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Difference cross sections of unpolarized SIDIS with transverse momentum dependence

Ekaterina Christova[*]

Institute for Nuclear Research and Nuclear Energy, Sofia 1784, Bulgaria

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Previously we showed that, based only on charge conjugation and isospin invariance of strong interactions, the difference cross sections of hadrons with opposite charge in semi-inclusive deep inelastic scattering (SIDIS) $e + N \to l + h + X$ are expressed solely in terms of the valence-quark densities and certain nonsinglet combinations of fragmentation functions (FFs). This allowed us to determine these quantities in a model-independent way. Now we extend this approach to processes when the transverse momentum of the final hadron is measured as well. We show that the difference cross sections of unpolarized SIDIS on proton and deuterium targets, $d\sigma_N^{h^+-h^-}$, $d\sigma_N^{\pi^+-\pi^-}$ and $d\sigma_N^{K^+-K^-}$, are expressed solely in terms of the transverse momentum-dependent (TMD) unpolarized valence-quark densities and FFs, and the valence-quark Boer-Mulders and Collins functions. This allows us to determine them separately and study the flavor dependence of the quark transverse momentum. Measurements on the deuterium target, $d\sigma_d^{h^+-h^-}$, $d\sigma_d^{\pi^+-\pi^-}$ and $d\sigma_d^{K^+-K^-}$, provide three independent measurements for the sum of the TMD valence-quark densities and Boer-Mulders functions: $(u_{1,V} + d_{1,V})$ and $(h^\perp_{1,uV} + h^\perp_{1,dV})$.

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I. INTRODUCTION

Now it seems quite well established that the simple collinear picture of the quark-parton model appears too simple to explain existing experimental data. The measured azimuthal asymmetries in the direction of the final hadrons show that the transverse momentum of the quarks should be necessarily taken into account. This leads to the following three main differences as compared to the collinear parton model: 1) the known parton distribution functions (PDFs) and fragmentation functions (FFs) depend not only on the longitudinal, but on the transverse momenta of the quarks as well—we start to deal with transverse momentum-dependent parton density functions (TMD PDFs) and fragmentation functions (TMD FFs), 2) a new type of TMD parton densities and FFs arise from correlations among the transverse components of quark momentum or spin, and the longitudinal components of the particles in the process and 3) the TMD PDFs and TMD FFs always enter the cross sections in convolutions over the quark transverse momenta.

This makes the problem of extracting the transverse momentum densities and FFs from experiment considerably more complicated. In order to simplify analysis, a lot of assumptions on the TMD functions, in addition to those on the collinear PDFs and FFs, are made for the transverse momentum dependence: it is factorized, it is flavor blind, it is hadron blind, etc. Though sometimes quite reasonable, these are *ad hoc* model assumptions, motivated mainly by simplicity, and do not follow from QCD theory of strong interactions and thus introduce uncontrolled uncertainties.

For these reasons it is important to find measurable quantities, that would extract TMD functions without or with fewer additional assumptions.

Previously this task was fulfilled for the collinear polarized PDFs. We showed [1] that, based only on charge conjugation (C) and isospin SU(2) invariance of strong interactions, the so called "difference asymmetries" in semi-inclusive deep inelastic scattering (SIDIS) of longitudinally polarized leptons on longitudinally polarized nucleons determine the polarized valence-quark PDFs in a model-independent way. Such measurements were fulfilled and the polarized valence-quark densities were determined directly [2].

Later, the same approach was used for the collinear FFs. We showed [3] that differences between the cross sections for producing hadrons and their antiparticles in unpolarized SIDIS allow us to determine nonsinglet combinations of the collinear FFs in a model-independent way and test most of the commonly used assumptions. Recently, this approach was applied to HERMES data and the nonsinglet combination of the pion fragmentation functions was determined with very good precision [4].

Now we extend this approach to the noncollinear picture of the parton model, when parton densities and fragmentation functions depend on the transverse momentum of the quarks as well. Transverse momentum of the quarks plays a crucial role when not only the energy, but the transverse momentum of the final hadron is measured.

