# *CP* violation in bilinear *R*-parity violation and its consequences for the early universe

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Supersymmetric models with bilinear *R*-parity violation (BRpV) provide a framework for neutrino masses and mixing angles to explain neutrino oscillation data. We consider *CP* violation within the new physical phases in BRpV and discuss their effect on the generation of neutrino masses and the decays of the lightest supersymmetric particle (LSP), being a light neutralino with mass ~100 GeV, at next-to-leading order. The decays affect the lepton and via sphaleron transitions the baryon asymmetry in the early universe. For a rather light LSP, asymmetries generated before the electroweak phase transition via e.g. the Affleck-Dine mechanism are reduced up to 2 orders of magnitude, but are still present. On the other hand, the decays of a light LSP themselves can account for the generation of a lepton and baryon asymmetry, the latter in accordance with the observation in our universe, since the smallness of the BRpV parameters allows for an out-of-equilibrium decay and sufficiently large *CP* violation is possible consistent with experimental bounds from the nonobservation of electric dipole moments.

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## I. INTRODUCTION

The observed baryonic component of the universe comes along with the question of why the universe consists of entirely matter with hardly any primordial antimatter [1]. Defining the baryon and antibaryon number density  $n_B$  and  $n_{\bar{B}}$  and the entropy *s* at temperature *T*, the baryon asymmetry can be expressed in terms of the quantity

$$\delta_B = \frac{n_B - n_{\bar{B}}}{s}.\tag{1}$$

The value of  $\delta_B$ , which is consistent with the primordial abundances of light elements originating from big bang nucleosynthesis [2,3], is  $\delta_B = (6.19 \pm 0.14) \times 10^{-10}$  extracted from the measurements of the acoustic peaks in the cosmic microwave background (CMB) [4,5].

The dynamical creation of the baryon asymmetry in the universe (baryogenesis) requires the implementation of the three Sakharov conditions [6]: violation of baryon number *B*, C symmetry and *CP* symmetry violation and departure from thermal equilibrium. Nonperturbative effects (sphalerons) [7] can give rise to processes, which conserve B - L with *L* being the lepton number of involved particles, but violate B + L. Thus, a generated lepton asymmetry can account for the observed baryon asymmetry as well (baryogenesis via leptogenesis [8–10]), in particular since lepton asymmetries are hardly constrained by experiments [11–13].

In bilinear R-parity violation, where L violating parameters allow for the generation of neutrino masses and mixing, the decays of the lightest supersymmetric particle

(LSP) can thus affect the lepton and baryon asymmetries in the early universe after inflation. Whereas L violation is explicitly given by the bilinear *R*-parity violation (BRpV) parameters, we incorporate CP violation by complex phases for those parameters. Lastly, the LSP decay widths are small enough to be out of equilibrium, if the BRpV parameters are chosen in agreement with neutrino masses and mixing and the LSP is rather light, e.g.  $m_{\tilde{\chi}_1^0} \lesssim 100$  GeV. We study the evolution of the number densities by solving numerically the corresponding Boltzmann equations. On the one hand existing asymmetries e.g. induced by the Affleck-Dine mechanism [14] are reduced by up to 2 orders of magnitude, but are still present. On the other hand we demonstrate that *CP* violating LSP decays can generate lepton asymmetries. Before the electroweak phase transition the latter asymmetries can be partially transferred to baryon asymmetries via sphaleron transitions in accordance with the observation.

Earlier works on leptogenesis in the context of BRpV [15–19] made use of complex gaugino masses, leaving the *R*-parity violating parameters real. In those cases only small lepton asymmetries below  $10^{-10}$  can be generated, if the LSP is supposed to decay out of equilibrium. However, nonholomorphic terms in combination with fixed particle spectra [16] induce small enough decay widths for the neutralino to be out of equilibrium and allow for large enough *CP* asymmetries in the decay products being a charged Higgs boson and a lepton. As it was pointed in Ref. [15] lepton number violating decays can also spoil existing lepton asymmetries.

We focus on BRpV parameters, which are in agreement with the observations of neutrino oscillations. In accordance with the global fit carried out in Refs. [20–23] the preferred and in our analysis employed ranges of the oscillation parameters at  $2\sigma$  (for a normal neutrino mass hierarchy) are given by Ref. [22]<sup>1</sup>:

$$2.30 \times 10^{-3} \text{ eV}^2 \leq \Delta m_{31}^2 \leq 2.59 \times 10^{-3} \text{ eV}^2$$
  

$$7.15 \times 10^{-5} \text{ eV}^2 \leq \Delta m_{21}^2 \leq 8.00 \times 10^{-5} \text{ eV}^2$$
  

$$0.376 \leq \sin^2 \theta_{23} \leq 0.506,$$
  

$$0.275 \leq \sin^2 \theta_{12} \leq 0.342,$$
  

$$0.0197 \leq \sin^2 \theta_{13} \leq 0.0276.$$
  
(2)

The explanation of neutrino masses and mixing within BRpV was widely discussed in the literature; for reviews we refer to Refs. [24–26]. In BRpV it is well known that lowest order in perturbation theory is not sufficient to generate the full neutrino spectrum; however loop corrections can nicely explain the mass hierarchy between the neutrino mass eigenstates [27–37]. We shortly repeat the discussion, but focus mainly on complex phases in the BRpV parameters, which additionally induce a Dirac *CP* phase in the lepton/neutrino mixing matrix. *CP* violation in the partial decay widths of the LSP occurs at the one-loop level [38]. Our calculation of LSP decays at next-to-leading order (NLO) is based on Refs. [39,40].

The remainder of this paper is organized as follows: In Sec. II we explain the theory behind BRpV for complex BRpV parameters. This discussion includes the generation of neutrino masses and the calculation of the LSP decay at NLO in the electromagnetic coupling. Moreover we provide the basics of number density evolution in the early universe by the introduction of Boltzmann equations. Afterwards we shortly present a simple description of the transition between a lepton and baryon asymmetry via sphaleron transitions. In Sec. III we show our numerical results starting again with neutrino masses and mixing and the neutralino decays. In case of *CP* conserving BRpV initial asymmetries can be reduced, being up to 2 orders of magnitude lesser in size. For *CP* violation instead the neutralino decays themselves provide a large lepton asymmetry, which is partially transformed to a baryon asymmetry due to sphaleron transitions. In the last subsection we elaborate on the effects of LSP annihilation to SM particles in more detail. Finally we conclude in Sec. IV and present the implemented Boltzmann equations in the Appendix.

## II. BILINEAR *R*-PARITY VIOLATION AND *CP* VIOLATION THEREIN

For BRpV, which was first discussed in Refs. [41–45], the superpotential is given by

$$W = \varepsilon_{ab} [Y_U^{ij} \hat{Q}_i^a \hat{U}_j^c \hat{H}_u^b + Y_D^{ij} \hat{Q}_i^b \hat{D}_j^c \hat{H}_d^a + Y_E^{ij} \hat{L}_i^b \hat{E}_j^c \hat{H}_d^a - \mu \hat{H}_d^a \hat{H}_u^b + \epsilon_i \hat{L}_i^a \hat{H}_u^b], \qquad (3)$$

where  $Y_U$ ,  $Y_D$  and  $Y_E$  are the  $(3 \times 3)$  Yukawa matrices and  $\varepsilon_{\alpha\beta}$  is the complete antisymmetric SU(2) tensor with  $\varepsilon_{12} = 1$ , whereas *i*, *j* denote the three generations of leptons and quarks. The last terms  $\varepsilon_i$  explicitly break lepton number *L* and are similar to the parameter  $\mu$ , which determines the mass of the Higgsinos, given in units of mass. Additionally the three soft- SUSY breaking parameters  $B_i$  are added to the minimal supersymmetric standard model (MSSM) soft-breaking Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{soft}} &= M_Q^{ij2} \tilde{Q}_i^{a*} \tilde{Q}_j^a + M_U^{ij2} \tilde{U}_i \tilde{U}_j^* + M_D^{ij2} \tilde{D}_i \tilde{D}_j^* + M_L^{ij2} \tilde{L}_i^{a*} \tilde{L}_j^a + M_E^{ij2} \tilde{E}_i \tilde{E}_j^* + m_{H_d}^2 H_d^{a*} H_d^a + m_{H_u}^2 H_u^{a*} H_u^a \\ &- \frac{1}{2} [M_1 \tilde{B}^0 \tilde{B}^0 + M_2 \tilde{W}^\gamma \tilde{W}^\gamma + M_3 \tilde{g}^{\gamma'} \tilde{g}^{\gamma'} + \text{H.c.}] \\ &+ \varepsilon_{ab} [T_U^{ij} \tilde{Q}_i^a \tilde{U}_j^* H_u^b + T_D^{ij} \tilde{Q}_i^b \hat{D}_j^* H_d^a + T_E^{ij} \tilde{L}_i^b \tilde{E}_j^* H_d^a - B_\mu \mu H_d^a H_u^b - B_i \varepsilon_i \tilde{L}_i^a H_u^b + \text{H.c.}], \end{aligned}$$
(4)

where a summation over  $a, b \in \{1, 2\}, \gamma \in \{1, 2, 3\}$  and  $\gamma' \in \{1, ..., 8\}$  and the generation indices *i* and *j* is implied. The vacuum structure induces vacuum expectation values (VEVs) for the neutral components of the Higgs fields  $\langle H_u^0 \rangle = v_u/\sqrt{2}$  and  $\langle H_d^0 \rangle = v_d/\sqrt{2}$  as well as the sneutrinos  $\langle \tilde{\nu}_i \rangle = v_i/\sqrt{2}$ . The latter VEVs together with the last term in Eq. (3) result in a mixing between the gauge eigenstates of the neutralinos  $\tilde{B}, \tilde{W}_3^0, \tilde{H}_d^0$  and  $\tilde{H}_u^0$  and the three left-handed neutrinos  $\nu_i$  at tree level, providing an

effective Majorana mass term for the neutrinos at tree level [32,36,46]. Moreover the charginos mix with the charged leptons and the scalars, pseudoscalars and charged scalar states have to be combined with the sneutrinos and sleptons respectively.

To study the effects of *CP* violation in BRpV we closely follow Ref. [38] and allow for complex parameters

$$\epsilon_i = \epsilon_i^R + i\epsilon_i^I, \qquad B_i = B_i^R + iB_i^I, \qquad B_\mu = B_\mu^R + iB_\mu^I \quad (5)$$

in the superpotential Eq. (3) and the soft-breaking terms in Eq. (4). Our phase convention is such that the gaugino mass parameter  $M_2$  is real and positive. To simplify our model,  $\mu$ 

<sup>&</sup>lt;sup>1</sup>Similar values are found by the most recent global fit in Ref. [23].

