Constraints on a charge in the Reissner-Nordström metric for the black hole at the Galactic Center

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Using an algebraic condition of vanishing discriminant for multiple roots of fourth-degree polynomials, we derive an analytical expression of a shadow size as a function of a charge in the Reissner-Nordström (RN) metric [1,2]. We consider shadows for negative tidal charges and charges corresponding to naked singularities $q = \mathcal{Q}^2/M^2 > 1$, where $\mathcal{Q}$ and M are black hole charge and mass, respectively, with the derived expression. An introduction of a negative tidal charge q can describe black hole solutions in theories with extra dimensions, so following the approach we consider an opportunity to extend the RN metric to negative $\mathcal{Q}^2$, while for the standard RN metric $\mathcal{Q}^2$ is always non-negative. We found that for $q > 9/8$, black hole shadows disappear. Significant tidal charges $q = -6.4$ (suggested by Bin-Nun [3–5]) are not consistent with observations of a minimal spot size at the Galactic Center observed in mm-band; moreover, these observations demonstrate that a Reissner-Nordström black hole with a significant charge $q \approx 1$ provides a better fit of recent observational data for the black hole at the Galactic Center in comparison with the Schwarzschild black hole.

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I. INTRODUCTION

Soon after the discovery of general relativity (GR), the first solutions corresponding to spherical symmetric black holes were found [1,2,6]; however, initially people were rather sceptical about possible astronomical applications of the solutions corresponding to black holes [7] (see also, for instance, one of the first textbooks on GR [8]). Even after an introduction to the black hole concept by Wheeler [9] (he used the term in his public lecture in 1967 [10]), we did not know too many examples where we really need GR models with strong gravitational fields that arise near black hole horizons to explain observational data. The cases where we need strong field approximation are very important since they give an opportunity to check GR predictions in a strong field limit; therefore, one could significantly constrain alternative theories of gravity.

One of the most important options to test gravity in the strong field approximation is analysis of relativistic line shape as it was shown in [11], with assumptions that a line emission is originated at a circular ring area of a flat accretion disk. Later on, such signatures of the Fe $K\alpha$ line have been found in the active galaxy MCG-6-30-15 [12]. Analyzing the spectral line shape, the authors concluded the emission region is so close to the black hole horizon that one has to use Kerr metric approximation [13] to fit observational data [12]. Results of simulations of iron $K\alpha$ line formation are given in [14,15] (where we used our approach [16]); see also [17] for a more recent review of the subject.

Now there are two basic observational techniques to investigate a gravitational potential at the Galactic Center, namely, (a) monitoring the orbits of bright stars near the Galactic Center to reconstruct a gravitational potential [18] (see also a discussion about an opportunity to evaluate black hole dark matter parameters in [19] and an opportunity to constrain some class of an alternative theory of gravity [20]) and (b) measuring in mm band, with VLBI technique, the size and shape of shadows around the black hole, giving an alternative possibility to evaluate black hole parameters. The formation of retro-lensing images (also known as mirages, shadows, or "faces" in the literature) due to the strong gravitational field effects nearby black holes has been investigated by several authors [21–24].

Theories with extra dimensions admit astrophysical objects (supermassive black holes in particular) which are rather different from standard ones. Tests have been proposed when it would be possible to discover signatures of extra dimensions in supermassive black holes since the gravitational field may be different from the standard one in the GR approach. So, gravitational lensing features are different for alternative gravity theories with extra dimensions and general relativity.

Recently, Bin-Nun [3–5] discussed the possibility that the black hole at the Galactic Center is described by the tidal Reissner-Nordström metric which may be admitted by the Randall-Sundrum II braneworld scenario [25]. Bin-Nun suggested an opportunity of evaluating the black hole

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metric analyzing (retro-)lensing of bright stars around the black hole in the Galactic Center. Doeleman *et al.* evaluated a size of the smallest spot for the black hole at the Galactic Center with VLBI technique in mm-band [26] (see constraints done from previous observations [27]). Theoretical studies showed that the size of the smallest spot near a black hole practically coincides with shadow size because the spot is the envelope of the shadow [23,24,28]. As it was shown [23,24], measurements of the shadow size around the black hole may help to evaluate parameters of black hole metric [29]. Sizes and shapes of shadows are calculated for different types of black holes and gravitational lensing in strong gravitational field has been analyzed in a number of papers [34].

