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Accretion onto a black hole in a string cloud background

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We examine the accretion process onto the black hole with a string cloud background, where the horizon of the black hole has an enlarged radius $r_H = 2M/(1-\alpha)$, due to the string cloud parameter $\alpha\,(0 \le \alpha < 1)$. The problem of stationary, spherically symmetric accretion of a polytropic fluid is analyzed to obtain an analytic solution for such a perturbation. Generalized expressions for the accretion rate $\dot{M}$, critical radius r_s, and other flow parameters are found. The accretion rate $\dot{M}$ is an explicit function of the black hole mass M, as well as the gas boundary conditions and the string cloud parameter α. We also find the gas compression ratios and temperature profiles below the accretion radius and at the event horizon. It is shown that the mass accretion rate, for both the relativistic and the nonrelativistic fluid by a black hole in the string cloud model, increases with increase in α.

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I. INTRODUCTION

Black holes are amongst the most striking predictions of Einstein's theory of general relativity. One of the most important effects of the black hole is its tendency to accrete, and hence several aspects of the spherical accretion onto the black hole have been actively investigated in detail over the past four decades (see [1] for a review). Accretion is the term used by astrophysicists to describe the inflow of matter towards a central gravitating object or towards the center of the mass of an extended system. It may be pointed out that accretion of matter onto black holes is one of the most promising ideas explaining the highly luminous active galactic nuclei and quasars. The first study of spherical accretion onto compact objects dates back more than 40 years in the seminal paper due to Bondi [2]. In this classic work, the hydrodynamics of polytropic flow is studied within the Newtonian framework, and it is found that either a settling or transonic solution is mathematically possible for the gas accreting onto compact objects. Note that the accretion rate is highest for the transonic solution. The relativistic version of the same problem was solved by Michel [3] 20 years later. Michel [3] investigated the steady state spherically symmetric flow of a test gas onto a Schwarzschild black hole in the framework of general relativity. He showed that accretion onto the black hole should be transonic. Michel's relativistic results attracted several researchers [4,5]. Spherical accretion and winds in the context of general relativity have also been analyzed using equations of state other than the polytrope. Other extensive studies include the calculation of the frequency and luminosity spectra [6], the influence of an interstellar magnetic field in ionized gases [7], and the changes in accreting processes when the black hole rotates [8]. Several radiative processes have been included by Blumenthal and Mathews [9], and Brinkmann [10]. In addition Malec [11] considered general relativistic spherical accretion with and without backreaction, and showed that relativistic effects increase mass accretion when backreaction is absent. Accretion of a perfect fluid with a general equation of state onto a Schwarzschild black hole has been investigated in [12,13], and a similar analysis for a charged black hole has been done in [14]. Accretion processes related to a charged black hole were analyzed in [3] and investigated further in [15–18]. The main aim of these studies is to obtain the net energy output emitted by infalling gas with application of black hole accretion to several classes of astrophysical sources. It is understood that accretion onto a black hole might be an important source of radiant energy. This may be related to the accretion rate $\dot{M}$, and we may expect that an increase in $\dot{M}$ should lead to an increase in the luminosity [6]. In this paper, we consider the steady state spherical accretion onto a black hole that has a cloud of strings in the background. We thereby generalize the previous work of Michel [3]. It may be noted that the study of Einstein's equations coupled with a *string cloud* may be very important as the relativistic strings at a classical level can be used to construct applicable models [19]. Also, the Universe can be represented as a collection of extended (nonpoint) objects and one-dimensional strings are the most popular candidate for such fundamental objects. Hence the study of the gravitational effects of matter, in the form of clouds of both cosmic and fundamental strings, has generated considerable attention [20].

Cosmic strings are a generic outcome of symmetry-breaking phase transitions in the early Universe [21], and

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further motivation comes from a potential role in large scale structure formation [22]. Strings may have been present in the early Universe, and they play a role in the seeding of density inhomogeneities [23]. The magnitude of such strings is determined by the dimensionless parameter

$$\frac{G\mu}{c^2} = \left(\frac{\eta}{m_{Pl}}\right)^2,$$

where η is the energy scale of string and $m_{Pl} = \sqrt{hc/G}$ is the Planck mass. For the Nambu-Goto string model, using the Planck data, it has been shown that a constraint on the string tension of $\frac{G\mu}{c^2} < 1.5 \times 10^{-7}$ at 95 percent confidence that can be improved to $\frac{G\mu}{c^2} < 1.3 \times 10^{-7}$ on inclusion of high-l CMB data [24].

It may be also pointed out that strings have become a very important ingredient in many physical theories, and the idea of strings is fundamental in superstring theories [25]. The apparent relationship between counting string states and the entropy of the black hole horizon [26,27] suggests an association of strings with black holes. Furthermore the intense level of activity in string theory has lead to the idea that many of the classic vacuum scenarios, such as the static Schwarzschild point black hole, may have atmospheres composed of a fluid or field of strings [28]. Many authors have found exact black hole solutions with string cloud backgrounds, for instance, in general relativity [19,29], in Einstein-Gauss-Bonnet models [30], and in Lovelock gravity [31], thereby generalizing the pioneering work of Letelier [19] who modified the Schwarzschild black hole for the string cloud model. Glass and Krisch [32] pointed out that allowing the Schwarzschild mass parameter to be a function of radial position creates an atmosphere with a string fluid stress-energy tensor around a static, spherically symmetric object.