In this paper we consider unpolarized SIDIS and show how, based only on the general symmetries of C and SU(2)

[*]echristo@inrne.bas.bg

invariance, information on certain combinations of the TMD PDFs and TMD FFs can be obtained in a model-independent way. The key experimental ingredients are the differences between cross sections for producing hadrons and producing their antiparticles, i.e. data on $d\sigma^{h-\bar{h}} \equiv d\sigma^h - d\sigma^{\bar{h}}$, for $h = h^\pm, \pi^\pm, K^\pm$.

The paper is organized as follows. In the next section we recall the general expression of the cross section and introduce the notation. In Secs. III, IV, and V we give the difference cross sections for any charged hadrons, for charged pions and charged kaons, respectively; Sec. V ends up with a brief summary of the obtained results for charged hadron production. In Sec. VI we present the difference cross section for charged and neutral kaons. In all cases we present the results for proton and deuterium targets. In Sec. VII we discuss the standard parametrizations and those appropriate to the considered approach and the possibilities to study flavor and Q^2 dependence in the quark transverse momenta in the TMDs. We end up with our comments and conclusions.

II. THE CROSS SECTION: GENERAL EXPRESSION

The cross section for SIDIS of unpolarized leptons l on unpolarized nucleons N

$$l(l^\mu) + N(P^\mu) \to l'(l'^\mu) + h(P_h^\mu) + X \tag{1}$$

exhibits a characteristic $\cos 2\phi_h$ and $\cos\phi_h$ azimuthal dependence in the kinematic region of low $P_T \simeq \Lambda_{\rm QCD} \ll Q$; ϕ_h is the azimuthal angle of the produced hadron h. The general expression for the cross section in the TMD factorization scheme [5], in the one-photon exchange approximation and in the LO of QCD reads [6,7]

$$\frac{d^5\sigma_p^h}{dx_B dQ^2 dz_h d^2\mathbf{P}_T} = \frac{2\pi\alpha_{em}^2}{Q^4}\{[1+(1-y)^2]F_{UU}^h + 2(1-y)\cos 2\phi_h F_{UU}^{\cos 2\phi,h} + 2(2-y)\sqrt{1-y}\cos\phi_h F_{UU}^{\cos\phi_h,h}\}. \tag{2}$$

Here $\mathbf{P}_T$ is the transverse momentum of the final hadron in the $\gamma^* - p$ c.m. frame, and x_B, z_h, Q^2 and y are the usual measurable SIDIS quantities:

$$x_B = \frac{Q^2}{2(P\cdot q)}, \qquad z_h = \frac{(P\cdot P_h)}{(P\cdot q)}, \qquad Q^2 = -q^2,$$
$$y = \frac{(P\cdot q)}{(P\cdot l)}, \qquad q = l - l'. \tag{3}$$

Throughout the paper we use the kinematic configuration and the results of [7]. However, we write the F_{UU}'s in a slightly different form—we indicate explicitly the indices of the quark flavors $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$ and the type of the produced hadron h, and we single out the quantities that are flavor and hadron-type independent. We have

$$F_{UU}^h = \sum_q e_q^2 f_{1q}\otimes D_{1q}^h,$$
$$F_{UU}^{\cos 2\phi,h} = \sum_q e_q^2\left[h_{1q}^\perp \otimes H_{1q}^{\perp,h}\otimes w_2^\perp + \frac{2}{Q^2} f_{1q}\otimes D_{1q}^h \otimes w_2\right],$$
$$F_{UU}^{\cos\phi,h} = -\frac{2}{Q}\sum_q e_q^2 [h_{1q}^\perp \otimes H_{1q}^{\perp,h}\otimes w_1^\perp + f_{1q}\otimes D_{1q}^h\otimes w_1], \tag{4}$$

where the convolutions are defined as follows:

$$f\otimes D\otimes w = \int d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp \delta^2(\mathbf{P}_T - z_h\mathbf{k}_\perp - \mathbf{p}_\perp) \times f(x_B, k_\perp)D(z_h, p_\perp)w(\mathbf{P}_T, \mathbf{k}_\perp). \tag{5}$$