### CP VIOLATION IN BILINEAR R-PARITY ...

and all other parameters in the soft-breaking Lagrangian are taken to be real, although additional complex phases are possible. In this way, the constraints on the electric dipole moments of electron, neutron and various atoms are satisfied if the parameters are chosen to fulfill neutrino data [38]. In case of *CP* violation scalars and pseudoscalars are indistinguishable. The resulting mass matrix is shown in Ref. [38]. We choose the VEVs  $v_d$ ,  $v_u$  and  $v_i$  to be real and determine the real and complex parts of  $B_{\mu}$  and  $B_i$  from the tadpole equations. Due to the real  $\mu$  parameter it yields  $B^I_{\mu} \propto \sum_i v_i \epsilon^I_i$  and  $B^I_i \propto \epsilon^I_i$ , such that the real BRpV model is restored in the limit  $\epsilon^I_i \rightarrow 0$ .

## A. Neutrino masses and mixing angles

In this subsection we discuss the neutralino sector of BRpV at tree level. We refer to Refs. [27-37,40] for studies related to neutrino masses in bilinear *R*-parity violation. If we make use of the basis

$$(\psi^0)^T = (\tilde{B}^0, \tilde{W}^0_3, \tilde{H}^0_d, \tilde{H}^0_u, \nu_1, \nu_2, \nu_3)$$
(6)

the mass matrices of the neutral fermions have the generic form

$$\mathcal{M}_{n}^{\text{tree}} = \begin{pmatrix} M_{H} & \hat{m} \\ \hat{m}^{T} & 0 \end{pmatrix} \tag{7}$$

and enter the Lagrangian density as follows:

$$\mathcal{L} \supset -\frac{1}{2} ((\psi^0)^T \mathcal{M}_n^{\text{tree}} \psi^0) - \frac{1}{2} ((\psi^0)^{\dagger} \mathcal{M}_n^{\text{tree}*} \psi^{0*}).$$
(8)

Therein the submatrix  $M_H$  is the usual MSSM neutralino mass matrix, whereas the submatrix  $\hat{m}$  includes the mixing with the left-handed neutrinos and contains the *R*-parity violating parameters. In detail the elements are

$$M_{H} = \begin{pmatrix} M_{1} & 0 & -\frac{1}{2}g'v_{d} & \frac{1}{2}g'v_{u} \\ 0 & M_{2} & \frac{1}{2}gv_{d} & -\frac{1}{2}gv_{u} \\ -\frac{1}{2}g'v_{d} & \frac{1}{2}gv_{d} & 0 & -\mu \\ \frac{1}{2}g'v_{u} & -\frac{1}{2}gv_{u} & -\mu & 0 \end{pmatrix},$$
$$\hat{m}^{T} = \begin{pmatrix} -\frac{1}{2}g'v_{1} & \frac{1}{2}gv_{1} & 0 & \epsilon_{1} \\ -\frac{1}{2}g'v_{2} & \frac{1}{2}gv_{2} & 0 & \epsilon_{2} \\ -\frac{1}{2}g'v_{3} & \frac{1}{2}gv_{3} & 0 & \epsilon_{3} \end{pmatrix}$$
(9)

with g and g' being the gauge couplings of  $SU(2)_L$  and  $U(1)_Y$  respectively. The mass eigenstates  $F_i^0$  are related to the gauge eigenstates  $\psi_s^0$  by  $F_i^0 = \mathcal{N}_{is}\psi_s^0$ , where the unitary matrix  $\mathcal{N}$  diagonalizes the full neutralino mass matrix  $\mathcal{M}_n$  in accordance with

$$\mathcal{M}_{n,\text{dia}} = \text{Diag}(m_{\tilde{\chi}_1^0}, \dots, m_{\tilde{\chi}_7^0}) = \mathcal{N}^* \mathcal{M}_n^{\text{tree}} \mathcal{N}^{\dagger}.$$
(10)

The second part of the Lagrangian density in Eq. (8) has to be diagonalized by  $\mathcal{NM}_n^{\text{tree*}}\mathcal{N}^T$  in case of *CP* violation. The mass eigenstates in Weyl notation can finally be built up to 4-component spinors by

$$\tilde{\chi}_i^0 = \begin{pmatrix} F_i^0 \\ F_i^{0\dagger} \end{pmatrix}.$$
(11)

The mixing of neutrinos with neutralinos gives rise to one massive neutrino at tree level. Its mass yields

$$m_{\nu_3} = \frac{g^2 M_1 + g'^2 M_2}{4 \det(M_H)} |\vec{\Lambda}|^2 \tag{12}$$

with the alignment parameter  $\Lambda_i = \mu v_i + \epsilon_i v_d$ . The atmospheric and the reactor mixing angle of the neutrinos can be expressed in terms of the alignment parameters  $\Lambda_i$  at tree level by

$$\tan^2 \theta_{23} = \left| \frac{\Lambda_2}{\Lambda_3} \right|^2, \qquad |U_{e3}|^2 \simeq \frac{|\Lambda_1|^2}{|\Lambda_2|^2 + |\Lambda_3|^2}.$$
(13)

At one-loop level  $|U_{e3}|$  can receive considerable corrections. In the complex case the absolute value of  $\Lambda_i$  is given by  $|\Lambda_i|^2 = |\mu v_i + v_d \epsilon_i^R|^2 + |v_d \epsilon_i^I|^2$ . Necessarily the size of  $|\epsilon_i^I| \sim |\Lambda_i| / v_d$  is fixed by the neutrino mass generated at tree level as shown in Eq. (12) and needs to be smaller than the value of  $\epsilon_i^R$  in the pure real case, where a cancellation between the two terms of  $\Lambda_i$  can be arranged. As a consequence neutrino data will restrict the size of possible complex phases  $\phi_i = \arctan(\epsilon_i^I / \epsilon_i^R)$ . This observation is in accordance with the discussion in Ref. [38], where the cancellation between the terms within  $\Lambda_i$  is used to constrain the complex phases. Setting the complex phase of  $M_2$  to zero allows for a phase  $\phi_{\mu}$  for the  $\mu$  parameter, which in turn permits larger complex phases for the parameters  $\epsilon_i$ . However,  $\phi_\mu$  is severely constrained by the nonobservation of electric dipole moments and within the allowed range for  $\phi_u$  the impact of this phase is small and does not lead to any new qualitative features.

In the following we discuss the effects of NLO corrections to the neutralino mass matrix, which allow for the explanation of the solar mass and mixing angle in accordance with Ref. [22] as well. Compared to existing work [27–37,40] we define  $\overline{DR}$  masses at NLO for the neutralino and neutrino sector in a slightly different way which is better suited for the study of *CP* violating effects. The fermionic self-energies can be decomposed as follows:

$$f_{j} = \Gamma_{ij} = \delta_{ij}(\not p - m_{fi}) + [\not p(P_L \hat{\Sigma}_{ij}^L(p^2) + P_R \hat{\Sigma}_{ij}^R(p^2))$$
(14)

$$+P_L \hat{\Sigma}_{ij}^{SL}(p^2) + P_R \hat{\Sigma}_{ij}^{SR}(p^2)], \qquad (15)$$

where the hat refers to  $\overline{\text{DR}}$  renormalized contributions and  $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$  are projection operators. They enter the Lagrangian density in the form  $-\frac{1}{2}\tilde{\chi}_i^0\Gamma_{ij}\tilde{\chi}_j^0$  including both terms of Eq. (8). In order to calculate  $\overline{\text{DR}}$  masses for the neutralinos, we have to respect that  $F_i$  and  $F_i^*$  are obtained from gauge eigenstates by  $\mathcal{N}$  and  $\mathcal{N}^*$  respectively. Taking the different rotations into account we define the  $\overline{\text{DR}}$  mass term to be added at NLO by

$$\mathcal{M}_n^{\text{tree}} \to \mathcal{M}_n^{\text{tree}} - \delta \mathcal{M}_n \quad \text{with}$$
 (16)

$$(\delta \mathcal{M}_n)_{ij}(p^2) = \sum_k \frac{1}{2} (\mathcal{M}_n^{\text{tree}})_{ik} (\mathcal{N}^{\dagger} \hat{\Sigma}^{R,T}(p^2) \mathcal{N})_{kj} + \frac{1}{2} (\mathcal{N}^T \hat{\Sigma}^R(p^2) \mathcal{N}^*)_{ik} (\mathcal{M}_n^{\text{tree}})_{kj} + \frac{1}{2} (\mathcal{N}^T \hat{\Sigma}^{SL,T}(p^2) \mathcal{N})_{ij} + \frac{1}{2} (\mathcal{N}^T \hat{\Sigma}^{SL}(p^2) \mathcal{N})_{ij}.$$
(17)

As we are dealing with Majorana particles, one finds

$$\begin{split} \Sigma_{ij}^{L}(p^{2}) &= \Sigma_{ji}^{R}(p^{2}), \qquad \Sigma_{ij}^{SL}(p^{2}) = \Sigma_{ji}^{SL}(p^{2}), \\ \Sigma_{ij}^{SR}(p^{2}) &= \Sigma_{ji}^{SR}(p^{2}) \end{split} \tag{18}$$

such that we are able to rewrite Eq. (17):

$$(\delta \mathcal{M}_n)_{ij}(p^2) = \sum_k \frac{1}{2} (\mathcal{M}_n^{\text{tree}})_{ik} (\mathcal{N}^{\dagger} \hat{\Sigma}^L(p^2) \mathcal{N})_{kj} + \frac{1}{2} (\mathcal{N}^T \hat{\Sigma}^R(p^2) \mathcal{N}^*)_{ik} (\mathcal{M}_n^{\text{tree}})_{kj} + (\mathcal{N}^T \hat{\Sigma}^{SL}(p^2) \mathcal{N})_{ij}.$$
(19)

Therein we perform the following replacement for the practical calculation

$$\hat{\Sigma}_{ij}(p^2) \to \frac{1}{2}(\hat{\Sigma}_{ij}(m_i^2) + \hat{\Sigma}_{ij}(m_j^2)).$$
 (20)

Equation (19) is similar to the formulas in Refs. [47,48], generalized for the left-hand part of Eq. (8). Adding the Goldstone tadpoles as done in Refs. [32,39,40] allows for the determination of gauge-independent neutralino/ neutrino masses at NLO. In principle the generalization of the on-shell neutralino and neutrino masses as shown in Ref. [40] is straightforward, but will not be presented in this context. Rather we point out the most important NLO contributions in the following: For large values of tan  $\beta$  *b*-(s)quark contributions are of quite importance. However, a major contribution always stems from loops involving a neutral or charged scalar, whereas loops with gauge bosons are less dominant.

