PHYSICAL REVIEW D **90,** 055023 (2014)

Inflation and majoron dark matter in the neutrino seesaw mechanism

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(Received 24 April 2014; published 24 September 2014)

We propose that inflation and dark matter have a common origin, connected to the neutrino mass generation scheme. As a model we consider spontaneous breaking of global lepton number within the seesaw mechanism. We show that it provides an acceptable inflationary scenario consistent with the recent cosmic microwave background B-mode observation by the BICEP2 experiment. The scheme may also account for the baryon asymmetry of the Universe through leptogenesis for reasonable parameter choices.

DOI: 10.1103/PhysRevD.90.055023 PACS numbers: 98.80.Cq, 14.60.Pq, 95.35.+d

I. INTRODUCTION

The need to account for neutrino mass [1,2] as well as cosmological issues such as the explanation of dark matter [3], inflation [4–6] and the baryon asymmetry [7] suggests that the standard model must be extended. The recent measurement by the BICEP2 experiment of the tensor-to-scalar ratio parameter $r = 0.20^{+0.07}_{-0.05}$ [8] of the primordial fluctuations of the cosmic microwave background (CMB) has caused tremendous interest, see for instance [9] and references therein. The possible discovery of gravity waves, if confirmed, would certainly count as one of the greatest in cosmology. Apart from such intrinsic significance, the measurement of nonzero r implies important constraints on inflationary models of the Universe. Here we consider the simplest type-I seesaw scenario [10–15][1] of neutrino mass generation in which lepton number is promoted to a spontaneously broken symmetry, within the standard $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ gauge framework [16,17]. In order to consistently formulate the spontaneous violation of lepton number within the $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ model, one requires the presence of a lepton-number-carrying complex scalar singlet, σ, coupled to the singlet "right-handed" neutrinos ν_R. The real part of σ drives inflation through a Higgs potential [18–22] while the imaginary part, which is the associated Nambu-Goldstone boson, is assumed to pick up a mass due to the presence of small explicit soft lepton number violation terms in the scalar potential, whose origin we need not specify at this stage. For suitable masses such a majoron can account for the dark matter [23], consistent with the CMB observations [24].

We show how, for reasonable parameter choices, this simplest scenario for neutrino masses provides an acceptable inflationary scenario. The scheme has also the potential to account for baryogenesis through leptogenesis. A previous attempt relating inflation to neutrinos can be found in [25] where a supersymmetric model was suggested in which the right-handed sneutrino drives chaotic inflation.

A. Preliminary considerations

Our model is the simplest type-I seesaw extension of the standard $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ model with a global lepton number symmetry. In addition to the standard model fields we add three generations of right-handed neutrinos and a complex singlet σ carrying two units of lepton number. The relevant invariant Yukawa interactions are

$$\mathcal{L}_y = -Y_D^{ij}\overline{\ell_L^j}\,\imath\tau_2\Phi^*\nu_R^i - \frac{1}{2}Y_N^i\sigma\overline{\nu_R^{ic}}\nu_R^i + \mathrm{H.c.}, \qquad (1)$$

where ℓ denotes the lepton doublet, Φ is the Higgs boson and τ_2 is the second Pauli matrix. After symmetry breaking characterized by the lepton number violation scale $v_\mathrm{L} = \langle\sigma\rangle$ [16,17] and the usual electroweak scale $\langle\Phi\rangle \equiv v_2$ the resulting seesaw scheme is characterized by singlet and doublet neutrino mass terms, described by

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & Y_D v_2 \\ {Y_D}^T v_2 & Y_N v_\mathrm{L} \end{bmatrix}, \qquad (2)$$

in the basis ν_L, ν_R.

The Yukawa coupling matrix Y_D generates the "Dirac" neutrino mass term, while Y_N gives the right-handed Majorana mass term. While the former is in principle

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[1]Note that in [15] this was called type II, just the opposite of what has become established.

