emerge when the constraint in terms of source current conservation (40) for the vector field multiplet $\boldsymbol{A}_\mu^p$ is required. In contrast to the previous case with the vanishing source current $\mathbb{J}^{p\mu}$, where structure of the constraint-bearing Lagrangian $\tilde{\mathbb{L}}$ was virtually established (35), (36) by the constraints (34) required, now this Lagrangian, despite the constraints (40) imposed, is still left quite arbitrary. However, if we additionally propose that, as in the previous case, the Lagrangian $\tilde{\mathbb{L}}$ depends on all the fields involved only through their length squared invariants, then all goes well and gauge invariance arises. Indeed, using the Lagrangian form determined above (35), (36) and corresponding expressions for vector and fermion field Eulerians (37), one can easily confirm that all "tilded" terms induced by the constraint-bearing Lagrangian $\tilde{\mathbb{L}}$ in the identity (39) are strictly canceled for obvious symmetry reasons. So, this identity acquires a form to which Noether's second theorem [39] can be directly applied in order to finally establish gauge invariance of the generic Lagrangian $\mathbb{L}_g$.

Eventually, for a minimal case with the mass squared dimension and dimensionless coupling constants, the whole emergent theory acquires a form

$$\mathbb{L}^{\mathrm{em}}(\boldsymbol{A}_\mu, \boldsymbol{\psi}; \boldsymbol{\lambda}_c, \mathbf{M}^2) = \mathbb{L}_{YM}(\boldsymbol{A}_\mu, \boldsymbol{\psi}) - \frac{\boldsymbol{\lambda}_c}{4}(\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} - \boldsymbol{n}^2 \mathbf{M}^2)^2, \tag{42}$$

where the first term is a conventional Yang-Mills Lagrangian arising from a generic Lagrangian $\mathbb{L}_g$, while the second term is a minimal constraint-bearing Lagrangian $\tilde{\mathbb{L}}_{\mathrm{min}}$ (35). In contrast to the previous case, we have obtained some gauge noninvariant extensions to Yang-Mills theory in the form of the potential with the mass and self-interaction terms for vector fields. Note that for the Abelian symmetry case, the emergent Lagrangian (42) turns to the bumblebee model (7) considered in Sec. II B. Interestingly, while the Lagrangian $\tilde{\mathbb{L}}$ taken above (35), (36) provides an emergence of gauge invariance in the generic Lagrangian $\mathbb{L}_g$ ($\mathbb{L}_g \to \mathbb{L}_{\mathrm{YM}}$), it breaks this gauge invariance by itself. In the simplest case ($\boldsymbol{\lambda}_c \to 0$, $\boldsymbol{\lambda}_c \mathbf{M}^2 \to \mathbf{M}_A^2$), one has the massive Yang-Mills theory where the constraint (40) is reduced to the spin-1 condition (33) for massive vector fields (having the mass $\mathbf{M}_A$). This particular case was thoroughly studied in its own right quite long ago [37].

One can conclude that the length-fixing vector field constraints (10), (23) seem really to be of fundamental significance for emergent gauge invariance. Actually, when constraints being put on the field system are determined by the underlying Lagrangian itself, rather than taken *ad hoc* through some Lagrange multiplier terms, the SLIV constraints (10), (23) appear strongly preferred over other ones. Indeed, as was shown, only the strictly vanishing vector field source current, $\mathbb{J}_\mu^p = 0$, that corresponds to the SLIV constraints (10), (23), leads to the full conversion of a starting global symmetry G of the total Lagrangian $\mathbb{L} = \mathbb{L}_g + \tilde{\mathbb{L}}$ into a local one G_{loc}. For nonzero current $\mathbb{J}_\mu^p$, on the other hand, when the vector field constraint is solely determined by the current conservation, $\partial^\mu \mathbb{J}_\mu^a = 0$, gauge symmetry does not emerge or, at best, may only be partial.

E. Gauge invariance versus spontaneous Lorentz violation

One can see that the gauge theory framework, be it taken from the outset or emerged, makes in turn spontaneous Lorentz violation physically unobservable in both the Abelian and non-Abelian symmetry cases. We referred to it above as the inactive SLIV, in contrast to the active SLIV case where physical Lorentz invariance could effectively occur. From the present standpoint, the only way for an active SLIV to occur would be if the emergent gauge symmetries presented above were slightly broken at small distances. This could naturally happen, for example, in a partially gauge invariant theory (42) which emerges due to the properly chosen constraint (40) being put on the physical field system, as was illustrated above. A more radical point of view would be that the considered field system could become unphysical at distances presumably controlled by quantum gravity. One could think that quantum gravity could in principle hinder the setting of the required initial conditions in the appropriate Cauchy problem, thus admitting a superfluous restriction of vector fields in terms of some high-order operators which occur at the Planck scale.

Recall in this connection that in the emergence equations (17) and (24), we have only considered the lowest-dimension terms which eventually provide an emergent gauge theory at a renormalizable level. All other terms [following from the expansion in (13)] contain field combinations and derivatives with higher mass dimension and must therefore have coefficients with an inverse mass dimension. We expect the mass scale associated with these coefficients should correspond to a large fundamental mass (e.g., the Planck mass M_P). Hence we may conclude that such higher-dimensional terms must be highly suppressed and can be neglected for the effective low-energy gauge invariant theory. However, these terms could lead to the breaking of an emergent gauge symmetry at high energies, just what is actually needed for SLIV to become active. This may be a place where the emergent vector field theories may significantly differ from conventional gauge theories that could have some observational evidence at low energies. Below we present some particular models to see more clearly how it may happen.

Looking for some appropriate examples of physical Lorentz violation in a QFT framework, one necessarily comes across a problem of proper suppression of gauge noninvariant high-dimension couplings where such violation can in principle occur. Remarkably enough, for QED-type theories with the supplementary vector field constraint (10) gauge symmetry breaking naturally appears only for five- and higher-dimensional couplings. Indeed, all dimension-4 couplings are generically gauge invariant, if the vector field kinetic term has a standard $F_{\mu\nu}F^{\mu\nu}$ and, apart from relativistic invariance, the restrictions related to the conservation of parity, charge-conjugation symmetry, and fermion number conservation are generally imposed on a theory [27]. With these restrictions taken, one can easily confirm that all possible dimension-5 couplings are also combined by themselves in some would-be gauge invariant form, provided that the vector field is constrained by the SLIV condition (10). Indeed, for charged matter fermions interacting with the vector field, such couplings generally amount to

$$L_{\dim 5}=\frac{1}{\mathcal{M}}\check{D}_{\mu}^{*}\bar{\psi}\cdot\check{D}^{\mu}\psi+\frac{\mathrm{G}}{\mathcal{M}}A_{\mu}A^{\mu}\bar{\psi}\psi,\quad A_{\mu}A^{\mu}=n^{2}\mathrm{M}^{2}. \tag{43}$$

Such couplings could presumably become significant at an ultraviolet scale $\mathcal{M}$, probably close to the Planck scale M_P. They, besides the covariant derivative terms, also include an independent "seagull" fermion-vector field term with the coupling constant G being in general of the order 1. The main point regarding the Lagrangian (43) is that, while it is gauge invariant in itself, the coupling constant ě in the covariant derivative $\check{D}^{\mu}=\partial^{\mu}+i\check{e}A^{\mu}$ differs in general from the coupling e in the covariant derivative $D^{\mu}=\partial^{\mu}+ieA^{\mu}$ in the standard Dirac Lagrangian (11):

$$L_{\mathrm{QED}}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}(i\gamma_{\mu}D^{\mu}-\mathrm{m})\psi. \tag{44}$$

Therefore, gauge invariance is no longer preserved in the total Lagrangian $L_{\mathrm{QED}}+L_{\dim 5}$. It is worth noting that, though the high-dimension Lagrangian part $L_{\dim 5}$ (43) usually only gives some small corrections to a conventional QED Lagrangian (44), the situation may drastically change when the vector field A_μ develops a VEV and SLIV occurs.

Actually, putting the SLIV parametrization (12) into the basic QED Lagrangian (44), one comes to the truly emergent model for QED being essentially nonlinear in the vector Goldstone modes a_μ associated with photons. This model contains, among other terms, the inappropriately large (while false, see below) Lorentz-violating fermion bilinear $-\mathrm{eM}\bar{\psi}\gamma_{\mu}n^{\mu}\psi$. This term appears when the effective Higgs mode expansion in Goldstone modes a_μ [as is given in the parametrization (12)] is applied to the fermion current interaction term $-\mathrm{e}\bar{\psi}\gamma_{\mu}A^{\mu}\psi$ in the QED Lagrangian (44). However, due to local invariance this bilinear term can be gauged away by making an appropriate redefinition of the fermion field $\psi\rightarrow e^{-ie\omega(x)}\psi$ with a gauge function $\omega(x)$ linear in coordinates, $\omega(x)=(x_{\mu}n^{\mu})\mathrm{M}$. Meanwhile, the dimension-5 Lagrangian $L_{\dim 5}$ (43) is substantially changed under this redefinition that significantly modifies fermion bilinear terms,

$$L_{\bar{\psi}\psi}=i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi+\frac{1}{\mathcal{M}}\partial_{\mu}\bar{\psi}\cdot\partial^{\mu}\psi-i\Delta\mathrm{e}\frac{\mathrm{M}}{\mathcal{M}}n_{\mu}\bar{\psi}\overleftrightarrow{\partial^{\mu}}\psi - \mathrm{m}_{f}\bar{\psi}\psi, \tag{45}$$

where we retained the notation ψ for the redefined fermion field and denoted, as usual, $\bar{\psi}\overleftrightarrow{\partial^{\mu}}\psi=\bar{\psi}(\partial^{\mu}\psi)-(\partial^{\mu}\bar{\psi})\psi$. Note that the extra fermion derivative terms given in (45) are produced just due to the gauge invariance breaking that is determined by the electromagnetic charge difference $\Delta\mathrm{e}=\check{\mathrm{e}}-\mathrm{e}$ in the total Lagrangian $L_{\mathrm{QED}}+L_{\dim 5}$. As a result, there appears the entirely new, SLIV inspired, dispersion relation for a charged fermion (taken with four-momentum p_μ) of type

$$p_{\mu}^{2}\cong[\mathrm{m}_{f}+2\delta(p_{\mu}n^{\mu})]^{2},\qquad \mathrm{m}_{f}=\mathrm{m}-\mathrm{GM}^{2}/\mathcal{M}-\delta^{2}n^{2}\mathcal{M}, \tag{46}$$

given to an accuracy of $O(\mathrm{m}_{f}^{2}/\mathcal{M}^{2})$ with a properly modified total fermion mass m_f. Here δ stands for the small characteristic, positive or negative, parameter $\delta=(\Delta e)\mathrm{M}/\mathcal{M}$ of physical Lorentz violation that reflects the joint effect as is given, from the one hand, by the SLIV scale M and, from the other, by the charge difference $\Delta\mathrm{e}$ being a measure of an internal gauge noninvariance. Notably, the spacetime in itself still possesses Lorentz invariance; however, fermions with SLIV contributing into their total mass m_f (46) propagate and interact in it in the Lorentz noncovariant way. At the same time, the photon dispersion relation is still retained undeformed in the order $1/\mathcal{M}$ considered.

So, it was shown that SLIV caused by the vector field VEV (8), while being superficial in a strictly gauge invariant theory, may become physically significant when this gauge invariance is broken at the SLIV scale M, close to the scale $\mathcal{M}$, which is proposed to be located near the Planck mass scale M_P. This may happen even at relatively low energies provided the gauge noninvariance caused by high-dimension couplings of matter and vector fields is not vanishingly small.

As a consequence, through special dispersion relations appearing for matter charged fermions, one is led to a new class of phenomena which could be of distinctive observational interest in particle physics and astrophysics [27]. They include a significant change in the Greizen-Zatsepin-Kouzmin cutoff for ultra-high energy cosmic-ray nucleons, stability of high-energy pions and W bosons, modification of nucleon beta decays, and some others just in the presently accessible energy area in cosmic ray physics.

However, though one could speculate about some generically broken or partial gauge symmetry in a QFT framework [27], this appears to be too high a price for an actual Lorentz violation which may stem from SLIV. And, what is more, should one insist on physical Lorentz violation, if emergent gauge fields are anyway generated through the "safe" inactive SLIV models which recover a conventional Lorentz invariance? As will be seen in later sections, emergent SUSY theories are most likely to give a negative answer to this question, thus favoring just an inactive SLIV version.

IV. EMERGENT SUSY THEORIES: A QED PRIMER

In contrast to theories probing physical Lorentz noninvariance, be it caused by generically broken gauge symmetry or external tensor-valued backgrounds, we are primarily focused here on a spontaneous Lorentz violation in an actual gauge invariant QFT framework related to the Standard Model rather than its hypothetical effective SME counterpart originated somewhere around the Planck scale. In essence, we try to extend emergent gauge theories with SLIV and an associated emergence of the SM gauge bosons as massless vector Nambu-Goldstone modes studied earlier [5,6,9,10,28] to their supersymmetric analogs. Generally speaking, it may turn out that SLIV is not the only reason why massless photons could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest may be related to supersymmetry, as was recently argued in [18]. Actually, the situation is changed remarkably in the SUSY inspired emergent models which, in contrast to non-SUSY theories, could naturally have some clear observational evidence. Indeed, as we discussed in Sec. III E, ordinary emergent theories admit some experimental verification only if gauge invariance is properly broken, caused by some high-dimension couplings. Their SUSY counterparts, and primarily emergent SUSY QED, generically appear with supersymmetry spontaneously broken in a visible sector to ensure stability of the theory. Therefore, the verification is now related to an inevitable emergence of a Goldstino-like photino state in the SUSY particle spectrum at low energies, while physical Lorentz invariance is still left intact.[9] In this sense, a generic trigger for a massless photon to appear may be spontaneously broken supersymmetry rather than physically manifested spontaneous Lorentz violation.

In this and subsequent sections, the supersymmetric emergent gauge theories, including their possible observational consequences, are considered in significant detail.