The unitary matrix  $\mathcal{N}$ , which diagonalizes the neutralino mass matrix, contains the block mixing the neutrino generations, which, together with the leptonic block in the chargino mixing matrices, forms the Pontecorvo-Maki-Nakagawa-Sakata (PMNS)  $U^{l\nu}$  matrix [49]. The latter contains at least one *CP* violating phase, namely the Dirac phase  $\delta$ , which enters the Jarlskog invariant [50] in the lepton/neutrino sector as follows:

$$J_{CP} = \operatorname{Im}(U_{23}^{l\nu}U_{13}^{l\nu*}U_{12}^{l\nu}U_{22}^{l\nu*}) = \frac{1}{8}\cos\theta_{13}\sin(2\theta_{12})\sin(2\theta_{23})\sin(2\theta_{13})\sin\delta.$$
(21)

We will demonstrate in Sec. III A that in our restricted range of phases  $|J_{CP}|$  can be sizable and close to the experimental bound.

#### **B.** Leptonic neutralino decays

We turn to the calculation of the decays of the lightest neutralino, which are dominated by L violating two-body decays for neutralino masses above  $m_W$ . We follow Refs. [39,40] regarding the evaluation of the decay widths. Even though the loop contributions, which generate eventually the lepton asymmetry, are finite, we do perform a complete one-loop analysis to ensure that the lifetime remains long enough such that the neutralino decays out of equilibrium. We start with a short discussion of the LO decay width, for which the relevant part of the Lagrangian density is given by

$$\mathcal{L} \supset \overline{\tilde{\chi}_{l}^{0}} \gamma^{\mu} (P_{L} O_{Llj}^{Z} + P_{R} O_{Rlj}^{Z}) \tilde{\chi}_{j}^{0} Z_{\mu} + (\overline{\tilde{\chi}_{l}^{-}} \gamma^{\mu} (P_{L} O_{Llj}^{W} + P_{R} O_{Rlj}^{W}) \tilde{\chi}_{j}^{0} W_{\mu}^{-} + \text{H.c.}) + \overline{\tilde{\chi}_{l}^{0}} (P_{L} O_{Llj}^{h^{0}} + P_{R} O_{Rlj}^{h^{0}}) \tilde{\chi}_{j}^{0} h^{0}.$$
(22)

It includes the coupling to the charged leptons  $l^{\pm}$ , which are part of the charginos  $\tilde{\chi}_l^{\pm}$ . The explicit form of the couplings can be taken from Ref. [40]. The widths for the channels involving a final state gauge boson can be written as follows:

$$\Gamma^{0} = \frac{\sqrt{\kappa(m_{i}^{2}, m_{o}^{2}, m_{V}^{2})}}{16\pi m_{i}^{3}} [(|O_{L}^{V}|^{2} + |O_{R}^{V}|^{2})f(m_{i}^{2}, m_{o}^{2}, m_{V}^{2}) - 6\operatorname{Re}(O_{L}^{V}O_{R}^{V*})m_{i}m_{o}]$$
(23)

with  $V \in \{W, Z\}$ , the masses of the mother (daughter) particle  $m_i$  ( $m_o$ ) and the functions

$$f(x, y, z) = \frac{x + y}{2} - z + \frac{(x - y)^2}{2z},$$
  

$$\kappa(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.$$
 (24)

In case of the channel with a final state Higgs boson the partial width is given by

$$\Gamma^{0} = \frac{\sqrt{\kappa(m_{i}^{2}, m_{o}^{2}, m_{h}^{2})}}{16\pi m_{i}^{3}} \left[ \frac{|O_{L}^{h^{0}}|^{2} + |O_{R}^{h^{0}}|^{2}}{2} (m_{i}^{2} + m_{o}^{2} - m_{h}^{2}) + 2\operatorname{Re}(O_{L}^{h^{0}}O_{R}^{h^{0}})m_{i}m_{o} \right].$$
(25)

The LO decay widths with leptons/neutrinos and antileptons/antineutrinos in the final state are identical. In order to observe CP violating effects with respect to the different final states we proceed as in Refs. [39,40] and calculate NLO contributions, which are all implemented in CNNDECAYS. The NLO decay widths can be written in the form

$$\Gamma^{1} = \Gamma^{0} + \frac{\sqrt{\kappa(m_{i}^{2}, m_{o}^{2}, m_{V}^{2})}}{16\pi m_{i}^{3}} \frac{1}{2} \sum_{\text{pol}} 2\text{Re}(M_{1}M_{0}^{\dagger}) \qquad (26)$$

with the tree-level amplitude  $M_0$  and the NLO amplitude  $M_1$ . The latter includes the NLO vertex corrections as well as the wave function corrections of in- and outgoing particles as discussed in Refs. [39,40]. For the decay  $\tilde{\chi}_1^0 \rightarrow l^{\pm} W^{\mp}$  real corrections by photon emission are added accordingly. For  $\tilde{\chi}_1^0 \rightarrow \nu(\bar{\nu})Z$  we distinguish neutrinos and antineutrinos as follows: We assign a lepton number +1(-1) to the left-handed (right-handed) part of the neutrino Dirac spinor, which due to the smallness of neutrino masses and the energies considered here is an extremely good approximation. For LSP masses above the lightest Higgs mass the decay channel  $\tilde{\chi}_1^0 \to \nu(\bar{\nu})h$  is relevant as well. We implemented the NLO virtual contributions for the latter decays to CNNDECAYS in order to estimate the CP asymmetry with respect to the different final states and find a similar asymmetry as in the decays involving heavy gauge boson final states.

In Fig. 1 we show the dominant NLO virtual contributions, which generate the *CP* asymmetry between the final states  $l^-W^+$  and  $l^+W^-$  in accordance with Ref. [38],  $\nu Z$  and  $\bar{\nu}Z$  as well as  $\nu h$  and  $\bar{\nu}h$ . In case of a light stau, the corresponding light stau loop contribution to final states containing an (anti)neutrino could be as important as the ones shown.

## C. Number density evolution via Boltzmann equations

Within this section we present the evolution of number densities in the universe at temperatures which correspond to energies around the electroweak scale. For a quantitative discussion we make use of Boltzmann equations, in which we take into account the decays as well as the inverse decays of the lightest neutralino. Moreover we add



FIG. 1. Dominant NLO virtual contribution generating a *CP* asymmetry between the different final states for (a)  $\tilde{\chi}_1^0 \rightarrow l^{\pm} W^{\mp}$ ; (b)  $\tilde{\chi}_1^0 \rightarrow \nu Z$ ; (c)  $\tilde{\chi}_1^0 \rightarrow \nu h$ .

*R*-parity conserving annihilation processes of the LSP, which are known to have an impact on the final particle densities [17]. Sphaleron transitions between baryon and lepton asymmetries are discussed in Sec. II D. However, we can neglect *R*-parity violating scattering processes changing the lepton number by one or two units, since those processes involve an intermediate neutrino or neutralino and are thus either suppressed by the small neutrino mass or a product of *R*-parity violating couplings. In addition *CP* violating scatterings affect final particle densities only slightly, if the neutralino density stays close to its equilibrium density as pointed out in Ref. [51]. Thus, the Boltzmann equations take the generic form

$$xH(x)\frac{dN_{\tilde{\chi}_{1}^{0}}}{dx} = -\sum_{i,j} \left[ \frac{K_{1}(x)}{K_{2}(x)} \left( N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} \to ij) - \frac{N_{i}N_{j}}{N_{i}^{eq}N_{j}^{eq}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(ij \to \tilde{\chi}_{1}^{0}) \right) + \hat{\sigma}(\tilde{\chi}_{1}^{0}\tilde{\chi}_{1}^{0} \to ij) \left( N_{\tilde{\chi}_{1}^{0}}^{2} - \frac{N_{i}N_{j}}{N_{i}^{eq}} N_{\tilde{\chi}_{1}^{0}}^{eq,2} \right) \right],$$

$$(27)$$

where *i*, *j* denote SM particles.  $K_1(x)$  and  $K_2(x)$  are modified Bessel functions, the parameter  $x = m_{\tilde{\chi}_1^0}/T$ denotes the inverse of the temperature and  $\Gamma$  is the usual decay width in the rest frame of the decaying particle.<sup>2</sup> Moreover we define the density  $N_i := N_i(x) = n_i(x)/s(x)$ per comoving volume element by the ratio of the particle density  $n_i(x)$  to the entropy s(x). The quantity  $\hat{\sigma}$  contains

<sup>&</sup>lt;sup>2</sup>We are using units where the Boltzmann constant  $k_B$  is set to 1.

the thermally averaged annihilation cross section  $\langle \sigma_{ij} v \rangle$  of the LSP [52]:

$$\hat{\sigma}(\tilde{\chi}_{1}^{0}\tilde{\chi}_{1}^{0} \to ij) = xH(x)\frac{m_{\tilde{\chi}_{1}^{0}}}{x^{2}}\sqrt{\frac{\pi g_{*}}{45}}M_{p}\langle\sigma_{ij}v\rangle$$
with  $H \coloneqq H(x) = \sqrt{\frac{4\pi^{3}g_{*}}{45}}\frac{m_{\tilde{\chi}_{1}^{0}}^{2}}{M_{p}}\frac{1}{x^{2}}.$  (28)

The latter formulas include the Planck mass  $M_p$  and the effective degrees of freedom  $g_*$ , which are taken as a function of x from the tabulated values in MICROMEGAS [53]. The thermally averaged cross section  $\langle \sigma_{ij} v \rangle$  can be calculated with the help of MICROMEGAS as the *R*-parity violating parameters are too small to impact on the MSSM annihilation cross sections.

Since on cosmological time scales the massive gauge bosons and the Higgs boson decay instantaneously, we directly elaborate the Boltzmann equations with the decay products, which assumes the validity of the narrow-width approximation. In turn Eq. (27) can be written in the form

$$xH\frac{dN_{\tilde{\chi}_{1}^{0}}}{dx} = -\frac{\mathbf{K}_{1}(x)}{\mathbf{K}_{2}(x)}\sum_{i,q,\bar{q}} \left[ N_{\tilde{\chi}_{1}^{0}}\Gamma(\tilde{\chi}_{1}^{0} \to \bar{\nu}_{i}Z)\mathrm{Br}(Z \to q\bar{q}) - \frac{N_{\bar{\nu}_{i}}}{N_{\nu}^{\mathrm{eq}}}\frac{N_{q}N_{\bar{q}}}{N_{q}^{\mathrm{eq}}}N_{\tilde{\chi}_{1}^{0}}^{\mathrm{eq}}\Gamma(\bar{\nu}_{i}Z \to \tilde{\chi}_{1}^{0})\mathrm{Br}(Z \to q\bar{q}) + \cdots \right] - \sum_{q,\bar{q}} \left[ \hat{\sigma}(\tilde{\chi}_{1}^{0}\tilde{\chi}_{1}^{0} \to q\bar{q}) \left( N_{\tilde{\chi}_{1}^{0}}^{2} - \frac{N_{q}N_{\bar{q}}}{N_{q}^{\mathrm{eq}}}N_{\tilde{\chi}_{1}^{0}}^{\mathrm{eq}} \right) \right] + \cdots,$$

$$(29)$$

where we have just presented the decay of the heavy gauge boson to a quark pair in combination with the LSP annihilation process to this specific final state. The complete set of formulas is given in the Appendix. To shorten our notation we sum up the generations of u- and d-type quarks and denote them  $q_1$  and  $q_2$  in our study. The Boltzmann equations for the number densities of the (anti)leptons, (anti)neutrinos and (anti)quarks are obtained similarly and presented in the Appendix as well.

