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Permutation on hybrid natural inflation

Christopher D. Carone,* Joshua Erlich,† Raymundo Ramos,‡ and Marc Sher§

High Energy Theory Group, Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA

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We analyze a model of hybrid natural inflation based on the smallest non-Abelian discrete group S_3. Leading invariant terms in the scalar potential have an accidental global symmetry that is spontaneously broken, providing a pseudo-Goldstone boson that is identified as the inflaton. The S_3 symmetry restricts both the form of the inflaton potential and the couplings of the inflaton field to the waterfall fields responsible for the end of inflation. We identify viable points in the model parameter space. Although the power in tensor modes is small in most of the parameter space of the model, we identify parameter choices that yield potentially observable values of r without super-Planckian initial values of the inflaton field.

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I. INTRODUCTION

Measurements of the anisotropy in the cosmic microwave background (CMB) have led to the development of a "standard model" of cosmology, with a cosmological constant, cold dark matter and a spectrum of initial CMB fluctuations that seed large scale structure [1]. It is widely believed that these initial fluctuations arise from an inflationary epoch, resulting in a nearly scale-invariant spectrum. More precise measurements of the CMB fluctuations, including polarization measurements, have been carried out by experiments such as WMAP [2], PLANCK [3] and BICEP2 [4]. These measurements provide information about initial metric perturbations that can severely constrain (or rule out) various inflationary models.

In order to satisfy the limits on the size of the CMB anisotropy fluctuations, the scalar self-coupling constant of the inflaton field must be very small, typically less than 10^{-12} in most realistic models [5]. While such a small coupling could be assumed, it would be aesthetically more desirable if it arose naturally. This is the case for theories in which the inflaton is identified with the pseudo-Goldstone boson of a spontaneously broken approximate global symmetry. Such "natural inflation" scenarios were proposed first by Freese, Frieman and Olinto [6]. If the scale of spontaneous symmetry breaking is f and if there is an explicit breaking of the global symmetry via an anomaly, the inflaton potential takes the form

$$V = V_0[1 \pm \cos(n\phi/f)], \tag{1.1}$$

where n is an integer. The model is consistent with measured values of the spectral index and its running, as well as constraints on the ratio of powers in tensor and scalar modes [7]. A concern about natural inflation is that the value of f must be very close to or above the Planck scale, so that quantum-gravitational corrections to the potential are not automatically under control.

A model that can result in a lower value of f is "hybrid natural inflation" [8–11]. The original hybrid inflation model, proposed by Linde [12], has a second scalar field which couples to the inflaton and ends the inflationary epoch. As the inflaton slowly rolls, the parameters of the potential of the second scalar field change due to the coupling, and at some point the second scalar field acquires a vacuum expectation value (VEV), ending inflation. This second scalar field was referred to as the "waterfall" field. Hybrid natural inflation models are natural inflation models in which inflation is terminated due to the dynamics of such additional fields.

An important question in any model based on the natural inflation idea is the origin of the approximate global symmetry. Global symmetries are not believed to be fundamental (for example, they are typically violated by quantum gravitational effects [13]), so it is desirable to arrange that these symmetries arise by accident, as a consequence of the form of the leading terms in the potential; these terms are restricted by the continuous or discrete gauge symmetries of the theory. While discrete gauge symmetries can be thought of as discrete remnants of a spontaneously broken continuous gauge symmetry [14], they also can be defined consistently without such an embedding [15]; in either case, they are preserved by quantum gravitational effects. Cohn and Stewart [8,9] showed that accidental global symmetries could easily be obtained in models with non-Abelian discrete gauge symmetries and illustrated their point with hybrid models based on the discrete group $\Delta(96)$. They note that many other models based on smaller discrete groups are likely possible. Nevertheless, the literature on such models is relatively sparse. Ross and Germán [10,11] have explored

*cdcaro@wm.edu
†jxerli@wm.edu
‡raramos@email.wm.edu
§mtsher@wm.edu

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hybrid natural inflation models based on the discrete group D_4. In their model, the inflaton potential takes the form

$$V = V_0[1 + a\cos(\phi/f)], \tag{1.2}$$

where a is a constant. This potential can generate phenomenologically acceptable inflation with f substantially smaller than the Planck mass, so that higher-order corrections are under control. Ross and Germán [10,11] point out that potentials of the form Eq. (1.2) should be expected in similar models based on other non-Abelian discrete groups.