Interestingly, it turns out that the mass accretion rate $\dot{M}$ increases with the string cloud as a background in comparison to the standard black hole. Also note that the mass accretion rate is affected by the presence of higher dimensions [33].

The paper is organized as follows: In Sec. II we review the action, the energy momentum tensor for a cloud of strings, and the corresponding black hole solution. In Sec. III the analytic general relativistic accretion onto a Schwarzschild black hole is appropriately generalized to model spherical steady state accretion onto a black hole surrounded by a cloud of strings. We calculate how the presence of a string cloud would affect the mass accretion rate $\dot{M}$ of a gas onto a black hole. We also determine analytic corrections to the critical radius, the critical fluid velocity and the sound speed, and subsequently to the mass accretion rate. We then obtain expressions for the asymptotic behavior of the fluid density and the temperature near the event horizon in Sec. IV. Finally we conclude in Sec. V.

We use the following values for the physical constants for numerical computations and plots: $c = 3.00 \times 10^{10}$ cm s^{-1}, $G = 6.674 \times 10^{-8}$ cm^3 g^{-1} s^{-2}, $k_B = 1.380 \times 10^{-16}$ erg K^{-1}, $M = M_\odot = 1.989 \times 10^{33}$ g, $m_b = m_p = 1.67 \times 10^{-24}$ g, $n_\infty = 1$ cm^{-3}, $T_\infty = 10^4$ K.

II. SCHWARZSCHILD BLACK HOLE IN A STRING CLOUD BACKGROUND

In this model, we assume a spherically symmetric metric, and steady state flow onto a nonrotating black hole of mass M at rest. Recall that the nonrelativistic model was discussed by Bondi [2], and the standard four-dimensional general relativistic version was developed by Michel [3]. The known analytic relativistic accretion solution onto the Schwarzschild black hole by Michel [3] is generalized by considering a cloud of strings in the background. To achieve this, we first briefly review the theory on a cloud of strings (see [19] for further details) and the corresponding modified Schwarzschild black hole.

The Nambu-Goto action of a string evolving in spacetime is given by

$$I_S = \int_\Sigma \mathcal{L} d\lambda^0 d\lambda^1, \qquad \mathcal{L} = m(\Gamma)^{-1/2},$$

where m is a positive constant that characterizes each string, (λ^0, λ^1) is a parametrization of the world sheet Σ with λ^0 and λ^1 being timelike and spacelike parameters [20], and Γ is the determinant of the induced metric on the string world sheet Σ given by

$$\Gamma_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b}, \tag{1}$$

and $\Gamma = \det \Gamma_{ab}$. Associated with the string world sheet we have the bivector of the form

$$\Sigma^{\mu\nu} = \epsilon^{ab} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b}, \tag{2}$$

where ϵ^{ab} denotes the two-dimensional Levi-Civita tensor given by $\epsilon^{01} = -\epsilon^{10} = 1$. Within this setup, the Lagrangian density becomes

$$\mathcal{L} = m\left[-\frac{1}{2}\Sigma^{\mu\nu}\Sigma_{\mu\nu}\right]^{1/2}.$$

Further, since $T^{\mu\nu} = 2\partial\mathcal{L}/\partial g^{\mu\nu}$, we obtain the energy momentum tensor for one string as

$$T^{\mu\nu} = m\Sigma^{\mu\rho}\Sigma_\rho{}^\nu/(-\Gamma)^{1/2}. \tag{3}$$

Hence, the energy momentum tensor for a cloud of string is

$$T^{\mu\nu} = \rho\Sigma^{\mu\sigma}\Sigma_\sigma{}^\nu/(-\Gamma)^{1/2}, \tag{4}$$

where ρ is the proper density of a string cloud. The quantity $\rho(\Gamma)^{-1/2}$ is the gauge invariant quantity called the gauge-invariant density.

The general solution of Einstein's equations for a string cloud in four dimensions takes the form

$$ds^2 = -\left(1 - \frac{2M}{r} - \alpha\right)dt^2 + \left(1 - \frac{2M}{r} - \alpha\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

where we have set $G = c = 1$ in this paper. Here M arises as an integration constant which is identified as the black hole mass and is not a function of α. The event horizon for the metric (5) has radius

$$r_H = \frac{2M}{1-\alpha}, \qquad \alpha \neq 1. \quad (6)$$

In the limit $\alpha \to 0$, we recover the Schwarzschild radius, and close to unity the event horizon radius tends to infinity. In general the string cloud parameter $\alpha \neq 1$. We note that the case of static spherical symmetry restricts the value of the gauge-invariant density to $\rho(-\Gamma)^{1/2} = \alpha/r^2$ [19], and thereby α is a positive constant. However, for the realistic model under consideration here the string cloud parameter is restricted to $0 < \alpha < 1$. On the other hand, the cloud of strings alone ($M = 0$) does not have a horizon; it generates only a naked singularity at $r = 0$. This solution was first obtained by Letelier [19] and the metric represents the black hole spacetime associated with a spherical mass M centered at the origin of the system of coordinates, surrounded by a spherical cloud of strings. Furthermore it can be interpreted as the metric associated with a global monopole. In the string cloud background, the Schwarzschild radius of the black hole is displaced by the factor $(1-\alpha)^{-1}$.