#### D. Baryogenesis via leptogenesis

The lepton asymmetry can be transformed into a baryon asymmetry and vice versa via sphaleron transitions [9,10,17]. However, the sphaleron rate is dramatically suppressed for temperatures below the electroweak scale, thus after the electroweak phase transition. We follow Refs. [54,55], which discuss the sphaleron rate in the light of the recent Higgs discovery with  $m_H \sim 125 \text{ GeV}$  leading to a critical temperature of  $T_c = 159 \pm 1 \text{ GeV}$ . Due to the fast drop of the sphaleron rate for temperatures below  $T_c$ , we can safely assume that the baryon asymmetry decouples from the lepton

asymmetry at this temperature. Effects resulting from the transition region down to temperatures of  $m_{\tilde{\chi}_1^0} \sim$ 100 GeV are tiny with respect to our qualitative discussion. Nonetheless we implemented formulas (1.10) and (1.11) of Ref. [54] and split them accordingly to particles and antiparticles. For this purpose we define the lepton asymmetry  $\delta_N$  as a sum over neutrino and lepton flavors by

$$\delta_N = \sum_{i=1,2,3} N_{l_i^-} - N_{l_i^+} + N_{\nu_i} - N_{\bar{\nu}_i}$$
(30)

and accordingly the baryon asymmetry in the form

$$\delta_B = N_{q_1} - N_{\bar{q}_1} + N_{q_2} - N_{\bar{q}_2}.$$
(31)

Formulas (1.10) and (1.11) of Ref. [54] then read e.g.

$$xH\frac{d(N_{q_1} - N_{\bar{q}_1})}{dx} = \frac{\gamma(x)}{2}[\delta_B + \eta(x)\delta_N], \qquad (32)$$

$$xH\frac{d(N_{l_i^-} - N_{l_i^+})}{dx} = \frac{\gamma(x)}{6}[\delta_B + \eta(x)\delta_N].$$
 (33)

The function  $\gamma(x)$  incorporates the strength of the sphaleron transitions and thus drops rapidly to zero for  $T < T_c$ , i.e.  $x > m_{\tilde{\chi}_1^0}/T_c$ . The function  $\eta(x)$  determines the ratio of  $\delta_B$  and  $\delta_N$  for  $x < T_c/m_{\tilde{\chi}_1^0}$ . We use  $\eta(x) = 0.5$ , which is a reasonable approximation for our study. The corresponding results of this procedure are presented in Sec. III D.

#### **III. NUMERICAL RESULTS**

In this section we show numerical results obtained with the previously discussed formulas and tools. We first stick to the *CP* conserving case of BRpV, before discussing the effects of *CP* violation on lepton asymmetries and baryon asymmetries in the early universe. Our discussion is based on the following low-energy SUSY points: The soft-breaking masses are set diagonal to  $M_L = M_E = 1$  TeV and  $M_Q =$  $M_U = M_D = 1.5$  TeV (generation independent) and the gaugino masses are fixed to  $M_1 = 100$  GeV,  $M_2 =$ 400 GeV,  $M_3 = 1.5$  TeV, which also ensures compatibility with the latest ATLAS results [56]. We choose the softbreaking couplings to be  $A_b = -1$  TeV and  $A_\tau =$ -500 GeV. We finally define two points with small and large value of tan  $\beta$ , namely,

Scenario 
$$P_1$$
:  $\tan \beta = 5$ ,  $A_t = 3 \text{ TeV}$   
and Scenario  $P_2$ :  $\tan \beta = 35$ ,  $A_t = 2.5 \text{ TeV}$ . (34)

Similar to the lepton sector the soft-breaking masses of the squark sector are set diagonal. In both cases we set  $\mu = 1$  TeV and  $m_A = 370$  GeV resulting in a lightest



FIG. 2 (color online). Values of *R*-parity violating parameters as a function of  $\phi := \phi_1 = \phi_2 = \phi_3$  in degrees for scenario  $P_2$ , where in (a) we give  $|\epsilon_i|$  in GeV with i = 1 (black, circle), 2 (blue, square) and 3 (red, diamond) and in (b)  $v_i$  in  $10^{-3}$  GeV with the same coding.

SM-like Higgs *h* with mass  $m_h$  close to 125 GeV and a lightest neutralino with mass close to 105 GeV.<sup>3</sup>

#### A. Neutrino masses and mixing angles

Neutrino data provide a constraint on the possible phases of the parameters  $\epsilon_i$ , which can be easily understood by  $|\Lambda_i|^2 = |\mu v_i + v_d \epsilon_i^R|^2 + |v_d \epsilon_i^I|^2$ . As discussed in Sec. II A in the real BRpV a cancellation in the sum  $\mu v_i + v_d \epsilon_i$ allows us to explain the atmospheric neutrino mass scale at tree level on the one hand together with an explanation of the solar mass scale at the loop level thanks to sufficiently large  $\epsilon_i$  on the other hand [32,36]. As a consequence purely imaginary  $\epsilon_i$  for all generations at the same time are impossible. Moreover as  $v_d$  decreases with increasing  $\tan \beta$  we expect that larger phases are possible for larger values of  $\tan \beta$ .

As an example we give in Fig. 2 the adjustment of  $v_i$  and the absolute values of  $\epsilon_i$  as a function of the phases  $\phi_i =$  $\arctan\left(\frac{\epsilon_{i}^{I}}{\epsilon_{i}^{R}}\right)$  for the case  $P_{2}$ . Here and in the following we will take the phases of all three  $\epsilon_i$  to be equal to maximize the effects. The width of the bands reflects the experimental uncertainty of the neutrino data and the upper bound of the complex phases is given by the requirement to obtain correctly both neutrino mass scales at the same time. In principle one could get a somewhat larger range be adjusting the soft parameters in the sbottom and in the stau sector [32,36]. However, as no new features show up for larger values of the complex phases we do not pursue this road. This can also be seen by checking the Jarlskog invariant  $J_{CP}$ of the PMNS matrix defined in Eq. (21) which we show in Fig. 3. Taking the current neutrino data leads to an upper bound  $|J_{CP}| \le 0.040$  assuming a maximal Dirac phase. Note that we reach this bound in case of scenario  $P_1$ .

#### **B.** Decay of the LSP

The smallness of the neutrino masses and in turn the smallness of the BRpV parameters imply a small decay



FIG. 3.  $J_{CP}$  as a function  $\phi \coloneqq \phi_1 = \phi_2 = \phi_3$  in degrees for (a)  $P_1$  and (b)  $P_2$ .

<sup>&</sup>lt;sup>3</sup>For completeness we note that we have calculated the Higgs masses in the *R*-parity conserving limit including two-loop effects using SPHENO [57,58] as the complete formulas for the Higgs masses including *R*-parity violation are not known. However, this is an excellent approximation as the corresponding couplings are much smaller than the *R*-parity conserving ones.

TABLE I. Standard choice of real *R*-parity violating parameters for the three scenarios such that neutrino data [22] are correctly explained.

Scenario	$P_1$	$P_2$	$P_1'$
$\epsilon_1$ (GeV)	$3.12 \times 10^{-2}$	$2.21 \times 10^{-2}$	$2.62 \times 10^{-2}$
$\epsilon_2$ (GeV)	$3.13 \times 10^{-2}$	$2.22 \times 10^{-2}$	$2.62 \times 10^{-2}$
$\epsilon_3$ (GeV)	$-3.13 \times 10^{-2}$	$-2.22 \times 10^{-2}$	$-2.62 \times 10^{-2}$
$v_1$ (GeV)	$-1.45 \times 10^{-3}$	$-1.20 \times 10^{-4}$	$-1.23 \times 10^{-3}$
$v_2$ (GeV)	$-1.27 \times 10^{-3}$	$7.70 \times 10^{-6}$	$-1.05 \times 10^{-3}$
$v_3$ (GeV)	$1.71 \times 10^{-3}$	$3.40 \times 10^{-4}$	$1.43 \times 10^{-3}$

width of the neutralino, such that it decays out of equilibrium in the early universe. Additionally the decays come with displaced vertices in collider experiments [59–61] allowing an experimental verification of this feature at the LHC. Taking the real values for the *R*-parity breaking parameters as provided in Table I we find for scenario  $P_1$  at NLO

$$\Gamma(\tilde{\chi}_1^0 \to e^{\pm} W^{\mp}) = 3.36 \times 10^{-16} \text{ GeV},$$
 (35)

$$\Gamma(\tilde{\chi}_1^0 \to \mu^{\pm} W^{\mp}) = 1.31 \times 10^{-14} \text{ GeV}$$
 (36)

$$\Gamma(\tilde{\chi}^0_1 \to \tau^{\pm} W^{\mp}) = 1.44 \times 10^{-14} \text{ GeV},$$
 (37)

$$\Gamma(\tilde{\chi}_1^0 \to \nu_3(\bar{\nu}_3)Z) = 3.27 \times 10^{-15} \text{ GeV}$$
 (38)

and for scenario  $P_2$  accordingly

$$\Gamma(\tilde{\chi}_1^0 \to e^{\pm} W^{\mp}) = 3.07 \times 10^{-16} \text{ GeV},$$
 (39)

$$\Gamma(\tilde{\chi}_1^0 \to \mu^{\pm} W^{\mp}) = 7.71 \times 10^{-15} \text{ GeV}$$
 (40)

$$\Gamma(\tilde{\chi}^0_1 \to \tau^{\pm} W^{\mp}) = 1.04 \times 10^{-14} \text{ GeV},$$
 (41)

$$\Gamma(\tilde{\chi}_1^0 \to \nu_3(\bar{\nu}_3)Z) = 2.35 \times 10^{-15} \text{ GeV}.$$
 (42)

The final states  $e^{\pm}W^{\mp}$  have considerably smaller decay widths than the others, the reason being the generation of the atmospheric scale of neutrino mixing at tree level. The decays in the two lightest neutrino flavors are both vanishing at tree level [62], but also at loop level. Due to  $\Gamma < H(T = m_{\tilde{\chi}_1^0}) \approx 2 \times 10^{-14}$  GeV the decay of the lightest neutralino at the temperature  $T = m_{\tilde{\chi}_1^0}$  occurs out of equilibrium. Further details with respect to the out-of-equilibrium decay can be found in the subsequent section. We emphasize that an outof-equilibrium decay only occurs for a light neutralino  $m_{\tilde{\chi}_1^0} \lesssim 100$  GeV. For heavier neutralinos  $\gtrsim 150$  GeV the decay widths start to exceed the Hubble parameter. Only a detailed description with Boltzmann equations can reveal the impact on lepton and baryon asymmetries.