Given the promise of the models considered in Refs. [8–11], and motivated by minimality, we explore in this paper a hybrid natural inflation model based on the smallest non-Abelian discrete group, the permutation group S_3. The discrete symmetry restricts both the inflaton potential and the couplings of the inflaton to the waterfall fields. The S_3 charge assignments in our model satisfy the requirements for a discrete gauge symmetry, as set out in Ref. [15]. In Sec. II, we review the group S_3 and its representations. The model is presented in Sec. III. After reviewing inflationary parameters in Sec. IV, we study a typical point in model parameter space in quantitative detail in Sec. V. Motivated by the potential signature in gravitational waves, we show in Sec. VI that the model can yield a potentially observable tensor-to-scalar ratio, without requiring super-Planckian values of the inflaton field, and we explain why this is not in conflict with the Lyth bound [16]. In Sec. VII, we discuss the cutoff of inflation and reheating. Finally, in Sec. VIII, we present our conclusions.

II. THE GROUP S_3

We base our model on S_3, the smallest non-Abelian discrete symmetry group. The group has six elements whose action can be identified with the permutation of three objects. A useful discussion of this symmetry in a model-building context can be found in Ref. [17].

S_3 has three irreducible representations: a two-dimensional representation $\mathbf{2}$ and two one-dimensional representations, $\mathbf{1}_A$ and $\mathbf{1}_S$. The $\mathbf{1}_S$ representation is the trivial singlet. The rules for group multiplication are given by $\mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_S \otimes \mathbf{1}_S = \mathbf{1}_S$, $\mathbf{1}_A \otimes \mathbf{1}_S = \mathbf{1}_A$ and $\mathbf{2} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{1}_A \oplus \mathbf{1}_S$. The product of two doublet representations can be decomposed into its irreducible components using Clebsch-Gordan matrices. Let ψ and η represent two-component column vectors that transform as doublets under S_3 and let σ^a denote the Pauli matrices. The products $\psi^T C_{\mathbf{1}_S}\eta$ and $\psi^T C_{\mathbf{1}_A}\eta$ transform in the $\mathbf{1}_S$ and $\mathbf{1}_A$ representations, respectively, where

$$C_{\mathbf{1}_S} = \mathbb{1} \quad \text{and} \quad C_{\mathbf{1}_A} = i\sigma^2. \tag{2.1}$$

Similarly, we can construct a doublet

$$\begin{bmatrix} \psi^T C_{\mathbf{2}}^{(1)}\eta \\ \psi^T C_{\mathbf{2}}^{(2)}\eta \end{bmatrix} \sim \mathbf{2}, \tag{2.2}$$

where

$$C_{\mathbf{2}}^{(1)} = \sigma^3 \quad \text{and} \quad C_{\mathbf{2}}^{(2)} = -\sigma^1. \tag{2.3}$$

The model we present in the next section includes an S_3 doublet field $\phi = (\phi_1, \phi_2)^T$, so it is useful to enumerate the S_3 invariants that can be constructed from products of ϕ, up to quartic order. The quadratic combination of fields that transforms in the $\mathbf{1}_S$ representation has the form

$$(\phi^2)_{\mathbf{1}_S} \equiv \phi^T C_{\mathbf{1}_S}\phi = \phi_1^2 + \phi_2^2. \tag{2.4}$$

While there are three $\mathbf{1}_S$ reps in the product $\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2}$, all such invariants constructed from a single ϕ have the same form:

$$(\phi^4)_{\mathbf{1}_S} = (\phi_1^2 + \phi_2^2)^2. \tag{2.5}$$

While Eqs. (2.4) and (2.5) follow from S_3 invariance, it is important to note that these expressions are also invariant under a continuous symmetry, SO(2), under which the ϕ field is also a doublet. However, this accidental symmetry is broken by the S_3 cubic invariant

$$(\phi^3)_{\mathbf{1}_S} = \phi_1(\phi_1^2 - 3\phi_2^2). \tag{2.6}$$

The model of the next section will identify the inflaton field θ with the pseudo-Goldstone boson of this accidental SO(2) symmetry; the soft breaking of this symmetry by the cubic invariant will be used to generate the inflaton potential. Notice, if we parameterize

$$\phi = (\rho + v)\begin{bmatrix} \cos(\theta/v) \\ \sin(\theta/v) \end{bmatrix}, \tag{2.7}$$

where v is the scale of spontaneous symmetry breaking and ρ is the massive radial excitation, then Eqs. (2.4) and (2.5) are independent of θ, indicating that these terms contribute nothing to the inflaton potential. (Note that in this parameterization the kinetic term for θ is canonically normalized.) On the other hand, Eq. (2.6) simplifies to

$$(\phi^3)_{\mathbf{1}_S} = (\rho + v)^3\cos(3\theta/v), \tag{2.8}$$

which can be used to lift the flat direction. In the next section we show how these ingredients can be combined to produce a viable model of hybrid natural inflation.