III. GENERAL EQUATIONS FOR SPHERICAL ACCRETION

We now present the basic relations in spherical symmetry with accreting matter, and describe the flow of gas into the modified Schwarzschild black hole (5). Also we probe how the string cloud background affects the accretion rate $\dot{M}$, the asymptotic compression ratio, and the temperature profiles. We consider the steady state radial inflow of gas onto a central mass M by following the approach of Michel [3] and Shapiro [5]. The gas is approximated as a perfect fluid described by the energy momentum tensor

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p g^{\mu\nu}, \quad (7)$$

where ρ and p are the fluid proper energy density and pressure respectively, and

$$u^\mu = \frac{dx^\mu}{ds} \quad (8)$$

is the fluid 4-velocity which obeys the normalization condition $u^\mu u_\mu = -1$. We also define the proper baryon number density n, and the baryon number flux $J^\mu = n u^\mu$. All these quantities are measured in the local inertial rest frame of the fluid. The spacetime curvature is dominated by the compact object and we ignore the self-gravity of the fluid. The accretion process is based on two important conservation laws. First, if no particles are created or destroyed then particle number is conserved and

$$\nabla_\mu J^\mu = \nabla_\mu(n u^\mu) = 0. \quad (9)$$

Second, the conservation law is that of energy momentum which is governed by

$$\nabla_\mu T^\mu_\nu = 0. \quad (10)$$

The non-null components of the 4-velocity are $u^0 = dt/ds$ and $v(r) = u^1 = dr/ds$. Since $u_\mu u^\mu = -1$, and the velocity components vanish for $\mu > 1$, we have

$$u^0 = \left[\frac{v^2 + 1 - \frac{2M}{r} - \alpha}{(1 - \frac{2M}{r} - \alpha)^2}\right]^{1/2}. \quad (11)$$

Equation (9) can be written as

$$\frac{1}{r^2}\frac{d}{dr}(r^2 n v) = 0. \quad (12)$$

Our assumptions of spherical symmetry and steady state flow make (10) comparatively easier to tackle. The $\nu = 0$ component is

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2(\rho + p)v\left(1 - \frac{2M}{r} - \alpha + v^2\right)^{1/2}\right] = 0. \quad (13)$$

The $\nu = 1$ component can be simplified to

$$v\frac{dv}{dr} = -\frac{dp}{dr}\left(\frac{1 - \frac{2M}{r} - \alpha + v^2}{\rho + p}\right) - \frac{M}{r^2}. \quad (14)$$

The above equations are a generalization of the results obtained for the standard Schwarzschild black hole [3,5].

A. Accretion onto a black hole

The accretion of matter onto black holes remains a classic problem of contemporary astrophysics, as it does on the related problems of active galactic nuclei and quasars, the mechanism of jets, and the nature of certain galactic x-ray source. Let us consider spherical steady state accretion onto a Schwarzschild black hole of mass M in a string cloud background to obtain the mass accretion rate from a qualitative analysis of (12) and (14). For an adiabatic fluid there is no entropy production and the conservation of mass energy is governed by

$$Tds = 0 = d\left(\frac{\rho}{n}\right) + pd\left(\frac{1}{n}\right), \tag{15}$$

which may be put in the form

$$\frac{d\rho}{dn} = \frac{\rho + p}{n}. \tag{16}$$

We define the adiabatic sound speed a via [5]

$$a^2 \equiv \frac{dp}{d\rho} = \frac{dp}{dn}\frac{n}{\rho + p}, \tag{17}$$

and we have used Eq. (16). Using (17), the baryon and momentum conservation equations can be written as

$$\frac{v'}{v} + \frac{n'}{n} + \frac{2}{r} = 0, \tag{18}$$

$$vv' + a^2\left(1 - \frac{2M}{r} - \alpha + v^2\right)\frac{n'}{n} + \frac{M}{r^2} = 0, \tag{19}$$

with $p' = (dp/dn)n'$ where a dash ($'$) denotes a derivative with respect to r. With the help of the above equations, we obtain the system

$$v' = \frac{N_1}{N}, \quad n' = -\frac{N_2}{N}, \tag{20}$$

where

$$N_1 = \frac{1}{n}\left[\left(1 - \frac{2M}{r} - \alpha + v^2\right)\frac{2a^2}{r} - \frac{M}{r^2}\right], \tag{21a}$$

