PHYSICAL REVIEW D **90,** 057501 (2014)

Modified model of top quark condensation

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(Received 20 May 2014; published 5 September 2014)

We develop the modification of the top-quark condensation scenario, in which the Higgs boson is composed of all Standard Model fermions. Within this scenario, we suggest the phenomenological model with nonlocal four-fermion interactions in which at the distances of the order of $\sim 1/100$ GeV the theory is represented in terms of only one Majorana spinor that carries the $U(12)$ index and, in addition, belongs to the spinor representation of $O(4)$. The Standard Model fermions are the components of this spinor. The symmetry $U(12) \otimes O(4)$ is responsible for the one-loop relation between the Higgs boson mass and the top-quark mass $M_H^2 = m_t^2/2$. At the distances $\gg 1/100$ GeV, the mentioned symmetry is broken, and the interaction term dominates that provides the nonzero mass of the top quark. Our phenomenological model should be considered in zeta or dimensional regularization. In conventional cutoff regularization, this is equivalent to the existence of the additional counterterms that cancel all quadratic divergences. As a result, the $1/N$ expansion may be applied.

DOI: 10.1103/PhysRevD.90.057501 PACS numbers: 14.65.Ha, 12.60.Fr, 12.60.-i

I. INTRODUCTION

The original idea of top-quark condensation [1–6] implied that the Higgs boson is composed of the top quark. In Ref. [7], we suggested the modification of this scenario, in which the 125 GeV h boson [8,9] is composed of all known quarks and leptons of the Standard Model (SM) (see also Ref. [10], where it was suggested that the Higgs boson is composed of the SM fermions). In our approach, the W and Z boson masses as well as the h-boson mass and SM fermion masses are determined by the condensate of the 125 GeV h boson according to the Higgs mechanism [11,12]. The important difference from the conventional models of top-quark condensation is that the new strong dynamics is rather complicated and is not described by the pointlike four-fermion interaction [13]. Unlike the conventional models, where the scale of the new dynamics is assumed to be at about $\Lambda \sim 10^{15}$ GeV, in our scenario, the scale of the new strong dynamics is supposed to be of the order of several TeV. The conventional top-quark condensation models typically predict the Higgs boson mass not very different from $2m_t \approx 350$ GeV and are excluded by the present experimental data [14]. It is worth mentioning that recently the modification of the top-condensation scenario was suggested [15], in which the 125 GeV Higgs boson appears as the pseudo-Goldstone boson. In this scenario, the value of the Higgs boson mass is suppressed naturally, but the inclusion of extra fermions is necessary.

In Ref. [7], it was shown that the nontrivial form factors for the interaction between the Higgs boson and the SM fermions are able to provide the composite Higgs boson mass $M_H = m_t/\sqrt{2} \approx 125$ GeV provided that *at the distances* $\sim 1/100$ GeV all SM fermions interact with the composite Higgs boson field in an equal way. Following this scenario, in the present paper, we suggest the particular model, in which all SM fermions are arranged within one Majorana spinor. This Majorana spinor carries the $U(12)$ index. In addition, it belongs to the four-dimensional spinor representation of $O(4)$. The fermions of different $O(4)$ chiralities may be transformed to each other with the emission of the 125 GeV Higgs boson. The Higgs boson in this model appears as the real - valued vector from the representation of $O(4) \simeq SU(2)_L \otimes SU(2)_R$. In the SM, only the $SU(2)_L$ component and the $U(1)_Y \subset SU(2)_R$ component of $O(4)$ are gauged. Using the $SU(2)_L$ transformations, we are able to bring the Higgs field to the simple form with only one nonvanishing real component (the analog of the unitary gauge of the Standard Model).

At distances much larger, than $1/100$ GeV, the interaction becomes more complicated. We consider the simplified scenario, in which the global symmetry $O(4) \otimes U(12)$ of the interaction between the composite Higgs boson and the fermions is broken, and one of the possible interaction terms dominates that provides the top-quark mass. The other fermions remain massless on this level of understanding. We assume that they acquire masses due to the perturbations above the considered pattern.