Before presenting our results for the particle densities in the early universe, let us briefly mention the effects of the PHYSICAL REVIEW D 90, 055012 (2014)

complex phases  $\phi_i$  on the partial decay widths and the individual *CP* asymmetries, the latter being defined as

$$\delta_{\Gamma} = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} \tag{43}$$

with  $\Gamma^{\pm} = \Gamma(\tilde{\chi}_1^0 \to l^{\pm} W^{\mp})$  or  $\Gamma^{\pm} = \Gamma(\tilde{\chi}_1^0 \to \nu(\bar{\nu})Z)$ . For both scenarios  $P_1$  and  $P_2$  Fig. 4 shows the decay widths  $\Gamma^+$ of the LSP in GeV for the various decay channels. The *CP* asymmetry  $\delta_{\Gamma}$  is presented in Fig. 5 for both scenarios: The *CP* violation in the final state involving electrons is effectively of the same size as in the other decay channels, if one takes into account the smallness of the decay widths in this particular final state. Changing the sign  $\sin \phi_i \to$  $-\sin \phi_i$  for all  $i \in \{1, 2, 3\}$  yields  $\delta_{\Gamma} \to -\delta_{\Gamma}$ .

In order to estimate the size of the asymmetry in decays  $\tilde{\chi}_1^0 \rightarrow \nu(\bar{\nu})h$  we add a scenario  $P'_1$ , where the gaugino mass  $M_1$  is shifted from 100 GeV to 150 GeV, such that the decay channel into the light Higgs *h* opens. A possible set of *R*-parity violating parameters fulfilling neutrino data is added to Table I and Fig. 6 presents the corresponding *CP* asymmetries. The *CP* asymmetry in the final state  $\nu_3(\bar{\nu}_3)h$ 



FIG. 4 (color online). Decay widths  $\Gamma^+$  for the final states  $e^+W^-$  (black, circle),  $\mu^+W^-$  (blue, square),  $\tau^+W^-$  (red, diamond),  $\bar{\nu}_3 Z$  (green, triangle) as a function of  $\phi := \phi_1 = \phi_2 = \phi_3$  in degrees for (a)  $P_1$  and (b)  $P_2$ .



FIG. 5 (color online). *CP* asymmetry  $|\delta_{\Gamma}|$  for the final states  $e^{\pm}W^{\mp}$  (black, circle),  $\mu^{\pm}W^{\mp}$  (blue, square),  $\tau^{\pm}W^{\mp}$  (red, diamond),  $\nu_3(\bar{\nu}_3)Z$  (green, triangle) defined in Eq. (43) as a function of  $\phi := \phi_1 = \phi_2 = \phi_3$  in degrees for (a)  $P_1$  and (b)  $P_2$ .



FIG. 6 (color online). *CP* asymmetry  $|\delta_{\Gamma}|$  for the final states  $e^{\pm}W^{\mp}$  (black, empty circle),  $\mu^{\pm}W^{\mp}$  (blue, empty square),  $\tau^{\pm}W^{\mp}$  (red, filled diamond),  $\nu_3(\bar{\nu}_3)Z$  (green, filled triangle),  $\nu_1(\bar{\nu}_1)h$  (yellow, filled circle),  $\nu_2(\bar{\nu}_2)h$  (purple, filled square) and  $\nu_3(\bar{\nu}_3)h$  (brown, empty triangle) defined in Eq. (43) as a function of  $\phi := \phi_1 = \phi_2 = \phi_3$  in degrees for  $P'_1$ .

is of a similar size as in case of the final states with the gauge boson, whereas the ones in the first two neutrino mass generations are negligible. The  $\nu_i(\bar{\nu}_i)h$  final states have branching fractions comparable to  $\mu^{\pm}/\tau^{\pm}W^{\mp}$  final states and thus their inclusion for  $m_{\tilde{\nu}_i} > m_h$  is advisable.

#### C. Lepton asymmetries in the *CP* conserving BRpV

In this subsection we discuss the effect of the LSP decays on the number densities and thus the lepton and baryon asymmetries in the early universe and we start with the case of *CP* conserving BRpV. The small decay rates of the neutralino can have a sizable impact on lepton asymmetries, which could for example be generated at an earlier stage of the universe by the Affleck-Dine mechanism [14]. If for the moment we ignore neutrino data and choose all decay widths slightly larger than the Hubble parameter  $\Gamma > H(T = m_{\tilde{\chi}_1^0})$ , a sizable washout of initial lepton asymmetries can occur [63]. This is explicitly demonstrated in Fig. 7, where all widths are set to  $5 \times 10^{-14}$  GeV and the annihilation cross sections are taken from scenario  $P_2$ . For a light neutralino  $m_{\tilde{\chi}_1^0} < T_c$  initial baryon asymmetries are



FIG. 7. (a) Lepton asymmetry  $|\delta_N|$  as defined in Eq. (30) as a function of x for *CP* conserving BRpV for all widths set to  $5 \times 10^{-14}$  GeV for different asymmetries  $\delta_N$  at  $x = 10^{-2}$ ; (b) LSP density as a function of x for the same cases.

only mildly affected, since for  $T < T_c$  they are decoupled from the lepton asymmetries—see Sec. III D. For numerical stability we choose all initial particle densities equal to their equilibrium density with (small) displacements to establish the shown asymmetries. The presented effects are independent of the sign of the initial lepton asymmetry  $\delta_N$ . Figure 7(b) shows the behavior of the LSP number density, which follows the equilibrium density and thus motivates the neglect of scattering processes.

In accordance with Refs. [64,65] the washout of an initial asymmetry driven by the backreaction of leptons, quarks, neutrinos and their antiparticles to the LSP and the decays of the LSP itself stays small, if just one of the flavor final states is suppressed with respect to the others and thus decays out of equilibrium. To confirm this statement Fig. 8 shows the small washout for different initial lepton asymmetries for both scenarios  $P_1$  and  $P_2$  with fulfilled neutrino data. Initially present lepton and thus also baryon asymmetries are almost conserved, if neutrino data are explained by the BRpV parameters.

## D. Baryogenesis via leptogenesis in the *CP* violating BRpV

In this subsection we discuss the impact of *CP* violation by complex BRpV parameters  $e_i$  on the decay widths of the lightest neutralino and the lepton and baryon asymmetries in the early universe. Before doing so let us briefly comment on the stringent bounds coming from the nonobservation of electric dipole moments, in particular the one of the electron has to be below  $\leq 10^{-28}$  ecm [66]. As we only consider *CP* phases in the *R*-parity violating parameters, the corresponding effect is small and in case of the slepton and sneutrino contributions is further suppressed by their heavy masses. The potentially most troublesome are the  $\tilde{\chi}_j^0$ -W contributions which are proportional to  $\text{Im}(O_{\text{Lej}}^W(O_{\text{Rej}}^W)^*)$ . Using an expansion in the



FIG. 8 (color online). Lepton asymmetry  $|\delta_N|$  as defined in Eq. (30) as a function of x for *CP* conserving BRpV for  $P_1$  (black) and  $P_2$  (red, dashed) for different asymmetries  $\delta_N$  at  $x = 10^{-2}$ .

*R*-parity violating parameters [40,62] one finds that this product is tiny for several reasons: (i) it is proportional to the *R*-parity violation couplings squared, (ii) it is suppressed by a factor  $Y_E^{11}v_d/\min(\mu, M_2)$  and (iii) it vanishes completely in case of a pure bino. Numerically we find that the induced electron dipole moment is always below  $\mathcal{O}(10^{-32} \text{ ecm})$  in our examples.

For relatively large phases  $\phi_i$  and thus *CP* asymmetries up to per-mile level the *CP* violating contributions can have sizable effects on lepton asymmetries in the universe. If the initial lepton asymmetry  $\delta_N$  is large, the effect of the BRpV induced washout dominates—as discussed in the previous subsection. But for initial lepton asymmetries being rather small  $|\delta_N| < 10^{-5}$  the *CP* violating contributions to the LSP decays come into the game. They induce a lepton asymmetry of  $|\delta_N| \sim 10^{-5}-10^{-3}$ , if the complex phases  $\phi_i$ are chosen large ~1 degree. Details can be taken from Fig. 9 for scenario  $P_1$  choosing a phase of 5 degrees and scenario  $P_2$  with a phase of 15 degrees. Figure 10 shows the obtained lepton asymmetry in the universe as a function of the *CP* phases  $\phi_i$  in the BRpV parameters for



FIG. 9 (color online). Lepton asymmetry  $|\delta_N|$  as defined in Eq. (30) for (a)  $P_1$  with phase  $\phi := \phi_1 = \phi_2 = \phi_3 = 5$  degrees; (b)  $P_2$  with phase  $\phi := \phi_1 = \phi_2 = \phi_3 = 15$  degrees in both cases for different initial asymmetries  $\delta_N$  at  $x = 10^{-2}$ .



FIG. 10. Resulting lepton asymmetry  $|\delta_N|$  at  $x = 10^2$  as a function of  $\phi := \phi_1 = \phi_2 = \phi_3$  for  $P_2$  for zero initial asymmetry  $\delta_N = 0$  at  $x = 10^{-2}$ .

scenario  $P_2$ , if initially no lepton asymmetry is present,  $\delta_N(x = 10^{-2}) = 0.$ 

As pointed out in the previous section  $\sin \phi_i \rightarrow -\sin \phi_i$ for all  $i \in \{1, 2, 3\}$  results in  $\delta_{\Gamma} \rightarrow -\delta_{\Gamma}$ , which induces  $\delta_N \rightarrow -\delta_N$ , if no initial asymmetry is present. This statement implies that for different signs in  $\sin \phi_i$  also smaller lepton asymmetries with different signs for the three generations can be accommodated. Additionally a cancellation between existing lepton asymmetries and the generated lepton asymmetries can be arranged.