III. THE MODEL

In addition to the doublet field ϕ described in the previous section, our model includes two real scalars, χ_1 and χ_2, each in the $\mathbf{1}_S$ representation of S_3. We assume a $\mathbb{Z}_2$ symmetry under which both χ fields are odd, which eliminates unwanted linear terms that would otherwise give the χ_i VEVs. The SO(2) invariant terms in the potential

$$V_{\mathrm{SO}(2)}(\phi, \chi_i) = -\frac{1}{2}m_\phi^2(\phi_1^2 + \phi_2^2) + \lambda_\phi(\phi_1^2 + \phi_2^2)^2 + \cdots \tag{3.1}$$

lead to the spontaneous breaking of the SO(2) symmetry due to the negative mass squared term for ϕ. The terms not shown include various $\phi^2\chi^2$ couplings as well as the potential for the χ_i fields by themselves. It is not hard to see that it is possible to choose parameters such that ϕ^2 develops a vacuum expectation value, while the χ_i do not. The details are not crucial for our purposes because the SO(2) invariant terms have no effect on the form of the inflaton potential. All that is relevant at this stage is that the spontaneous symmetry breaking is consistent with the parameterization in Eq. (2.7), with the Goldstone boson θ identified as the inflaton.

In the spirit of a perturbative expansion, we now introduce smaller terms which violate the accidental SO(2) symmetry. At the renormalizable level, we can include a term of the form $m_0(\phi^3)_{\mathbf{1}_s}$; the dimensionful coefficient m_0 parameterizes the breaking of the SO(2) symmetry. We could simply assume a small value of m_0 as a fine-tuning in the model (after all, we have to accept the same for the Higgs boson mass in any nonsupersymmetric theory). However, we can do better if we allow an additional $\mathbb{Z}_2'$ symmetry under which the ϕ doublet and χ_1 are odd and treat m_0 consistently as a soft $\mathbb{Z}_2'$-breaking parameter. Since the $\mathbb{Z}_2'$ symmetry is restored in the limit of vanishing m_0, there can be no large radiative corrections and a small m_0 will be natural following the criterion of 't Hooft [18]. We will adopt this assumption henceforth. The only other term that we include that violates the SO(2) symmetry is of the form $\chi_1\chi_2(\phi^3)_{\mathbf{1}_s}$. Identifying the χ fields as the waterfall fields of a hybrid inflation model, such couplings are responsible for ending inflation in the model. In the present case, this SO(2)-breaking term is Planck suppressed for sub-Planckian field values.

We now consider the effective theory below the SO(2)-breaking scale (the scale of the ρ mass). With the particle content and the symmetries of the theory as we have specified them, the scalar potential for the θ, χ_1 and χ_2 fields is somewhat cumbersome for a general analysis. We will therefore adopt a simplifying assumption in our parameter choices to demonstrate most simply that viable cosmological solutions exist. Additional solutions are possible for less restrictive choices of model parameters.

We study the following simplified form for the scalar potential:

$$V(\theta,\chi_i) = V_0 + c_1\frac{v^3}{M_P}\chi_1\chi_2\cos(3\theta/v) - m_0 v^3\cos(3\theta/v) + \frac{1}{2}m_\chi^2(\chi_1^2+\chi_2^2) + (\lambda\chi_1^4 + \lambda_{12}\chi_1^2\chi_2^2 + \lambda\chi_2^4). \tag{3.2}$$

Here V_0 is a constant, c_1, λ and λ_{12} are couplings, and m_χ is a common χ_i field mass. The second and third terms are SO(2)-breaking interactions discussed previously. For definiteness, we assume $c_1 > 0$. In contrast to the most general case, we have assumed symmetry under $\chi_1 \leftrightarrow \chi_2$. This simplifying assumption has no effect on the shape of the inflaton potential (which is obtained by setting $\chi_i = 0$) but substantially streamlines our presentation. If one relaxes this assumption, one has to contend with minimization conditions that are cubic; this complicates the analysis but does not affect our conclusions qualitatively. Note also that we have omitted the $(\chi_1^2+\chi_2^2)\cos(3\theta/v)$ and $\chi_1\chi_2^3 + \chi_2\chi_1^3$ interactions, which are $\mathbb{Z}_2'$ odd. Since the $\mathbb{Z}_2'$ symmetry is broken only by m_0, these are suppressed by m_0/M_P relative to the second and fifth terms in Eq. (3.2), respectively, making them negligible.[1] We set the cosmological constant to zero at the global minimum of the potential by choice of the parameter V_0.