The low-energy effective model with the four-fermion interaction is not renormalizable. It is the effective theory only, and its output strongly depends on the regularization scheme. We imply the use of zeta regularization (see, for example, Refs. [16,17]) or dimensional regularization (see, for example, Ref. [18]) to give sense to the expressions for the observables. The $1/N$ expansion works good enough in the effective theory because of the chosen regularization. This is well known, that the ultraviolet divergences break the $1/N$ expansion in the Nambu-Jona-Lasinio (NJL) models defined in ordinary cutoff

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regularization (see, for example, Ref. [19]). However, in zeta regularization, the ultraviolet divergences do not appear at all, while in dimensional regularization, only the logarithmic divergences appear while the quadratic ones are absent. This is the reason why the leading-order $1/N$ approximation to various quantities does gives the reasonable estimates. The zeta/dimensionally regularized NJL model is equivalent to the NJL model in ordinary cutoff regularization with the additional counterterms that cancel all quadratic divergences. As a result of this subtraction, in particular, the sign of the four-fermion coupling constant is formally changed. For the relation between the values of this coupling constant before and after the subtraction, see, for example, Ref. [20], Appendix, Sec. 4.2. That is why the attractive four-fermion interaction in the bare Lagrangian of the zeta/dimensionally regularized model has the unusual sign (that naively looks like that of the repulsive interaction). There, presumably, exists the class of renormalizable theories with attractive interaction between the fermions that are approximated well by such NJL models defined in zeta/dimensional regularization. We imply that the theory standing behind the SM belongs to this class of theories. Therefore, while considering the four-fermion approximation to this unknown theory in the present paper, we rely on zeta/dimensional regularization. Notice that the existence of such theories, in which the quadratic divergences of the NJL approximation are to be subtracted, was mentioned in Ref. [21], in which this subtraction was related to the existence of a certain stability principle similar to that of the condensed matter theories in which the divergences in the vacuum energy of the hydrodynamic description are subtracted by the complete theory due to the thermodynamical stability of vacuum (see Refs. [22–25] and recent review [26]).

II. MODEL AT THE DISTANCES ~1/100 GeV: THE STANDARD MODEL FERMIONS AS THE COMPONENTS OF THE ONLY MAJORANA SPINOR

We adopt the notations used in Ref. [7]. For completeness, we describe them here briefly. Left-handed doublets and right-handed doublets of quarks are denoted by $L^{\mathbf{a}}_K$ and $R^{\mathbf{a}}_K$, where $\mathbf{a}$ is the generation index while K is the color index. The left-handed doublets and the right-handed doublets of leptons are $\mathcal{L}^{\mathbf{a}}$ and $\mathcal{R}^{\mathbf{a}}$, respectively. It will be useful to identify the lepton of each generation as the fourth component of the colored quark. Then, $L^{\mathbf{a}}_{a,4} = \mathcal{L}^{\mathbf{a}}_a$ and $R^{\mathbf{a}}_{a,4} = \mathcal{R}^{\mathbf{a}}_a$. So, later, we consider the lepton number as the fourth color in the symmetric expressions. We define the analog of the Nambu–Gorkov spinor

$$\mathbf{L}^{\mathbf{a}A}_{aiU} = \begin{pmatrix} L^{\mathbf{a}A}_{ai} \\ \bar{L}^{\mathbf{a}B}_{c'i}\epsilon_{c'a}\epsilon^{BA} \end{pmatrix}, \qquad \mathbf{R}^{\mathbf{a}A}_{aiU} = \begin{pmatrix} \bar{R}^{\mathbf{a}B}_{bi}\epsilon_{ba}\epsilon^{BA} \\ R^{\mathbf{a}A}_{a,i} \end{pmatrix},$$