A. Spontaneous SUSY violation

Precisely speaking, since gauge invariance is not generically assumed in an emergent approach, some essential gauge-noninvariant couplings inevitably occur in the theory in a preemergent phase. They, as seen above, are basically related to the vector field self-interaction terms, triggering an emergence process in non-SUSY theories. Starting from this standpoint, we consider a conventional supersymmetric QED being similarly extended by an arbitrary polynomial potential of a general vector superfield $V(x,\theta,\bar{\theta})$ which in the standard parametrization [40] has a form

$$V(x,\theta,\bar{\theta}) = C + i\theta\chi - i\bar{\theta}\bar{\chi} + \frac{i}{2}\theta\theta S - \frac{i}{2}\bar{\theta}\bar{\theta}S^{*} - \theta\sigma^{\mu}\bar{\theta}A_{\mu} + i\theta\theta\bar{\theta}\bar{\lambda}' - i\bar{\theta}\bar{\theta}\theta\lambda' + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D', \tag{47}$$

where its vector field component A_{μ} is usually associated with a photon. Note that, apart from an ordinary photino field λ and an auxiliary D field, the superfield (47) contains in general some additional degrees of freedom in terms of the dynamical C and χ fields and nondynamical complex scalar field S [we have used the brief notations, $\lambda' = \lambda + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\bar{\chi}$ and $D' = D + \frac{1}{2}\partial^{2}C$ with $\sigma^{\mu} = (1,\vec{\sigma})$ and $\bar{\sigma}^{\mu} = (1,-\vec{\sigma})$]. The corresponding Lagrangian can be written as

$$\mathcal{L} = L_{\mathrm{SQED}} + \frac{1}{2}D^{2} + \sum_{k=1} b_{k}V^{k}|_{D}, \tag{48}$$

where, besides a standard SUSY QED part, new potential terms are presented in the sum by corresponding D-term expansions $V^{k}|_{D}$ of the vector superfield (47) into the component fields (b_{k} are some constants). It can readily be checked that the first term in this expansion is the known Fayet-Iliopoulos D term, while other terms only contain bilinear, trilinear, and quartic combinations of the superfield components A_{μ}, S, λ, and χ, respectively.[10] Actually,

[9]Of course, physical Lorentz violation will also appear if one admits some gauge noninvariance in the emergent SUSY theory as well. This may happen, for example, through high-dimension couplings being supersymmetric analogs of the couplings (43).

[10]Without loss of generality, we may restrict ourselves to the third degree superfield polynomial in the Lagrangian $\mathcal{L}$ (48) to eventually have a theory with dimensionless coupling constants for component fields. However, for the sake of completeness, it seems better to proceed with a general case. As we have recently learned, a similar self-interaction polynomial for the vector superfield [see also below the Lagrangian (51)] had been first considered quite long ago [41] to get some kind of economic Higgs model in a massive SUSY QED.

the higher-degree terms only appear for the scalar field component $C(x)$. Expressing them all in terms of the C field polynomial,

$$P(C) = \sum_{k=1} \frac{k}{2} b_k C^{k-1}(x), \tag{49}$$

and its first three derivatives

$$P'_C \equiv \frac{\partial P}{\partial C}, \qquad P''_C \equiv \frac{\partial^2 P}{\partial C^2}, \qquad P'''_C \equiv \frac{\partial^3 P}{\partial C^3}, \tag{50}$$

one has for the whole Lagrangian $\mathcal{L}$,

$$\begin{aligned}\mathcal{L} = L_{\mathrm{SQED}} + \frac{1}{2}D^2 + P\left(D + \frac{1}{2}\partial^2 C\right) \\ + P'_C\left(\frac{1}{2}SS^* - \chi\lambda' - \bar{\chi}\bar{\lambda}' - \frac{1}{2}A_\mu A^\mu\right) \\ + \frac{1}{2}P''_C\left(\frac{i}{2}\bar{\chi}\bar{\chi}S - \frac{i}{2}\chi\chi S^* - \chi\sigma^\mu\bar{\chi}A_\mu\right) + \frac{1}{8}P'''_C(\chi\chi\bar{\chi}\bar{\chi}),\end{aligned} \tag{51}$$

where, for more clarity, we still omitted in L_{SQED} matter superfields, reserving them for Sec. VI. As one can see, extra degrees of freedom related to the C and χ component fields in a general vector superfield $V(x,\theta,\bar{\theta})$ appear through the potential terms in (51), rather than from the properly constructed supersymmetric field strengths, as they appear for the vector field A_μ and its gaugino companion λ.

Note that all terms in the sum in (48) except the Fayet-Iliopoulos D term explicitly break gauge invariance. However, as we will see in the next section, the special gauge invariance constrained by some gauge condition will be recovered in the Lagrangian in the broken SUSY phase. Furthermore, as is seen from (51), the vector field A_μ may only appear with a bilinear mass term in the polynomially extended superfield Lagrangian (48), in sharp contrast to the non-SUSY theory case where, apart from the vector field mass term, some high-linear stabilizing terms necessarily appear in a similar polynomially extended Lagrangian. This means in turn that physical Lorentz invariance is still preserved in the theory. Actually, only supersymmetry appears to be spontaneously broken, as mentioned above.

Indeed, varying the Lagrangian $\mathcal{L}$ with respect to the D field we come to

$$D = -P(C), \tag{52}$$

which finally gives the following potential energy for the field system considered:

$$U(C) = \frac{1}{2}[P(C)]^2. \tag{53}$$

The potential (53) may lead to spontaneous SUSY breaking in the visible sector, provided that the polynomial P (49) has no real roots, while its first derivative has

$$P \neq 0, \qquad P'_C = 0. \tag{54}$$

This requires $P(C)$ to be an even degree polynomial with properly chosen coefficients b_k in (49) that will force its derivative P'_C to have at least one root, $C = C_0$, in which the potential (53) is minimized. Therefore, supersymmetry is spontaneously broken and the C field acquires the VEV:

$$\langle C \rangle = C_0, \qquad P'_C(C_0) = 0. \tag{55}$$

As an immediate consequence that one can readily see from the Lagrangian $\mathcal{L}$ (51), a massless photino λ, being a Goldstone fermion in the broken SUSY phase, makes all the other component fields in the superfield $V(x,\theta,\bar{\theta})$, including the photon, also become massless. However, the question then arises whether this masslessness of the photon will be stable against radiative corrections, since gauge invariance is explicitly broken in the Lagrangian (51). We show below that it could be the case if the vector superfield $V(x,\theta,\bar{\theta})$ would appear to be properly constrained.

B. Instability of superfield polynomial potential

Let us first analyze possible vacuum configurations for the superfield components in the polynomially extended QED case taken above. In general, besides the "standard" potential energy expression (53) determined solely by the scalar field component $C(x)$ of the vector superfield (47), one also has to consider other field component contributions into the potential energy. A possible extension of the potential energy (53) seems to appear only due to the pure bosonic field contributions, namely, due to couplings of the vector and auxiliary scalar fields, A_μ and S, in (51),

$$\mathcal{U} = \frac{1}{2}P^2 + \frac{1}{2}P'(A_\mu A^\mu - SS^*), \tag{56}$$

rather than due to the potential terms containing the superfield fermionic components.[11] It can be immediately seen that these new couplings in (56) can make the potential unstable since the vector and scalar fields mentioned may in general develop any arbitrary VEVs. This happens, as emphasized above, due to the fact that their bilinear term contributions are not properly compensated by appropriate four-linear field terms which are generically absent in a SUSY theory context.

For more detail we consider the extremum conditions for the entire potential (56) with respect to all fields involved:

[11] Actually, this restriction is not essential for what follows and is taken just for simplicity. Generally, the fermion bilinears involved could also develop VEVs.

C, A_μ, and S. They are given by the appropriate first partial derivative equations

$$\mathcal{U}'_C = PP' + \frac{1}{2}P''(A_\mu A^\mu - SS^*) = 0,$$
$$\mathcal{U}'_{A_\mu} = P'A^\mu = 0, \qquad \mathcal{U}'_S = -P'S^* = 0, \tag{57}$$

where and hereafter all the VEVs are denoted by the corresponding field symbols (supplied below with the lower index 0). One can see that there can occur a local minimum for the potential (56) with the unbroken SUSY solution,[12]

$$C = C_0, \qquad P(C_0) = 0, \qquad P'(C_0) \neq 0;$$
$$A_{\mu 0} = 0, \qquad S_0 = 0, \tag{58}$$

with the vanishing potential energy

$$\mathcal{U}^s_{\min} = 0, \tag{59}$$

provided that the polynomial P (49) has some real root $C = C_0$. Otherwise, a local minimum with the broken SUSY solution can occur for some other C field value (though denoted by the same letter C_0),

$$C = C_0, \qquad P(C_0) \neq 0, \qquad P'(C_0) = 0;$$
$$A_{\mu 0} \neq 0, \qquad S_0 \neq 0, \qquad A_{\mu 0}A^\mu_0 - S_0 S^*_0 = 0. \tag{60}$$

In this case one has the nonzero potential energy

$$\mathcal{U}^{as}_{\min} = \frac{1}{2}[P(C_0)]^2, \tag{61}$$

as directly follows from the extremum equations (57) and potential energy expression (56).

However, as the standard second partial derivative test shows, the fact is that the local minima mentioned above are minima with respect to the C field VEV (C_0) only. Actually, for all three fields' VEVs included, the potential (56) has indeed saddle points with "coordinates" indicated in (58) and (60), respectively. For testing convenience, this potential can be rewritten in the form

$$\mathcal{U} = \frac{1}{2}P^2 + \frac{1}{2}P' g^{\Theta\Theta'} B_\Theta B_{\Theta'},$$
$$g^{\Theta\Theta'} = \mathrm{diag}(1, -1, -1, -1, -1, -1), \tag{62}$$

with only two variable fields C and B_Θ, where the new field B_Θ unifies the A_μ and S field components, $B_\Theta = (A_\mu, S_\alpha)$ ($\Theta = \mu, a$; $\mu = 0, 1, 2, 3$; $\alpha = 1, 2$).[13] The complex S field is now taken in a real basis,

$$S_1 = (S + S^*)/2, \qquad S_2 = (S - S^*)/2i, \tag{63}$$

so that the "vector" B_Θ field has one time and five space components. As a result, one finally comes to the following Hessian 7×7 matrix [being in fact the second-order partial derivatives matrix taken in the extremum point (C_0, $A_{\mu 0}$, S_0) (58)]

$$H(\mathcal{U}^s) = \begin{bmatrix} [P'(C_0)]^2 & 0 \\ 0 & P'(C_0)g^{\Theta\Theta'} \end{bmatrix},$$
$$|H(\mathcal{U}^s)| = -[P'(C_0)]^8. \tag{64}$$

This matrix clearly has the negative determinant $|H(\mathcal{U}^s)|$, as is indicated above, that confirms that the potential definitely has a saddle point for the solution (58). This means the VEVs of the A_μ and S fields can take in fact any arbitrary value making the potential (56), (62) be unbounded from below in the unbroken SUSY case that is certainly inaccessible.

One might think that in the broken SUSY case the situation would be better, since due to the conditions (60) the B_Θ term completely disappears from the potential $\mathcal{U}$ (56), (62) in the ground state. Unfortunately, the direct second partial derivative test in this case is inconclusive, since the determinant of the corresponding Hessian 7×7 matrix appears to vanish:

$$H(\mathcal{U}^{as}) = \begin{bmatrix} P(C_0)P''(C_0) & P''(C_0)g^{\Theta\Theta'}B_{\Theta'} \\ P''(C_0)g^{\Theta\Theta'}B_{\Theta'} & 0 \end{bmatrix},$$
$$|H(\mathcal{U}^{as})| = 0. \tag{65}$$

Nevertheless, since in general the B_Θ term can take both positive and negative values in small neighborhoods around the vacuum point ($C_0, A_{\mu 0}, S_0$) where the conditions (60) are satisfied, this point also turns out to be a saddle point. Thus, the potential $\mathcal{U}$ (56), (62) appears generically unstable in both the SUSY invariant and SUSY broken phases.

C. Stabilization of vacuum by constraining vector superfield

The only possible way to stabilize the ground states (58) and (60) seems to be seeking the proper constraints on the superfield component fields (C, A_μ, S) themselves rather than on their expectation values. Indeed, if such (potential

[12]Hereafter, by $P(C_0)$ and $P'(C_0)$ are meant the C field polynomial P (49) and its functional derivative P' (50) taken in the potential extremum point C_0.

[13]Interestingly, the B_Θ term in the potential (62) possesses the accidental $SO(1,5)$ symmetry. This symmetry, though it is not shared by kinetic terms, appears in fact to be stable under radiative corrections, since the S field is nondynamical and, therefore, can always be properly arranged.

bounding) constraints are physically realizable, the vacua (58) and (60) will be automatically stabilized. Besides, as we confirm, instead of gauge symmetry broken in the extended QED Lagrangian (51) some special gauge invariance is recovered in (51) at the SUSY breaking minimum of the potential (53).

Let us try to understand first what such constraints may look like. We will expand the action around the vacuum (55) by writing

$$C(x) = C_0 + c(x), \tag{66}$$

which gives for the C field polynomial $P(C)$ (49) and its derivatives (50) to the lowest order in the Higgs-like field $c(x)$

$$\begin{aligned} &P(C) \simeq P(C_0) + \frac{1}{2}P''_C(C_0)c^2, \qquad P'_C(C) \simeq P''_C(C_0)c, \\ &P''_C(C) \simeq P''_C(C_0) + P'''_C(C_0)c, \\ &P'''_C(C) \simeq P'''_C(C_0) + P''''_C(C_0)c, \end{aligned} \tag{67}$$

with $P'_C(C_0) = 0$ taken at the minimum point C_0, as is determined in (55). Now, combining the equations of motion for $c(x)$ and for some other component field, say $S(x)$, both derived by varying the Lagrangian (51), one has

$$A_\mu A^\mu - SS^* = O(c, c\partial^2 c), \qquad \chi\chi = O(c), \tag{68}$$

where we have used approximate equalities (67) with typical nonzero values of all $P(C_0)$, $P''_C(C_0)$, $P'''_C(C_0)$, $P''''_C(C_0)$ taken at the minimum point C_0. For the vanishingly small Higgs-like mode $c(x)$ in (66) and (68), one eventually comes to the necessary constraints which have to be put on the V superfield components to provide stability of the total potential (56).

These pure heuristic arguments can be also realized in a more rigorous way by properly constraining the vector superfield $V(x, \theta, \bar{\theta})$ from the outset. In a SUSY context, a constraint can only be put on an entire superfield rather than individually on its field components. Actually, one can constrain the vector superfield (47) by analogy with the constrained vector field in the nonlinear QED model (11). This will be done again through some invariant Lagrange multiplier coupling simply by adding its D term to the above Lagrangian (48), (51):

$$\mathcal{L}^{\rm tot} = \mathcal{L} + \frac{1}{2}\Lambda(V - C_0)^2|_D, \tag{69}$$

where $\Lambda(x, \theta, \bar{\theta})$ is some auxiliary vector superfield, while C_0 is the constant background value of the C field which minimizes the potential U (53). Accordingly, the potential vanishes for the supersymmetric minimum or acquires some positive value corresponding to the SUSY breaking minimum (54) in the visible sector. We will consider both cases simultaneously using the same notation C_0 for either of the background values of the C field.