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arbitrary, the matrix Y_N characterizing the coupling of σ to the right-handed neutrinos is symmetric and can be taken diagonal and with real positive entries without loss of generality. The effective light neutrino mass, obtained by perturbative diagonalization of Eq. (2) is of the form

$$m_\nu \simeq Y_D Y_N{}^{-1} Y_D{}^T \frac{v_2{}^2}{v_\mathrm{L}}. \tag{3}$$

This relation is consistent with tiny neutrino masses of order 10^{-1} electron volt. For example, assuming Y_D of $\mathcal{O}(1)$, one needs $v_\mathrm{L} \gtrsim 10^{14}$ GeV

$$Y_N \approx \frac{10^{14}\ \mathrm{GeV}}{v_\mathrm{L}}. \tag{4}$$

B. Scalar potential

We now turn to the dynamical justification of this scenario,[2] starting from the scalar potential. The tree level Higgs potential associated with the singlet and doublet scalar multiplets σ and Φ is a simple extension of that which characterizes the standard model,

$$V_\mathrm{tree} = \lambda\left(\sigma^\dagger\sigma - \frac{v_{\mathrm{L}^2}}{2}\right)^2 + \lambda_\mathrm{mix}(\sigma^\dagger\sigma)(\Phi^\dagger\Phi) + V_\Phi, \tag{5}$$

where V_Φ is the SM potential. As will become clear later, inflation and neutrino masses require that $\langle\sigma\rangle \gg \langle\Phi\rangle$. We also consider λ_mix to be negligible in order to use the small decay width approximation [22]. The inflaton is identified with the real part of σ

$$\rho \equiv \sqrt{2}\Re[\sigma], \tag{6}$$

and we parametrize the effective potential in the leading-log approximation, with the renormalization scale fixed at v_L, as [26]

$$V = \lambda\left[\frac{1}{4}(\rho^2 - v_\mathrm{L}^2)^2 + a \log\left[\frac{\rho}{v_\mathrm{L}}\right]\rho^4 + V_0\right], \tag{7}$$

where $a = \frac{\beta_\lambda}{16\pi^2\lambda}$ and the coefficient β_λ is given as

$$\beta_\lambda = 20\lambda^2 + 2\lambda\left(\sum_i (Y_N^i)^2\right) - \sum_i (Y_N^i)^4.$$
$$\simeq -\sum_i (Y_N^i)^4. \tag{8}$$

The last approximation $\lambda \ll Y_N$ will be justified later. An analysis of the potential reveals that $a \gtrsim -0.2$ ensures a consistent local minimum.

[2] For simplicity, we take a one-generation neutrino seesaw scheme with 0.1 eV mass scale in the analysis of our proposed inflationary scenario.

C. Inflation scenarios

Here we consider the radiatively corrected ρ^4 potential. Inflation takes places as the inflaton slowly rolls down to the potential minimum either from above ($\sigma > v_\mathrm{L}$) or from below ($\sigma < v_\mathrm{L}$). The inflationary slow-roll parameters are given by

$$\epsilon(\rho) = \frac{1}{2}M_P^2\left(\frac{V'}{V}\right)^2, \qquad \eta(\rho) = M_P^2\left(\frac{V''}{V}\right),$$
$$\zeta^2(\rho) = M_P^4\left(\frac{V'V'''}{V^2}\right), \tag{9}$$

where prime denotes a derivative with respect to ρ and $M_P = 2.4 \times 10^{18}$ is the (reduced) Planck mass. The slow-roll approximation is valid as long as the conditions $\epsilon, |\eta|, \zeta^2 \ll 1$ hold. In this case, the scalar spectral index n_s, the tensor-to-scalar ratio r, and the running of the spectral index α are given by

$$n_s \simeq 1 - 6\epsilon + 2, \qquad r \simeq 16\epsilon,$$
$$\alpha \equiv \frac{dn_s}{d\ln k} \simeq 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2. \tag{10}$$