As discussed in Sec. II D we add sphaleron transitions to our Boltzmann equations in order to determine the baryon asymmetry generated from the lepton asymmetry and vice versa. We therefore start once with initial and once without initial lepton and baryon asymmetries and examine how both evolve as a function of the temperature parametrized by x. Figure 11(a) shows the corresponding results for  $P_2$  again for a phase of  $\phi \coloneqq \phi_1 = \phi_2 = \phi_3 = 15$  degrees. For numerical stability we choose  $\delta_B = -\eta(x)\delta_L$  at  $x = 10^{-2}$ in case of given initial asymmetries. As expected, the baryon asymmetry  $\delta_B$  freezes at temperatures  $x = m_{\tilde{z}_c^0}/T_c \approx 0.66$ , whereas the lepton asymmetry  $\delta_N$  evolves, further driven by the neutralino decays. Thus, if an initial baryon asymmetry is present, it is hardly affected by CP violating decays of light neutralinos. The reason is that for  $x \ge 0.66$ , where the change of the lepton asymmetry is strongest, the sphaleron process is close to being frozen out. On the other hand, even in case of no initial baryon asymmetry it can be generated at small x. We translate Fig. 10 to the corresponding baryon asymmetry obtained at  $x \approx 0.66$  and present the result in Fig. 11(b). Since the baryon asymmetry remains constant for lower temperatures, it yields  $\delta_B(x = 10^2) = \delta_B(x = 0.66)$ . A baryon asymmetry of order  $10^{-10}$  as observed in the universe can be generated from neutralino decays having a mass of  $m_{\tilde{\chi}_1^0} \sim 100$  GeV in BRpV in case of rather small complex phases,  $\phi_i \sim 10^{-3}$  degrees for the *R*-parity breaking parameters. The generated lepton asymmetry is approximately 2 orders of magnitude larger. Alternatively larger



FIG. 11 (color online). (a) Lepton asymmetry  $|\delta_N|$  (solid) and baryon asymmetry  $|\delta_B|$  (dashed) for scenario  $P_2$  with phase  $\phi := \phi_1 = \phi_2 = \phi_3 = 15$  degrees for two different initial asymmetries  $\delta_N, \delta_B$  at  $x = 10^{-2}$ ; (b) Resulting baryon asymmetry  $|\delta_B(x = 0.66)|$  as a function of  $\phi := \phi_1 = \phi_2 = \phi_3$  for  $P_2$  for zero initial asymmetry  $\delta_N = \delta_B = 0$  at  $x = 10^{-2}$ .

phases are possible, if a cancellation between various *CP* violating contributions occurs.

#### E. LSP annihilation to SM particles

As it was pointed out in Ref. [17] *R*-parity conserving scattering processes, which lead to an annihilation of the LSP, can impact on the densities of the LSP and the SM particles. Therefore all our discussion included the R-parity conserving annihilation processes. Within this section we want to discuss their impact in more detail, neglecting sphaleron transitions for simplicity. For scenario  $P_2$ Fig. 12(a) shows the thermally averaged cross sections  $\langle \sigma_{ii} v \rangle$  for different final states in 1/GeV<sup>2</sup> as obtained by MICROMEGAs. Figure 12(b) presents for comparison  $\hat{\sigma}(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to b \bar{b}) N^{\text{eq},2}$  with  $N^{\text{eq}}$  being the neutralino equilibrium density versus  $K_1/K_2 N^{eq}\Gamma$  with  $\Gamma = 10^{-14}$  GeV and the ratio  $K_1/K_2$  of modified Bessel functions and thus reflects the right-hand side terms entering the Boltzmann equations (27). The annihilation processes dominate the evolution of the Boltzmann equations for most



FIG. 12 (color online). (a) Cross section  $\langle \sigma_{ij}v \rangle$  in  $1/\text{GeV}^2$  as a function of x for  $P_2$  for different final states:  $ij = d\bar{d}/s\bar{s}/b\bar{b}$  (black),  $u\bar{u}/c\bar{c}/t\bar{t}$  (black, dashed),  $\tau^+\tau^-$  (red, dot-dashed),  $e^+e^-/\mu^+\mu^-$  (red, dotted); (b)  $K_1/K_2N^{\text{eq}}\Gamma$  in GeV with  $\Gamma = 10^{-14}$  GeV (red, dashed) and  $\hat{\sigma}(\tilde{\chi}_1^0\tilde{\chi}_1^0 \to b\bar{b})N^{\text{eq},2}$  in GeV (black) as a function of x.

temperatures; their relative importance drops below the decay processes only for very low temperatures  $T < m_{\tilde{\chi}_1^0}$ .

The impact of the annihilation versus the decay processes in the Boltzmann equations is further elaborated in Fig. 13 for scenario  $P_1$ . Scenario  $P_2$  is visually indistinguishable from  $P_1$ . Figure 13(a) shows the neutralino density  $N_{\tilde{\chi}_1^0}$  and its difference to the corresponding equilibrium value  $|N_{\tilde{\chi}_1^0} - N_{\tilde{\chi}_1^0}^{\text{eq}}|$  for three different scenarios: In case of only annihilation processes, which corresponds to a scenario of the MSSM with *R*-parity conservation, the wellknown freeze-out of the lightest SUSY particle yielding a constant neutralino density is observed at low temperatures. If just *CP* violating decays of the neutralinos are included, the neutralino density vanishes for large *x*. In case of annihilation and decay processes the neutralino density closely follows the case of only annihilation processes, but all neutralinos tend to decay at low temperatures.

Figure 13(b) presents the evolving lepton asymmetry, if no initial lepton asymmetry  $\delta_N = 0$  at  $x = 10^{-2}$  is assumed.



FIG. 13 (color online). (a) LSP number density  $|N_{\tilde{\chi}_1^0} - N_{\tilde{\chi}_1^0}^{eq}|$  (solid) and  $N_{\tilde{\chi}_1^0}$  (dashed) as a function of *x* for only annihilation (green), only decays (red), annihilation and decays (black) for  $P_1$  with phase  $\phi_i = 5$  degrees. (b) Evolution of  $\delta_N$  as defined in Eq. (30) as a function of *x* for only decays (red, dashed), and annihilation and decays (black) for the same case.

 $10^{0}$ 

x

(b)

 $10^{1}$ 

 $10^{-1}$ 

10

 $10^2$ 

A difference between just decay processes and decay as well as annihilation processes can be observed. However, the order of magnitude of the generated lepton asymmetry remains unchanged. The inclusion of sphaleron transitions will distort the observations only minimally.

#### **IV. CONCLUSION**

We examined the effects of LSP decays, the LSP being a light neutralino  $m_{\tilde{\chi}_1^0} \sim 100$  GeV, in bilinear *R*-parity violation (BRpV) with complex BRpV parameters on the lepton and baryon asymmetries in the early universe. We presented a description of the neutralino sector at NLO for complex BRpV parameters and calculated the LSP decays at NLO. In this way we get both the total width and the induced *CP* asymmetries between leptonic and antileptonic final states. With respect to the evolution of number densities in the early universe our discussion includes—apart from the mentioned LSP decays and their inverse counterparts—the LSP annihilation to SM particles, which we assume to be very close to the minimal supersymmetric

## CP VIOLATION IN BILINEAR R-PARITY ...

standard model. In order to describe the transition between lepton and baryon asymmetries we add a simple description of sphaleron processes, which however get frozen out at the mass scale of the neutralino if its mass is below the electroweak phase transition.

Our conclusion is two sided: Initial lepton and baryon asymmetries are preserved by the LSP decay, if neutrino data are described correctly by the BRpV parameters. On the other hand, lepton and baryon asymmetries can also be generated in the complex BRpV model, the latter being in accordance with the observation in our universe. Clearly, both statements hold for different values of the *CP* violating phases in case of a light neutralino LSP. Last but not least we note that in both regimes the electric dipole moment of the electron induced via those couplings is well below the sensitivity of present and future experiments.

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## APPENDIX: FORMULAS: BOLTZMANN EQUATIONS

In this section we present the full set of Boltzmann equations for the neutralino and the final state particles resulting from the gauge bosons/Higgs decays, namely leptons, neutrinos (antineutrinos) and quarks (antiquarks). To be brief we just add the decay mode  $h \rightarrow q\bar{q}$  for the Higgs boson. Moreover to shorten the subsequent formulas, we leave the distinction of  $q_1$  and  $q_2$  to the reader. It is clear that e.g. the decays of the W boson involve two different quark types, whereas the decays of the Z boson result in identical quark types  $q_i \bar{q}_i$ . Accordingly Eq. (A5) and Eq. (A6) have to be doubled for both quarks  $q_1$  and  $q_2$ . For details with respect to the sphaleron transitions we refer to Sec. II D. To shorten our notation we write  $\Gamma$ ,  $Br(X|YZ) \coloneqq \Gamma$ ,  $Br(X \rightarrow YZ)$  and  $\hat{\sigma}(XX|YZ) \coloneqq \hat{\sigma}(XX \to YZ)$ . We also define  $N_X$  to be the ratio of the number density of the particle  $X \in$  $\{\ell_i^{\pm}, \nu, \bar{\nu}, q, \bar{q}\}$  and its value in the thermal equilibrium:

$$\tilde{N}_X = \frac{N_X}{N_X^{\rm eq}} \tag{A1}$$

$$\begin{split} tH \frac{dN_{\tilde{Z}_{1}^{0}}}{dx} &= -\frac{\mathbf{K}_{1}(x)}{\mathbf{K}_{2}(x)} \sum_{i,j} \sum_{q,\bar{q}} [N_{\tilde{\chi}_{1}^{0}} (\Gamma(\tilde{\chi}_{1}^{0}|\ell_{i}^{+}W^{-}) + \Gamma(\tilde{\chi}_{1}^{0}|\ell_{i}^{-}W^{+})) \mathrm{Br}(W^{\pm}|\ell_{j}^{\pm}\nu(\bar{\nu})_{j}) \\ &\quad -\tilde{N}_{\ell_{1}^{*}} \tilde{N}_{\ell_{j}^{*}} \tilde{N}_{\ell_{j}} N_{\tilde{\chi}_{1}^{0}}^{\mathrm{eq}} \Gamma(\ell_{i}^{+}W^{-}|\tilde{\chi}_{1}^{0}) \mathrm{Br}(W^{-}|\ell_{j}^{-}\bar{\nu}_{j}) \\ &\quad -\tilde{N}_{\ell_{1}^{*}} \tilde{N}_{\ell_{j}^{*}} \tilde{N}_{\ell_{j}} N_{\tilde{\chi}_{1}^{0}}^{\mathrm{eq}} \Gamma(\ell_{i}^{-}W^{+}|\tilde{\chi}_{1}^{0}) \mathrm{Br}(W^{+}|\ell_{j}^{+}\nu_{j}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} (\Gamma(\tilde{\chi}_{1}^{0}|\ell_{i}^{-}W^{+}) + \Gamma(\tilde{\chi}_{1}^{0}|\ell_{i}^{+}W^{-})) \mathrm{Br}(W^{\pm}|q\bar{q}) \\ &\quad -\tilde{N}_{q} \tilde{N}_{q} N_{\tilde{\chi}_{1}^{0}}^{\mathrm{eq}} (\tilde{N}_{\ell_{1}^{*}} \Gamma(\ell_{i}^{-}W^{+}|\tilde{\chi}_{1}^{0}) + \tilde{N}_{\ell_{1}^{*}} \Gamma(\ell_{i}^{+}W^{-}|\tilde{\chi}_{1}^{0})) \mathrm{Br}(W^{\pm}|q\bar{q}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} (\Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0}|Z\bar{\nu}_{i})) \mathrm{Br}(Z|\ell_{j}^{-}\ell_{j}^{+}) \\ &\quad -\tilde{N}_{\ell_{j}} \tilde{N}_{\ell_{1}^{*}} N_{\tilde{\chi}_{1}^{eq}}^{\mathrm{eq}} (\tilde{N}_{\ell_{1}^{*}} (Z\nu_{i}|\tilde{\chi}_{1}^{0}) + \tilde{N}_{\tilde{\nu}_{1}} \Gamma(Z\bar{\nu}_{i}|\tilde{\chi}_{1}^{0})) \mathrm{Br}(Z|\ell_{j}^{-}\ell_{j}^{+}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} (\Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0}|Z\bar{\nu}_{i})) \mathrm{Br}(Z|q\bar{q}) \\ &\quad -\tilde{N}_{q} \tilde{N}_{q} N_{\tilde{\chi}_{1}^{eq}} (\tilde{N}_{\ell_{1}} (Z\nu_{\ell}|\tilde{\chi}_{1}^{0}) + \tilde{N}_{\tilde{\nu}_{1}} \Gamma(Z\bar{\nu}_{i}|\tilde{\chi}_{1}^{0})) \mathrm{Br}(Z|q\bar{q}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} (\Gamma(\tilde{\chi}_{1}^{0}|h\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0}|h\bar{\nu}_{i})) \mathrm{Br}(h|q\bar{q}) \\ &\quad -\tilde{N}_{q} \tilde{N}_{q} N_{\tilde{\chi}_{1}^{eq}} (\tilde{N}_{\nu_{1}} (h\nu_{i}|\tilde{\chi}_{1}^{0}) + \tilde{N}_{\tilde{\nu}_{1}} \Gamma(h\bar{\nu}_{i}|\tilde{\chi}_{1}^{0})) \mathrm{Br}(A|q\bar{q}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|2\nu_{i}) \mathrm{Br}(Z|\nu_{j}\bar{\nu}_{j}) - \tilde{N}_{\nu_{i}} \tilde{N}_{\nu_{i}} \tilde{N}_{\tilde{\chi}_{1}^{eq}} \Gamma(Z\nu_{i}|\tilde{\chi}_{1}^{0}) \mathrm{Br}(Z|\nu_{j}\bar{\nu}_{j})) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{i}) \mathrm{Br}(Z|\nu_{j}\bar{\nu}_{j}) - \tilde{N}_{\tilde{\nu}_{1}} \tilde{N}_{\nu} \tilde{N}_{\tilde{\chi}_{1}^{eq}} \Gamma(Z\bar{\nu}_{i}|\tilde{\chi}_{1}^{0}) \mathrm{Br}(Z|\nu_{j}\bar{\nu}_{j})] \\ &\quad -\tilde{N}_{i} \frac{1}{q_{q}^{q}} \Gamma(\tilde{\chi}_{1}^{0}|\ell_{\ell}^{-}\ell_{\ell}^{0}) (N_{\tilde{\chi}_{1}^{0}}^{2} - \tilde{N}_{\ell} \tilde{N}_{\ell}^{eq}} N_{\tilde{\chi}_{1}^{eq}}^{eq}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{i}) \mathrm{Br}(Z|\nu_{j}\bar{\nu}_{j}) - \tilde{N}_{\tilde$$

$$\begin{aligned} xH\frac{dN_{\nu_{j}}}{dx} &= \frac{K_{1}(x)}{K_{2}(x)} \sum_{i} \sum_{q,\bar{q}} [N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | \ell_{i}^{-} W^{+}) \operatorname{Br}(W^{+} | \ell_{j}^{+} \nu_{j}) \\ &\quad - \tilde{N}_{\ell_{i}^{-}} \tilde{N}_{\ell_{j}^{+}} \tilde{N}_{\nu_{j}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(\ell_{i}^{-} W^{+} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{+} | \ell_{j}^{+} \nu_{j}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | Z \nu_{j}) (\operatorname{Br}(Z | \ell_{i}^{-} \ell_{i}^{+}) + \operatorname{Br}(Z | \nu_{i} \bar{\nu}_{i}) + \operatorname{Br}(Z | q \bar{q})) \\ &\quad - \tilde{N}_{\nu_{j}} \tilde{N}_{\ell_{i}^{-}} \tilde{N}_{\ell_{i}^{+}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(Z \nu_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \ell_{i}^{-} \ell_{i}^{+}) - \tilde{N}_{\nu_{j}} \tilde{N}_{q} \tilde{N}_{q} \tilde{N}_{q} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(Z \nu_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | q \bar{q}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | h \nu_{j}) \operatorname{Br}(h | q \bar{q}) - \tilde{N}_{\nu_{j}} \tilde{N}_{q} \tilde{N}_{q} \tilde{N}_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(h \nu_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(h | q \bar{q}) \\ &\quad - \tilde{N}_{\nu_{j}} \tilde{N}_{\nu_{i}} \tilde{N}_{\tilde{\nu}_{i}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(Z \nu_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \nu_{i} \bar{\nu}_{i}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | Z \bar{\nu}_{j}) \operatorname{Br}(Z | \nu_{i} \bar{\nu}_{i}) - \tilde{N}_{\nu_{j}} \tilde{N}_{\nu_{i}} \tilde{N}_{\tilde{\nu}_{i}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(Z \bar{\nu}_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \nu_{i} \bar{\nu}_{i}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | Z \nu_{i}) \operatorname{Br}(Z | \nu_{j} \bar{\nu}_{j}) - \tilde{N}_{\nu_{i}} \tilde{N}_{\nu_{j}} \tilde{N}_{\tilde{\nu}_{i}} \tilde{N}_{\tilde{\chi}_{1}^{0}} \Gamma(Z \bar{\nu}_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \nu_{i} \bar{\nu}_{i}) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | Z \nu_{i}) \operatorname{Br}(Z | \nu_{j} \bar{\nu}_{j}) - \tilde{N}_{\nu_{i}} \tilde{N}_{\nu_{j}} \tilde{N}_{\tilde{\nu}_{1}} \tilde{N}_{1} \Gamma(Z \bar{\nu}_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \nu_{i} \bar{\nu}_{i}) ] \\ &\quad + \frac{1}{2} \hat{\sigma}(\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0} | \nu_{j} \bar{\nu}_{j}) (N_{\tilde{\chi}_{1}^{0}^{-}} - \tilde{N}_{\nu_{j}} \tilde{N}_{\tilde{\nu}_{1}} N_{\tilde{\chi}_{1}}^{eq}) + \frac{\gamma(x)}{12} [\delta_{B} + \eta(x) \delta_{L}] \end{aligned}$$

$$\begin{aligned} xH\frac{dN_{\bar{\nu}_{j}}}{dx} &= \frac{K_{1}(x)}{K_{2}(x)} \sum_{i} \sum_{q,\bar{q}} [N_{\bar{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|\ell_{i}^{+}W^{-}) \operatorname{Br}(W^{-}|\ell_{j}^{-}\bar{\nu}_{j}) \\ &\quad - \tilde{N}_{\ell_{i}^{+}} \tilde{N}_{\ell_{j}^{-}} \tilde{N}_{\bar{\nu}_{j}} N_{\bar{\chi}_{1}^{0}}^{eq} \Gamma(\ell_{i}^{+}W^{-}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{-}|\ell_{j}^{-}\bar{\nu}_{j}) \\ &\quad + N_{\bar{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|Z\bar{\nu}_{j}) (\operatorname{Br}(Z|\ell_{i}^{-}\ell_{i}^{+}) + \operatorname{Br}(Z|\nu_{i}\bar{\nu}_{i}) + \operatorname{Br}(Z|q\bar{q})) \\ &\quad - \tilde{N}_{\bar{\nu}_{j}} \tilde{N}_{\ell_{i}^{-}} \tilde{N}_{\ell_{i}^{+}} N_{\bar{\chi}_{1}^{0}}^{eq} \Gamma(Z\bar{\nu}_{j}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(Z|\ell_{i}^{-}\ell_{i}^{+}) - \tilde{N}_{\bar{\nu}_{j}} \tilde{N}_{q} \tilde{N}_{q} N_{\bar{\chi}_{1}^{0}}^{eq} \Gamma(Z\bar{\nu}_{j}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(Z|q\bar{q}) \\ &\quad + N_{\bar{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|h\bar{\nu}_{j}) \operatorname{Br}(h|q\bar{q}) - \tilde{N}_{\bar{\nu}_{j}} \tilde{N}_{q} \tilde{N}_{q} N_{\bar{\chi}_{1}^{0}}^{eq} \Gamma(h\bar{\nu}_{j}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(h|q\bar{q}) \\ &\quad - \tilde{N}_{\bar{\nu}_{j}} \tilde{N}_{\nu_{i}} \tilde{N}_{\bar{\chi}_{1}^{0}} \Gamma(Z\bar{\nu}_{j}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(Z|\bar{\nu}_{i}\bar{\nu}_{i}) \\ &\quad + N_{\bar{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{j}) \operatorname{Br}(Z|\nu_{i}\bar{\nu}_{i}) - \tilde{N}_{\nu_{j}} \tilde{N}_{\nu_{i}} \tilde{N}_{\bar{\chi}_{1}^{0}} \Gamma(Z\nu_{j}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(Z|\nu_{i}\bar{\nu}_{i}) \\ &\quad + N_{\bar{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{j}) \operatorname{Br}(Z|\nu_{i}\bar{\nu}_{i}) - \tilde{N}_{\nu_{j}} \tilde{N}_{\nu_{j}} \tilde{N}_{\nu_{j}} \tilde{N}_{\bar{\chi}_{1}^{0}} \Gamma(Z\bar{\nu}_{i}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(Z|\nu_{i}\bar{\nu}_{i}) \\ &\quad + N_{\bar{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{j}) \operatorname{Br}(Z|\nu_{i}\bar{\nu}_{j}) - \tilde{N}_{\bar{\nu}_{i}} \tilde{N}_{\nu_{j}} \mathrm{Br}(Z|\nu_{i}\bar{\nu}_{i})] \\ &\quad + \frac{1}{2} \hat{\sigma}(\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0}|\nu_{j}\bar{\nu}_{j}) (N_{\tilde{\chi}_{1}^{0}^{2}} - \tilde{N}_{\nu_{j}} \tilde{N}$$