$$N_2 = \frac{1}{v}\left(\frac{2v^2}{r} - \frac{M}{r^2}\right), \tag{21b}$$

$$N = \frac{v^2 - (1 - \frac{2M}{r} - \alpha + v^2)a^2}{vn}. \tag{21c}$$

In the stationary accretion of gas onto the black hole, the amount of infalling matter per unit time $\dot{M}$, and other parameters are determined by the gas properties and the gravitational field at large distances. For large r, the flow is subsonic i.e., $v < a$ and since the sound speed must be subluminal, i.e., $a < 1$, we have $v^2 \ll 1$. The denominator (21b) is therefore

$$N \approx \frac{v^2 - a^2(1 - \alpha)}{vn}, \tag{22}$$

and so $N < 0$ as $r \to \infty$ if we demand $v^2 < a^2(1 - \alpha)$. At the event horizon $r_H = 2M/(1 - \alpha)$, and we have

$$N = \frac{v(1 - a^2)}{n}. \tag{23}$$

Under the causality constraint $a^2 < 1$, we have $N > 0$. Therefore N should pass through a critical point r_s where it goes to zero. As the flow is assumed to be smooth everywhere, so N_1 and N_2 should also vanish at r_s, i.e., to avoid discontinuities in the flow, we must have $N = N_1 = N_2 = 0$ at the radius r_s. This is nothing but the so-called *sonic condition*. Hence, the flow must pass through a critical point outside the event horizon, i.e., $r_H < r_s < \infty$. At the critical point the system (21) satisfies the condition

$$v_s^2 = \frac{a_s^2(1 - \alpha)}{1 + 3a_s^2} = \frac{M}{2r_s}, \tag{24}$$

where $v_s \equiv v(r_s)$ and $a_s \equiv a(r_s)$. The quantities with a subscript s are defined at the critical point or the sonic points of the flow. It can be clearly seen that the critical velocity in this model is modified by the factor $(1 - \alpha)$, and the physically acceptable solution $v_s^2 > 0$ is ensured since $0 \leq \alpha < 0$.

To calculate the mass accretion rate, we integrate (12) over a four-dimensional volume and multiply by m_b, the mass of each baryon, to obtain

$$\dot{M} = 4\pi r^2 m_b n v, \tag{25}$$

where $\dot{M}$ is an integration constant, independent of r, having dimensions of mass per unit time. It is similar to the Schwarzschild case. Equations (12) and (13) can be combined to yield

$$\left(\frac{\rho + p}{n}\right)^2\left(1 - \frac{2M}{r} - \alpha + v^2\right) = \left(\frac{\rho_\infty + p_\infty}{n_\infty}\right)^2, \tag{26}$$

which is the modified relativistic Bernoulli equation for the steady state accretion onto black holes surrounded by a cloud of strings. Equations (25) and (26) are the basic equations that characterize accretion onto a black hole with parameter α where we have ignored the backreaction of matter. In the limit $\alpha = 0$, our results reduce to those obtained in [3,5] for the standard Schwarzschild black hole.

B. The polytropic solution

In order to calculate $\dot{M}$ explicitly and all the fundamental characteristics of the flow, (25) and (26) must be supplemented with an equation of state which is a relation that characterizes the state of matter of the gas. Following Bondi [2] and Michel [3], we introduce a polytropic equation of state

$$p = Kn^{\gamma}, \tag{27}$$

where K and the adiabatic index γ are constants. On inserting (27) into the energy equation (15) and integrating, we obtain

$$\rho = \frac{K}{\gamma - 1} n^{\gamma} + m_b n, \tag{28}$$

where m_b is an integration constant obtained by matching with the total energy density equation $\rho = m_b n + U$, where $m_b n$ is the rest-mass energy density of the baryons and U is the internal energy density. Equations (27) and (28) give

$$\gamma K n^{\gamma - 1} = \frac{a^2 m_b}{(1 - \frac{a^2}{\gamma - 1})}. \tag{29}$$

Using (28) and (29) we can easily rewrite the Bernoulli equation (26) as

$$\left(1 + \frac{a^2}{\gamma - 1 - a^2}\right)^2 \left(1 - \frac{2M}{r} - \alpha + v^2\right) = \left(1 + \frac{a_\infty^2}{\gamma - 1 - a_\infty^2}\right)^2. \tag{30}$$

At the critical radius r_s, using the relation (24) and inverting the above equation, we get

$$(1 + 3a_s^2)\left(1 - \frac{a_s^2}{\gamma - 1}\right)^2 = \left(1 - \frac{a_\infty^2}{\gamma - 1}\right)^2. \tag{31}$$

It must be noted that, in general, the Bernoulli equation is modified due to a string cloud background. However at the critical radius, the form remains unchanged from the Schwarzschild case [5].