Inflation occurs as the field θ slow rolls toward the origin, between initial and final field values that lie within the interval $0 < 3\theta/v < \pi$. During inflation, the effective χ_i masses are positive and the χ fields remain at the origin. Inflation ends via the waterfall mechanism when θ is such that

$$c_1\frac{v^3}{M_P}\cos(3\theta/v) > m_\chi^2. \tag{3.3}$$

At this point, the χ_i potential is destabilized and the χ fields develop VEVs.[2] Within a Hubble time, the fields reach a global minimum, and inflation abruptly ends. Oscillations of the waterfall fields about this minimum leads to reheating. Given the inequality in Eq. (3.3), we find that the locations of the degenerate global minima in our model are given by

$$\theta = 0, \tag{3.4}$$

$$\chi_1 = -\chi_2, \tag{3.5}$$

and

$$\chi_1^2 = \frac{1}{2(2\lambda+\lambda_{12})}\left[c_1\frac{v^3}{M_P} - m_\chi^2\right]. \tag{3.6}$$

Setting the cosmological constant to zero at any of these minima determines the constant V_0 in Eq. (3.2):

$$V_0 = m_0 v^3 + \frac{1}{4}\frac{1}{(2\lambda+\lambda_{12})}\left(c_1\frac{v^3}{M_P} - m_\chi^2\right)^2. \tag{3.7}$$

[1] If one prefers to dispense with the softly broken $\mathbb{Z}_2'$ symmetry and allow m_0 to be fine-tuned, then these terms can be omitted as a parametric simplification. The effect of including a $c_2\frac{v^3}{M_P}(\chi_1^2+\chi_2^2)\cos(3\theta/v)$ term, with $c_2 > 0$, is to change Eq. (3.3) by replacing $c_1 \to c_1 - c_2$ and Eqs. (3.6) and (3.7) by $m_\chi^2 \to m_\chi^2 + 2c_2\frac{v^3}{M_P}$. If one adds a $\lambda_3(\chi_1^3\chi_2 + \chi_2^3\chi_1)$ term, then the only change in these equations is $\lambda_{12} \to \lambda_{12} - 2\lambda_3$. These changes do not affect our results qualitatively.

[2] As we will see in Sec. V, $\cos(3\theta/v) > 0$ when inflation ends, as has been assumed in Eq. (3.3).

With this result in hand, the form of the inflaton potential during the period of slow roll is fixed in term of the model parameters:

$$V(\theta) = V_0[1 - \xi\cos(3\theta/v)], \tag{3.8}$$

where $\xi \equiv m_0 v^3/V_0$ and V_0 is given by Eq. (3.7). Our parameter choices in the next sections have $\xi < 1$.

Equation (3.8) is amenable to the standard analysis of a single-field inflation model until the end of inflation. We review the quantities of interest in such an analysis in the next section and explore numerical results for a number of benchmark points in our model's parameter space. For these points, we will also present estimates to justify that the shutoff of inflation via the waterfall mechanism is sufficiently fast.

IV. INFLATION PARAMETERS

In terms of the inflaton potential $V(\theta)$, the slow-roll parameters may be written [3]

$$\epsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2, \qquad \eta \equiv \frac{M_P^2}{8\pi}\frac{V''}{V},$$
$$\text{and} \quad \gamma \equiv \frac{M_P^4}{64\pi^2}\frac{V'V'''}{V^2}. \tag{4.1}$$

In a generic single-field model, $\epsilon = 1$ is usually chosen to define the end of inflation; in the present case, ϵ remains small throughout the period of slow roll until inflation is terminated by the destabilization of the effective χ potential. The number of e-folds of inflation N may be expressed as [1]

$$N = \frac{2\sqrt{\pi}}{M_P}\int_{\theta_f}^{\theta_i}\frac{1}{\sqrt{\epsilon}}d\theta, \tag{4.2}$$

where θ_i and θ_f are the initial and final inflaton field values, respectively. We will evaluate this quantity in our model to assure that sufficient inflation is achieved.

A number of cosmic microwave background parameters can be expressed conveniently in terms of the slow-roll parameters, as we now summarize [3,19]. All are evaluated at values of the inflaton field corresponding to ~60 e-folds before the end of inflation, when scales of order the current Hubble radius exited the horizon. The amplitude of the tensor power spectrum in the slow-roll approximation is

$$\Delta_T^2(k) = \frac{128}{3}\frac{V}{M_P^4}, \tag{4.3}$$

while the amplitude of the scalar power spectrum is

$$\Delta_R^2(k) = \frac{128\pi}{3M_P^6}\frac{V^3}{V'^2} = \frac{8}{3M_P^4}\frac{V}{\epsilon}. \tag{4.4}$$

The ratio of the tensor-to-scalar amplitudes is then

$$r = 16\epsilon. \tag{4.5}$$

The scalar spectral index and its running are given by

$$n_s(k) = 1 - 6\epsilon + 2\eta \quad \text{and} \quad n_r = 16\epsilon\eta - 24\epsilon^2 - 2\gamma. \tag{4.6}$$