We derive an analytic expression for the black hole shadow size as a function of the tidal charge for the Reissner-Nordström metric. We conclude that observational data concerning shadow size measurements are not consistent with significant negative charges, in particular, the significant tidal charge $q = \mathcal{Q}^2/M^2 = -6.4$ [35], discussed in [3–5], where the author used slightly different notations, namely $q' = q/4$, is practically ruled out with a very high probability (the tidal charge is roughly speaking is far beyond 9σ confidence level). We also show a smaller shadow sizes in respect to estimates obtained with the Schwarzschild black hole model can be explained with the Reissner–Nordström metric with a significant charge. It was found a critical q value for shadow existence, namely for $q \le 9/8$, Reissner–Nordström black holes have shadows while for $q > 9/8$ the shadows do not exist. Interestingly, the same critical value is responsible for a qualitative different behavior of quasinormal modes for the scattering [36] and for existence of circular orbits of neutral test particles [37].

As J. A. Wheeler coined "Black holes have no hair": it means that a black hole is characterized by only three parameters, its mass M, angular momentum J and charge $\mathcal{Q}$ (see, e.g., [38,39] or [40] for a more recent review). Therefore, in principle, charged black holes can be formed, although astrophysical conditions that lead to their formation may look rather problematic. Nevertheless, one could not claim that their existence is forbidden by theoretical or observational arguments. Moreover, we will show below that observations give a hint about an existence of a significant charge, but its origin is not clear at the moment.

Charged black holes are also object of intensive studies in quantum gravity, since a static, spherically symmetrical solution of Yang-Mills-Einstein equations with fairly natural requirements on asymptotic behavior of the solutions gives a Reissner-Nordström metric [41]. Thus, the metric describes a spherically symmetric black hole with a color charge (and (or) a magnetic monopole). Later on, color charges have been found for rotating black holes as well [42].

II. BASIC EQUATIONS

The expression for the Reissner-Nordström metric in natural units ($G = c = 1$) has the form

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{\mathcal{Q}^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{\mathcal{Q}^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

Applying the Hamilton-Jacobi method to the problem of geodesics in the Reissner-Nordström metric, the motion of a test particle in the r coordinate can be described by following equation (see, for example, [38])

$$r^4(dr/d\lambda)^2 = R(r), \quad (2)$$

where λ is the affine parameter [38] and

$$R(r) = P^2(r) - \Delta(\mu^2r^2 + L^2),$$
$$P(r) = Er^2 - e\mathcal{Q}r,$$
$$\Delta = r^2 - 2Mr + \mathcal{Q}^2. \quad (3)$$

Here, the constants μ, E, L and e are associated with the particle, i.e., μ is its mass, E is energy at infinity, L is its angular momentum at infinity, and e is the particle's charge.

We shall consider the motion of uncharged particles ($e = 0$) below. In this case, the expression for the polynomial $R(r)$ takes the form

$$R(r) = (E^2 - \mu^2)r^4 + 2M\mu r^3 - (\mathcal{Q}^2\mu^2 + L^2)r^2 + 2ML^2r - \mathcal{Q}^2L^2. \quad (4)$$

Depending on the multiplicities of the roots of the polynomial $R(r)$, we can have three types of motion in the r coordinate [43]. In particular, by defining the outer event horizon as usual $r_+ = M + \sqrt{M^2 - \mathcal{Q}^2}$ [38], we have

(1) if the polynomial $R(r)$ has no roots for $r \ge r_+$, a test particle is captured by the black hole;
(2) if $R(r)$ has roots and $(\partial R/\partial r)(r_{\max}) \ne 0$ with $r_{\max} > r_+$ ($r_{\max}$ is the maximal root), a particle is scattered after approaching the black hole;
(3) if $R(r)$ has a root and $R(r_{\max}) = (\partial R/\partial r)(r_{\max}) = 0$, the particle now takes an infinite proper time to approach the surface $r = \text{const}$.