For large but finite values of r, i.e., $r \geq r_s$ the baryons will be nonrelativistic, i.e., $T \ll mc^2/k = 10^{13}$K for neutral hydrogen. In this regime we should have $a \leq a_s \ll 1$. Expanding (31) up to second order in a_s and a_∞, we obtain

$$\begin{aligned} a_s^2 &\approx \frac{2}{5 - 3\gamma} a_\infty^2, \qquad \gamma \neq \frac{5}{3}, \\ &\approx \frac{2}{3} a_\infty, \qquad \gamma = \frac{5}{3}. \end{aligned} \tag{32}$$

We thus obtain the critical radius r_s in terms of the black hole mass M and the boundary condition a_∞ from (24) and (32):

$$\begin{aligned} r_s &\approx \frac{5 - 3\gamma}{4} \frac{M}{a_\infty^2 (1 - \alpha)}, \qquad \gamma \neq \frac{5}{3} \\ &\approx \frac{3}{4} \frac{M}{a_\infty (1 - \alpha)}, \qquad \gamma = \frac{5}{3}. \end{aligned} \tag{33}$$

Also, for $a^2/(\gamma - 1) \ll 1$, we get from (29)

$$\frac{n}{n_\infty} \approx \left(\frac{a}{a_\infty}\right)^{2/(\gamma - 1)}. \tag{34}$$

We are now in a position to evaluate the accretion rate $\dot{M}$. Since $\dot{M}$ is independent of r, (25) must also hold for $r = r_s$. We use the critical point to determine the Bondi accretion rate $\dot{M} = 4\pi r_s^2 m_b n_s v_s$. By virtue of Eqs. (24), (32), (33), and (34) the accretion rate becomes

$$\dot{M} = \frac{4\pi}{(1 - \alpha)^{3/2}} \lambda_s M^2 m_b n_\infty a_\infty^{-3}, \tag{35}$$

where we have defined the dimensionless accretion eigenvalue

$$\lambda_s = \left(\frac{1}{2}\right)^{(\gamma+1)/2(\gamma-1)} \left(\frac{5 - 3\gamma}{4}\right)^{-(5-3\gamma)/2(\gamma-1)}. \tag{36}$$

From (35), it is evident that the mass accretion in a string cloud background is increased by the factor $(1 - \alpha)^{-3/2}$, which may result in a more luminous source. However, the accretion rate still scales as $\dot{M} \sim M^2$ which is similar to that of the Newtonian model [2] as well as the relativistic case [3,5]. In the limiting case $\alpha = 0$, we obtain the well-known relations derived in [3,5] for the Schwarzschild black hole. In Fig. 1, we have plotted the logarithm of the accretion rate $\dot{M}$ against the string cloud parameter α for various polytropic indices γ. Here $\dot{M}$ is calculated in ergs/sec. We see that $\dot{M}$ increases rapidly with increasing α ($0 \leq \alpha < 1$), and interestingly $\dot{M} \to \infty$ as $\alpha \to 1$.

C. Some numerical results

The radial motion of the relativistic fluid accreting onto the black hole in a strings cloud background is governed by (12) and (30). These equations are difficult to solve analytically and we solve them numerically as in Ref. [14]. We consider only the case of the relativistic fluid with $\gamma = \frac{4}{3}$ to study the radial velocity of the flow. Following [14], we introduce dimensionless variables, the radial distance in terms of the gravitational radius ($x = (r/2M)$) and the particle number density with respect to its value at infinity ($y = n/n_\infty$). Now considering $a \ll 1$, Eq. (30) can be rewritten, in terms of a new variable, as

$$\left(1 + \frac{a_\infty^2}{\gamma - 1} y^{\gamma - 1}\right)^2 \left(1 - \frac{1 - \alpha}{x} - \alpha + v^2\right) = \left(1 + \frac{a_\infty^2}{\gamma - 1}\right)^2. \tag{37}$$

On the other hand, using the same notation, the baryon conservation equation (12) can be recast as

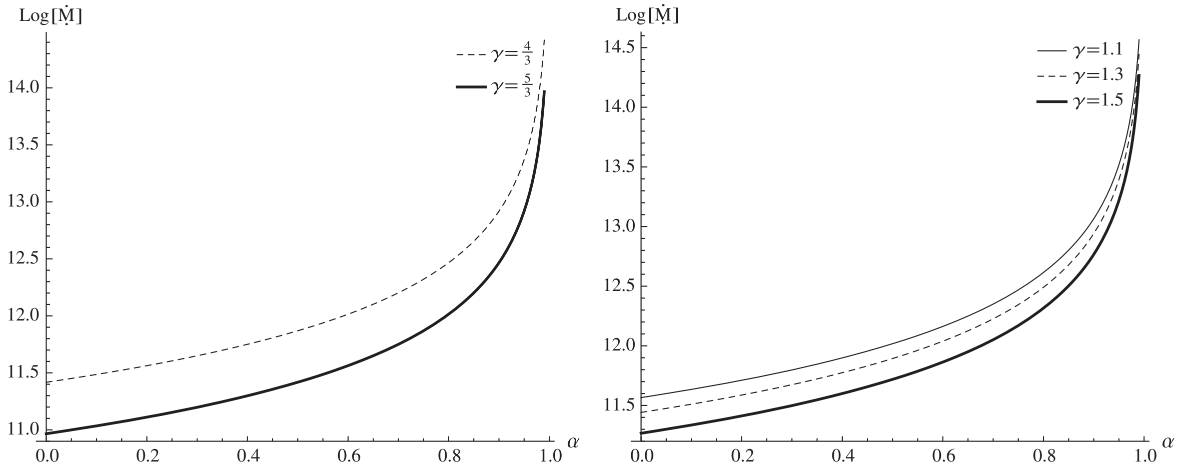

FIG. 1. Plots showing the logarithm of the accretion rate $\dot{M}$ as a function of α for different values of γ.