The predictions following from our model for the parameters summarized in this section can easily be computed starting with Eqs. (3.7) and (3.8). For example, the slow-roll parameters take the form

$$\epsilon = \frac{9\xi^2 M_P^2 \sin^2(\frac{3\theta}{v})}{16\pi v^2(1 - \xi\cos(\frac{3\theta}{v}))^2}, \tag{4.7}$$

$$\eta = \frac{9\xi M_P^2 \cos(\frac{3\theta}{v})}{8\pi v^2(1 - \xi\cos(\frac{3\theta}{v}))}, \tag{4.8}$$

$$\gamma = -\frac{81\xi^2 M_P^4 \sin^2(\frac{3\theta}{v})}{64\pi^2 v^4(1 - \xi\cos(\frac{3\theta}{v}))^2}. \tag{4.9}$$

The parameters n_s, n_r, r and Δ_R^2 can then be evaluated using these expressions, with θ set to θ_i as determined from Eq. (4.2) with $N = 60$. We will follow this procedure in our quantitative analysis in the following section. The measured values of the cosmological parameters that we use in this analysis are $n_s = 0.9603 \pm 0.0073$, $n_r = -0.013 \pm 0.009$, $r < 0.12$(95% C.L.) and $\Delta_R^2 = 2.2 \times 10^{-9}$ [3]. Note that the recent observation by the BICEP2 experiment of B-mode polarization in the CMB would imply $r = 0.20^{+0.07}_{-0.05}$ if the signal is interpreted as cosmological in origin [4]. However, the contribution of foreground dust to the BICEP2 signal is currently uncertain, so one cannot draw a reliable conclusion on the value of r from this measurement at present [20,21].

V. NUMERICAL ANALYSIS

In this section, we present the numerical analysis corresponding to a typical, benchmark point in the model parameter space. We will find in this example that the primordial gravitational wave signal is small. In the next section, we show that for a careful choice of parameters, a larger value of r can be obtained.

Working with the generic potential, Eq. (3.8), let us focus first on two quantities: the spectral index

$$n_s - 1 = -\frac{9}{16\pi}\frac{M_p^2}{v^2} \times \left\{\frac{2\xi^2[2 + \sin^2(3\theta_i/v)] - 4\xi\cos(3\theta_i/v)}{[1 - \xi\cos(3\theta_i/v)]^2}\right\} \tag{5.1}$$

and the amplitude of the scalar power spectrum,

$$\Delta_R^2 = \frac{128\pi}{27}\frac{V_0 v^2}{M_p^6}\frac{1}{\xi^2}\frac{[1-\xi\cos(3\theta_i/v)]^3}{\sin^2(3\theta_i/v)}. \tag{5.2}$$

Both are evaluated at the initial field value θ_i, corresponding to 60 e-folds before the end of inflation. The number of e-folds, following from Eq. (4.2), is given by

$$N = \frac{8\pi}{9}\frac{v^2}{M_P^2}\frac{1}{\xi}\left[(1-\xi)\ln\left(\frac{\sin(3\theta_i/v)}{\sin(3\theta_f/v)}\right) - \ln\left(\frac{1+\cos(3\theta_i/v)}{1+\cos(3\theta_f/v)}\right)\right]. \tag{5.3}$$

Let us define $x_{i,f} \equiv \cos(3\theta_{i,f}/v)$, as well as

$$N_0 \equiv \frac{1}{[\frac{9}{4\pi}\frac{M_P^2}{v^2}\xi]} \quad \text{and} \quad y \equiv \frac{\sqrt{V_0}}{M_p v}, \tag{5.4}$$

and temporarily work in units where $M_p = 1$. Working in the approximation $\xi \ll 1$, which will be accurate for the parameter choices that we consider, we choose $N = 60$, $n_s = 0.9603$ and $\Delta_R = 4.69 \times 10^{-5}$. Then, Eqs. (5.1)–(5.3) lead to the constraints

$$0.9603 = 1 + \frac{1}{N_0}x_i, \tag{5.5}$$

$$4.69 \times 10^{-5} = \frac{2\sqrt{6}}{\sqrt{\pi}}\frac{yN_0}{\sqrt{1-x_i^2}}, \tag{5.6}$$

$$60 = 2N_0 \ln\left[\sqrt{\frac{(1-x_i)(1+x_f)}{(1+x_i)(1-x_f)}}\right]. \tag{5.7}$$

The parameter x_f is set by the scale m_χ and can be chosen freely, provided that the magnitude of the cosine is less than one. For this example, we choose $x_f = 0.8$. Now the three equations above can be solved for the three unknowns, N_0, x_i and y. We find

$$\begin{aligned} x_i &= -0.64, \\ N_0 &= 16, \\ y &= 8.0 \times 10^{-7}. \end{aligned} \tag{5.8}$$