If we are considering a photon ($\mu = 0$), its motion in the r-coordinate depends on the root multiplicity of the polynomial $\hat{R}(\hat{r})$

$$\hat{R}(\hat{r}) = R(r)/(M^4E^2) = \hat{r}^4 - \xi^2\hat{r}^2 + 2\xi^2\hat{r} - \hat{\mathcal{Q}}^2\xi^2, \quad (5)$$

where $\hat{r} = r/M, \xi = L/(ME)$ and $\hat{\mathcal{Q}} = \mathcal{Q}/M$. Below we do not write the hat symbol for these quantities.

One could see from Eqs. (5) and (3) as well that the black hole charge may influence substantially the photon motion at small radii ($r \approx 1$), while the charge effect is almost negligible at large radial coordinates of photon trajectories ($r \gg 1$). In the last case we should keep in mind that the charge may cause only small corrections on photon motion.

III. DERIVATION OF SHADOW SIZE AS A FUNCTION OF CHARGE

Let us consider the problem of the capture cross section of a photon by a charged black hole. It is clear that the critical value of the impact parameter for a photon to be captured by a Reissner-Nordström black hole depends on the multiplicity root condition of the polynomial $R(r)$. This requirement is equivalent to the vanishing discriminant condition [44]. To find the critical value of impact parameter for Schwarzschild and RN metrics the condition has been used for corresponding cubic and quartic equations [45–47]. In particular, it was shown that for these cases the vanishing discriminant condition approach is more powerful in comparison with the procedure excluding $r_{\max}$ from the following system

$$R(r_{\max}) = 0, \tag{6}$$

$$\frac{\partial R}{\partial r}(r_{\max}) = 0, \tag{7}$$

as it was done, for example, by Chandrasekhar [33] (and earlier by Darwin [48]) to solve similar problems, because $r_{\max}$ is automatically excluded in the condition for vanishing discriminant.

Introducing the notation $\xi^2 = l$, $\mathcal{Q}^2 = q$, we obtain

$$R(r) = r^4 - lr^2 + 2lr - ql. \tag{8}$$

We remind basic algebraic definitions and relations. If we consider an arbitrary polynomial $f(X)$ with degree n,

$$f(X) = X^n + a_1X^{n-1} + \cdots + a_{n-1}X + a_n, \tag{9}$$

the elementary symmetric polynomials s_k have the following form, where $X_1, \ldots X_n$ are roots of the polynomial $f(X)$ [44],

$$s_k(X_1, \ldots X_n) = \sum_{1 \le i_1 < i_2 < \ldots < i_k \le n} X_{i_1}X_{i_2}\ldots X_{i_k}, \tag{10}$$

where $k = 1, 2, \ldots, n$. The symmetrical k-power sum polynomial p_k have the following expression

$$p_k(X_1, \ldots X_n) = X_1^k + X_2^k + \cdots + X_n^k, \quad \text{for } k \ge 0. \tag{11}$$

To express p_k through s_k one can use Newton's equations

$$p_k - p_{k-1}s_1 + p_{k-2}s_2 + \cdots + (-1)^{k-1}p_1s_{k-1} + (-1)^k k s_k = 0, \quad \text{for } 1 \le k \le n; \tag{12}$$

$$p_k - p_{k-1}s_1 + p_{k-2}s_2 + \cdots + (-1)^{n-1}p_{k-n+1}s_{n-1} + (-1)^n p_{k-n}s_n = 0, \quad \text{for } k > n. \tag{13}$$

We introduce the following polynomial

$$\Delta_n(X_1, \ldots X_n) = \prod_{1 \le i < j \le n} (X_i - X_j), \tag{14}$$

which can be represented as the Vandermonde determinant

$$\Delta_n(X_1, \ldots X_n) = \begin{vmatrix} 1 & 1 & \ldots & 1 \\ X_1 & X_2 & \ldots & X_n \\ \ldots & \ldots & \ldots & \ldots \\ X_1^{n-1} & X_2^{n-1} & \ldots & X_n^{n-1} \end{vmatrix}. \tag{15}$$