$$\begin{aligned} xH\frac{dN_{q}}{dx} &= \frac{K_{1}(x)}{K_{2}(x)} \sum_{i} \sum_{\bar{q}} [N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | \ell_{i}^{-} W^{+}) \operatorname{Br}(W^{+} | q\bar{q}) \\ &\quad - \tilde{N}_{\ell_{i}^{-}} \tilde{N}_{q} \tilde{N}_{\bar{q}} N_{\tilde{\chi}_{1}^{0}}^{\operatorname{eq}} \Gamma(\ell_{i}^{-} W^{+} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{+} | q\bar{q}) + N_{\tilde{\chi}_{1}^{0}} \operatorname{Br}(Z | q\bar{q}) (\Gamma(\tilde{\chi}_{1}^{0} | Z\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0} | Z\bar{\nu}_{i})) \\ &\quad - \tilde{N}_{q} \tilde{N}_{\bar{q}} N_{\tilde{\chi}_{1}^{0}}^{\operatorname{eq}} \operatorname{Br}(Z | q\bar{q}) (\tilde{N}_{\nu_{i}} \Gamma(Z\nu_{i} | \tilde{\chi}_{1}^{0}) + \tilde{N}_{\bar{\nu}_{i}} \Gamma(Z\bar{\nu}_{i} | \tilde{\chi}_{1}^{0})) \\ &\quad + N_{\tilde{\chi}_{1}^{0}} \operatorname{Br}(h | q\bar{q}) (\Gamma(\tilde{\chi}_{1}^{0} | h\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0} | h\bar{\nu}_{i})) \\ &\quad - \tilde{N}_{q} \tilde{N}_{\bar{q}} N_{\tilde{\chi}_{1}^{0}}^{\operatorname{eq}} \operatorname{Br}(h | q\bar{q}) (\tilde{N}_{\nu_{i}} \Gamma(h\nu_{i} | \tilde{\chi}_{1}^{0}) + \tilde{N}_{\bar{\nu}_{i}} \Gamma(h\bar{\nu}_{i} | \tilde{\chi}_{1}^{0}))] \\ &\quad + \sum_{\bar{q}} \left[ \frac{1}{2} \hat{\sigma} (\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0} | q\bar{q}) (N_{\tilde{\chi}_{1}^{0}}^{2} - \tilde{N}_{q} \tilde{N}_{\bar{q}} N_{\tilde{\chi}_{1}^{0}}^{\operatorname{eq}2}) \right] + \frac{\gamma(x)}{4} [\delta_{B} + \eta(x)\delta_{L}] \end{aligned} \tag{A5}$$

$$xH\frac{dN_{\bar{q}}}{dx} = \frac{K_{1}(x)}{K_{2}(x)} \sum_{i} \sum_{q} [N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0}|\ell_{i}^{+}W^{-}) \operatorname{Br}(W^{-}|q\bar{q}) \\ - \tilde{N}_{\ell_{i}^{+}} \tilde{N}_{q} \tilde{N}_{q} N_{\bar{\chi}_{1}^{0}}^{\operatorname{eq}} \Gamma(\ell_{i}^{+}W^{-}|\tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{-}|q\bar{q}) + N_{\tilde{\chi}_{1}^{0}} \operatorname{Br}(Z|q\bar{q}) (\Gamma(\tilde{\chi}_{1}^{0}|Z\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0}|Z\bar{\nu}_{i})) \\ - \tilde{N}_{q} \tilde{N}_{q} N_{\bar{\chi}_{1}^{0}}^{\operatorname{eq}} \operatorname{Br}(Z|q\bar{q}) (\tilde{N}_{\nu_{i}} \Gamma(Z\nu_{i}|\tilde{\chi}_{1}^{0}) + \tilde{N}_{\bar{\nu}_{i}} \Gamma(Z\bar{\nu}_{i}|\tilde{\chi}_{1}^{0})) \\ + N_{\tilde{\chi}_{1}^{0}} \operatorname{Br}(h|q\bar{q}) (\Gamma(\tilde{\chi}_{1}^{0}|h\nu_{i}) + \Gamma(\tilde{\chi}_{1}^{0}|h\bar{\nu}_{i})) \\ - \tilde{N}_{q} \tilde{N}_{q} N_{\bar{\chi}_{1}^{0}}^{\operatorname{eq}} \operatorname{Br}(h|q\bar{q}) (\tilde{N}_{\nu_{i}} \Gamma(h\nu_{i}|\tilde{\chi}_{1}^{0}) + \tilde{N}_{\bar{\nu}_{i}} \Gamma(h\bar{\nu}_{i}|\tilde{\chi}_{1}^{0}))] \\ + \sum_{q} \left[ \frac{1}{2} \hat{\sigma} (\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0}|q\bar{q}) (N_{\tilde{\chi}_{1}^{0}}^{2} - \tilde{N}_{q} \tilde{N}_{\bar{q}} N_{\tilde{\chi}_{1}^{0}}^{\operatorname{eq}2}) \right] - \frac{\gamma(x)}{4} [\delta_{B} + \eta(x)\delta_{L}]$$
 (A6)

$$xH \frac{dN_{\ell_i^-}}{dx} = \frac{K_1(x)}{K_2(x)} \sum_j \sum_{q,\bar{q}} [N_{\tilde{\chi}_1^0} \Gamma(\tilde{\chi}_1^0 | \ell_i^- W^+) \operatorname{Br}(W^+ | \ell_j^+ \nu_j) - \tilde{N}_{\ell_i^-} \tilde{N}_{\ell_j^+} \tilde{N}_{\nu_j} N_{\tilde{\chi}_1^0}^{eq} \Gamma(\ell_i^- W^+ | \tilde{\chi}_1^0) \operatorname{Br}(W^+ | \ell_j^+ \nu_j) + N_{\tilde{\chi}_1^0} \Gamma(\tilde{\chi}_1^0 | \ell_i^- W^+) \operatorname{Br}(W^+ | q\bar{q}) - \tilde{N}_{\ell_i^-} \tilde{N}_q \tilde{N}_{\bar{q}} N_{\tilde{\chi}_1^0}^{eq} \Gamma(\ell_i^- W^+ | \tilde{\chi}_1^0) \operatorname{Br}(W^+ | q\bar{q}) + N_{\tilde{\chi}_1^0} \Gamma(\tilde{\chi}_1^0 | \ell_j^+ W^-) \operatorname{Br}(W^- | \ell_i^- \bar{\nu}_i) - \tilde{N}_{l_j^+} \tilde{N}_{\ell_i^-} \tilde{N}_{\nu_i} N_{\tilde{\chi}_1^0}^{eq} \Gamma(l_j^+ W^- | \tilde{\chi}_1^0) \operatorname{Br}(W^- | \ell_i^- \bar{\nu}_i) + N_{\tilde{\chi}_1^0} \Gamma(\tilde{\chi}_1^0 | Z\nu_j) \operatorname{Br}(Z | \ell_i^- \ell_i^+) - \tilde{N}_{\nu_j} \tilde{N}_{\ell_i^-} \tilde{N}_{\ell_i^+} N_{\tilde{\chi}_1^0}^{eq} \Gamma(Z\nu_j | \tilde{\chi}_1^0) \operatorname{Br}(Z | \ell_i^- \ell_i^+) + N_{\tilde{\chi}_1^0} \Gamma(\tilde{\chi}_1^0 | Z\bar{\nu}_j) \operatorname{Br}(Z | \ell_i^- \ell_i^+) - \tilde{N}_{\nu_j} \tilde{N}_{\ell_i^-} \tilde{N}_{\ell_i^+} N_{\tilde{\chi}_1^0}^{eq} \Gamma(Z\bar{\nu}_j | \tilde{\chi}_1^0) \operatorname{Br}(Z | \ell_i^- \ell_i^+)] + \hat{\sigma}(\tilde{\chi}_1^0 \tilde{\chi}_1^0 | \ell_i^+ \ell_i^-) (N_{\tilde{\chi}_1^0}^2 - \tilde{N}_{\ell_i^+} \tilde{N}_{\ell_i^-} N_{\tilde{\chi}_1^0}^{eq2}) + \frac{\gamma(x)}{12} [\delta_B + \eta(x)\delta_L]$$
 (A7)

$$xH \frac{dN_{\ell_{i}^{+}}}{dx} = \frac{K_{1}(x)}{K_{2}(x)} \sum_{j} \sum_{q,\bar{q}} [N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | \ell_{i}^{+} W^{-}) \operatorname{Br}(W^{-} | \ell_{j}^{-} \bar{\nu}_{j}) - \tilde{N}_{l_{i}^{+}} \tilde{N}_{\ell_{j}^{-}} \tilde{N}_{\bar{\nu}_{j}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(l_{i}^{+} W^{-} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{-} \to \ell_{j}^{-} \bar{\nu}_{j}) + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | \ell_{i}^{+} W^{-}) \operatorname{Br}(W^{-} | \bar{q}q) - \tilde{N}_{\ell_{i}^{+}} \tilde{N}_{\bar{q}} \tilde{N}_{q} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(l_{i}^{+} W^{-} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{-} | \bar{q}q) + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | \ell_{j}^{-} W^{+}) \operatorname{Br}(W^{+} | \ell_{i}^{+} \nu_{i}) - \tilde{N}_{\ell_{j}^{-}} \tilde{N}_{l_{i}^{+}} \tilde{N}_{\nu_{i}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(\ell_{j}^{-} W^{+} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(W^{+} | \ell_{i}^{+} \nu_{i}) + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | Z \nu_{j}) \operatorname{Br}(Z | \ell_{i}^{-} \ell_{i}^{+}) - \tilde{N}_{\nu_{j}} \tilde{N}_{\ell_{i}^{-}} \tilde{N}_{\ell_{i}^{+}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(Z \bar{\nu}_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \ell_{i}^{-} \ell_{i}^{+}) + N_{\tilde{\chi}_{1}^{0}} \Gamma(\tilde{\chi}_{1}^{0} | Z \bar{\nu}_{j}) \operatorname{Br}(Z | \ell_{i}^{-} \ell_{i}^{+}) - \tilde{N}_{\bar{\nu}j} \tilde{N}_{\ell_{i}^{-}} \tilde{N}_{\ell_{i}^{+}} N_{\tilde{\chi}_{1}^{0}}^{eq} \Gamma(Z \bar{\nu}_{j} | \tilde{\chi}_{1}^{0}) \operatorname{Br}(Z | \ell_{i}^{-} \ell_{i}^{+})] + \hat{\sigma}(\tilde{\chi}_{1}^{0} \tilde{\chi}_{1}^{0} | \ell_{i}^{+} \ell_{i}^{-}) (N_{\tilde{\chi}_{1}^{0}}^{2} - \tilde{N}_{\ell_{i}^{+}} \tilde{N}_{\ell_{i}^{-}} N_{\tilde{\chi}_{1}^{0}}^{eq2}) - \frac{\gamma(x)}{12} [\delta_{B} + \eta(x) \delta_{L}].$$

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