Note that first of all, the Lagrange multiplier term in (69) has in fact the simplest possible form that leads to some nontrivial constrained superfield $V(x, \theta, \bar{\theta})$. The alternative minimal forms, such as the bilinear form $\Lambda(V - C_0)$ or trilinear one $\Lambda(V^2 - C_0^2)$, appear too restrictive. One can easily confirm that they eliminate most component fields in the superfield $V(x, \theta, \bar{\theta})$, including the physical photon and photino fields that are definitely inadmissible. As to appropriate nonminimal high-linear multiplier forms, they basically lead to the same consequences as follow from the minimal multiplier term taken in the total Lagrangian (69). Writing down its invariant D term through the component fields, one finds

$$\begin{aligned} \Lambda(V - C_0)^2|_D = {}& C_\Lambda\left[\tilde{C}D' + \left(\frac{1}{2}SS^* - \chi\lambda' - \bar{\chi}\bar{\lambda}' - \frac{1}{2}A_\mu A^\mu\right)\right] \\ &+ \chi_\Lambda[2\tilde{C}\lambda' + i(\chi S^* + i\sigma^\mu\bar{\chi}A_\mu)] + \bar{\chi}_\Lambda[2\tilde{C}\bar{\lambda}' - i(\bar{\chi}S - i\chi\sigma^\mu A_\mu)] \\ &+ \frac{1}{2}S_\Lambda\left(\tilde{C}S^* + \frac{i}{2}\bar{\chi}\bar{\chi}\right) + \frac{1}{2}S^*_\Lambda\left(\tilde{C}S - \frac{i}{2}\chi\chi\right) \\ &+ 2A^\mu_\Lambda(\tilde{C}A_\mu - \chi\sigma_\mu\bar{\chi}) + 2\lambda'_\Lambda(\tilde{C}\chi) + 2\bar{\lambda}'_\Lambda(\tilde{C}\bar{\chi}) + \frac{1}{2}D'_\Lambda\tilde{C}^2, \end{aligned} \tag{70}$$

where

$$C_\Lambda, \chi_\Lambda, S_\Lambda, A^\mu_\Lambda, \lambda'_\Lambda = \lambda_\Lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}_\Lambda, \qquad D'_\Lambda = D_\Lambda + \frac{1}{2}\partial^2 C_\Lambda \tag{71}$$

are the component fields of the Lagrange multiplier superfield $\Lambda(x, \theta, \bar{\theta})$ in the standard parametrization (47) and $\tilde{C}$ stands for the difference $C(x) - C_0$. Varying the Lagrangian (69) with respect to these fields and properly combining their equations of motion,

$$\frac{\partial\mathcal{L}^{\rm tot}}{\partial(C_\Lambda, \chi_\Lambda, S_\Lambda, A^\mu_\Lambda, \lambda_\Lambda, D_\Lambda)} = 0, \tag{72}$$

we find the constraints which appear to be put on the V superfield components,[14]

$$C = C_0, \qquad \chi = 0, \qquad A_\mu A^\mu = SS^*. \tag{73}$$

They also determine the corresponding D term (52), $D = -P(C_0)$, for the spontaneously broken supersymmetry. Again, as in the non-SUSY case (11), we only take a solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (71). This will provide, as before, a ghost-free theory with a positive Hamiltonian.[15]

Remarkably, the constraints (73) do not touch the physical degrees of freedom of the superfield $V(x,\theta,\bar{\theta})$ related to photon and photino fields. The point is, however, that apart from the constraints (73), one has the equations of motion for all fields involved in the basic superfield $V(x,\theta,\bar{\theta})$. With vanishing multiplier component fields (71), as was proposed above, these equations appear in fact as extra constraints on components of the superfield $V(x,\theta,\bar{\theta})$. Indeed, equations of motion for the fields C, S, and χ received by the corresponding variations of the total Lagrangian $\mathcal{L}^{\rm tot}$ (69), (51) turn out to be, respectively,

$$P(C_0)P'(C_0)=0, \quad S(x)P'(C_0)=0, \quad \lambda(x)P'(C_0)=0, \tag{74}$$

where the basic constraints (73) emerging at the potential extremum point $C = C_0$ have also been used. One can immediately see now that these equations turn to trivial identities in the broken SUSY case, in which the factor $P'(C_0)$ in each of them appears to identically vanish, $P'(C_0) = 0$ (60). In the unbroken SUSY case, in which the potential (53) vanishes instead, i.e., $P(C_0) = 0$ and $P'(C_0) \neq 0$ (58), the situation is drastically changed. Indeed, though the first equation in (74) still automatically turns into an identity at the extremum point $C(x) = C_0$, the other two equations require that the auxiliary field S and the photino field λ identically vanish as well. This causes in turn that the photon field should also vanish according to the basic constraints (73). Besides, the D field component in the vector superfield also vanishes in the unbroken SUSY case according to Eq. (52), $D = -P(C_0) = 0$. Thus, one is ultimately left with a trivial superfield $V(x,\theta,\bar{\theta})$ which only contains the constant C field component C_0, which is unacceptable. So, we have to conclude that the unbroken SUSY fails to provide stability of the potential (56), even by constraining the superfield $V(x,\theta,\bar{\theta})$. In contrast, in the spontaneously broken SUSY case, extra constraints do not appear at all, and one has a physically meaningful theory that we basically consider in what follows.

Finally, implementing the constraints (73) into the total Lagrangian $\mathcal{L}^{\rm tot}$ (69), (51) through the Lagrange multiplier terms for component fields, we come to the emergent SUSY QED appearing in the broken SUSY phase:

$$\mathcal{L}^{\rm em} = \mathcal{L}_{\rm SQED} + P(C)D + \frac{D_\Lambda}{4}(C-C_0)^2 - \frac{C_\Lambda}{4}(A_\mu A^\mu - SS^*). \tag{75}$$

The last two terms with the component multiplier functions C_Λ and D_Λ of the auxiliary superfield Λ (71) provide the vacuum stability condition of the theory. In essence, one does not need now to postulate from the outset gauge invariance for the physical SUSY QED Lagrangian $\mathcal{L}_{\rm SQED}$. Rather, one can derive it following the emergence conjecture specified for Abelian theories in Sec. III B. Indeed, due to the constraints (73), the Lagrangian $\mathcal{L}_{\rm SQED}$ is only allowed to have a conventional gauge invariant form:

$$\mathcal{L}_{\rm SQED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{2}D^2. \tag{76}$$

Thus, for the constrained vector superfield involved,

$$\hat{V}(x,\theta,\bar{\theta}) = C_0 + \frac{i}{2}\theta\theta S - \frac{i}{2}\bar{\theta}\bar{\theta}S^* - \theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \tag{77}$$

we have the almost standard SUSY QED Lagrangian with the same states—a photon, a photino, and an auxiliary scalar D field—in its gauge supermultiplet, while another auxiliary complex scalar field S only gets involved in the vector field constraint. The linear (Fayet-Iliopoulos) D term with the effective coupling constant $P(C_0)$ in (75) shows that supersymmetry in the theory is spontaneously broken, due to which the D field acquires the VEV, $D = -P(C_0)$. Taking the nondynamical S field in the constraint (73) to be some constant background field (for a more formal discussion, see below), we come to the SLIV constraint (10) which we discussed above regarding an ordinary nonsupersymmetric QED theory (Sec. II C). As is seen from this constraint in (75), one may only have the timelike SLIV in a SUSY framework but never the spacelike one. There also may be a lightlike SLIV, if the S field

[14]Indeed, the equations $\partial\mathcal{L}_{\rm tot}/\partial D_\Lambda = 0$ and $\partial\mathcal{L}_{\rm tot}/\partial S_\Lambda = 0$ immediately give the constraints $C = C_0$ and $\chi = 0$, respectively, while the equation $\partial\mathcal{L}_{\rm tot}/\partial C_\Lambda = 0$ leads to the constraint $A_\mu A^\mu = SS^*$ once the previous two constraints are satisfied. They coincide, as expected, with constraints arisen for the vanishingly small Higgs-like mode $c(x)$ in Eqs. (66) and (68).

[15]As in the nonsupersymmetric case discussed above (see also footnote 3), this solution with all vanishing components of the basic Lagrangian multiplier superfield $\Lambda(x,\theta,\bar{\theta})$ can be reached by introducing in the total Lagrangian (69) an appropriate extra Lagrange multiplier term of the type $\Sigma\Lambda^2$, where $\Sigma(x)$ is a new multiplier superfield.

vanishes.[16] So, any possible choice for the S field corresponds to the particular gauge choice for the vector field A_μ in an otherwise gauge invariant theory. So, the massless photon appearing first as a companion of a massless photino (being a Goldstone fermion in the visible broken SUSY phase) remains massless due to this recovering gauge invariance in the emergent SUSY QED. At the same time, the “built-in" nonlinear gauge condition in (75) allows us to treat the photon as a vector Goldstone boson induced by an inactive SLIV.

D. Constrained vector superfield: A formal view

We proceed by showing that our extended Lagrangian $\mathcal{L}^{\text{tot}}$ (69), (51), underlying the emergent QED model, is SUSY invariant, and also the constraints (73) on the field space appearing due to the Lagrange multiplier term in (69) are consistent with supersymmetry. The first part of this assertion is somewhat immediate, since the Lagrangian $\mathcal{L}^{\text{tot}}$, aside from the standard supersymmetric QED part L_{SQED} (48), only contains D terms of various vector superfield products. They are, by definition, invariant under conventional SUSY transformations [40] which for the component fields (47) of a general superfield $V(x,\theta,\bar{\theta})$ (47) are written as

$$\begin{aligned}
&\delta_\xi C = i\xi\chi - i\bar{\xi}\bar{\chi}, \qquad \delta_\xi\chi = \xi S + \sigma^\mu\bar{\xi}(\partial_\mu C + iA_\mu),\\
&\frac{1}{2}\delta_\xi S = \bar{\xi}\,\bar{\lambda} + \bar{\sigma}_\mu\partial^\mu\chi,\\
&\delta_\xi A_\mu = \xi\partial_\mu\chi + \bar{\xi}\partial_\mu\bar{\chi} + i\xi\sigma_\mu\bar{\lambda} - i\lambda\sigma_\mu\bar{\xi},\\
&\delta_\xi\lambda = \frac{1}{2}\xi\sigma^\mu\bar{\sigma}^\nu F_{\mu\nu} + \xi D,\\
&\delta_\xi D = -\xi\sigma^\mu\partial_\mu\bar{\lambda} + \bar{\xi}\sigma^\mu\partial_\mu\lambda.
\end{aligned} \tag{78}$$

However, there may still be left a question about whether supersymmetry remains in force when the constraints (73) on the field space are “switched on,” thus leading to the final Lagrangian $\mathcal{L}^{\text{em}}$ (75) in the broken SUSY phase with both dynamical fields C and χ eliminated. This Lagrangian appears similar to the standard supersymmetric QED taken in the Wess-Zumino gauge, except that supersymmetry is spontaneously broken in our case. In both cases, the photon stress tensor $F_{\mu\nu}$, the photino λ, and the nondynamical scalar D field form an irreducible representation of the supersymmetry algebra [the last two lines in (78)]. Nevertheless, any reduction of component fields in the vector superfield is not consistent in general with the linear superspace version of supersymmetry transformations, whether it is the Wess-Zumino gauge case or our constrained superfield (77). Indeed, a general SUSY transformation does not preserve the Wess-Zumino gauge: a vector superfield in this gauge,

$$V_{WZ}(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\,\bar{\lambda} - i\bar{\theta}\,\bar{\theta}\,\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\,\bar{\theta}\,D, \tag{79}$$

acquires all possible extra terms when being SUSY transformed. The same also occurs with our constrained superfield $\hat{V}$ (77). The point, however, is that in both cases a total supergauge transformation,

$$V \to V + \frac{i}{2}(\Omega - \Omega^*), \tag{80}$$

where Ω is an arbitrary chiral superfield transformation parameter [40],

$$\begin{aligned}
\Omega = \varphi + \sqrt{2}\theta\psi + \theta\theta F + i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi - \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\sigma^\mu\partial_\mu\psi\\
-\frac{1}{4}\theta\theta\bar{\theta}\,\bar{\theta}\,\partial^2\varphi,
\end{aligned} \tag{81}$$

can always restore the vector superfield initial (restricted) form (77) or (79), respectively. In a conventional supersymmetric QED in the Wess-Zumino supergauge taken, an ordinary gauge freedom is left untouched. This means that the nontrivial part of the V_{WZ} superfield transformation amounts to

$$V_{WZ} \to V_{WZ} - \theta\sigma^\mu\bar{\theta}\partial_\mu\varphi, \qquad A_\mu \to A_\mu - \partial_\mu\varphi, \tag{82}$$

where the scalar component φ in the SUSY transformation parameter Ω (81) is used. In contrast, in the emergent SUSY QED (75), the ordinary gauge is fixed by the vector field constraint (73). However, this constraint remains under supergauge transformation (80) applied to our superfield $\hat{V}$ (77). Indeed, the essential part of this transformation which directly acts on the constraint (73) has the form

$$\hat{V} \to \hat{V} + \frac{i}{2}\theta\theta F - \frac{i}{2}\bar{\theta}\,\bar{\theta}\,F^* - \theta\sigma^\mu\bar{\theta}\partial_\mu\varphi, \tag{83}$$

where the real and complex scalar field components, φ and F, in a chiral superfield parameter Ω, are properly activated. As a result, the vector and scalar fields, A_μ and S, in the supermultiplet $\hat{V}$ (77) transform as

$$A_\mu \to a_\mu = A_\mu - \partial_\mu\varphi, \qquad S \to s = S + F. \tag{84}$$

It can be immediately seen that our basic Lagrangian $\mathcal{L}^{\text{em}}$ (75), (76), being gauge invariant and containing no scalar S field, is automatically invariant under either of these two transformations individually. In contrast, the

[16] Indeed, this case, first mentioned in [10], may also mean spontaneous Lorentz violation with a nonzero VEV $\langle A_\mu\rangle = (\widetilde{M},0,0,\widetilde{M})$ and Goldstone modes $A_{1,2}$ and $(A_0+A_3)/2 - \widetilde{M}$. The “effective” Higgs mode $(A_0 - A_3)/2$ can be then expressed through Goldstone modes so the lightlike condition $A_\mu A^\mu = 0$ can be satisfied.

supplementary vector field constraint (73) also turns out to be invariant under supergauge transformations (84), but only if they act jointly. Indeed, for any choice of the scalar φ in (84) there can always be found such a scalar F (and vice versa) that the constraint remains invariant. In other words, the vector field constraint is invariant under supergauge transformations (84) but is not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess-Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the scalar field bilinear SS^* to some constant background value and find the final equation for the gauge function $\varphi(x)$. It is convenient to come to real field basis (63) S_α and F_α ($\alpha, \beta, \ldots = 1, 2$) and choose the parameter fields F_α as

$$F_\alpha = r_\alpha(\mathrm{M} + f), \qquad r_\alpha^2 = 1, \tag{85}$$

so that the old S_α fields in (84) are related to the new ones s_α in the following way:

$$S_\alpha = s_\alpha - r_\alpha(\mathrm{M} + f), \qquad r_\alpha S_\alpha = 0,$$
$$S_\alpha S_\alpha = s_\alpha s_\alpha + (\mathrm{M} + f)^2, \tag{86}$$

where M is a mass parameter, $f(x)$ is some Higgs field–like function, while r_α is a two-component unit vector orthogonal to the scalar "doublet" S_α. Actually, the parametrization (86) formally looks as if the old fields S_α would develop the VEV, $\langle S_\alpha \rangle = -r_\alpha \mathrm{M}$, due to which some related $SO(2)$ symmetry was spontaneously violated and corresponding zero modes in terms of the new fields s_α could be consequently produced. Eventually, for the properly chosen "Higgs field" f,

$$f = -\mathrm{M} + \sqrt{\mathrm{M}^2 - s_\alpha s_\alpha}, \tag{87}$$

we come to

$$A_\mu A^\mu = \mathrm{M}^2, \tag{88}$$

which is nothing but our old constraint (10) taken for the timelike SLIV. Recall that this constraint, as was thoroughly discussed in Sec. II C, does not physically break gauge invariance. It rather fixes the gauge to which such a gauge function $\varphi(x)$ has to satisfy. Actually, comparing the relation between the old and new vector fields in (84) with a conventional SLIV parametrization (12), one can find a simple expression for this gauge function,

$$\varphi = \int^x d(n_\mu x^\mu)\sqrt{\mathrm{M}^2 - n^2 a^2}, \tag{89}$$

that explicitly demonstrates that this gauge condition is possible, at least in the case when the new vector fields in (84) are taken in terms of the Lorentz breaking zero modes ($a^2 = a_\mu a^\mu, n_\mu a^\mu = 0$).