According to the discriminant Dis definition we have the $\mathrm{Dis}(s_1, \ldots, s_n)$ polynomial

$$\mathrm{Dis}(s_1, \ldots, s_n) = \Delta_n^2(X_1, \ldots X_n) = \prod_{1 \le i < j \le n} (X_i - X_j)^2, \tag{16}$$

one can find [44]

$$\mathrm{Dis}(s_1, \ldots s_n) = \begin{vmatrix} n & p_1 & p_2 & \ldots & p_{n-1} \\ p_1 & p_2 & p_3 & \ldots & p_n \\ p_2 & p_3 & p_4 & \ldots & p_{n+1} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ p_{n-1} & p_n & p_{n+1} & \ldots & p_{2n-2} \end{vmatrix}. \tag{17}$$

Clearly, that the vanishing discriminant condition is equivalent to an existence of multiple roots among roots $X_1, \ldots X_n$. We apply this technique for the quartic polynomial $R(r)$ in Eq. (8). So that the symmetric k-power polynomials for $n = 4$ have the form

$$p_k = X_1^k + X_2^k + X_3^k + X_4^k, k \ge 0. \tag{18}$$

The symmetric elementary polynomials for $n = 4$ have the form

$$\begin{aligned} s_1 &= X_1 + X_2 + X_3 + X_4, \\ s_2 &= X_1X_2 + X_1X_3 + X_1X_4 + X_2X_3 + X_2X_4 + X_3X_4, \\ s_3 &= X_1X_2X_3 + X_2X_3X_4 + X_1X_3X_4 + X_1X_2X_4, \\ s_4 &= X_1X_2X_3X_4. \end{aligned} \tag{19}$$

We calculate the discriminant of the family X_1, X_2, X_3, X_4

$$\mathrm{Dis}(s_1,s_2,s_3,s_4)=\begin{vmatrix}1&1&1&1\\X_1&X_2&X_3&X_4\\X_1^2&X_2^2&X_3^2&X_4^2\\X_1^3&X_2^3&X_3^3&X_4^3\end{vmatrix}^2=\begin{vmatrix}4&p_1&p_2&p_3\\p_1&p_2&p_3&p_4\\p_2&p_3&p_4&p_5\\p_3&p_4&p_5&p_6\end{vmatrix}. \quad (20)$$

Expressing the polynomials $p_k(1\le k\le 6)$ in terms of the polynomials $s_k(1\le k\le 4)$ and using Newton's equations we calculate the polynomials and discriminant of the family X_1,X_2,X_3,X_4 in roots of the polynomial $R(r)$; we obtain

$$\begin{aligned}&p_1=s_1=0,\qquad p_2=-2s_2,\qquad p_3=3s_3,\\&p_4=2s_2^2-4s_4,\qquad p_5=-5s_3s_2,\\&p_6=-2s_2^3+3s_3^2+6s_4s_2,\end{aligned} \quad (21)$$

where $s_1=0, s_2=-l, s_3=-2l, s_4=-ql$, corresponding to the polynomial $R(r)$ in Eq. (8). The discriminant Dis of the polynomial $R(r)$ has the form

$$\begin{aligned}\mathrm{Dis}(s_1,s_2,s_3,s_4)&=\begin{vmatrix}4&0&2l&-6l\\0&2l&-6l&2l(l+2q)\\2l&-6l&2l(l+2q)&-10l^2\\-6l&2l(l+2q)&-10l^2&2l^2(l+6+3q)\end{vmatrix}\\&=16l^3[l^2(1-q)+l(-8q^2+36q-27)-16q^3].\end{aligned} \quad (22)$$

The polynomial $R(r)$ thus has a multiple root if and only if

$$l^3[l^2(1-q)+l(-8q^2+36q-27)-16q^3]=0. \quad (23)$$

Excluding the case $l=0$, which corresponds to a multiple root at $r=0$, we find that the polynomial $R(r)$ has a multiple root for $r\ge r_+$ if and only if

$$l^2(1-q)+l(-8q^2+36q-27)-16q^3=0. \quad (24)$$

If $q=0$, we obtain the well-known result for a Schwarzschild black hole [38,39,49], $l_{\rm cr}=27$, or $\xi_{\rm cr}=3\sqrt{3}$ [where $l_{\rm cr}$ is the positive root of Eq. (24)]. If $q=1$, then $l=16$, or $\xi_{\rm cr}=4$, which also corresponds to numerical results given in paper [50]. The photon capture cross section for an extreme charged black hole turns out to be considerably smaller than the capture cross section of a Schwarzschild black hole. The critical value of the impact parameter, characterizing the capture cross section for a RN black hole, is determined by the equation