Here $\mathbf{k}_\perp$ is the transverse momentum of the quark in the target nucleon, $k_\perp = |\mathbf{k}_\perp|$; $\mathbf{p}_\perp$ is the transverse momentum of the final hadron with respect to the direction of the fragmenting quark, $p_\perp = |\mathbf{p}_\perp|$; at the order $(k_\perp/Q)$, for the measured transverse momentum of the final hadron, we have $\mathbf{P}_T = z_h\mathbf{k}_\perp + \mathbf{p}_\perp$.

The functions w_i and $w_i^\perp$ are flavor and hadron-type independent, and contain only kinematic factors:

$$w_1 = (\hat{\mathbf{P}}_T\mathbf{k}_\perp),$$
$$w_2 = 2(\hat{\mathbf{P}}_T\mathbf{k}_\perp)^2 - k_\perp^2,$$
$$w_1^\perp = \frac{k_\perp^2(P_T - z_h(\hat{\mathbf{P}}_T\mathbf{k}_\perp))}{z_h M_h M},$$
$$w_2^\perp = \frac{(\mathbf{P}_T\mathbf{k}_\perp) - 2z_h(\hat{\mathbf{P}}_T\mathbf{k}_\perp)^2 + z_h k_\perp^2}{z_h M_h M},$$
$$\hat{\mathbf{P}}_T = \frac{\mathbf{P}_T}{|\mathbf{P}_T|}, \qquad P_T = |\mathbf{P}_T|. \tag{6}$$

The only dependence of w_i and $w_i^\perp$ on the final hadron h is through M_h. However, this is irrelevant for us, as we shall consider the production of h and its antiparticle $\bar{h}$, for which $M_h = M_{\bar{h}}$.

In (4) $f_{1q}(x, k_\perp)$ and $D_{1q}^h(z, p_\perp)$ are the unpolarized TMD parton distribution and fragmentation functions, respectively, $h_{1q}^\perp(x, k_\perp)$ are the Boer-Mulders distribution functions [8] that describe the probability to find a transversely polarized quark q in an unpolarized proton, and $H_{1,q}^{\perp,h}(z, p_\perp)$ are the Collins fragmentation functions [9], that describe the probability for a transversely polarized quark q to produce an unpolarized hadron h with a fraction z of the longitudinal momentum and transverse momentum $p_\perp$ with respect to the momentum of the fragmenting quark.

The first term in (2), with F_{UU}^h, describes the ϕ_h-independent cross section; it is expressed through $f_{1q}(x,k_\perp)$ and $D_{1q}^h(z,p_\perp)$.

Two mechanisms generate the azimuthal $\cos\phi_h$ and $\cos 2\phi_h$ dependence.

(1) The Cahn effect [10], which is a purely kinematic effect, generated by the intrinsic transverse-quark momenta. It is described by the unpolarized TMD functions f_{1q} and $D_{1,q}^h$, and is a subleading effect: $1/Q^2$ contribution to $F_{UU}^{\cos 2\phi_h}$ and $1/Q$ contribution to $F_{UU}^{\cos\phi_h}$.

(2) The Boer-Mulders effect [8], which points to the existence of nonzero transverse polarization of the quarks and is described by the TMD functions with transversely polarized quarks: $h_{1q}^\perp$ and $H_{1q}^\perp$. The induced $\cos 2\phi_h$ dependence is a leading (twist-2) effect—the first term in $F_{UU}^{\cos 2\phi_h}$ in Eq. (4), the $\cos\phi_h$ dependence, is a subleading $1/Q$ effect.

Note that in (4) we have included the Cahn contribution to the $\cos 2\phi_h$ term, though it is of higher $1/Q^2$ order and the other terms of the same order are not included. We think this gives a more clear physical picture of the different contributions to the cross sections, though it is irrelevant for the discussions in the paper.