Once v is specified, we can solve for the parameters V_0 and m_0 (the latter given by the definition of ξ.). In this example, we choose $v = M_p/100$. Then we find (including the input mass scales, for comparison)

$$\begin{aligned} M_P &= 1.2 \times 10^{19}\ \text{GeV}, \\ v &= M_P/100, \\ V_0 &= (1.1 \times 10^{15}\ \text{GeV})^4, \\ m_0 &= 6.7\ \text{TeV}. \end{aligned} \tag{5.9}$$

In our fundamental theory, Eq. (3.2), V_0 is fixed by Eq. (3.7). We find that the value for V_0 shown in Eq. (5.9) is obtained for the dimensionless parameter choices[3] $\lambda = 0.1$, $\lambda_{12} = 0.2$ and $c_1 = 0.051$. Notice that none of the fundamental dimensionless couplings is forced to be unnaturally small, unlike the nonsupersymmetric model based on the group D_4 that appeared in Ref. [10]; the D_4 symmetry in that proposal allows marginal SO(2)-violating quartic self-couplings for the inflaton doublet, which necessitates a fine-tuning, while the S_3 symmetry prevents such operators and avoids this outcome. Given our choice of x_f, it follows from Eq. (3.3) that $m_\chi = 2.5 \times 10^{15}$ GeV. Since this is a nonsupersymmetric model, tuning of scalar masses is unavoidable; however, the χ mass is at a relatively high scale, so the largest tuning required is still that of the Higgs boson mass, as in the standard model.

Now we can summarize the values of the remaining cosmological parameters:

$$\begin{aligned} \epsilon &= 7.9 \times 10^{-8}, \\ r &= 1.3 \times 10^{-6}, \\ n_r &= 1.1 \times 10^{-3}. \end{aligned} \tag{5.10}$$

These are consistent with the current bounds, assuming that one conservatively accepts the Planck upper bound on r. An observable primordial gravitational wave signal, if confirmed, would rule out this parameter choice. Therefore, we next consider how one could obtain a solution with larger r.

VI. ENHANCING PRIMORDIAL GRAVITY WAVES

In the slow-roll approximation, by Eqs. (4.1) and (4.5),

$$r = 16\epsilon = \frac{M_P^2}{\pi}\left(\frac{V'}{V}\right)^2. \tag{6.1}$$

On the other hand, the scalar spectral index was given in Eq. (4.6):

$$n_s(k) = 1 - 6\epsilon + 2\eta, \tag{6.2}$$

with the value $n_s = 0.9603 \pm 0.0073$, from Ref. [3]. In order to increase r with fixed n_s in our model, we need to increase the values of both ϵ and η at the time that the

[3]Given our normalization of the quartic couplings, perturbativity requires that they be $\ll (4\pi)^2/4! \approx 6.6$, which is easily satisfied.

fluctuations were created, which we take to be 60 e-folds prior to the end of inflation. We must therefore increase $|V'/V|$ while V''/V becomes less negative; this suggests that the inflaton in our model should minimize $|\cos(3\theta_i/v)|$ in order to obtain large r. Although we find that it is challenging to obtain 60 e-folds of inflation while satisfying observational constraints beginning with such a small magnitude of $\cos(3\theta_i/v)$, we find nonetheless that there are points in parameter space where a primordial gravitational wave signal is large enough to be potentially observable in upcoming experiments. These points require a relatively small separation between v and M_P, pushing the limits of effective field theory.

The Lyth bound [16] relates the number of e-folds of inflation to the change in the inflaton field θ during the same period and suggests that in a wide class of models it is not possible to obtain a sizable gravitational wave signal without a change in the inflaton field during inflation that is much larger than M_P. Such large field values would be problematic for the effective-field-theory interpretation of the model. Using the inflaton equation of motion and the relations for the power spectra of scalar and tensor modes in the slow-roll approximation, one obtains the relation [16]

$$\left(\frac{d\theta}{dN}\right)^2 = \frac{M_P^2}{64\pi} r. \tag{6.3}$$

If r is roughly constant during the last 60 e-folds of inflation, then one obtains

$$\Delta\theta = \frac{1}{8\sqrt{\pi}} N\sqrt{r} M_P, \tag{6.4}$$

which exceeds M_P for $N\sqrt{r} > 8\sqrt{\pi}$. In particular, this will be the case if $N = 60$ and r is of a typical observable value, for example, $r \sim 0.1$. We refer to Eq. (6.4) as the Lyth bound. Hybrid natural inflation models, including the one presented here, can evade the Lyth bound if the inflaton rolls from a steep point in the potential to near the bottom of the potential prior to the end of inflation [22,23], as sketched in Fig. 1. In that case r varies significantly during inflation, which violates the assumption of nearly constant r that fed into the bound.