where A is the usual spin index, U is the Nambu–Gorkov spin index ($U = 1$, 2 and $A = 1$, 2), i is the $SU(4)$ Pati–Salam color index (the lepton number is the fourth color), $\mathbf{a}$ is the generation index, and a, b are the $SU(2)_L$, $SU(2)_R$ indices. Both $\mathbf{R}^{\mathbf{a}A}_{aiU}$ and $\mathbf{L}^{\mathbf{a}A}_{aiU}$ for the fixed values of a, i, and $\mathbf{a}$ compose the four-component Dirac spinors $\mathbf{R}^{\mathbf{a}}_{ai}$ and $\mathbf{L}^{\mathbf{a}}_{ai}$. These spinors for the fixed value of a have $(N_c+1)\times N_g =$ 12 components. Both $\mathbf{R}$ and $\mathbf{L}$ belong to the fundamental representation of $U((N_c+1)\times N_g)$, where $N_c = 3$ is the number of colors and $N_g = 3$ is the number of generations. Notice, that $\mathbf{L}$ and $\bar{\mathbf{L}}$ ($\mathbf{R}$ and $\bar{\mathbf{R}}$) are not independent: $\bar{\mathbf{L}}^{\mathbf{a}}_{ai} = \epsilon_{ab}(\mathbf{L}^{\mathbf{a}}_{bi})^T i\gamma^2\gamma^5\gamma^0$, $\bar{\mathbf{R}}^{\mathbf{a}}_{ai} = \epsilon_{ab}(\mathbf{R}^{\mathbf{a}}_{bi})^T i\gamma^2\gamma^5\gamma^0$.

Next, we arrange the Dirac spinors $\mathbf{L}^{\mathbf{a}}_{ai}$, $\mathbf{R}^{\mathbf{a}}_{ai}$ in the $SO(4)$ spinor Ψ:

$$\Psi^{\mathbf{a}}_i = \begin{pmatrix} \mathbf{L}^{\mathbf{a}}_{ai} \\ \mathbf{R}^{\mathbf{a}}_{ai} \end{pmatrix}.$$

We introduce the Euclidean $SO(4)$ gamma matrices Γ^a (in chiral representation). The action of the SM gauge fields $e^{i\theta} \in U(1)_Y \subset SU(2)_R$, $U^{(L)} \in SU(2)_L$,

$$U^{(R)} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \in SU(2)_R,$$

and

$$V = \begin{pmatrix} Qe^{i\theta/3} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \in SU(4)_{\text{Pati Salam}} \subset U(12)$$

[where $Q \in SU(3)$] on the given Majorana spinor is

$$\Psi^{\mathbf{a}}_i \to \left(V_{ij}\frac{1+\Gamma^5\gamma^5}{2} + \bar{V}_{ij}\frac{1-\Gamma^5\gamma^5}{2}\right)\begin{pmatrix} U^{(L)} & 0 \\ 0 & U^{(R)} \end{pmatrix}\Psi^{\mathbf{a}}_j.$$

Thus, $U^{(L)}$, $U^{(R)}$ realize the representation of $O(4) \simeq SU(2)_L \otimes SU(2)_R$, while V realizes the representation of the subgroup $SU(4)$ of $U(12)$. The action of the element $R \in U(12)$ of the latter group on the spinor $\Psi^{\mathbf{a}}_i$ is $\Psi^{\mathbf{a}}_i \to (R^{\mathbf{ab}}_{ij}\frac{1+\Gamma^5\gamma^5}{2} + \bar{R}^{\mathbf{ab}}_{ij}\frac{1-\Gamma^5\gamma^5}{2})\Psi^{\mathbf{b}}_j$.