To summarize, it was shown that the constraints on the allowed configurations of the vector-superfield component fields (73), that provide the potential energy stability in a general polynomially extended Lagrangian (69), are entirely consistent with supersymmetry. One might think that, unlike the gauge invariant linear (Fayet-Iliopoulos) superfield term, the quadratic and higher-order superfield terms in the starting Lagrangian (69) would seem to break gauge invariance. However, the fear proves groundless. In the broken SUSY phase, one eventually comes to the standard SUSY QED-type Lagrangian (75) being supplemented by the vector field constraint invariant under supergauge transformations. Thus, the gauge noninvariance mentioned above amounts to the gauge-fixing condition with a gauge function which can be explicitly calculated (89).

V. ON EMERGENT SUSY STANDARD MODELS AND GUTS

A. Potential of Abelian and non-Abelian vector superfields

In this section we extend our discussion to the non-Abelian internal symmetry case given by some group G with generators t^p (21). This case may correspond in general to some grand unified theory which includes the Standard Model and its possible extensions. For definiteness, we will be further focused on the $U(1) \times SU(N)$ symmetrical theories, though any other non-Abelian group in place of $SU(N)$ is also admissible. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case, supersymmetry does not get spontaneously broken in a visible sector, which makes it inappropriate for an outgrowth of an emergence process.[17] So, the theory now contains the Abelian vector superfield V, as is given in (47), and non-Abelian superfield multiplet $\boldsymbol{V}^p$:

$$\boldsymbol{V}^p(x,\theta,\bar\theta) = \boldsymbol{C}^p + i\theta\boldsymbol{\chi}^p - i\bar\theta\bar{\boldsymbol{\chi}}^p + \frac{i}{2}\theta\theta\boldsymbol{S}^p - \frac{i}{2}\bar\theta\bar\theta\boldsymbol{S}^p - \theta\sigma^\mu\bar\theta\boldsymbol{A}_\mu^p + i\theta\theta\bar\theta\bar{\boldsymbol{\lambda}}'^p - i\bar\theta\bar\theta\theta\boldsymbol{\lambda}'^p + \frac{1}{2}\theta\theta\bar\theta\bar\theta\boldsymbol{D}'^p, \tag{90}$$

where its vector field components $\boldsymbol{A}_\mu^p$ are usually associated with an adjoint gauge field multiplet, $(\boldsymbol{A}_\mu)^i_j \equiv (\boldsymbol{A}_\mu^p t^p)^i_j$ ($i, j, k = 1, 2, \ldots, N;\ p, q, r = 1, 2, \ldots, N^2 - 1$). Note that, apart from the conventional gaugino multiplet $\boldsymbol{\lambda}^p$ and the auxiliary fields $\boldsymbol{D}^p$, the superfield $\boldsymbol{V}^p$ contains in general the additional degrees of freedom in terms of the dynamical

[17] In principle, SUSY may be spontaneously broken in the visible sector even in the pure non-Abelian symmetry case, provided that the vector superfield potential includes some essential high-dimension couplings.

scalar and fermion field multiplets $\boldsymbol{C}^p$ and $\boldsymbol{\chi}^p$ and nondynamical complex scalar field $\boldsymbol{S}^p$. Note that for the non-Abelian superfield components we use hereafter the bold symbols and take again the brief notations, $\boldsymbol{\lambda}'^p = \boldsymbol{\lambda}^p + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\boldsymbol{\chi}}^p$ and $\boldsymbol{D}'^p = \boldsymbol{D}^p + \frac{1}{2}\partial^2\boldsymbol{C}^p$.

Augmenting the SUSY and $U(1)\times SU(N)$ invariant GUT by some polynomial potential of vector superfields V and $\boldsymbol{V}^p$, one comes to

$$\mathfrak{L} = L_{\mathrm{SGUT}} + \frac{1}{2}D^2 + \frac{1}{2}\boldsymbol{D}^p\boldsymbol{D}^p + [\xi V + b_1V^3/3 + b_2V(\boldsymbol{VV}) + b_3(\boldsymbol{VVV})/3]_D, \tag{91}$$

where ξ and $b_{1,2,3}$ stand for coupling constants, and the last term in (91) contains products of the Abelian superfield V and the adjoint $SU(N)$ superfield multiplet $\boldsymbol{V}^i_j \equiv (\boldsymbol{V}^p t^p)^i_j$. The round brackets denote hereafter traces for the superfield $\boldsymbol{V}^i_j$,

$$(\boldsymbol{VV}\ldots) \equiv \mathrm{Tr}(\boldsymbol{VV}\ldots), \tag{92}$$

and its field components (see below). For simplicity, we restricted ourselves to the third degree superfield terms in the Lagrangian $\mathfrak{L}$ to eventually have a theory at a renormalizable level. Furthermore, we have only taken the odd power superfield terms that provide, as we see below, an additional discrete symmetry of the potential with respect to the scalar field components in the V and $\boldsymbol{V}^p$ superfields:

$$C \to -C, \qquad \boldsymbol{C}^p \to -\boldsymbol{C}^p. \tag{93}$$

Finally, by eliminating the auxiliary D and $\boldsymbol{D}^p$ fields in the Lagrangian $\mathfrak{L}$, we come to the total potential for all superfield bosonic field components written in terms of traces mentioned above (92):

$$\mathfrak{U}_B = \mathfrak{U}_B(C,\boldsymbol{C}) + \frac{1}{2}b_1C(A_\mu A^\mu - S_\alpha S_\alpha) + \frac{1}{2}b_2C[(\boldsymbol{A}_\mu\boldsymbol{A}^\mu) - (\boldsymbol{S}_\alpha\boldsymbol{S}_\alpha) + \frac{1}{2}b_2[A_\mu(\boldsymbol{A}^\mu\boldsymbol{C}) - S_\alpha(\boldsymbol{S}_\alpha\boldsymbol{C})] + \frac{1}{2}b_3[(\boldsymbol{A}_\mu\boldsymbol{A}^\mu\boldsymbol{C}) - (\boldsymbol{S}_\alpha\boldsymbol{S}_\alpha\boldsymbol{C})], \tag{94}$$

where the potential terms depending only on scalar fields C and $\boldsymbol{C}^i_j \equiv (\boldsymbol{C}^a t^a)^i_j$ are collected in

$$\mathfrak{U}_B(C,\boldsymbol{C}) = \frac{1}{8}[\xi + b_1C^2 + b_2(\boldsymbol{CC})]^2 + \frac{1}{2}\left[b_2^2C^2(\boldsymbol{CC}) + b_2b_3C(\boldsymbol{CCC}) + \frac{1}{4}b_3^2(\boldsymbol{CCCC})\right], \tag{95}$$

and complex scalar fields S_α and $\boldsymbol{S}^p_\alpha$ are now taken in the real field basis (63). One can see that all these terms are invariant under the discrete symmetry (93), whereas the vector field couplings in $\mathfrak{U}_B$ break it. However, they vanish when the V and $\boldsymbol{V}^p$ superfields are properly constrained, which we actually confirm in the next section.

As in the SUSY QED case (Sec. IV B), consider first the pure scalar field potential $\mathfrak{U}_B(C,\boldsymbol{C})$. The corresponding extremum conditions for C and $\boldsymbol{C}^a$ fields are

$$\mathfrak{U}'_C = b_1(\xi + b_1C^2)C + b_2(b_1 - 2b_2)C(\boldsymbol{CC}) = 0, \quad \mathrm{Tr}(\mathfrak{U}'_{\boldsymbol{C}^i_j}) = 3b_2C(\boldsymbol{CC}) + b_3(\boldsymbol{CCC}) = 0, \tag{96}$$

respectively.[18] As the second partial derivative test shows, the simplest solution to the above equations,

$$C_0 = 0, \qquad \boldsymbol{C}^i_j = 0, \tag{97}$$

provides, under conditions put on the potential parameters,

$$\xi, b_1 > 0, b_2 \geq 0 \quad \text{or} \quad \xi, b_1 < 0, b_2 \leq 0, \tag{98}$$

its global minimum

$$\mathfrak{U}_B(C,\boldsymbol{C})^{as}_{\min} = \frac{1}{8}\xi^2. \tag{99}$$

This minimum corresponds to the broken SUSY phase with the unbroken internal symmetry $U(1)\times SU(N)$ that is just what one would want to trigger an emergence process. This minimum appears in fact due to the Fayet-Iliopoulos linear term in the superfield polynomial in (91). As can easily be confirmed, in the absence of this term, namely, for $\xi = 0$ and any arbitrary values of all other parameters, there is only the SUSY symmetrical solution with unbroken internal symmetry:

$$\mathfrak{U}_B(C,\boldsymbol{C})^{\mathrm{sym}}_{\min} = 0. \tag{100}$$

Interestingly, the symmetrical solution corresponding to the global minimum (100) may appear for the nonzero parameter ξ as well,

$$C_0^{(\pm)} = \pm\sqrt{-\xi/b_1}, \qquad \boldsymbol{C}^i_j = 0, \tag{101}$$

provided that

$$\xi b_1 < 0. \tag{102}$$

[18] In more detail, we first calculated here the variations $\mathfrak{U}'_C = 0$ and $\mathfrak{U}'_{\boldsymbol{C}^i_j} = 0$, then took the trace from the second one [thus properly simplifying it due to the traceless condition for the adjoint $SU(N)$ multiplet $\mathrm{Tr}(\boldsymbol{C}^i_j) = 0$], and finally substituted it into the first one.

However, as we saw in the QED case, in the unbroken SUSY case one comes to the trivial constant superfield when all factual constraints are included into consideration [see Eqs. (74) and the subsequent discussion] and, therefore, this case is in general of little interest.[19]

B. Constrained vector supermultiplets

Let us now take the vector fields A_μ and $\boldsymbol{A}_\mu^p$ into consideration that immediately reveals that, in contrast to the pure scalar field part (95), $\mathfrak{U}_B(C, \boldsymbol{C})$, the vector field couplings in the total potential (94) make it unstable. This happens, as was emphasized before, due to the fact that bilinear term VEV contributions of the vector fields A_μ and $\boldsymbol{A}_\mu^p$, as well as the auxiliary scalar fields S_α and $\boldsymbol{S}_\alpha^p$, are not properly compensated by appropriate four-linear field terms which are generically absent in a supersymmetric theory framework.

Again, as in the supersymmetric QED case considered above, the only possible way to stabilize the ground state (97)–(99) seems to be seeking the proper constraints on the superfields' component fields (C, $\boldsymbol{C}^p$; A_μ, $\boldsymbol{A}^p$; S_α, $\boldsymbol{S}^p$) themselves rather than on their expectation values. Provided that such constraints are physically realizable, the required vacuum will be automatically stabilized. This will be done again through some invariant Lagrange multiplier couplings simply by adding their D terms to the above Lagrangian (91):

$$\mathfrak{L}^{\rm tot} = \mathfrak{L} + \frac{1}{2}\Lambda(V - C_0)^2|_D + \frac{1}{2}\Pi(\boldsymbol{V}\boldsymbol{V})|_D, \qquad (103)$$

where $\Lambda(x,\theta,\bar\theta)$ and $\Pi(x,\theta,\bar\theta)$ are auxiliary vector superfields. Note that C_0 presented in the first multiplier coupling is just the constant background value of the C field for which the potential part $\mathfrak{U}_B(C,\boldsymbol{C})$ in (94) vanishes as appears for the supersymmetric minimum (100) or has some nonzero value corresponding to the SUSY breaking minimum (99) in the visible sector. We will consider both cases simultaneously using the same notation C_0 for either of the potential minimizing values of the C field. The second multiplier coupling in (103) provides, as we will soon see, the vanishing background value for the non-Abelian scalar field, $\boldsymbol{C}^a = 0$, due to which the underlying internal symmetry $U(1)\times SU(N)$ is left intact in both unbroken and broken SUSY phases. As was emphasized before, the Lagrange multiplier terms presented in (103) have in fact the simplest possible form that leads to some nontrivial constrained superfields $V(x,\theta,\bar\theta)$ and $\boldsymbol{V}^p(x,\theta,\bar\theta)$. By writing down their invariant D terms through the component fields, one finds precisely the same expression (70) as in the SUSY QED case for the Abelian superfield V and the slightly modified one for the non-Abelian superfield $\boldsymbol{V}^a$:

$$\Pi(\boldsymbol{V}\boldsymbol{V})|_D = C_\Pi\left[\boldsymbol{C}\boldsymbol{D}' + \left(\frac{1}{2}\boldsymbol{S}\boldsymbol{S}^* - \boldsymbol{\chi}\boldsymbol{\lambda}' - \bar{\boldsymbol{\chi}}\bar{\boldsymbol{\lambda}}' - \frac{1}{2}\boldsymbol{A}_\mu\boldsymbol{A}^\mu\right)\right] + \chi_\Pi[2\boldsymbol{C}\boldsymbol{\lambda}' + i(\boldsymbol{\chi}\boldsymbol{S}^* + i\sigma^\mu\bar{\boldsymbol{\chi}}\boldsymbol{A}_\mu)] + \bar\chi_\Pi[2\boldsymbol{C}\bar{\boldsymbol{\lambda}}' - i(\bar{\boldsymbol{\chi}}\boldsymbol{S} - i\boldsymbol{\chi}\sigma^\mu\boldsymbol{A}_\mu)]$$
$$+ \frac{1}{2}S_\Pi\left(\boldsymbol{C}\boldsymbol{S}^* + \frac{i}{2}\bar{\boldsymbol{\chi}}\bar{\boldsymbol{\chi}}\right) + \frac{1}{2}S_\Pi^*\left(\boldsymbol{C}\boldsymbol{S} - \frac{i}{2}\boldsymbol{\chi}\boldsymbol{\chi}\right) + 2A_\Pi^\mu(\boldsymbol{C}\boldsymbol{A}_\mu - \boldsymbol{\chi}\sigma_\mu\bar{\boldsymbol{\chi}}) + 2\lambda_\Pi'(\boldsymbol{C}\boldsymbol{\chi}) + 2\bar\lambda_\Pi'(\boldsymbol{C}\bar{\boldsymbol{\chi}}) + \frac{1}{2}D_\Pi'(\boldsymbol{C}\boldsymbol{C}), \quad (104)$$

where the bold field symbols grouped into pairs mean hereafter the $SU(N)$ scalar products of the component field multiplets (for instance, $\boldsymbol{C}\boldsymbol{D}' = \boldsymbol{C}^p\boldsymbol{D}'^p$, and so forth), and

$$C_\Pi, \chi_\Pi, S_\Pi, A_\Pi^\mu, \qquad \lambda_\Pi' = \lambda_\Pi + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi_\Pi,$$
$$D_\Pi' = D_\Pi + \frac{1}{2}\partial^2 C_\Pi \qquad (105)$$

are the component fields of the Lagrange multiplier superfield $\Pi(x,\theta,\bar\theta)$ in the standard parametrization (90). Varying the total Lagrangian (103) with respect to the component fields of both multipliers, (71) and (105), and properly combining their equations of motion, we find the constraints which appear to be put on the V and $\boldsymbol{V}^a$ superfields components (in the same way[14] for both Abelian and non-Abelian superfield cases),

$$C = C_0, \qquad \chi = 0, \qquad A_\mu A^\mu = S_\alpha S_\alpha,$$
$$\boldsymbol{C}^p = 0, \qquad \boldsymbol{\chi}^p = 0, \qquad (\boldsymbol{A}_\mu\boldsymbol{A}^\mu) = (\boldsymbol{S}_\alpha\boldsymbol{S}_\alpha),$$
$$\alpha = 1, 2. \qquad (106)$$