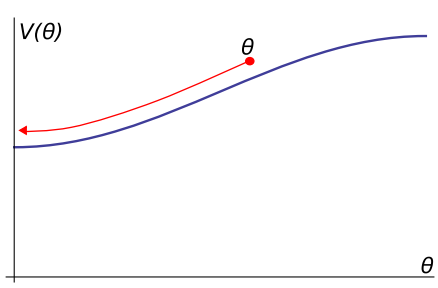

FIG. 1 (color online). The Lyth bound is evaded if the inflaton slowly rolls from a steep point in the potential to near the minimum before the waterfall fields turn on.

In order to obtain 60 e-folds of inflation in this enhanced-gravity-wave scenario, we need inflation to end near the bottom of the inflaton potential, which is possible if the waterfall fields have large diagonal masses m_χ. After fixing the parameters to the well-measured values of Δ_R and n_s, we find that the less-well-measured running of the scalar tilt, $n_r = -0.013 \pm 0.009$ from the Planck experiment [3], in fact provides the greatest obstacle to rolling from near the steepest point of the potential. A viable parameter choice within 2σ of the measured n_r is obtained by setting $x_f = 0.995$ and $v = M_P/2$, in which case we find[4]

$$\begin{aligned} x_i &= -0.32, \\ N_0 &= 9.3, \\ y &= 1.70 \times 10^{-6}. \end{aligned} \tag{6.5}$$

The physical mass scales in this case are given by

$$\begin{aligned} M_P &= 1.2 \times 10^{19}\ \text{GeV}, \\ v &= M_P/2, \\ V_0 &= (1.12 \times 10^{16}\ \text{GeV})^4, \\ m_0 &= 2.6 \times 10^6\ \text{GeV}. \end{aligned} \tag{6.6}$$

In terms of the fundamental potential, Eq. (3.2), the scale V_0 can be reproduced in this case with the choices $\lambda = 0.1$, $\lambda_{12} = 0.2$ and $c_1 = 0.0017$. In that case, from Eq. (3.3) we find $m_\chi = 1.8 \times 10^{17}$ GeV.

The cosmological parameters evaluated at $\theta = \theta_i$ are now

$$\begin{aligned} \epsilon &= 8.9 \times 10^{-4}, \\ r &= 0.014, \\ n_r &= 4.8 \times 10^{-3}. \end{aligned} \tag{6.7}$$

For this point in parameter space, a primordial gravitational wave signal could be within the reach of future CMB polarization measurements. With the same value of x_f but with $v = M_P/3$ rather than $M_P/2$, r decreases to 0.0066. We have assumed that the cutoff of the theory is M_P, where quantum gravity effects are expected to become strong, rather than the reduced Planck mass $M_* = M_P/\sqrt{8\pi}$ that normalizes the gravitational coupling. If we assume $v = M_*/2$ with the same value of x_f as above, we obtain $r = 0.00061$. For comparison, the upcoming primordial inflation polarization explorer (PIPER) experiment expects a sensitivity to measure r as low as 0.007 [24].

[4]For this point in parameter space, there is a more substantial difference in the second significant digit between the exact results and those obtained using the small-ξ approximations in Eqs. (5.5)–(5.7). Hence, we show the exact results in this section.

VII. INFLATION SHUTOFF AND REHEATING

In this section, we consider the end of inflation and reheating. We first present estimates that indicate the end of inflation happens abruptly, less than a Hubble time after the χ fields develop vacuum expectation values.

Our estimates follow the arguments of Ref. [12]. Consider the evolution of the inflaton field θ during $\Delta t = H^{-1}$ after the critical time t_c, where Eq. (3.3) is an exact equality. At the very end of slow roll, $3H\dot{\theta} \approx -V'(\theta_f)$; hence the change in the inflaton field during the subsequent Δt is given by

$$\Delta\theta = -\frac{3M_P^2}{8\pi v}\frac{\xi\sin(3\theta_f/v)}{[1-\xi\cos(3\theta_f/v)]}. \tag{7.1}$$

At t_c, the χ mass matrix has a zero eigenvalue, so the magnitude of the negative mass squared term that emerges Δt later is determined by $\Delta\theta$. To assure a rapid evolution of the χ fields, we require that the magnitude of this negative squared mass is larger than H^2:

$$3|c_1|\frac{v^2}{M_P}\sin(3\theta_f/v)|\Delta\theta| > H^2, \tag{7.2}$$

which, in the notation of the previous section, leads to the inequality

$$\frac{27}{64\pi^2}|c_1|\xi(1-x_f^2)\frac{v}{M_P} > \frac{V_0}{M_P^4}. \tag{7.3}$$