Again, Ψ and $\bar{\Psi}$ are not independent: $\bar{\Psi}^{\mathbf{a}}_i = (\Psi^{\mathbf{a}}_i)^T i\gamma^2\gamma^5\gamma^0\Gamma^4\Gamma^2\Gamma^5$. The partition function for the SM fermions in the presence of the SM gauge fields has the form $Z = \int D\Psi e^{iS}$. The action $S = S_K + S^{(4)}_I$ contains two terms. The first one is the kinetic term

$$S_K = \frac{i}{2}\int d^4x(\bar{\Psi}^{\mathbf{a}}_i\gamma^\mu\nabla_\mu\Psi^{\mathbf{a}}_i). \tag{1}$$

One can check that this term being written in terms of the original SM fermions is reduced to the conventional SM fermion action (without the mass term). Here, ∇_μ is the covariant derivative that includes the gauge field of the model.

$S_I^{(4)}$ is the four-fermion interaction term

$$S_I^{(4)} = \frac{1}{16M_I^2}\int d^4x(\bar{\Psi}_i^{\mathbf{a}}\gamma^5\Gamma^5\Gamma^K\Psi_i^{\mathbf{a}})(\bar{\Psi}_j^{\mathbf{b}}\gamma^5\Gamma^5\Gamma^K\Psi_j^{\mathbf{b}}). \tag{2}$$

The given four-fermion interaction describes the dynamics at the electroweak scale but already is not relevant at the energies much smaller than 100 GeV.

As usual, the auxiliary scalar field of the Higgs boson may be introduced. In our case, it appears in the form

$$\mathbf{H} = \sum_{K=1,2,3,4}\mathbf{h}_K\Gamma^K,$$

where $\mathbf{h}^K \in \mathcal{R}$. Thus, in our model, the Higgs boson is the four-component real vector that is transformed under the action of $O(4) \simeq SU(2)_L \otimes SU(2)_R$. As a result, we have the action that consists of three terms $S = S_K + S_I + S_H$, where

$$S_I = \frac{1}{2}\int d^4x(\bar{\Psi}_i^{\mathbf{a}}\gamma^5\Gamma^5\mathbf{H}\Psi_i^{\mathbf{a}}), \tag{3}$$

while the pure bare scalar field action is

$$S_H = -\int d^4x\frac{M_I^2}{4}\mathrm{Tr}\mathbf{H}^2. \tag{4}$$

It will be seen below that the valuable kinetic term for the scalar field arises dynamically through the integration over fermions. We may rewrite the interaction term as follows:

$$S_I = -\frac{1}{2}\int d^4x(\bar{L}_{bi}^{\mathbf{a}A}R_{ai}^{\mathbf{a}A}H_{ab} + \bar{R}_{ci}^{\mathbf{a}A}L_{c'i}^{\mathbf{a}A}\epsilon_{c'b}\epsilon_{ca}H_{ab} + (\mathrm{H.c.})). \tag{5}$$

Here, the scalar field is represented in the form

$$H_{ab} = \mathbf{h}^4\delta_{ab} + i\sum_{K=1,2,3}\mathbf{h}^K\tau_{ab}^K = HU_{\mathbf{h}}$$
$$U_{\mathbf{h}} = \left[\hat{\mathbf{h}}^4\mathbf{1} + i\sum_{K=1,2,3}\hat{\mathbf{h}}^K\tau^K\right]_{ab} \in SU(2),$$
$$H = \sqrt{\sum_{k=1,2,3,4}\mathbf{h}_k^2}, \qquad \hat{\mathbf{h}}^K = \frac{1}{H}\mathbf{h}^K, \tag{6}$$

where τ^K are the Pauli matrices. Using local $SU(2)_L$ transformation $L_{ai}^{\mathbf{a}A} \to [U_{\mathbf{h}}]_{ab}L_{bi}^{\mathbf{a}A}$, we fix the gauge, in which

$$\mathbf{H} = H\Gamma^4, \qquad H_{ab} = H\delta_{ab}, \qquad H = v + h \in \mathcal{R}, \tag{7}$$

where h is the real-valued field of the 125 GeV Higgs boson while v is the condensate. Since the obtained expressions are to be considered for the momenta of all particles involved of the order of M_H, we may omit the condensate in the interaction term (as it contributes to the zero momentum component $H_{p=0}$) and arrive at

$$S_I = -\int d^4x(\bar{L}_{1i}^{\mathbf{a}A}R_{1i}^{\mathbf{a}A}h + \bar{R}_{2i}^{\mathbf{a}A}L_{2i}^{\mathbf{a}A}h + (\mathrm{H.c.})).$$

In this form, the interaction term coincides with that of Ref. [7] written in the unitary gauge for small distances ($\sim 1/M_H$).