The amplitude of the curvature perturbation $\Delta_\mathcal{R}$ is

$$\Delta_\mathcal{R}^2 = \frac{V}{24\pi^2 M_P^4 \epsilon}\bigg|_{k_0}, \tag{11}$$

and is taken as $\Delta_\mathcal{R}^2 = 2.215 \times 10^{-9}$ to fit PLANCK CMB anisotropy measurements [27], with the pivot scale chosen at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$. Finally, the number of e-folds realized during inflation is

$$N = \frac{1}{\sqrt{2}M_P}\int_{\rho_e}^{\rho_0} \frac{d\rho}{\sqrt{\epsilon(\rho)}}, \tag{12}$$

where ρ_0 is the field value that corresponds to k_0 and ρ_e denotes the value of ρ at the end of inflation, i.e. when $\epsilon(\rho_e) \approx 1$.

At this stage we have four parameters (Y_D, a, v_L and λ) for five observables (m_ν, r, n_s, α and $\Delta_\mathcal{R}^2$). Once we calculate ρ_e and ρ_0, λ is fixed from the constrain on $\Delta_\mathcal{R}^2$ and we find that $\lambda \approx 10^{-17} - 10^{-12}$ in the parameter space of the model, which justifies the approximation made in Eq. (8). We are then left with a (i.e. Y_N), Y_D and v_L and neutrino masses further constrain the relation between Y_N and Y_D. The predicted values of r, n_s and α are therefore predicted for fixed values of a and v_BL.

We will consider two limits: $v_\mathrm{L} > M_P$, the so-called Higgs inflation as well as $v_\mathrm{L} \ll M_P$ when the scalar potential considered in Eq. (7) reduces to the radiatively corrected quartic inflation [28].

1. Higgs inflation

This scenario requires trans-Planckian vevs. The seesaw relation, Eq. (4) imposes $Y_N \ll 1$ in order to suppress the

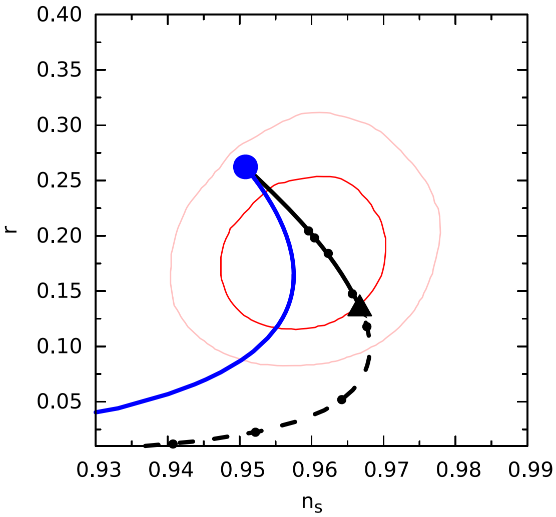

FIG. 1 (color online). Majoron inflation: The tensor-to-scalar ratio r is shown vs the spectral index n_s. Black line is the Majoron inflation scenario with $v_L > M_P$. The small black points on each branch, from left to right, indicate the values $v_L/M_P = 12, 14, 20$ and 100. The dashed branch corresponds to $\sigma < v_L$ and the solid one to $\sigma > v_L$. The point and the triangle are the quartic and quadratic inflation predictions, respectively. The blue (gray) line is for $v_L \ll M_P$. The contours are the 68% and 95% CL allowed region, combining PLANCK, WP, highL and BICEP2, given in [8] and N is taken to be 60.

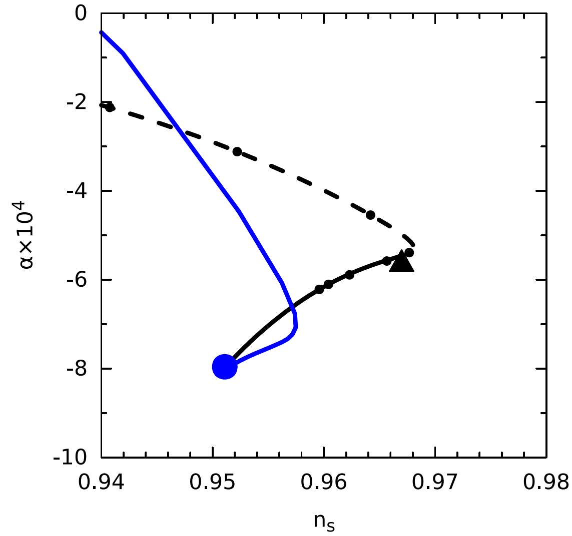

FIG. 2 (color online). Majoron inflation: α vs n_s for various v_{BL} values. See caption of Fig. 1 for more details.