$$yv = \left(\frac{x_s}{x}\right)^2 a_\infty \left(\frac{2}{5-3\gamma}\right)^{\gamma+1/2(\gamma-1)} (1-\alpha)^{1/2}, \tag{38}$$

where the constant of integration is calculated by applying baryon conservation at the critical point. Observe that (37) and (38) are corrected equations for the string cloud model and when $\alpha \to 0$ we recover the familiar model of Michel [3]. Clearly, Eqs. (37) and (38) form a nonlinear system of algebraic equations which is solved numerically for the fluid velocity v given in terms of the velocity of light and y. The parameters defining the flow are the sound velocity at infinity a_∞, the adiabatic coefficient γ and the string cloud parameter α. The velocity profile of the flow as a function of the dimensionless variable x for different values of the parameter α is plotted in Fig. 2. The solution is obtained by assuming an asymptotic temperature at infinity of $10^{-9} m_p c^2/k_B$ for the relativistic case, i.e., $\gamma = \frac{4}{3}$.

The event horizons for the model are located at $x = 1/(1-\alpha)$, and hence the event horizon varies with

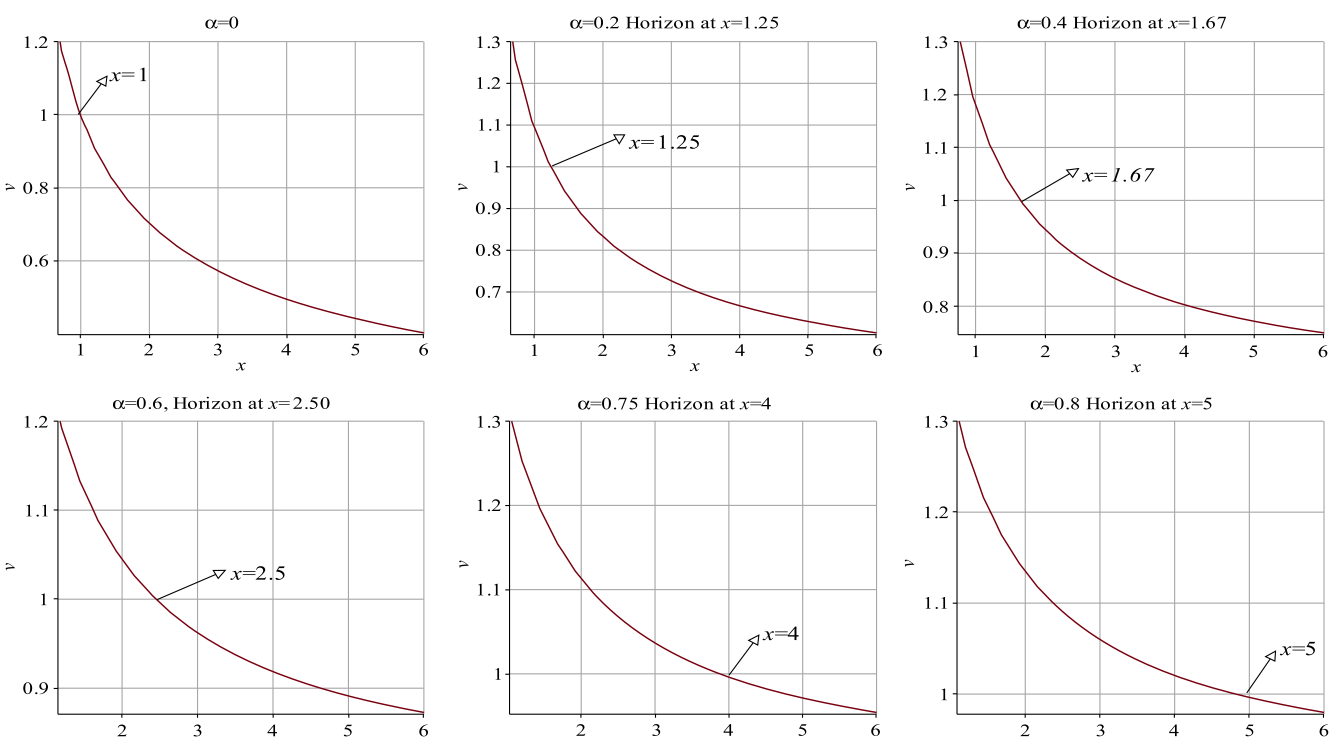

FIG. 2 (color online). The radial velocity profile (v) for a relativistic fluid $\gamma = 4/3$ accreting onto the black hole as a function of the dimensionless radius $x = (r/2M)$ for different values of the string cloud parameter α.