$$l_{\rm cr}=\frac{(8q^2-36q+27)+\sqrt{D_1}}{2(1-q)}, \quad (25)$$

where $D_1=(8q^2-36q+27)^2+64q^3(1-q)=-512(q-\frac{9}{8})^3$. It is clear from the last relation that there are circular unstable photon orbits only for $q\le\frac{9}{8}$ (see also results in [37] about the same critical value). Substituting Eq. (25) into the expression for the coefficients of the polynomial $R(r)$ it is easy to calculate the radius of the unstable circular photon orbit (which is the same as the minimum periastron distance). The orbit of a photon moving from infinity with the critical impact parameter, determined in accordance with Eq. (25) spirals into circular orbit. To find a radius of photon unstable orbit we will solve Eq. (7) substituting $l_{\rm cr}$ in the relation. From trigonometric formula for roots of cubic equation we have

$$r_{\rm crit}=2\sqrt{\frac{l_{\rm cr}}{6}}\cos\frac{\alpha}{3}, \quad (26)$$

where

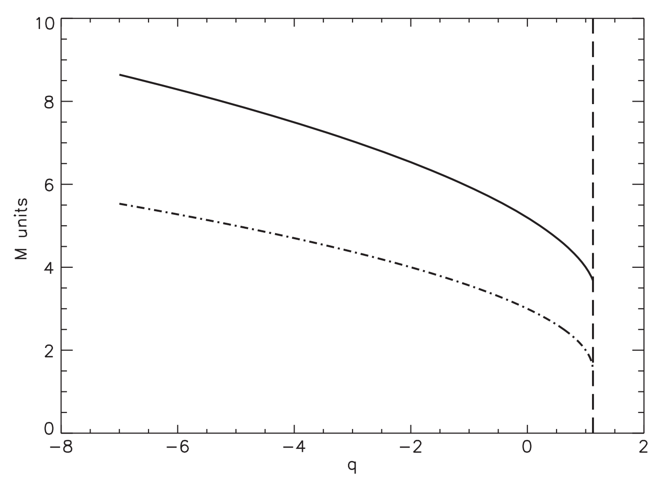

FIG. 1. Shadow (mirage) radius (solid line) and radius of the last circular unstable photon orbit (dot-dashed line) in M units as a function of q. The critical value $q=9/8$ is shown with dashed vertical line.

$$\cos\alpha = -\sqrt{\frac{27}{2l_{\mathrm{cr}}}}. \quad (27)$$

As it was explained in [24] this leads to the formation of shadows described by the critical value of ξ_{cr} or, in other words, in the spherically symmetric case, shadows are circles with radii ξ_{cr}. Therefore, by measuring the shadow size, one could evaluate the black hole charge in black hole mass units M. In Fig. 1 a shadow radius and a radius of last unstable orbit for photons as a function of q are given as a function of charge (including possible tidal charge with a negative q and superextreme charge $q > 1$).

IV. CONSEQUENCES

A. A disappearance of shadows for naked singularities

In spite of the cosmic censorship hypothesis [51] that a singularity has to be shielded by a horizon, properties of naked singularities are a subject of intensive theoretical studies. As usual spherical symmetrical cases are easier for analysis and RN metrics with super extreme charge $q > 1$ are investigated in a number of papers (see, for instance, [52] and references therein).

So, if we assume that $q > 1$, we can see from Eq. (25) that for $q \le 9/8$ we have shadows, while for $q > 9/8$ the shadows do not exist. For these charges ($q > 9/8$) incoming photons always scattering by black holes for $l \neq 0$ because the polynomial $R(r)$ has no multiple roots but it has a single positive root (it means scattering) since for great positive r we have $R(r) > 0$ while $R(0) < 0$. The degenerate case of radial trajectories of photons ($l = 0$) can be ignored as the case with "zero measure" or the structural unstable case using the Poincaré-Pontryagin-Andronov-Anosov-Arnold terminology [53]. It means that in any small vicinity a behavior of the geodesics other than the radial ones is qualitatively different; therefore, such objects cannot be observed in nature. Therefore, shadows exist only for $q \le 9/8$. So, $q = 9/8$ is critical value which is characterized "catastrophe" [54] or the qualitatively different behavior of the system (the appearance and the disappearance of shadows).