right handed neutrino mass. For instance for $v_L = 10^3$ Mp, one gets $Y_N \approx 10^{-6}$, a value similar to the electron Yukawa coupling. The Coleman-Weinberg radiative corrections are negligible in this case and we consider only the tree level potential. Black lines in Fig. 1 show the predicted values of r and n_s obtained by varying v_L and taking the number of e-foldings $N = 60$. The allowed 68% and 95% CL contours are indicated. The dashed line is when the inflaton rolls from "below" ($\rho < v_{BL}$) while the solid one is for the opposite case. Both branches converge toward quadratic (indicated by a triangle) inflation in the limit $\rho \to \infty$, $(n_s, r) = (0.967, 0.132)$. We show various values of v_L as small circles. The small vev limit, depicted by a big circle corresponds to the textbook quartic inflation potential, $(n_s, r) = (0.951, 0.262)$. The running of the spectral index, α, is depicted in Fig. 2. In Fig. 3 we show the connection between inflation and neutrino masses, in the plane Y_N vs v_L. The black lines are upper bounds on Y_N for a given Dirac coupling Y_D. We also show some values of a corresponding to each Y_N and v_L for completeness. The numerical results for this case are displayed in Table I.

2. *Quartic inflation*

The sub-Planckian inflationary scenario $v_L \ll M_P$, in principle physically more attractive, is well approximated by the quartic potential. In this case, Y_N can be large so that the radiative corrections to the ρ^4 potential should be taken into account. The quantum corrections allow us to depart from the fixed textbook prediction of quartic inflation to lie closer to the BICEP2 region. Figure 1 and Fig. 2 show the effect of the coupling of the inflaton to right handed neutrinos on the inflationary observables. The blue line, departing from the quartic inflation prediction is obtained by varying a, and consequently Y_N in the range [−0.2, 0] corresponding to a variation of Y_N around $\approx 10^{-3}$. If v_L is taken to lie around 10^{14} GeV then $Y_N \approx 10^{-2}$ reproduces the correct neutrino mass scale. We display in Table II the numerical results for this case.

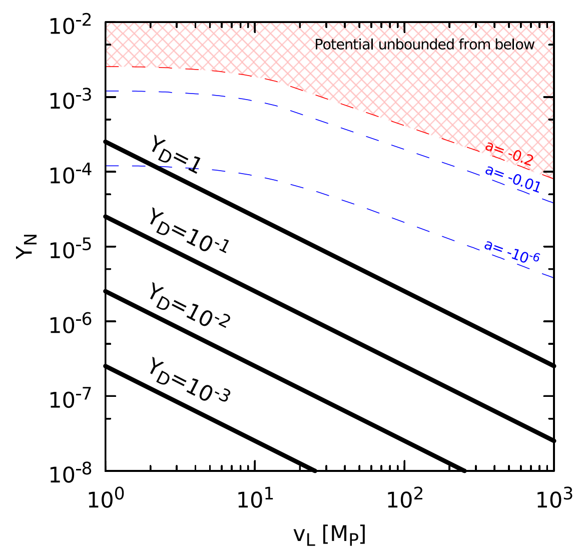

FIG. 3 (color online). Majoron inflation: Y_N vs v_L for various Y_D. Dashed lines show some values of the coefficient a of the Coleman-Weinberg term in the potential. Solid black lines are upper bounds on Y_N for the corresponding Dirac neutrino Yukawa coupling Y_D.