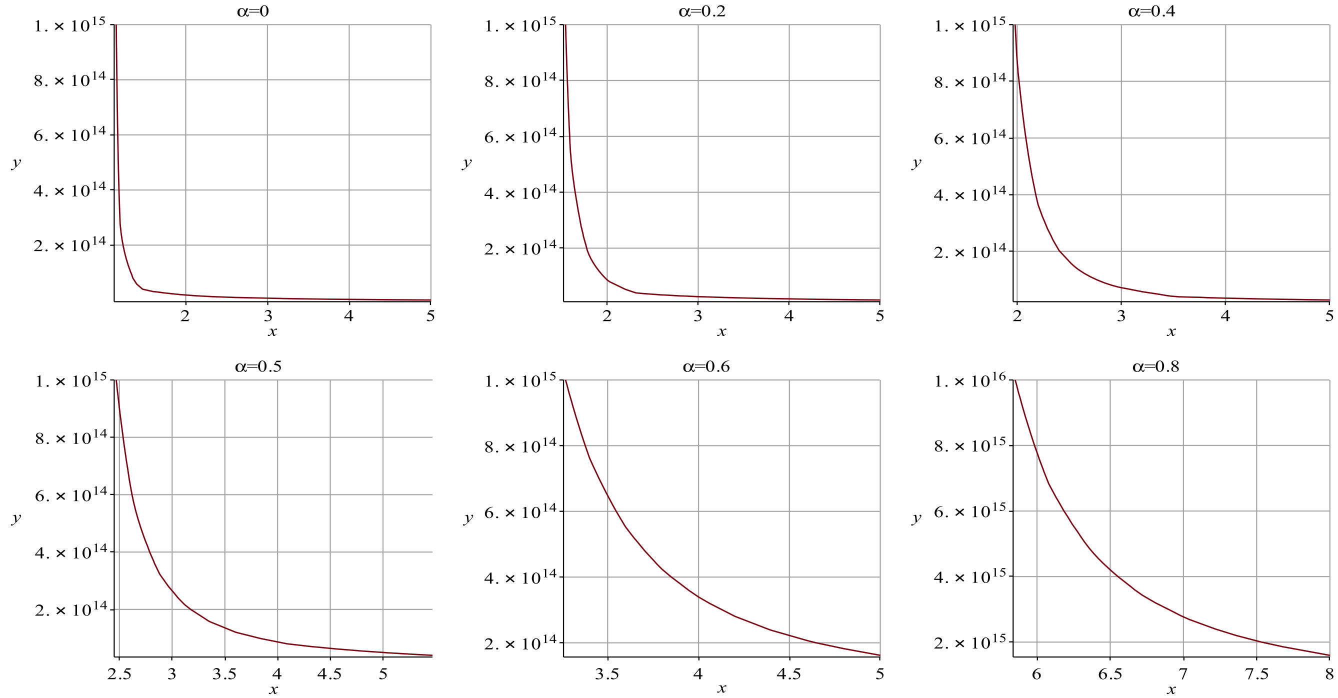

FIG. 3 (color online). The compression factor (y) for a relativistic fluid $\gamma = 4/3$ accreting onto the black hole as a function of the dimensionless radius $x = (r/2M)$ for different values of the string cloud parameter α.

α. Interestingly a string cloud in the background makes a profound influence on the radial velocity, and the result is strikingly different from the Schwarzschild case ($\alpha = 0$). In the familiar Schwarzschild case ($\alpha = 0$, Fig. 2), we note that the flow speed of the accreting gas crosses the event horizon at the speed of light. This feature is consistent with the treatment of de Freitas [14] who considered relativistic accretion onto a charged black hole. The critical radius is far away from the event horizon ($x_c = 1.25 \times 10^8$) where the flow velocity is much less the value at the event horizon. To conserve space we have plotted velocity profile for $\gamma = 4/3$, as the radial velocity v for other values of γ have similar profiles. The velocity profiles are plotted for some specific values of string cloud parameter $\alpha = 0$, 0.2, 0.4, 0.6, 0.75 and 0.8, respectively for which the event horizons are located at $r = 1$, 1.25, 1.67, 2.5, 4 and 5. It is clear from Fig. 2, the fluid always crosses the event horizon with the velocity of light for all values of α.

We have also plotted the compression ratio y as a function of radial coordinate for a relativistic accreting gas with $\gamma = \frac{4}{3}$ in Fig. 3 for different values of string cloud parameter, more specifically for $\alpha = 0$, 0.2, 0.4, 0.6, 0.75 and 0.8. This graph shows that the compression factor profiles also affected by a change in the strings cloud parameter α is in contrast to the analogous compression factor of an accreting charged black hole [14]. The compression ratio for black hole in string cloud background increases with increase in α. In general, it may attain the value of the order of 10^{14}–10^{16}.