For the critical $q = 9/8$ we have the smallest shadow with $l = 27/2$ and a shadow size $\xi = \sqrt{13.5} \approx 3.674$ (in M units) or 37.5 μas in diameter for the black hole at the Galactic Center. For this impact parameter we have corresponding circular unstable orbit for photons with $r = 1.5$ (in M units).

B. Observational constraints on a charge of the black hole at the Galactic Center

If we adopt the distance toward the Galactic Center $d_* = 8.3 \pm 0.4$ kpc (or $d_* = 8.35 \pm 0.15$ kpc [55]) and mass of the black hole $M_{BH} = (4.3 \pm 0.4) \times 10^6 M_\odot$ [56,57] (a significant part of black hole mass uncertainty is connected with a distance determination uncertainty [57]), then we have the angle 10.45 μas for the corresponding Schwarzschild radius $R_g = 2.95 * \frac{M_{\mathrm{BH}}}{M_\odot} * 10^5$ cm roughly with 10% uncertainty of black hole mass and distance estimations, so a shadow size for the Schwarzschild black hole is around 53 μas, for a black hole with a tidal charge ($q = -6.4$) suggested by Bin-Nun [3–5] a shadow size is about 86.1 μas, while for the extreme charge ($q = 1$) and critical charge ($q = 9/8$) the shadow sizes are 40.9 μas and 37.5 μas, respectively. Uncertainties of angular shadow size evaluations are at a level around 10% which corresponds to an uncertainty of black hole mass evaluation.

C. Comparison with observations

A couple of year ago Doeleman *et al.* [26] claimed that the intrinsic diameter of Sgr A^* is 37^{+16}_{-10} μas at the 3σ confidence level. If we believe in GR and the central object is a black hole, then we have to conclude that a shadow practically coincides with the intrinsic diameter, so in spite of the fact that a Schwarzschild black hole is marginally consistent with observations, a Reissner-Nordström black hole provides much better fit of a shadow size, while a black hole with a significant tidal charge ($q = -6.4$) is out of a more 9σ level interval. Later on, the accuracy of intrinsic size measurements was significantly improved, so Fish *et al.* [58] gave $41.3^{+5.4}_{-4.3}$ μas (at 3σ level) on day 95, $44.4^{+3.0}_{-3.0}$ μas on day 96, and $42.6^{+3.1}_{-2.9}$ μas on day 97, so a tidal charge ($q = -6.4$) is out of 26σ level for day 95 and even less probable for other observations.

The black hole in the elliptical galaxy M87 looks also perspective to evaluate shadow size [59] (probably even its shape in the future to estimate a black hole spin) because the distance toward the galaxy is 16 ± 0.6 Mpc [60], black hole mass is $M_{M87} = (6.2 \pm 0.4) \times 10^9 M_\odot$ [61], so that an angle (7.3 ± 0.5) μas corresponds to the Schwarzschild radius [59], so the angle is comparable with the corresponding value considered earlier for our Galactic Center case. Recently, it was reported that smallest shadow size is $5.5 \pm 0.4 R_{\mathrm{SCH}}$ with 1σ errors (where $R_{\mathrm{SCH}} = 2GM_{M87}/c^2$) [59], so that at the moment the shadow size is consistent with the Schwarzschild metric for the object.

V. CONCLUSIONS

Based on observations [26,58] one can say that for the Schwarzschild black hole model we have tensions between evaluations of black hole mass done with observations of bright star orbits near the Galactic Center and the evaluated shadow size. To reduce tensions between estimates of the black hole mass and the intrinsic size measurements, one can use the Reissner-Nordström metric with a significant charge which is comparable with the critical one. We do not claim that the corresponding charge has an electric origin because an interstellar environment is electrically neutral, so the corresponding charge may be induced (like a tidal charge induced by extra dimension) and has a nonelectric origin. Charge estimates for the Reissner-Nordström metric given from geodesic trajectories for orbital motions are given in [62].

Recent estimates of the smallest structure in the M87 published in paper [59] do not need an introduction of charge (tidal or normal) to fit observational data because sizes of the smallest spot near the black hole at the object are still consistent with the shadow size evaluated for the Schwarzschild metric.

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