III. MODEL AT THE DISTANCES $\gg 1/100$ GeV: THE APPEARANCE OF THE TOP-QUARK MASS

We suppose that the interaction between the fermions and the composite Higgs boson of the form of Eq. (3) works for the distances $\sim 1/M_H$, i.e., for the momenta squared of all three participating fields (Higgs field and the two fermionic fields) $|p^2| \sim M_H^2$. At larger distances $\gg 1/M_H$, the global $U(12) \otimes O(4)$ symmetry is broken. In addition to the interaction term of the form of Eq. (2), in particular, the interaction term

$$S_I^{(4)\prime} = \frac{\alpha^2}{M_I^2}\int d^4x(\bar{L}_{aK}^{(tb)A}t_{R,K}^A)(\bar{t}_{R,N}^B L_{aN}^{(tb)B}), \tag{8}$$

is allowed, where α is a dimensionless constant.

$$L_{aK}^{(tb)A} = \begin{pmatrix} t_{L,K}^A \\ b_{L,K}^A \end{pmatrix}$$

and $t_{R,K}^A$ are the left-handed doublet of the top and bottom quarks and the right-handed singlet of the top quark. $K = 1, 2, 3$ is the color index. Equation (8) appears as a result of the integration over the Higgs boson field in the theory with action $S = S_K + S_H + S_I'$, where

$$S_I' = \frac{\alpha}{2}\int d^4x(\bar{\Psi}_K^{\mathbf{3}}\gamma^5\Gamma^5(\hat{\Pi}_+\mathbf{H} + \mathbf{H}\hat{\Pi}_-(\Psi_K^{\mathbf{3}}). \tag{9}$$

Here, $\hat{\Pi}_\pm$ is the projector that distinguishes the components of Ψ corresponding to the right-handed top quark t_R:

$$\hat{\Pi}_\pm = \frac{1 \pm \frac{1}{2i}[\Gamma^1, \Gamma^2]\gamma^5}{2} \times \frac{1 - \Gamma^5}{2}.$$

We assume that at the distances much larger than $1/M_H$ the interaction term of the form of Eq. (9) dominates, while at the distances of the order of $1/M_H$, the interaction term of Eq. (3) dominates. We are able to use the action that interpolates between the two,

$$S_I = \frac{1}{2}\int d^4xd^4yd^4z\bar{\Psi}_i^{\mathbf{a}}(x)\gamma^5\Gamma^5\mathbf{H}(z)\Psi_i^{\mathbf{a}}(y)G(x,y,z)$$
$$+\frac{1}{2}\int d^4xd^4yd^4z\bar{\Psi}_K^{\mathbf{3}}(x)\gamma^5\Gamma^5(\hat{\Pi}_+\mathbf{H}(z) + \mathbf{H}(z)\hat{\Pi}_-)$$
$$\times \Psi_K^{\mathbf{3}}(y)(\delta(x-z)\delta(y-z) - G(x,y,z)), \tag{10}$$