IV. ASYMPTOTIC BEHAVIOR

In the last section, we found that the accretion rate at some sonic point $r = r_s$ far away from event horizon, i.e., $r_s \gg 2M$ is not influenced by nonlinear gravity. Next we estimate the flow characteristics for $r_H < r \ll r_s$ and at the event horizon $r = r_H$.

A. Sub-Bondi radius $r_H < r \ll r_s$

At distances below the Bondi radius the gas is supersonic so that $v > a$ when $r_H < r \ll r_s$. From (30) we find the upper bound on the radial dependence of the gas velocity

$$v^2 \approx \frac{2M}{r}, \qquad \gamma \neq \frac{5}{3}. \tag{39}$$

We can now estimate the gas compression on these scales. With the help of (25), (35) and (39) we obtain

$$\frac{n(r)}{n_\infty} \approx \frac{\lambda_s}{\sqrt{2}(1-\alpha)^2}\left(\frac{M}{a_\infty^2 r}\right)^{3/2}. \tag{40}$$

For a Maxwell-Boltzmann gas, $p = nk_BT$, we generate the adiabatic temperature profile

$$\frac{T(r)}{T_\infty} = \left(\frac{n(r)}{n_\infty}\right)^{\gamma-1} \approx \left[\frac{\lambda_s}{\sqrt{2}(1-\alpha)^2}\left(\frac{M}{a_\infty^2 r}\right)^{3/2}\right]^{\gamma-1}, \tag{41}$$

using (27) and (40).

B. Event horizon

At the event horizon we have $r = r_H = 2M/(1-\alpha)$. As the flow is supersonic since we are well below the Bondi radius, it is reasonable to assume that the fluid velocity is approximated by $v^2 \approx \frac{2M}{r}$. At r_H, $v_H^2 \equiv v^2(r_H) \approx 1-\alpha$, i.e., the flow speed at the horizon is always less than the speed of light. Therefore, using the fact $M/r_H = (1-\alpha)/2$, we obtain the gas compression at the event horizon from (40):

$$\frac{n_H}{n_\infty} \approx \frac{\lambda_s}{4(1-\alpha)^{1/2}}\left(\frac{c}{a_\infty}\right)^3. \qquad (42)$$

Again assuming the presence of a Maxwell-Boltzmann gas, $p = nk_BT$, we find the adiabatic temperature profile at the event horizon using (41) and the horizon assumption:

$$\frac{T_H}{T_\infty} \approx \left[\frac{\lambda_s}{4(1-\alpha)^{1/2}}\left(\frac{c}{a_\infty}\right)^3\right]^{\gamma-1}, \qquad (43)$$

where, following [5], we have reintroduced the speed of light c in the above expressions. The limit $\alpha \to 0$ in the above equation gives us the corresponding result of accretion of the fluid onto the Schwarzschild black hole [5].

V. CONCLUSIONS

Historically the accretion problem with a polytropic equation of state was addressed by Bondi [2]. He showed that subsonic flow far from a black hole will inevitably become supersonic, and that the requirement of a smooth traversal of the sonic surface uniquely specifies the accretion rate as a function of two thermodynamic variables, namely the density and temperature of the gas at infinity. The relativistic version of the same problem was solved by Michel [3] 20 years later, after the discovery of celestial x-ray sources. He showed that accretion onto the black hole should be transonic. Accretion onto compact objects such as black holes and neutron stars is the most efficient method of releasing energy; up to 40 percent of the rest-mass energy of the matter accreting on the black hole is liberated. Recent developments in the theory of accretion are significant steps toward understanding various astronomical sources that are believed to be powered by the accretion onto black holes. Spherical accretion onto a black hole is generally specified by the mass accretion rate $\dot{M}$ which is a key parameter, and there is evidence that a higher accretion rate can provide higher luminosity values. In view of this, we analyzed the steady state and spherical accretion of a fluid onto the Schwarzschild black hole in a string cloud background. We determined exact expressions for the mass accretion rate at the critical radius. It turns out that this quantity is modified so that $\dot{M} \approx M^2/(1-\alpha)^{3/2}$ with $r_s \approx M/(1-\alpha)$. Thus the accretion rate by the black hole in a string cloud background is higher than that for a Schwarzschild black hole. Thus the parameter α can be introduced in the problem of accretion onto black hole to extend the work of Michel [3], and this quantity determines the accretion rate and other flow parameters. In principle, the accretion rate and other parameters still have same characteristics as in the Schwarzschild black hole; in this sense we may conclude that the familiar steady state spherical accretion solution onto the Schwarzschild black hole is stable. In the limit $\alpha \to 0$, our results reduce exactly to those obtained in [3,5] for the standard Schwarzschild black hole.

We can attempt to work out the effect of string cloud background on the luminosity, the frequency spectrum and the energy conversion efficiency of the accretion flow. It is possible to deviate from spherical symmetry, e.g., include rotation, which may lead to a higher accretion rate. These and other related issues are currently under investigation.

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