TABLE I. Higgs inflation scenario (no radiative corrections): The values of parameters for number of e-folds $N = 60$.

Solutions above the VEV ($\rho > v_L$)							
$v_L(M_P)$	$\log_{10}(\lambda)$	n_s	r	$\alpha(10^{-4})$	$V^{1/4}(10^{16}$ GeV)	$\rho_0(M_P)$	$\rho_e(M_P)$
1.	−12.8521	0.951168	0.260263	−7.96468	2.30678	22.2218	3.14626
5.	−13.0093	0.954908	0.237136	−7.05625	2.25373	24.2634	6.61037
10	−13.2351	0.958581	0.211972	−6.37463	2.1914	28.1285	11.5137
20.	−13.599	0.962148	0.184081	−5.89025	2.11546	37.1396	21.4642
50.	−14.2262	0.964453	0.159253	−5.80242	2.04021	66.1458	48.6058
100	−14.7789	0.965456	0.147557	−5.72255	2.00167	115.805	98.5958
500.	−16.1392	0.966211	0.137189	−5.66368	1.96554	515.506	498.588
1000.	−16.7367	0.9663	0.135828	−5.6565	1.96065	1015.47	998.587

Solutions below the VEV ($\rho < v_L$)							
$v_L(M_P)$	$\log_{10}(\lambda)$	n_s	r	$\alpha(10^{-4})$	$V^{1/4}(10^{16}$ GeV)	$\rho_0(M_P)$	$\rho_e(M_P)$
8.	−13.9086	0.87488	0.000385304	−0.150585	0.452484	0.111018	6.70982
9.	−13.5255	0.900769	0.00148882	−0.460638	0.6344	0.27599	7.69622
10.	−13.3033	0.918822	0.00377031	−0.949789	0.800289	0.541141	8.68529
15.	−13.1004	0.95579	0.0279442	−3.49461	1.32046	3.17548	13.6523
20.	−13.2562	0.964198	0.0518562	−4.54129	1.54118	7.05055	18.6357
30.	−13.5959	0.967596	0.0798131	−5.09597	1.71661	16.0451	28.6191
50.	−14.0675	0.96807	0.102141	−5.30133	1.8258	35.3404	48.6058
500.	−16.1213	0.966555	0.131662	−5.63496	1.94544	484.653	501.416
1000.	−16.7278	0.966472	0.133065	−5.64214	1.9506	984.613	1001.42

TABLE II. Radiatively corrected quartic potential: The values of parameters for number of e-folds $N = 60$.

Small solutions ($0.01 \lesssim r \lesssim 0.02$)								
a	$\|Y_N\|$	$\log_{10}(\|\lambda\|)$	n_s	r	$\alpha(10^{-4})$	$V^{1/4}(10^{16}$ GeV)	$\rho_0(M_P)$	$\rho_e(M_P)$
−0.01307	0.00135604	−11.7856	0.890248	0.0100493	7.9222	1.02256	15.1923	2.49121
−0.01305	0.00142537	−11.6983	0.899145	0.0137211	7.32328	1.10535	15.5053	2.49559
−0.01304	0.00145721	−11.6596	0.903321	0.0158434	6.92563	1.14582	15.6575	2.49774
−0.01303	0.00148709	−11.624	0.907307	0.0181559	6.47185	1.18552	15.8065	2.49987
−0.01302	0.00151498	−11.5914	0.911098	0.0206547	5.97014	1.22435	15.9522	2.50198