As before in the SUSY QED case, one may only have the timelike SLIV in a supersymmetric $U(1)\times SU(N)$ framework but never the spacelike one (there also may be a lightlike SLIV, if the S and $\boldsymbol{S}$ fields vanish). Also note that we only take the solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (71)

[19] It is worth noting that for nonzero b_1 values there are also lots of local and global SUSY breaking minima with both nonzero scalar field VEVs C_0 and $(\boldsymbol{C}_j^i)_0$ in some parameter area ($b_{1,3} > 0$ ($b_{1,3} < 0$), $b_2 < 0$ ($b_2 > 0$). This means that the $SU(N)$ symmetry is also spontaneously broken in this case that otherwise (when $b_1 = 0$) would not be happen in itself, as is clearly seen from the extremum conditions (96).

and (105) that will provide a ghost-free theory with a positive Hamiltonian.[20]

Again, apart from the constraints (106), one has the equations of motion for all fields involved in the basic superfields $V(x,\theta,\bar{\theta})$ and $\boldsymbol{V}^p(x,\theta,\bar{\theta})$. With vanishing multiplier component fields (71) and (105), as was proposed above, these equations appear in fact as extra constraints on components of the V and $\boldsymbol{V}^p$ superfields. Indeed, equations of motion for the S_α, χ, and C fields on the one hand and for the $\boldsymbol{S}^p_\alpha$, $\boldsymbol{\chi}^p$, and $\boldsymbol{C}^p$ fields on the other are obtained by the corresponding variations of the total Lagrangian $\mathfrak{L}^{\rm tot}$ (103), including the potential (94). They turn out to be, respectively,

$$S_\alpha C_0 = 0, \qquad \lambda C_0 = 0, \qquad (\xi + b_1 C_0^2) C_0 = 0,$$
$$\boldsymbol{S}^p_\alpha C_0 = 0, \qquad \boldsymbol{\lambda}^p C_0 = 0,$$
$$b_2[A_\mu \boldsymbol{A}^\mu{}^i_j - S_\alpha \boldsymbol{S}_\alpha{}^i_j] + b_3[(\boldsymbol{A}_\mu \boldsymbol{A}^\mu)^i_j - (\boldsymbol{S}_\alpha \boldsymbol{S}_\alpha)^i_j] = 0, \qquad (107)$$

where the basic constraints (106) emerging at the potential $\mathfrak{U}_B(C,\boldsymbol{C})$ extremum point (C_0, $\boldsymbol{C}^p_0 = 0$) have been also used for both broken and unbroken SUSY cases. Note also that the equations for gauginos λ and $\boldsymbol{\lambda}^p$ in (107) are received by variation of the potential terms in (91) containing fermion field couplings,

$$\mathfrak{U}_F = b_1 C(\chi\lambda' + \bar{\chi}\bar{\lambda}') + b_2 C[(\boldsymbol{\chi}\boldsymbol{\lambda}') + (\bar{\boldsymbol{\chi}}\bar{\boldsymbol{\lambda}}')]$$
$$+ \frac{1}{2} b_2[\chi(\boldsymbol{\lambda}'\boldsymbol{C}) + \bar{\chi}(\bar{\boldsymbol{\lambda}}'\boldsymbol{C}) + \lambda'(\boldsymbol{\chi}\boldsymbol{C}) + \bar{\lambda}'(\bar{\boldsymbol{\chi}}\boldsymbol{C})]$$
$$+ b_3(\boldsymbol{\chi}\boldsymbol{\lambda}'\boldsymbol{C}) + (\bar{\boldsymbol{\chi}}\bar{\boldsymbol{\lambda}}'\boldsymbol{C})]. \qquad (108)$$

One can immediately see now that all equations in (107) but the last equation system[21] turn into trivial identities in the broken SUSY case (97) in which the corresponding C field value appears to be identically vanished, $C_0 = 0$. In the unbroken SUSY case (101), this field value is definitely nonzero, $C_0 = \pm\sqrt{-\xi/b_1}$, and the situation is radically changed. Indeed, as follows from the equations (107), the auxiliary fields $S(x)$ and $\boldsymbol{S}^p$, as well as the gaugino fields $\lambda(x)$ and $\boldsymbol{\lambda}^p(x)$, have to identically vanish. This causes in turn that the gauge vector fields A_μ and $\boldsymbol{A}^p_\mu$ should also vanish according to the basic constraints (106). So, we have to conclude, as in the SUSY QED case, that the unbroken SUSY fails to provide stability of the potential (56) even by constraining the superfields V and $\boldsymbol{V}^p$ and, therefore, only the spontaneously broken SUSY case could in principle lead to a physically meaningful emergent theory.

C. Broken SUSY phase: An emergent $U(1) \times SU(N)$ theory

With the constraints (106) providing vacuum stability for the total Lagrangian $\mathfrak{L}^{\rm tot}$ (103), we eventually come to the emergent theory with a local $U(1) \times SU(N)$ symmetry that appears in the broken SUSY phase (97). Actually, implementing these constraints into the Lagrangian through the Lagrange multiplier terms for component fields, one has

$$\mathfrak{L}^{\rm em} = \mathfrak{L}_{\rm SGUT} + \frac{1}{2}\xi D + \frac{D_\Lambda}{4}(C - C_0)^2 - \frac{C_\Lambda}{4}(A_\mu A^\mu - SS^*)$$
$$+ \frac{D_\Pi}{4}(\boldsymbol{C}\boldsymbol{C}) - \frac{C_\Pi}{4}(\boldsymbol{A}_\mu \boldsymbol{A}^\mu - \boldsymbol{S}\boldsymbol{S}^*), \qquad (109)$$

with the multiplier component functions C_Λ and D_Λ of the auxiliary superfield Λ (71) and component functions C_Π and D_Π of the auxiliary superfield Π (105) presented in the Lagrangian (103). Again, with these constraints and the emergence conjecture specified for non-Abelian theories in Sec. III C, one does not need to postulate gauge invariance for the physical SUSY GUT Lagrangian $\mathfrak{L}_{\rm SGUT}$ from the outset. Instead, one can derive it. Indeed, even if the Lagrangian $\mathfrak{L}_{\rm SGUT}$ is initially taken to only possess the global $U(1) \times SU(N)$ symmetry, it will tend to uniquely acquire a standard gauge invariant form

$$\mathfrak{L}_{\rm SGUT} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{2}D^2$$
$$- \frac{1}{4}\boldsymbol{F}^{p\mu\nu}\boldsymbol{F}^p_{\mu\nu} + i\boldsymbol{\lambda}^p\sigma^\mu\mathcal{D}_\mu\bar{\boldsymbol{\lambda}}^p + \frac{1}{2}\boldsymbol{D}^p\boldsymbol{D}^p, \qquad (110)$$

where the conventional gauge field strengths for both $U(1)$ and $SU(N)$ parts and terms with proper covariant derivatives for gaugino fields $\boldsymbol{\lambda}^p$ necessarily appear. Again as in the pure Abelian case, for the respectively constrained vector superfields V and $\boldsymbol{V}^p$, we come in fact to a conventional SUSY GUT Lagrangian with a standard gauge supermultiplet containing gauge bosons A_μ and $\boldsymbol{A}^p$, gauginos λ and $\boldsymbol{\lambda}^p$, and auxiliary scalar D and $\boldsymbol{D}^p$ fields, whereas other auxiliary scalar fields S_α and $\boldsymbol{S}^p_\alpha$ get solely involved in the Lagrange multiplier terms (110). Actually, the only remnant of the polynomial potential of vector superfields V and $\boldsymbol{V}^p$ (91) that survived in the emergent theory (109) appears to be the Fayet-Iliopoulos D term, which shows that supersymmetry in the theory is indeed spontaneously broken and the D field acquires the VEV, $D = -\frac{1}{2}\xi$.

Let us show now that this theory is in essence gauge invariant and the constraints (106) on the field space appearing due to the Lagrange multiplier terms in (103) are consistent with supersymmetry. Namely, as was argued above (Sec. IV D), though restricted vector superfields are

[20] As in the nonsupersymmetric case discussed above (see footnote 3), this solution with all vanishing components of the basic Lagrangian multiplier superfields $\Lambda(x,\theta,\bar{\theta})$ and $\Pi(x,\theta,\bar{\theta})$ can be reached by introducing some extra Lagrange multipliers.

[21] This equation system is not at all dependent on the critical C field value. It allows us, as we will see in the next section, to eliminate the auxiliary scalar fields S_α and $\boldsymbol{S}^a_\alpha$ from the theory, thus properly expressing them through the vector fields A_μ and $\boldsymbol{A}^a_\mu$.

not strictly compatible with the linear superspace version of SUSY transformations, their supermultiplet structure can be restored by appropriate supergauge transformations. Following the same argumentation, one can see that these transformations keep invariant the constraints (106) put on the vector fields A_μ and $\boldsymbol{A}^p$. Leaving aside the $U(1)$ sector considered above in significant detail, we will now focus on the $SU(N)$ symmetry case with the constrained superfield $\boldsymbol{V}^p$ transformed as

$$\boldsymbol{V}^p \to \boldsymbol{V}^p + \frac{i}{2}(\Omega - \Omega^*)^p. \tag{111}$$

The essential part of this transformation which directly acts on the vector field constraint,

$$\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} = \boldsymbol{S}^p \boldsymbol{S}^{*p}, \tag{112}$$

has the form

$$\boldsymbol{V}^p \to \boldsymbol{V}^p + \frac{i}{2}\theta\theta \boldsymbol{F}^p - \frac{i}{2}\bar{\theta}\bar{\theta}\boldsymbol{F}^{*p} - \theta\sigma^\mu\bar{\theta}\partial_\mu \boldsymbol{\varphi}^p, \tag{113}$$

where the real and complex scalar field components, $\boldsymbol{\varphi}^p$ and $\boldsymbol{F}^p$, in a chiral superfield parameter Ω^p are properly activated. As a result, the corresponding vector and scalar component fields, $\boldsymbol{A}_\mu^p$ and $\boldsymbol{S}_\alpha^p$, in the constrained supermultiplet $\boldsymbol{V}^p$ transform as

$$\boldsymbol{A}_\mu^p \to \boldsymbol{a}_\mu^p = \boldsymbol{A}_\mu^p - \partial_\mu \boldsymbol{\varphi}^p, \quad \boldsymbol{S}^p \to \boldsymbol{s}^p = \boldsymbol{S}^p + \boldsymbol{F}^p. \tag{114}$$

One can readily see that our basic Lagrangian $\mathfrak{L}^{\mathrm{em}}$ (109), being gauge invariant and containing no auxiliary scalar fields $\boldsymbol{S}^p$, is automatically invariant under either of these two transformations individually. In contrast, the supplementary vector field constraint (112) also turns out to be invariant under supergauge transformations (114), but only if they act jointly. Indeed, for any choice of the scalar $\boldsymbol{\varphi}^p$ in (114), there can always be found such a scalar $\boldsymbol{F}^a$ (and vice versa) that the constraint remains invariant. In other words, the vector field constraint is invariant under supergauge transformations (114) but is not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess-Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the scalar field bilinear $\boldsymbol{S}^p\boldsymbol{S}^{*p}$ to some constant background value and find a final equation for the gauge function $\boldsymbol{\varphi}^p(x)$. It is convenient to come to the real field basis (63) for scalar fields $\boldsymbol{S}_\alpha^p$ and $\boldsymbol{F}_\alpha^p$ ($\alpha = 1, 2$) and choose the parameter fields $\boldsymbol{F}_\alpha^a$ as

$$\boldsymbol{F}_\alpha^p = r_\alpha \boldsymbol{\epsilon}^p(\mathbf{M} + \boldsymbol{f}), \quad r_\alpha \boldsymbol{s}_\alpha^p = 0, \quad r_\alpha^2 = 1, \quad \boldsymbol{\epsilon}^p\boldsymbol{\epsilon}^p = 1, \tag{115}$$

so that the old $\boldsymbol{S}_\alpha^p$ fields in (114) are related to the new ones $\boldsymbol{s}_\alpha^p$ in the following way:

$$\boldsymbol{S}_\alpha^p = \boldsymbol{s}_\alpha^p - r_\alpha \boldsymbol{\epsilon}^p(\mathbf{M} + \boldsymbol{f}), \qquad r_\alpha \boldsymbol{s}_\alpha^p = 0,$$
$$\boldsymbol{S}_\alpha^p \boldsymbol{S}_\alpha^p = \boldsymbol{s}_\alpha^p \boldsymbol{s}_\alpha^p + (\mathbf{M} + \boldsymbol{f})^2. \tag{116}$$

where $\mathbf{M}$ is a new mass parameter, $\boldsymbol{f}(x)$ is some Higgs field–like function, r_α is again the two-component unit vector chosen to be orthogonal to the scalar $\boldsymbol{s}_\alpha^p$, while $\boldsymbol{\epsilon}^p$ is the unit $SU(N)$ adjoint vector. Again, this parametrization for the old fields $\boldsymbol{S}_\alpha^p$ formally looks as if they develop the VEV, $\langle \boldsymbol{S}_\alpha^p \rangle = -r_\alpha \boldsymbol{\epsilon}^p \mathbf{M}$, due to which the related $SO(2) \times SU(N)$ symmetry would be spontaneously violated and corresponding zero modes in terms of the new fields $\boldsymbol{s}_\alpha^p$ could be consequently produced (indeed, they never appear in the theory). Eventually, for an appropriate choice of the Higgs field–like function $\boldsymbol{f}(x)$ in (116),

$$\boldsymbol{f} = -\mathbf{M} + \sqrt{\mathbf{M}^2 - \boldsymbol{s}_\alpha^p \boldsymbol{s}_\alpha^p}, \tag{117}$$

we come in (112) to the condition

$$\boldsymbol{A}_\mu^p \boldsymbol{A}^{p\mu} = \mathbf{M}^2, \tag{118}$$

conforming with a general non-Abelian vector field constraint (23) established above in Sec. III C. As the vector field constraint (88) for the $U(1)$ symmetry case, this constraint also leads exclusively to the timelike SLIV. Again, one can calculate the gauge function $\boldsymbol{\varphi}^p(x)$ by comparing the relation between the old and new vector fields in (114) with a conventional SLIV parametrization for non-Abelian vector fields (30),

$$\boldsymbol{\varphi}^p = \boldsymbol{\epsilon}^p \int^x d(n_\mu x^\mu)\sqrt{\mathbf{M}^2 - \boldsymbol{n}^2 \boldsymbol{a}^2}, \tag{119}$$

expressing it through the Goldstone and pseudo-Goldstone modes $\boldsymbol{a}_\mu^p$ involved ($\boldsymbol{a}^2 \equiv \boldsymbol{a}_\mu^p \boldsymbol{a}^{p\mu}$).