where the form factor G is introduced. We require that in momentum space it is given by $G(p,k,q) = \int dxdydzG(x,y,z)e^{ipx+iky+iqz} = (2\pi)^4\delta(q+p+k)g(p^2, k^2, q^2)$ with the function g that tends to 1 at $|p^2| \sim |q^2| \sim |k^2| \sim M_H^2$ and to zero if the absolute value of at least one of the three arguments is much smaller than $[100\text{ GeV}]^2$. For example, we may choose $G(p,k,q) = (2\pi)^4\delta(q+p+k)\times \frac{p^2}{p^2+M^2}\frac{k^2}{k^2+M^2}\frac{q^2}{q^2+M^2}$, where $M \ll 100$ GeV. In coordinate space, this form factor depends on three scalar parameters $W_1 = (x-z)^2$, $W_2 = (y-z)^2$, and $W_3 = (x-y)^2$. Function $G(x,y,z)$ is concentrated within the region $MW_1 \sim MW_2 \sim MW_2 \sim 1$ and decreases fast at $M|W_1|, M|W_2|, M|W_2| \to \infty$. It is worth mentioning that the gauge $(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y)$ is to be fixed to define the particular form of the function $G(x,y,z)$. Notice that we impose the requirement that the constant α of Eq. (8) is equal to unity. As a result the interaction between the top quark and the Higgs boson is given by Eq. (9) with $\alpha = 1$ at any distances. At the same time, the interaction of the other fermions with the Higgs boson is concentrated at the distances $\sim 1/100$ GeV.

In the following, we assume that the unitary gauge Eq. (7) that gives $H^{ab} = (v+h)\delta^{ab}$ is fixed, where v is the vacuum average of the scalar field H. We denote the Dirac four-component spinors in this gauge corresponding to the SM fermions by ψ. We omit angle degrees of freedom to be eaten by the gauge bosons. We take into account only the top-quark mass. It follows from Eq. (8) that $m_t = v$. The value of v is to be calculated using the gap equation that is the extremum condition for the effective action as a function of h. The action can be rewritten as

$$S = \int d^4x(\bar{\psi}(x)(i\partial\gamma - M)\psi(x) - \bar{\psi}(x)\hat{G}_h\psi(x) - M_I^2(v+h(x))^2), \quad (11)$$

where M is the mass matrix. It is diagonal, with the only nonzero component m_t. By G_h the h-depending operator is denoted. Its action is given by $[\hat{G}_h\xi](x_1) = \int d^4zd^4x_2\xi(x_2)G(x_1,x_2,z)h(z)$ for all fermions except for the top quark. For the top quark, $\hat{G}_h \equiv h$.

In Ref. [7], the theory with the action of the type of Eq. (11) was analyzed. The one-loop effective action for the Higgs boson is given by Eq. (3.5) of Ref. [7]. In our notations, we have

$$S[H] = \int d^4x\left(\frac{Z_h^2}{2}\mathrm{Tr}H^+(x)(-D^2w(D^2))H(x) - \frac{Z_h^2}{8}(|H|^2 - v^2)^2\right), \quad (12)$$

where $|H|^2 = \frac{1}{2}\mathrm{Tr}H^+H$, and H_{ab} is the 2×2 matrix in the representation of Eq. (6). Here, the gauge fields of the SM are already restored. As a result, the usual derivative ∂ of the field H is substituted by the covariant one $D = \partial + iA$ with the Standard Model $SU(2)_L \otimes U(1)_Y$ gauge field $A = (A^K_{SU(2)}\tau^K, A_{U(1)})$. Function w obeys

$$w(-p^2) \to 1(|p^2| \sim [90\text{ GeV}]^2);$$
$$w(-p^2) \to N_c/N_{\text{total}} = 1/8(|p^2| \ll [90\text{ GeV}]^2). \quad (13)$$

Constant Z_h is given by $Z_h^2 = \frac{N_{\text{total}}}{16\pi^2}\log\frac{\mu^2}{m_t^2}$.

Notice that in our case the effective action Eq. (12) is valid in the leading order in $1/N_{\text{total}} = 1/24$ for the energies $\gg 100$ GeV and in the leading order in $1/N_c = 1/3$ at low energies $\ll 100$ GeV. The expressions for various quantities in zeta regularization (or dimensional regularization) contain scale parameter μ. We identify this parameter with the typical scale of the interaction that is responsible for the formation of the composite Higgs boson. The vacuum value v of H satisfies gap equation $\frac{\delta}{\delta h}S[h] = 0$. It gives [7] $M_I^2 = -\frac{N_c}{8\pi^2}m_t^2\log\frac{\mu^2}{m_t^2} = -\frac{2N_c}{N_{\text{total}}}m_t^2Z_h^2$.