III. THE DIFFERENCE CROSS SECTIONS WITH $h^\pm$

Here we shall consider the difference of the cross sections for producing a hadron h and its antiparticle $\bar h$, when the type of the hadrons is not specified and they are distinguished only by their charge:

$$d\sigma_N^{h-\bar h} \equiv \frac{d^5\sigma_N^h}{dx_B dQ^2 dz_h d^2\mathbf{P}_T} - \frac{d^5\sigma_N^{\bar h}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T}, \quad (7)$$

where N stands for a proton or a neutron target, $N=p,n$.

Charge conjugation invariance of strong interactions implies the following relations on the unpolarized TMD and Collins FFs:

$$D_{1,q}^h = D_{1,\bar q}^{\bar h}, \qquad D_{1,\bar q}^h = D_{1,q}^{\bar h},$$
$$H_{1,q}^{\perp,h} = H_{1,\bar q}^{\perp,\bar h}, \qquad H_{1,\bar q}^{\perp,h} = H_{1,q}^{\perp,\bar h}. \quad (8)$$

Using these relations, from (2) and (7), we obtain the difference cross section $d\sigma_N^{h-\bar h}$. It is easily shown that the azimuthal dependence in $d\sigma_N^{h-\bar h}$ remains the same as in $d\sigma_N^h$, but the expressions for $F_{UU}^{h-\bar h}$ considerably simplify. We show that, based only on the general properties of charge conjugation invariance of strong interactions, only the contributions of the largest TMD valence-quark densities survive in $F_{UU}^{h-\bar h}$, $F_{UU}^{\cos 2\phi, h-\bar h}$ and $F_{UU}^{\cos\phi, h-\bar h}$. Below we give the expressions for the $F_{UU}^{h-\bar h}$'s for proton and deuterium targets separately.

As usual, subindex 1 indicates for a transverse momentum dependence $f_{1q}(x,k_\perp)\equiv q_1(x,k_\perp)$ and so on.

A. On proton target

The expression for the difference cross section on a proton target $d\sigma_p^{h-\bar h}$ is analogous to $d\sigma_p^h$, (2), in which F_{UU}^h are replaced by the corresponding $F_{UU}^{h-\bar h}$ as given below:

$$F_{UU}^{h-\bar h} = e_u^2 u_{1,V}\otimes D_{1,uV}^h + e_d^2 d_{1,V}\otimes D_{1,dV}^h,$$
$$F_{UU}^{\cos 2\phi_h, h-\bar h} = [e_u^2 h_{1,uV}^\perp \otimes H_{1,uV}^{\perp h} + e_d^2 h_{1,dV}^\perp\otimes H_{1,dV}^{\perp h}]\otimes w_2^\perp + \frac{2}{Q^2}[e_u^2 u_{1,V}\otimes D_{1,uV}^h + e_d^2 d_{1,V}\otimes D_{1,dV}^h]\otimes w_2,$$
$$F_{UU}^{\cos\phi_h, h-\bar h} = -\frac{2}{Q}\{[e_u^2 h_{1,uV}^\perp\otimes H_{1,uV}^{\perp h} + e_d^2 h_{1,dV}^\perp \otimes H_{1,dV}^{\perp h}]\otimes w_1^\perp + [e_u^2 u_{1,V}\otimes D_{1,uV}^h + e_d^2 d_{1,V}\otimes D_{1,dV}^h]\otimes w_1\}. \quad (9)$$

In these expressions we have neglected terms proportional to $s_{1V}\equiv s_1-\bar s_1$, which are small being proportional to $s-\bar s$, on which a strong bound from neutrino experiments exists, $|s-\bar s|\le 0.025$ [11]. We have also neglected the terms proportional to $h_{sV}^\perp \equiv h_{1s}^\perp - h_{1\bar s}^\perp$, which are small due to the positivity condition $h_{sV}^\perp \le s_{1V}$.