Remarkably, thanks to the generic high symmetry of the constraint (118), one can apply the emergence conjecture with dynamically produced massless gauge modes to any non-Abelian internal symmetry case as well, though SLIV itself could produce only one zero vector mode. The point is, as was presented in significant detail in Sec. III C, that although we only propose Lorentz invariance $SO(1,3)$ and internal symmetry $U(1) \times SU(N)$ of the Lagrangian $\mathfrak{L}^{\mathrm{em}}$ (109), the emerged constraint (118) in fact possesses a much higher accidental symmetry $SO(\Upsilon, 3\Upsilon)$ determined by the dimension $\Upsilon = N^2 - 1$ of the $SU(N)$ adjoint representation to which the vector fields $\boldsymbol{A}_\mu^p$ belong.[22] This symmetry is indeed spontaneously broken at a scale $\mathbf{M}$, leading exclusively to the timelike SLIV case (28), as is

[22] Actually, the total symmetry is even higher if one keeps in mind both constraints (10) and (118) put on the vector fields A_μ and $\boldsymbol{A}_\mu^a$, respectively. As long as they are independent, the related total symmetry is in fact $SO(1,3) \times SO(\Upsilon, 3\Upsilon)$ until it starts breaking.

determined by the positive sign in the SUSY SLIV constraint (118). The emerging pseudo-Goldstone vector bosons, as was thoroughly explained in Sec. III C, may be in fact considered as candidates for non-Abelian gauge fields, which together with the true vector Goldstone boson entirely complete the adjoint multiplet of the internal symmetry group $SU(N)$. Remarkably, they remain strictly massless, protected by the simultaneously generated non-Abelian gauge invariance. When expressed in these zero modes, the theory looks essentially nonlinear and contains many Lorentz- and CPT-violating couplings. However, as in the SUSY QED case, they do not lead to physical SLIV effects which due to simultaneously generated gauge invariance appear to be strictly canceled out.

Finally, it is worth noting that with the parametrizations (12), (30), (86), (116), (117) taken above for Abelian and non-Abelian vector and scalar field components, one comes to the following relations between them:

$$s_\alpha \boldsymbol{s}_{\alpha}{}^i_j + \boldsymbol{\epsilon}^i_j \sqrt{\mathrm{M}^2 - s^2}\sqrt{\mathbf{M}^2 - \boldsymbol{s}^2} + \frac{b_3}{b_2}[(\boldsymbol{s}_\alpha \boldsymbol{s}_\alpha)^i_j - (\boldsymbol{\epsilon\epsilon})^i_j \boldsymbol{s}^2]$$
$$= a_\mu \boldsymbol{a}_{\mu}{}^i_j + \boldsymbol{\epsilon}^i_j \sqrt{\mathrm{M}^2 - a^2}\sqrt{\mathbf{M}^2 - \boldsymbol{a}^2} + \frac{b_3}{b_2}[(\boldsymbol{a}_\mu \boldsymbol{a}^\mu)^i_j - (\boldsymbol{\epsilon\epsilon})^i_j \boldsymbol{a}^2)], \qquad (120)$$

as is determined by the equation system in (107) (with a full contraction of the field indices in $s^2 \equiv s_\alpha s_\alpha$, $\boldsymbol{s}^2 \equiv \boldsymbol{s}^p_\alpha \boldsymbol{s}^p_\alpha$, $a^2 \equiv a_\mu a^\mu$, and $\boldsymbol{a}^2 \equiv \boldsymbol{a}^p_\mu \boldsymbol{a}^{p\mu}$). They allow us to express the auxiliary scalar fields s_α and $\boldsymbol{s}^i_{\alpha j}$ through the vector zero modes a_μ and $\boldsymbol{a}^p_\mu$, thus completely excluding the former from the theory.

D. Some immediate outcomes

Quite remarkably, an obligatory split symmetry form $U(1) \times SU(N)$ [or $U(1) \times G$, in general] of plausible emergent theories which could exist beyond the prototype QED case, leads us to the standard electroweak theory with the $U(1) \times SU(2)$ symmetry as the simplest possibility. The potential of type (91) written for the corresponding superfields requires spontaneous SUSY breaking in the visible sector to avoid the vacuum instability in the theory. Eventually, this requires the SLIV-type constraints to be put on the hypercharge and weak isospin vector fields, respectively,

$$B_\mu B^\mu = \mathrm{M}^2, \qquad \boldsymbol{W}^p_\mu \boldsymbol{W}^{p\mu} = \mathbf{M}^2 \quad (p = 1, 2, 3). \quad (121)$$

These constraints are independent from each other and possess, as was generally argued above, the total symmetry $SO(1,3) \times SO(3,9)$ which is much higher than the actual Lorentz invariance and electroweak $U(1) \times SU(2)$ symmetry in the theory. Thanks to this fact, one Goldstone and three pseudo-Goldstone zero vector modes b_μ and $\boldsymbol{w}^p_\mu$ are generated to eventually complete the gauge multiplet of the Standard Model,

$$B_\mu = b_\mu + n_\mu \sqrt{\mathrm{M}^2 - b_\mu b^\mu}, \qquad n_\mu b_\mu = 0,$$
$$\boldsymbol{W}^p_\mu = \boldsymbol{w}^p_\mu + n_\mu \boldsymbol{\epsilon}^p \sqrt{\mathbf{M}^2 - \boldsymbol{w}^q_\mu \boldsymbol{w}^{q\mu}}, \qquad n_\mu \boldsymbol{w}^{p\mu} = 0, \quad (122)$$

where the unit vectors n_μ and $\boldsymbol{\epsilon}^p$ are defined in accordance with a rectangular unit matrix $\boldsymbol{n}^p_\mu$ taken in the two-vector form (32). The true vector Goldstone boson appears to be some superposition of the zero modes b_μ and $\boldsymbol{w}^3_\mu$. This superposition is in fact determined by the conventional Higgs doublet in the model since just through the Higgs field couplings these modes are only mixed. Thus, when the electroweak symmetry gets spontaneously broken, an accidental degeneracy related to the total symmetry of constraints mentioned above is lifted. As a consequence, the vector pseudo-Goldstones acquire masses and only the photon, being the true vector Goldstone boson in the model, is left massless.[23] In this sense, there is not much difference for the photon in emergent QED and SM: it emerges as a true vector Goldstone boson in both frameworks.

Going beyond the Standard Model, we unavoidably come to the flipped $SU(5)$ GUT [42] as a minimal and in fact distinguished possibility. Indeed, the $U(1)$ symmetry part being mandatory for emergent theories now naturally appears as a linear combination of a conventional electroweak hypercharge and another hypercharge belonging to the standard $SU(5)$. The flipped $SU(5)$ GUT has several advantages over the standard $SU(5)$ one—the doublet-triplet splitting problem is resolved with use of only minimal Higgs representations and protons are naturally long lived, neutrinos are necessarily massive, and supersymmetric hybrid inflation can easily be implemented successfully. Also in string theory, the flipped $SU(5)$ model is of significant interest for a variety of reasons. In essence, the above-mentioned natural solution to the doublet-triplet splitting problem without using large GUT representations is in the remarkable conformity with string theories where such representations are typically unavailable. Also, in weakly coupled heterotic models, the flipped $SU(5)$ allows us to achieve gauge coupling unification at the string scale 10^{17} GeV if some extra vectorlike particles are added. They are normally taken to transform in the 10 and $\overline{10}$ representations, which is easy to engineer in string theory.

So, supersymmetric emergent theories look attractive both theoretically and phenomenologically whether they are considered at low energies in terms of the Standard Model or at very high energies as the flipped $SU(5)$ GUTs inspired by superstrings. However, their most generic manifestations seem to be related to a spontaneous SUSY violation in the visible sector that we discuss in the next section.

[23]More details on how the zero vector modes can acquire masses in both emergent QED and SM can be found in [22,27].

VI. PHENOMENOLOGICAL IMPLICATIONS: PHOTINO AS PSEUDO-GOLDSTINO

Let us now turn to the matter sector described by chiral matter superfields which have not yet been included in both QED and the Standard Model. In their presence, the SUSY breaking in the tree approximation we have used above is in fact phenomenologically ruled out by the well-known supertrace sum rule [40]. In a supersymmetric QED, it looks especially simple:

$$\mathrm{STr}\mathcal{M}^2 \equiv \sum_{\mathrm{J}} (-1)^{2\mathrm{J}}(2\mathrm{J}+1)\mathrm{Tr}(\mathrm{m}_{\mathrm{J}}^2) = 2Tr\mathrm{Q}\langle D\rangle, \quad (123)$$

where $\mathrm{m_J}$ is the mass matrix for spin J fields, Q is the electric charge matrix of the chiral superfields under consideration, and $\langle D\rangle$ is the VEV of the gauge superfield D component. One can easily confirm that for all realistic cases requiring $\mathrm{TrQ} = 0$ to cancel the anomalies related to $U(1)_{em}$, this sum rule leads to some unacceptably light superpartners in the theory.[24]

Usually, the solution to this problem is related to a softly broken SUSY [40] that in our case would be inaccessible. Indeed, inclusion of direct soft mass terms for superpartners in the model would mean that the visible SUSY is explicitly, rather than spontaneously, broken, which would immediately invalidate the whole idea of the emergent nature of QED and SM. Therefore, we need models where SUSY spontaneously breaks, at least partially, in the visible sector as well. Actually, in the presence of a hidden sector, an additional visible SUSY breaking is not forbidden phenomenologically. Below, we will also consider a class of the pure visible SUSY breaking models, where supersymmetry is solely broken at tree level. Since this section is largely concerned with the phenomenological aspects of emergent SUSY theories, it is reasonable to consider them in a context of the entire $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ Standard Model, rather than in the pure QED framework.

A. Two-sector SUSY breaking

According to a conventional two-sector paradigm, supersymmetry breaking entirely occurs in a hidden sector and then this breaking is mediated to the visible sector by some indirect interactions whose nature depends on a particular mediation scenario [40]. An emergent approach for QED and SM advocated here requires some modification of this idea. While a hidden sector is largely responsible for supersymmetry breaking, providing a reliable solution to the problem of superpartner masses in the theory, supersymmetry itself can also be spontaneously broken in the visible sector that ultimately leads to a double spontaneous SUSY breaking pattern. As a result, the simplified picture discussed above in the SUSY QED case (Sec. IV) is properly changed: a strictly massless fermion eigenstate, a true Goldstino ζ_g, should now be some mix of the visible sector photino λ and the hidden sector Goldstino κ',

$$\zeta_g = \frac{\langle D\rangle\lambda + \langle F'\rangle\kappa'}{\sqrt{\langle D\rangle^2 + \langle F'\rangle^2}}, \quad (124)$$

where $\langle D\rangle$ and $\langle F'\rangle$ are the corresponding D- and F-term VEVs in the visible and hidden sectors, respectively (we use the primed letters for the hidden sector entities).[25] We have also proposed that spontaneous SUSY breaking in the hidden sector goes basically through the F-term VEVs and, in addition, we neglected possible mixing in (124) with other neutralinos in both visible and hidden sectors. So, the orthogonal combination of these states, which may be referred to as a pseudo-Goldstino, is

$$\zeta_{pg} = \frac{\langle F'\rangle\lambda - \langle D\rangle\kappa'}{\sqrt{\langle D\rangle^2 + \langle F'\rangle^2}}. \quad (125)$$

In the supergravity context, a true Goldstino ζ_g is eaten through the super-Higgs mechanism to form the longitudinal component of a massive gravitino ζ_G, while a pseudo-Goldstino ζ_{pg} gets some mass whose value depends on the particular mediation scenario taken. However, in any case, due to large soft masses that are required to be mediated, one may generally expect that SUSY is broken more strongly in the hidden sector than in the visible one, $\langle F'\rangle \gg \langle D\rangle$. This means in turn that the pseudo-Goldstino (125) is largely given by the pure photino state,

$$\zeta_{pg} \simeq \lambda. \quad (126)$$

These pseudo-Goldstone photinos seem to be of special observational interest in the model that, apart from some indication of the SM emergent nature, may shed light on SUSY breaking physics. The possibility that the supersymmetric SM visible sector might also spontaneously break SUSY, thus giving rise to some pseudo-Goldstino state was also considered, though in a different context, in [43,44]. Though this idea may be implemented in supersymmetric QED or SM with practically any hidden sector SUSY breaking scenario, we choose the gauge-mediated scheme. This scenario allows for a natural suppression of flavor violations in the supersymmetric sector [40] and has very distinctive phenomenological features.

Let us note first of all that our polynomially extended QED and SM Lagrangians (48) and (91) are not only SUSY

[24] Even worse, because particles with different electric charges cannot mix, the supertrace (123) vanishes separately in each charge sector, thus leading to light sparticles for all types of charges individually.