(Here and below we work to lowest order in $\xi \ll 1$.) In addition, the nonzero χ VEVs after t_c generate a contribution to the θ mass squared which we also require to be greater than H^2:

$$9|c_1|\frac{v}{M_P}\langle\chi_1\chi_2\rangle > H^2, \tag{7.4}$$

which reduces to

$$\frac{27}{16\pi}\frac{c_1^2}{2\lambda+\lambda_{12}}(1-x_f)\left(\frac{v}{M_P}\right)^4 > \frac{V_0}{M_P^4}. \tag{7.5}$$

For the two points in parameter space studied in Secs. V and VI, respectively, we find numerically that the inequalities in Eqs. (7.3) and (7.5) are satisfied by between 4 and 6 orders of magnitude. This suggests that the fields will be driven to their global minimum sufficiently quickly, bringing inflation to an end.

The reheat temperature is sensitive to whether there is substantial preheating and depends on details of the couplings of the waterfall fields to matter, but for an estimate we assume reheating through a Higgs portal due to the quartic coupling,

$$V_{\chi^2H^2} = \frac{\lambda_{\chi H}}{2}(\chi_1^2+\chi_2^2)H^\dagger H \supset \frac{\lambda_{\chi H}}{2}\chi^2 H^\dagger H, \tag{7.6}$$

where the waterfall field $\chi \equiv (\chi_1-\chi_2)/\sqrt{2}$ oscillates during reheating about the minimum of $V(\theta,\chi_i)$, which was determined in Eq. (3.6). We neglect the mixing with the orthogonal combination of $(\chi_1+\chi_2)/\sqrt{2}$ and the inflaton field θ in this simplified analysis. The Higgs-portal coupling contains the term $\lambda_{\chi H}\langle\chi\rangle\chi H^\dagger H$, where $\langle\chi\rangle/\sqrt{2} = \langle\chi_1\rangle = -\langle\chi_2\rangle$, with the $\langle\chi_i\rangle$ determined by Eq. (3.6). This coupling leads to the χ decay rate

$$\Gamma_\chi = \frac{\lambda_{\chi H}^2\langle\chi_1\rangle^2}{4\pi m_{\chi\mathrm{eff}}}. \tag{7.7}$$

The effective χ mass at the minimum of the potential is given by

$$m_{\chi\mathrm{eff}}^2 = \frac{2c_1v^3}{M_P} - 2m_\chi^2. \tag{7.8}$$

In most scenarios the reheat temperature is within an order of magnitude of [25]

$$T_{rh} \sim \sqrt{M_P\Gamma_\chi} = \frac{\lambda_{\chi H}}{4}\sqrt{\frac{M_P m_{\chi\mathrm{eff}}}{(2\lambda+\lambda_{12})\pi}}. \tag{7.9}$$

With parameters as in Secs. V and VI we find a generically high reheat temperature[5] $T_{rh} \sim 10^{17}\lambda_{\chi H}$ GeV.

VIII. CONCLUSIONS

We have analyzed a model of inflation based on the non-Abelian discrete group S_3. The mass term and quartic self-coupling of a doublet of scalar fields preserve an accidental SO(2) symmetry. The SO(2) is spontaneously broken, giving rise to a pseudo-Goldstone boson which plays the role of the inflaton, as in natural inflation. After the inflaton rolls sufficiently, the coupling of the inflaton to two additional scalar fields generates an instability in a linear combination of those fields, ending inflation and reheating the universe as in hybrid inflation. We studied constraints on the model due to the slow-roll conditions, the requirement of at least 60 e-folds of inflation, the measured magnitude of cosmic density perturbations, the measured scalar spectral index and its running. The model has a viable parameter space with technically natural couplings

[5]Note that the coupling $\lambda_{\chi H}$ first affects the flatness of the inflaton effective potential at two loops, but only if a Planck-suppressed inflaton-Higgs coupling is present. Such a coupling can be taken small independently so that the range of $\lambda_{\chi H}$ is not restricted from this consideration. All other effects on the inflaton potential involving $\lambda_{\chi H}$ occur at three or more loops.

and can accommodate potentially observable power in tensor modes without super-Planckian field values during inflation, with $r \sim 0.01$.

Our work has been motivated in part by the minimality of S_3, which is the smallest possible non-Abelian discrete gauge group. However, it also is worth pointing out that the group S_3 has been used successfully in flavor model building [17]. Such models include substantial scalar sectors (the flavons) that are restricted by the discrete symmetry. It would be interesting in future work to see if the model described here could be incorporated into the flavor-symmetry-breaking sector of a flavor model involving S_3 symmetry. In addition, the present model was constructed in a nonsupersymmetric framework, for the sake of simplicity. A study of a supersymmetric S_3 natural hybrid inflation model, which would also stabilize the electroweak scale, will be discussed elsewhere [26].

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