To derive the Higgs boson mass from Eq. (12), we should expand H_{ab} around its vacuum average $H_{ab} = (v+h)\delta_{ab}$ (the angular Goldstone modes and the gauge fields are to be disregarded). Then, up to the terms quadratic in h, we have the effective Lagrangian $L_h \approx Z_h^2h((-\partial^2)w(\partial^2) - m_t^2/2)h$. From Eq. (13), it follows that the propagator $(p^2w(-p^2) - m_t^2/2)^{-1}$ has the only pole at $p^2 = m_t^2/2$. This gives

$$M_H^2 \approx m_t^2/2 \approx 125\text{ GeV}. \quad (14)$$

To evaluate the gauge boson masses in the given model, we should substitute H in the form of Eq. (7) to Eq. (12) and cannot neglect nontrivial dependence of $w(\|A\|^2)$ on A (we denote $\|A\|^2 = (A^3_{SU(2)} - A_{U(1)})^2 + [A^1_{SU(2)}]^2 + [A^2_{SU(2)}]^2$). The typical value of A is given by M_Z. As a result, we may substitute $w(M_Z^2) \approx 1$ instead of $w(\|A\|^2)$. This gives the effective potential for the field A:

$$S_A \approx \frac{\eta^2}{2}\|A\|^2 \approx \frac{N_{\text{total}}}{16\pi^2}m_t^2\log\frac{\mu^2}{m_t^2}\|A\|^2.$$

That is why we arrive at the following expression for the renormalized vacuum average of the Higgs field η (which is to be equal to $\eta = 246$ GeV in order to provide the observed masses of the gauge bosons): $\eta^2 \approx 2Z_h^2v^2 = \frac{2N_{\text{total}}}{16\pi^2}m_t^2\log\frac{\mu^2}{m_t^2}$. From here, we obtain $\mu \sim 5$ TeV and $M_I^2 \approx -[90\text{ GeV}]^2$.

IV. CONCLUSIONS

We suggest that there is the hidden $U(12)\otimes O(4)$ symmetry behind the formation of the 125 GeV Higgs boson. It gives rise to the $U(12) \otimes O(4)$ symmetric four-fermion interaction term at the momenta transfer $\sim$100 GeV. The main output of the present paper is the *hypothesis* (which is supported by the considered model) that the ultraviolet completion of the Standard Model manifests itself already at the distances $\sim 1/M_H$, and the theory may be represented as the theory of only one Majorana spinor Ψ interacting with the Higgs boson field. The interaction between the fermions and the scalar field at the distances $\sim$1/100 GeV receives the form of Eq. (3).

In the opposite limit at the distances $\gg 1/100$ GeV, the interaction term of Eq. (9) with $\alpha = 1$ that provides the nonzero mass for the top quark dominates. The other fermions on this level of understanding are massless. We assume that their masses appear as the perturbations over the suggested pattern. Altogether, this construction provides the effective action, which gives rise to the relation between the Higgs boson mass M_H and the top-quark mass m_t of Eq. (14). It is worth mentioning that the requirement $\alpha = 1$ is essential for this relation to be valid. This prompts that there is, possibly, more symmetry behind the formation of the 125 GeV Higgs boson than is noticed in the present paper.

It is also worth mentioning that, according to the calculations of Sec. 3.C of Ref. [7], the particular form of the form factor G entering Eq. (10) may be chosen in such a way that the Higgs boson branching ratios and its production cross section calculated using the given model match the present experimental data.

ACKNOWLEDGMENTS

The author kindly acknowledges numerous discussions with V. A. Miransky and G. E. Volovik. The work is supported by the Natural Sciences and Engineering Research Council of Canada.

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