[25] Note that what we call photino in QED is the linear combination of bino and neutral wino in the SM framework. Thus, the term photino means hereafter the "photino content" of the neutralino states involved, rather than the pure photino state.

invariant but also generically possesses continuous R symmetry $U(1)_R$ [40]. Indeed, vector superfields always have zero R charge, since they are real. Accordingly, it follows that the physical components in the constrained vector superfield $\hat{V}$ (77) transform as

$$A_\mu \to A_\mu, \qquad \lambda \to e^{i\alpha}\lambda, \qquad D \to D, \tag{127}$$

and, therefore, they have R charges 0, 1, and 0, respectively. Along with that, we assume a suitable R-symmetric matter superfield setup as well, making a proper R-charge assignment for the basic fermions and scalars (and messenger fields) involved. This will lead to the light pseudo-Goldstone matter in the gauge-mediated scenario [43,44]. Normally, if the visible sector possesses the R symmetry which is preserved in the course of the mediation, then the masslessness of a photino (a gaugino, in general) is protected up to the supergravity effects which violate R symmetry.[26] As a result, our pseudo-Goldstino will acquire the mass being proportional to the gravitino mass. The latter can be typically estimated as

$$\mathrm{m}_{3/2} \simeq \langle F' \rangle / \mathrm{M}_P \tag{128}$$

(where we omitted the negligible D-term VEV contribution from the visible sector) that simply follows from dimensional analysis, since this mass must vanish in the limits when supersymmetry is restored ($\langle F' \rangle \to 0$) and when gravity is turned off ($\mathrm{M}_P \to \infty$). Once the gravitino mass is fixed by the properly chosen scale $\langle F' \rangle$ of the hidden sector SUSY breaking, it is straightforward to calculate the supergravity contribution to the pseudo-Goldstino mass (see [44] and references therein). It appears that in theories with both F-term and D-term visible sector breakings, the pseudo-Goldstino acquires a mass which is always lighter (much lighter in most of the parameter space) than twice the gravitino mass, $\mathrm{m}_{pg} < 2\mathrm{m}_{3/2}$. This means that the pseudo-Goldstino ζ_{pg}, being practically the visible sector photino λ (126), is in fact the lightest supersymmetric particle (LSP) in the model considered. Taking the mass $\mathrm{m}_{3/2}$ to be much smaller than the weak scale, say being of the keV order or less, one naturally comes to a possible solution for both gravitino and pseudo-Goldstino overproduction problems in the early Universe [44].

Apart from cosmological problems, many other sides of new physics related to pseudo-Goldstinos appearing through the multiple SUSY breaking were also studied recently (see [43,44,46] and references therein). The point is, however, that nonvanishing F terms have been used exclusively as the only mechanism of visible sector SUSY breaking.[27] In this connection, our pseudo-Goldstone photinos caused by a nonvanishing D term in the visible sector SUSY may lead to somewhat different observational consequences.

One interesting difference concerns the R-symmetry role in these approaches, though they both may typically start with an R-invariant setup, as we discussed above. However, for an appreciable R-symmetry violation due to the SUSY breaking mediation, one would come to dramatic consequences in the F-term visible sector SUSY breaking case that are basically determined by the superpotential mentioned above.[27] The reason is that even after coupling of the visible sector to a hidden source of SUSY breaking, a light pseudo-Goldstino persists as a remnant of the original visible SUSY breaking dynamics [44]. Its tree-level mass is suppressed because it is only induced by small mixings with gauginos, while at one loop its mass is still protected by the visible sector R symmetry. Actually, though R-violating mediation causes in general some rise of the pseudo-Goldstino mass, it is always one loop factor suppressed relative to the weak scale and typically located in the cosmologically dangerous range $O(10\ \mathrm{MeV}\text{–}1\ \mathrm{GeV})$. As to interactions, the pseudo-Goldstino inherits rather small couplings to supersymmetric SM fields through the mixing with gauginos and Higgsinos that determines its lifetime, which is typically longer than a second, the time at which big bang nucleosynthesis begins. As a result, one is unavoidably led to the conclusion that the visible sector pseudo-Goldstino is generically overproduced in the early Universe, unless R symmetry remains. In contrast, in the

[26]Note that Majorana masses for gauginos always break a continuous R symmetry, as is clearly seen from transformations (127). For R invariance, one might properly extend a field content in the theory so as to achieve Dirac gaugino masses (that is not yet assumed in our case). Remarkably, the properly arranged R symmetry in the theory supplemented by additional matter and Higgs chiral supermultiplets may lead to a very efficient suppression of flavor-changing effects [45].

[27]We briefly consider below this case to make clear a significant difference between the F-term visible sector SUSY breaking with our D-term breaking (see below). In the framework of supersymmetric SM, some minimal setup [43] of the visible sector F-term SUSY breaking includes, together with ordinary Yukawa interactions for quarks and leptons, a simple O'Raifeartaigh-type superpotential. So, the total superpotential is

$$W = W_{\mathrm{Yuk}} + fX(H_uH_d - \eta) + \mu_u H_u R_u + \mu_d H_d R_d,$$

where, apart from the standard Higgs doublets $H_{u,d}$, the new Higgs doublets $R_{u,d}$ appear and also, like the next-to-minimal supersymmetric SM, there is a gauge singlet field X (f, η, $\mu_{u,d}$ stand for some coupling constants and mass parameters). This superpotential possesses R symmetry with R charges 0, 1, and 2 for standard Higgs doublets $H_{u,d}$, quarks and leptons (Q, U^c, D^c, L, E^c) and extra superfields (R_u, R_d, X), respectively. Remarkably, in the absence of gauge interactions, this superpotential on its own is an example of a Wess-Zumino model having, as argued in [47], the persistent zero mode which remains for the arbitrary scalar field configurations that emerge. In the entire framework of supersymmetric SM with a hidden sector included, this mode appears as a massless (at tree level) pseudo-Goldstino mode that can be cosmologically safe or dangerous depending on whether R symmetry is exact or appreciably broken.

D-term visible sector SUSY breaking case, nothing dramatic would happen if R symmetry were really violated in the course of the mediation. Depending on the particular type of this violation, the pseudo-Goldstino, which now is essentially the visible sector photino λ (being properly mixed with other neutralinos), could in principle become the next-to-lightest supersymmetric particle (NLSP), which then decays into a gravitino and photon (see the next section).

Another, and more tangible, difference belongs to the Higgs boson decays in the supersymmetric SM framework. For the light pseudo-Goldstino and gravitino, these decays are appreciably modified. Actually, for the F-term visible sector SUSY breaking,[27] the dominant channel becomes [43,44] a conversion of the Higgs boson (say, the lighter CP-even Higgs boson h^0) into a conjugated pair of the corresponding pseudo-Sgoldstinos ϕ_{pg} and $\bar{\phi}_{pg}$,

$$h^0 \to \phi_{pg} + \bar{\phi}_{pg} \tag{129}$$

being superpartners of pseudo-Goldstinos ζ_{pg} and $\bar{\zeta}_{pg}$, respectively. If this decay is kinematically allowed, one may conclude that the Higgs boson could dominantly decay invisibly. By contrast, for the D-term SUSY breaking case considered here, the roles of a pseudo-Goldstino and a pseudo-Sgoldstino are just played by a photino and a photon, respectively, that could make the standard two-photon decay channel of the Higgs boson be even somewhat enhanced. In light of the recent discovery of a Higgs-like state [48] just through its visible decay modes, the F-term SUSY breaking in the visible sector seems to be disfavored by data, while D-term SUSY breaking is not yet in trouble with them.

B. Pure visible sector SUSY breaking scenario

Let us consider now the pure visible sector SUSY breaking models which, contrary to conventional lore, can also be constructed (see [49] and references therein). They appear to include some relatively low-scale extra hypercharge $U(1)_{\mathrm{Y}'}$ gauge symmetry which, when properly assigned to quarks and leptons and their superpartners, allows us to construct some phenomenologically viable supersymmetric SM extensions. So, for the tree-level supertrace equation (123) one has on its right-hand side

$$\mathrm{STr}\mathcal{M}^2 = 2[\mathrm{g_Y}\mathrm{Tr}(\mathrm{Y})\langle D\rangle + \mathrm{g_{Y'}}\mathrm{Tr}(\mathrm{Y'})\langle D'\rangle], \tag{130}$$

where $\mathrm{g_Y}$ and $\mathrm{g_{Y'}}$ are the corresponding gauge coupling constants. The first term in the bracket related to the standard $U(1)$ hypercharge symmetry will vanish since the quark and lepton representations are chosen to be anomaly free, which leads to the traceless condition $\mathrm{Tr}(\mathrm{Y}) = 0$. However, if in the second term in (130) the D-term VEV $\langle D'\rangle$ is nonvanishing and the trace $\mathrm{Tr}(\mathrm{Y'})$ over quarks and leptons is separately nonzero, as is the case when all quark and lepton superfields (as well as Higgs superfields) are given Y′ hypercharges of the same sign,[28] then all the sparticles can receive large masses. Normally, the extra $U(1)_{\mathrm{Y}'}$ hypercharge gauge symmetry is broken at tree level and the corresponding gauge boson Z' acquires a mass. Its lower bound has been recently pushed up to $\mathrm{M}_{Z'} > 2.33$ TeV at LHC [50]. In general, the Z' boson is mixed with an ordinary Z boson of the SM. As of now, for the $\mathrm{M}_{Z'}$ bound value mentioned, their mixing angle appears well below its experimental upper limit [50].

Generally, such models [49] are indeed rather complicated. They, apart from gauge and matter superfields of the conventional minimal supersymmetric Standard Model (MSSM), contain several exotic chiral superfields with SM quantum numbers: an $SU(3)_{\mathrm{C}}$ octet superfield, an $SU(2)_{\mathrm{L}}$ triplet superfield, two vectorlike pairs of the $U(1)_{\mathrm{Y}}$ hypercharged superfields, and several MSSM singlet fields only charged under $U(1)_{\mathrm{Y}'}$. These fields are introduced to cancel all the anomalies related to $SU(3)^2_{\mathrm{C}}U(1)_{\mathrm{Y}'}$, $SU(2)^2_{\mathrm{L}}U(1)_{\mathrm{Y}'}$, $U(1)^2_{\mathrm{Y}}U(1)_{\mathrm{Y}'}$, and others. Supersymmetry is spontaneously broken at tree level by Fayet-Iliopoulos terms for both $U(1)$ and $U(1)_{\mathrm{Y}'}$ hypercharges leading to the D- and D'-term VEVs shown above in the supertrace equation (130). Apart from that, a special O'Raifeartaigh-type superpotential is introduced to break SUSY and the $U(1)_{\mathrm{Y}'}$ spontaneously at tree level by generating the proper F-term VEVs (referred to as the F'-term VEVs for what follows). Due to this F'-term breaking, all of the MSSM matter superpartners (squarks and sleptons) and gauginos receive soft-breaking diagonal masses,

$$\begin{aligned} \mathrm{m}^2_{sq/sl} &\simeq \mathrm{g}^2_{\mathrm{Y}'}\langle D'\rangle^2 + (\Delta\mathrm{m})^2_{1-\mathrm{loop}}, \\ \mathrm{M}_{\mathrm{gaugino}} &\simeq (\Delta\mathrm{M})_{1-\mathrm{loop}}, \end{aligned} \tag{131}$$

at tree level and one loop, respectively. Remarkably, not only the universal tree-level SUSY breaking contribution related to the extra $U(1)_{\mathrm{Y}'}$ symmetry but also all radiative corrections implied in (131) turn out to be "flavor blind." Actually, spontaneous SUSY breaking caused by a generic $U(1)_{\mathrm{Y}'}$ symmetry mechanism is transmitted to superparticles according to some gauge-mediated–like scenario with the $\mathrm{SM}\times U(1)_{\mathrm{Y}'}$ gauge bosons playing the role of messenger fields.

In order to generate one-loop gaugino masses in (131) which are large enough to satisfy current experimental bounds (e.g., $\mathrm{m}_{\tilde{g}} > 800$ GeV [50] for the gluino mass), the heavy sector F'- and D'-term VEVs must be of order $(30\ \mathrm{TeV})^2$. This is in fact a single input scale in the theory. Note that due to the same sign Y′ hypercharges assigned to

[28]The simplest choice would be to assign positive Y' hypercharges ($Y' = +1$) to all quark and lepton superfields and negative Y' hypercharges ($Y' = -2$) to the Higgs superfields $H_{u,d}$ (for some earlier discussions, see the first paper in [40] and references therein).

all the quarks and leptons, the bare μ term is forbidden in the theory, but an effective μ term is generated once the $U(1)_{Y'}$ symmetry is spontaneously broken at the input scale mentioned. To obtain the proper electroweak scale, one has to require a single tree-level tuning of the Fayet-Iliopoulos parameters in $U(1)_Y$ and $U(1)_{Y'}$ sectors. However, it is not a fine-tuning in the ordinary sense, since radiative corrections to the Higgs boson masses are appreciably suppressed in the theory. Thus, these masses naturally remain the tree-level order values which are chosen to be of the electroweak-scale order. As some immediate outcome, the theory predicts relatively light gauginos and quite heavy squarks and sleptons with masses around 7–8 TeV for the input scale indicated. Such heavy squarks and sleptons may not be easily observable at the LHC in the foreseen future. One of the most attractive features of the theory is, as mentioned above, that flavor changing processes are naturally suppressed, similar to those in gauge-mediated SUSY theories. For more details on this class of models, we refer the reader to the original paper [49] and only consider here some of their generic predictions concerning the Goldstino phenomenology.

Indeed, all models of low-energy supersymmetry breaking predict that the gravitino may be the LSP, as is determined in the entire supergravity framework where the gravitino acquires a mass by eating the Goldstino through the super-Higgs mechanism. This Goldstino in the model considered is mostly made of heavy sector fields. This is in fact a combination of the respective $U(1)_{Y'}$ gaugino and chiral fermions underlying the abovementioned O'Raifeartaigh-type superpotential which breaks SUSY and the $U(1)_{Y'}$ at tree level. In addition, it also may have some small Higgsino content, which might be relevant for a subsequent gravitino phenomenology. The mass of the gravitino can be estimated this time as (the standard F- and D-term VEV contributions are neglected)

$$\mathrm{m}_{3/2} \simeq \frac{\sqrt{\langle F' \rangle^2 + \langle D' \rangle^2}}{\mathrm{M}_P}, \tag{132}$$

where the relatively low F'- and D'-term VEVs mentioned above give for its value $\mathrm{m}_{3/2} \sim 0.07$ eV, which is definitely safe for cosmology [40]. The gravitino, by absorbing the Goldstino, inherits its nongravitational interactions and so can play an important role in collider physics.

The generic interactions of the Goldstino ζ_g (being the longitudinal part of a massive gravitino ζ_G) follow, as usual [40], from the total supercurrent conservation that determines its effective low-energy Lagrangian as

$$L_{\mathrm{eff}} = -i\bar{\zeta}_g \bar{\sigma}^\mu \partial_\mu \zeta_g - \frac{1}{\sqrt{\langle F' \rangle^2 + \langle D' \rangle^2}} (\zeta_g \partial_\mu j^\mu + \mathrm{H.c.}), \tag{133}$$

where j^μ is the supercurrent which includes contributions from all matter and gauge supermultiplets involved. As a consequence, one has the basic Goldstino-scalar-chiral fermion vertex,

$$\zeta_g^w \partial_\mu (\sigma^\nu \bar{\sigma}^\mu \psi_i)_w \partial_\nu \varphi^{*i}, \tag{134}$$

and Goldstino-gaugino-gauge boson vertex,

$$-\zeta_g^w \partial_\mu [(\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \bar{\lambda}^p)_w F^p_{\nu\rho}]/2\sqrt{2}, \tag{135}$$

in the theory (here w stands for a spinor index, while indices i and p belong to the SM group representations for matter and gauge supermultiplets, respectively). Since this derivation depends only on the total supercurrent conservation, the Lagrangian (133) holds independently of the details of supersymmetry breaking. It universally determines the decay rate of any sparticle $\tilde{X}$ into its superpartner X plus the Goldstino/gravitino (ζ_g/ζ_G) whether (X, $\tilde{X}$) is a chiral superfield pair (φ, ψ) or a vector superfield pair (A_μ, λ), respectively.