In our approach, naturally the TMD densities of the valence quarks $q_V = q-\bar q$ appear. They fragment into the final hadrons and the TMD valence-quark FFs appear: $D_{qV}^h = D_{q-\bar q}^h = D_q^{h-\bar h}$ (not to be confused with favored FFs!). We use the notation:

$$u_{1V} = u_1 - \bar u_1, \qquad d_{1V} = d_1 - \bar d_1,$$
$$h_{1,uV}^\perp = h_{1,u}^\perp - h_{1,\bar u}^\perp, \qquad h_{1,dV}^\perp = h_{1,d}^\perp - h_{1,\bar d}^\perp,$$
$$D_{1,uV}^h \equiv D_{1,u}^h - D_{1,\bar u}^h, \qquad D_{1,dV}^h \equiv D_{1,d}^h - D_{1,\bar d}^h,$$
$$H_{1,uV}^{\perp h} \equiv H_{1,u}^{\perp h} - H_{1,\bar u}^{\perp h}, \qquad H_{1,dV}^{\perp h} \equiv H_{1,d}^{\perp h} - H_{1,\bar d}^{\perp h}. \quad (10)$$

Instead of the sum over all quark flavors $q=u,\bar u,d,\bar d,s,\bar s$ in $d\sigma_p^h$, in $d\sigma_p^{h-\bar h}$ we have a sum over the two valence u_V and d_V quarks only. The sea quarks do not contribute.

In addition, the FFs that survive $D_{1,uV}^h$, $D_{1,dV}^h$ and $H_{1,uV}^h$, $H_{1,dV}^h$ couple to the large valence-quark densities $q_{1,V}$ and

$h^{\perp}_{1,qV}$. The strange-quark TMD FFs $D^h_{1,sV}$ and $H^h_{1,sV}$ are suppressed by the small factor $(s-\bar{s})$ and we safely neglect them.

B. On deuterium target

SU(2) invariance implies that the cross section on a neutron target is obtained from (9) with the replacements of the u and d parton densities:

$$u_{1V} \leftrightarrow d_{1V}, \qquad s_{1V} \to s_{1V},$$
$$h^{\perp}_{1,uV} \leftrightarrow h^{\perp}_{1,dV}, \qquad h^{\perp}_{1,sV} \to h^{\perp}_{1,sV}. \tag{11}$$

Then, for the contributions to the cross section $d\sigma_d^{h-\bar{h}}$ on a deuterium target

$$d\sigma_d^{h-\bar{h}} = d\sigma_p^{h-\bar{h}} + d\sigma_n^{h-\bar{h}}, \tag{12}$$

we obtain

$$F_{UU}^{h-\bar{h}}(d=p+n) = (u_{1,V}+d_{1,V})\otimes(e_u^2 D^h_{1,uV}+e_d^2 D^h_{1,dV}),$$
$$F_{UU}^{\cos 2\phi_h,h-\bar{h}}(d=p+n) = (h^{\perp}_{1,uV}+h^{\perp}_{1,dV})\otimes(e_u^2 H^{h\perp}_{1,uV}+e_d^2 H^{h\perp}_{1,dV})\otimes w_2^{\perp}+\frac{2}{Q^2}(u_{1,V}+d_{1,V})\otimes(e_u^2 D^h_{1,uV}+e_d^2 D^h_{1,dV})\otimes w_2,$$
$$F_{UU}^{\cos\phi_h,h-\bar{h}}(d=p+n) = -\frac{2}{Q}\{(h^{\perp}_{1,uV}+h^{\perp}_{1,dV})\otimes(e_u^2 H^{h\perp}_{1,uV}+e_d^2 H^{h\perp}_{1,dV})\otimes w_1^{\perp}+(u_{1,V}+d_{1,V})\otimes(e_u^2 D^h_{1,uV}+e_d^2 D^h_{1,dV})\otimes w_1\}. \tag{13}$$

Note that only two combinations of TMD valence-quark densities $(u_{1,V}+d_{1,V})$ and $(h^{\perp}_{1,uV}+h^{\perp}_{1,dV})$ and only two combinations of TMD FFs $(e_u^2 D^h_{1,uV}+e_d^2 D^h_{1,dV})$ and $(e_u^2 H^{h\perp}_{1,uV}+e_d^2 H^{h\perp}_{1,dV})$ enter. In addition, TMD PDFs and TMD FFs do not mix and each one can be parametrized separately.