Large solutions ($0.1 \lesssim r \lesssim 0.2$)								
a	$\|Y_N\|$	$\log_{10}(\|\lambda\|)$	n_s	r	$\alpha(10^{-4})$	$V^{1/4}(10^{16}$ GeV)	$\rho_0(M_P)$	$\rho_e(M_P)$
−0.01279	0.00172752	−11.3556	0.952953	0.101404	−4.68889	1.82249	18.3706	2.54494
−0.01265	0.00167379	−11.4057	0.957019	0.141706	−6.48511	1.98152	19.1795	2.56674
−0.01261	0.00165322	−11.4258	0.957343	0.150727	−6.71294	2.01234	19.3554	2.57247
−0.01256	0.00162674	−11.4521	0.957507	0.160678	−6.9129	2.04476	19.5497	2.57934
−0.0125	0.00159495	−11.4843	0.957484	0.170937	−7.07347	2.07664	19.7519	2.5872
−0.0124	0.00154397	−11.5373	0.957174	0.184759	−7.2355	2.1174	20.0299	2.59943
−0.0123	0.00149676	−11.5877	0.956735	0.195481	−7.33264	2.14748	20.2527	2.61069
−0.0122	0.00145363	−11.635	0.956276	0.20395	−7.39978	2.17037	20.4349	2.62107
−0.0121	0.00141436	−11.679	0.95584	0.210759	−7.45154	2.18826	20.5865	2.63068
−0.0119	0.00134587	−11.7579	0.955081	0.220938	−7.53147	2.21422	20.8243	2.64788
−0.0116	0.00126256	−11.8579	0.954217	0.230944	−7.62064	2.23887	21.0753	2.66959

D. Dark matter and leptogenesis

In the limit where lepton number is an exact symmetry of the Lagrangian, lepton number violation is purely spontaneous so that the associated Nambu-Goldstone boson, i.e. the majoron, given as the imaginary part of σ, is strictly massless. However soft explicit lepton number violation may arise from a variety of sources, including quantum gravity effects [29,30]. Motivated by these considerations in fact the KeV majoron has been suggested as a viable dark matter candidate [23] much before the precise CMB observations from WMAP and PLANCK were available. Being a Goldstone boson associated with the spontaneous breaking of ungauged lepton number, the massive majoron will decay to a pair of neutrinos through a small coupling dictated by Noether's theorem to be proportional to the small neutrino mass [17]. The existence of this two–neutrino decay mode modifies the power spectrum of the cosmic microwave background temperature anisotropies [24]. One can determine the majoron lifetime and mass values required by the CMB observations in order for the majoron dark matter picture of the Universe to be consistent. It has been shown that the majoron provides an acceptable decaying dark matter scenario for suitably chosen mass values [31] which depend on whether or not the majorons are thermal or not. If the majoron production cannot be thermal, as it may be the case in the first inflationary scenario we considered, due to the smallness of the Y_N and λ_{mix} couplings, one can still consider nonthermal mechanisms such as freeze-in [32] or scalar field oscillations [33,34]. Moreover, in such nonthermal case, the mass of the majoron is not constrained to be of $\mathcal{O}$ (KeV) and can lie in a large range depending on the details of the mechanism under consideration.

Turning now to leptogenesis [35] we note that after spontaneous lepton number violation occurs at the scale v_{L} the type I seesaw mechanism is generated and the Universe reheats at the same time. The presence of right-handed neutrinos with direct couplings to the inflaton field is an important ingredient for leptogenesis [36].

II. CONCLUSIONS

We have suggested that neutrino masses, inflation and dark matter may have a common origin. We have illustrated this with the simplest type-I seesaw model with spontaneous breaking of global lepton number. The resulting inflationary scenario is consistent with the recent CMB B-mode observation by the BICEP2 experiment. On the other hand, the scheme may also account for majoron dark matter and possibly also leptogenesis induced through the out-of-equilibrium decays of the right-handed neutrinos, for reasonable parameter values. If supersymmetry is invoked, then one has a majoron version of the supersymmetric type I seesaw, in which lepton flavor violation processes may be within the reach of future experiments.

ACKNOWLEDGMENTS

The work of SB and JV was supported by MINECO Grant No. FPA2011-22975 and Multidark Consolider No. CSD2009-00064. S.M. thanks DFG Grant No. WI 2639/4-1. Q.S. acknowledges support provided by the DOE Grant No. DE-FG02-12ER41808 and thanks Jose Valle and members of the particle theory group for their hospitality.

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