Remarkably, an orthogonal combination to the Goldstino ζ_g, namely, the pseudo-Goldstino ζ_{pg}, happens to be mostly a bino,[29] or a photino (126) if we turn to a pure QED framework. In the SM context, this bino is a NLSP having the electroweak-scale order mass. As a consequence, the photino being the linear combination of the bino and the neutral wino will dominantly decay into the photon and the gravitino with a decay rate entirely determined by the interaction vertex (135):

$$\Gamma(\tilde{\gamma} \to \gamma + \zeta_G) \simeq \frac{\mathrm{m}^5_{\tilde{\gamma}} k_{\tilde{\gamma}}}{16\pi(\langle F' \rangle^2 + \langle D' \rangle^2)}, \tag{136}$$

where $k_{\tilde{\gamma}}$ is the pure photino content of the pseudo-Goldstino ζ_{pg} in the supersymmetric SM. For typical values $k_{\tilde{\gamma}} \sim 0.15$, $m_{\tilde{\gamma}} \sim 100$ GeV in the model, and the heavy sector VEVs $\langle F' \rangle \sim \langle D' \rangle \sim 30$ TeV taken above, one has for the photino lifetime $\tau_{\tilde{\gamma}} \sim 2 \times 10^{-15}$ s that could make its mean decay length reach up to 0.5 μm under LHC energies.

To summarize, the emergent Standard Models with spontaneous SUSY breaking, which only occurs in the visible sector, seem not to violate any current phenomenological constraint. In general, these models predict light gauginos and quite heavy squarks and sleptons which may not be observable at the LHC. The LSP is a stable very light

[29] For a typical range of parameters in the model considered in [49], this pseudo-Goldstino has a content

$$\zeta_{pg} = -0.9999\tilde{B} - 0.003\tilde{W}^0 - 0.002\tilde{H}^0_u + 0.004\tilde{H}^0_d,$$

including, apart from the bino, the vanishingly small admixtures of the wino $\tilde{W}^0$ and the Higgsinos $\tilde{H}^0_{u,d}$.

gravitino with a significant Higgsino admixture, while the NLSP is mostly a bino. Apart from that, it is worth noticing some other advantages of these low-scale models thoroughly described in [49]. Proton decay is sufficiently and naturally suppressed, even for a rather low cutoff scale about 10^8 GeV. The strong *CP* problem is naturally solved through the Nelson-Barr mechanism [51]. In addition, an introduction of the extra $U(1)_{Y'}$ helps to sufficiently suppress the B- and L-violating interactions. An interesting generic cold dark matter candidate is also found. This is the lightest particle among several SM singlet fields introduced in the theory heavy sector to cancel all possible anomalies related to the $U(1)_{Y'}$ symmetry. Although it typically has the TeV-scale order mass, it appears absolutely stable due to some surviving discrete symmetry of the appropriate O'Raifeartaigh superpotential taken.

VII. SUMMARY AND CONCLUSIONS

As we argued above, spontaneous Lorentz violation in a vector field theory framework may be active, as in the composite and potential-based models, leading to physical Lorentz violation, or inactive, as in the constraint-based models, resulting in the nonlinear gauge choice in an otherwise Lorentz invariant theory. Remarkably, between these two basic SLIV versions, SUSY unambiguously chooses the inactive SLIV case. Indeed, SUSY theories only admit the bilinear mass term in the vector field potential energy. As a result, without stabilizing quartic vector field terms, the physical spontaneous Lorentz violation never occurs in SUSY theories. Hence, it follows that the composite and potential-based SLIV models can in no way be realized in the SUSY context. This may have far-reaching consequences in that supergravity and superstring theories could also disfavor such models in general.

Nevertheless, even in the case when SLIV is not physical it inevitably leads to the generation of massless photons as vector NG bosons provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance. To see how this idea might work we considered a supersymmetric QED model extended by an arbitrary polynomial potential of a general vector superfield that induces spontaneous SUSY violation in the visible sector, and gauge invariance gets broken as well. Notably, massless photons at this point are related to spontaneously broken supersymmetry (SBS) itself rather than gauge invariance. Actually, SBS only provides the tree-level masslessness of a photon (as a photino companion) but cannot protect it against radiative corrections since its generic massless mode is only a photino rather than a whole gauge supermultiplet. Nevertheless, though gauge invariance is explicitly broken by the superfield potential, the special gauge invariance is in fact recovered in the broken SUSY phase that universally protects the photon masslessness. This invariance is only restricted by the nonlinear gauge condition (73) put on the vector field. The point, however, is that this length-fixing gauge condition happens at the same time to be the SLIV-type constraint which treats in turn the physical photon as the Lorentzian NG mode. So, figuratively speaking, the photon passes through three evolution stages, being initially the massive vector field component of a general vector superfield (51), then the tree-level massless companion of the Goldstone photino in the broken SUSY stage (55), and finally the generically massless state as the emergent Lorentzian NG mode in the inactive SLIV stage (73).

All basic arguments developed in SUSY QED were then generalized to Standard Model and grand unified theories. Remarkably, thanks to a generic high symmetry of the length-fixing SLIV constraint put on the vector fields, the emergence conjecture with dynamically produced massless gauge modes can be applied to any non-Abelian internal symmetry case. Specifically, one can argue that in a theory with an internal symmetry group G, not only the pure Lorentz symmetry $SO(1,3)$, but also the larger accidental symmetry $SO(\Upsilon, 3\Upsilon)$ of the SLIV constraint (118) in itself appears to be spontaneously broken (Υ is a dimension of the group G). As a result, although the pure Lorentz violation on its own still generates only one genuine Goldstone vector boson, the accompanying pseudo-Goldstone vector bosons related to the $SO(\Upsilon, 3\Upsilon)$ breaking also come into play, properly completing the whole gauge multiplet of the internal symmetry group G taken. Remarkably, they appear to be strictly massless as well, being protected by the simultaneously generated non-Abelian gauge invariance. For definiteness, we focused on the $U(1)\times SU(N)$ symmetrical theories. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case one only has the SUSY invariant phase in the theory that would make it inappropriate for an outgrowth of an emergence process. As briefly discussed, supersymmetric emergent theories look attractive both theoretically and phenomenologically whether they are considered at low energies in terms of the Standard Model or at very high energies as the flipped $SU(5)$ GUTs inspired by superstrings.

However, their most generic manifestations seem to be related to a spontaneous SUSY violation in the visible sector that we finally considered. The photino emerging due to this violation will be then mixed with another Goldstino which stems from a spontaneous SUSY violation in the hidden sector. Eventually, it essentially turns into a light pseudo-Goldstino whose physics seems to be of special interest. Such a pseudo-Goldstone photino appears typically as the eV-scale stable LSP or the electroweak-scale long-lived NLSP, being accompanied by a very light gravitino in both cases, that can be considered as some observational signature of the class of models where SUSY breaks, at least partially, in the visible sector as well. This is the only class of models where emergent supersymmetric QED or the Standard Model can be successfully realized. So, in contrast to non-SUSY analogs, the emergent SUSY theories even with the Lorentz-preserving inactive SLIV could naturally have some clear observational signal. Its validation, apart from some indication of an emergent nature of gauge symmetries, could shed considerable light on the SUSY breaking physics that has been actively studied in recent years.

We conclude with a general remark that supersymmetry with its well-known advantages, such as naturalness, grand unification, and dark matter candidate, seems to possess one more attractive feature: it may trigger, through its own spontaneous violation, a dynamical generation of massless gauge fields as massless NG modes during which physical Lorentz invariance itself is generically preserved. An extension of this idea to the local supersymmetry case, which could presumably underlie an emergent supergravity theory unifying all elementary forces, seems to be especially interesting and worth pursuing.

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[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); J. Goldstone, Nuovo Cimento **19**, 154 (1961).
[2] J. D. Bjorken, Ann. Phys. (N.Y.) **24**, 174 (1963); I. Bialynicki-Birula, Phys. Rev. **130**, 465 (1963); G. Guralnik, Phys. Rev. **136**, B1404 (1964).
[3] P. R. Phillips, Phys. Rev. **146**, 966 (1966); H. C. Ohanian, Phys. Rev. **184**, 1305 (1969); D. Atkatz, Phys. Rev. D **17**, 1972 (1978).
[4] T. Eguchi, Phys. Rev. D **14**, 2755 (1976); H. Terazava, Y. Chikashige, and K. Akama, Phys. Rev. D **15**, 480 (1977); M. Suzuki, Phys. Rev. D **37**, 210 (1988).
[5] J. D. Bjorken, arXiv:hep-th/0111196.
[6] P. Kraus and E. T. Tomboulis, Phys. Rev. D **66**, 045015 (2002).
[7] A. Jenkins, Phys. Rev. D **69**, 105007 (2004).
[8] V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989).
[9] V. A. Kostelecky, Phys. Rev. D **69**, 105009 (2004); R. Bluhm and V. A. Kostelecky, Phys. Rev. D **71**, 065008 (2005).
[10] Y. Nambu, Prog. Theor. Phys. Suppl. Extra, 190 (1968).
[11] A. T. Azatov and J. L. Chkareuli, Phys. Rev. D **73**, 065026 (2006).
[12] J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, Nucl. Phys. **B821**, 65 (2009).
[13] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
[14] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998); V. A. Kostelecky and R. Lehnert, Phys. Rev. D **63**, 065008 (2001).
[15] M. S. Berger and V. A. Kostelecky, Phys. Rev. D **65**, 091701 (2002).
[16] S. Groot Nibbelink and M. Pospelov, Phys. Rev. Lett. **94**, 081601 (2005).
[17] P. A. Bolokhov, S. G. Nibbelink, and M. Pospelov, Phys. Rev. D **72**, 015013 (2005).
[18] J. L. Chkareuli, Phys. Lett. B **721**, 146 (2013); Eur. Phys. J. C **74**, 2906 (2014).
[19] R. Bluhm, N. L. Gagne, R. Potting, and A. Vrublevskis, Phys. Rev. D **77**, 125007 (2008).
[20] A. Hashimoto, J. High Energy Phys. 08 (2008) 040.
[21] R. Righi and G. Venturi, Lett. Nuovo Cimento **19**, 633 (1977); R. Righi, G. Venturi, and V. Zamiralov, Nuovo Cimento Soc. Ital. Fis. **47A**, 518 (1978).
[22] J. L. Chkareuli and Z. R. Kepuladze, Phys. Lett. B **644**, 212 (2007).
[23] J. L. Chkareuli and J. G. Jejelava, Phys. Lett. B **659**, 754 (2008).
[24] O. J. Franca, R. Montemayor, and L. F. Urrutia, Phys. Rev. D **85**, 085008 (2012).
[25] J. L. Chkareuli, J. G. Jejelava, and G. Tatishvili, Phys. Lett. B **696**, 124 (2011).
[26] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 2000), Vol. 2.
[27] J. L. Chkareuli, Z. Kepuladze, and G. Tatishvili, Eur. Phys. J. C **55**, 309 (2008); J. L. Chkareuli and Z. Kepuladze, Eur. Phys. J. C **72**, 1954 (2012).
[28] J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, Phys. Rev. Lett. **87**, 091601 (2001); J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, Nucl. Phys. **B609**, 46 (2001).
[29] S. Chadha and H. B. Nielsen, Nucl. Phys. **B217**, 125 (1983).
[30] S. Coleman and S. L. Glashow, Phys. Lett. B **405**, 249 (1997); Phys. Rev. D **59**, 116008 (1999).
[31] T. Jacobson and D. Mattingly, Phys. Rev. D **64**, 024028 (2001).
[32] D. Colladay and P. McDonald, arXiv:1008.1279.
[33] O. Pujolas and S. Sibiryakov, J. High Energy Phys. 01 (2012) 062.
[34] J. L. Chkareuli, C. D. Froggatt, J. G. Jejelava, and H. B. Nielsen, Nucl. Phys. **B796**, 211 (2008).
[35] J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, Nucl. Phys. **B848**, 498 (2011).

[36] J. L. Chkareuli, in *Low Dimensional Physics and Gauge Principles* (World Scientific, Singapore, 2011), pp. 80–92.
[37] V. I. Ogievetsky and I. V. Polubarinov, Ann. Phys. (N.Y.) **25**, 358 (1963); Nucl. Phys. **76**, 677 (1966).
[38] S. Weinberg, Phys. Rev. **138**, B988 (1965).
[39] E. Noether, Nachr. K. Ges. Wiss. Göttingen, Math.-Phys. Kl. **2**, 235 (1918); translated in arXiv:physics/0503066.
[40] H. P. Nilles, Phys. Rep. **110**, 1 (1984); S. P. Martin, arXiv:hep-ph/9709356; H. Baer and X. Tata, *Weak Scale Supersymmetry* (Cambridge University Press, Cambridge, England, 2006).
[41] P. Fayet, Nuovo Cimento Soc. Ital. Fis. **31A**, 626 (1976); see also Nucl. Phys. **B90**, 104 (1975); Phys. Lett. **84B**, 416 (1979).
[42] S. M. Barr, Phys. Lett. **112B**, 219 (1982); J. P. Derendinger, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. **139B**, 170 (1984); for recent presentations, see E. Kuflik and J. Marsano, J. High Energy Phys. 03 (2011) 020; J. Jiang, T. Li, and D. V. Nanopoulos, Nucl. Phys. **B772**, 49 (2007).
[43] K. Izawa, Y. Nakai, and T. Shimomura, J. High Energy Phys. 03 (2011) 007.
[44] D. Bertolini, K. Rehermann, and J. Thaler, J. High Energy Phys. 04 (2012) 130.
[45] G. D. Kribs, E. Poppitz, and N. Weiner, Phys. Rev. D **78**, 055010 (2008).
[46] C. Cheung, Y. Nomura, and J. Thaler, J. High Energy Phys. 03 (2010) 073; N. Craig, J. March-Russell, and M. McCullough, J. High Energy Phys. 10 (2010) 095; H.-C. Cheng, W.-C. Huang, I. Low, and A. Menon, J. High Energy Phys. 03 (2011) 019; R. Argurio, Z. Komargodski, and A. Mariotti, Phys. Rev. Lett. **107**, 061601 (2011).
[47] M. Visser, J. Phys. A **18**, L979 (1985).
[48] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).
[49] P. Kumar and J. D. Lykken, J. High Energy Phys. 07 (2004) 001.
[50] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
[51] A. Nelson, Phys. Lett. **136B**, 387 (1984); S. M. Barr, Phys. Rev. D **30**, 1805 (1984).