These expressions are further simplified when the final hadrons are specified, which will be done in the next sections.

IV. THE DIFFERENCE CROSS SECTIONS WITH $\pi^{\pm}$

When the final hadrons are $\pi^{\pm}$, SU(2) invariance of strong interactions implies

$$D^{\pi^+}_{1,uV} \equiv D^{\pi^+}_{1,u} - D^{\pi^+}_{1,\bar{u}} = -D^{\pi^+}_{1,dV} \tag{14}$$

and similarly for the Collins FFs $H^{\perp\pi^{\pm}}_{1,q}$:

$$H^{\perp\pi^+}_{1,uV} = -H^{\perp\pi^+}_{1,dV}. \tag{15}$$

Then from (9) and (13) we obtain the difference cross sections $d\sigma_N^{\pi^+-\pi^-}$. We present the expressions for proton and deuterium targets separately.

A. On proton target

From (9), for the contributions to $d\sigma_p^{\pi^+-\pi^-}$ we obtain

$$F_{UU}^{\pi^+-\pi^-} = (e_u^2 u_{1,V}-e_d^2 d_{1,V})\otimes D^{\pi^+}_{1,uV},$$
$$F_{UU}^{\cos 2\phi_h,\pi^+-\pi^-} = (e_u^2 h^{\perp}_{1,uV}-e_d^2 h^{\perp}_{1,dV})\otimes H^{\perp\pi^+}_{1,uV}\otimes w_2^{\perp} + \frac{2}{Q^2}(e_u^2 u_{1,V}-e_d^2 d_{1,V})\otimes D^{\pi^+}_{1,uV}\otimes w_2,$$
$$F_{UU}^{\cos\phi_h,\pi^+-\pi^-} = -\frac{2}{Q}\{(e_u^2 h^{\perp}_{1,uV}-e_d^2 h^{\perp}_{1,dV})\otimes H^{\perp\pi^+}_{1,uV}\otimes w_1^{\perp} + (e_u^2 u_{1,V}-e_d^2 d_{1,V})\otimes D^{\pi^+}_{1,uV}\otimes w_1\}. \tag{16}$$

B. On deuterium target

From (13) for the contributions to $d\sigma_d^{\pi^+-\pi^-}$ we obtain

$$F_{UU}^{\pi^+-\pi^-} = (e_u^2-e_d^2)(u_{1,V}+d_{1,V})\otimes D^{\pi^+}_{1,uV},$$
$$F_{UU}^{\cos 2\phi_h,\pi^+-\pi^-} = (e_u^2-e_d^2)\{(h^{\perp}_{1,uV}+h^{\perp}_{1,dV})\otimes H^{\perp\pi^+}_{1,uV}\otimes w_2^{\perp} + \frac{2}{Q^2}(u_{1,V}+d_{1,V})\otimes D^{\pi^+}_{1,uV}\otimes w_2\},$$
$$F_{UU}^{\cos\phi_h,\pi^+-\pi^-} = -\frac{2}{Q}(e_u^2-e_d^2)\{(h^{\perp}_{1,uV}+h^{\perp}_{1,dV})\otimes H^{\perp\pi^+}_{1,uV}\otimes w_1^{\perp} + (u_{1,V}+d_{1,V})\otimes D^{\pi^+}_{1,uV}\otimes w_1\}. \tag{17}$$

It is just one TMD FF for unpolarized $D^{\pi^+}_{1,uV}$ and one for polarized quarks $H^{\perp\pi^+}_{1,uV}$ that enter, which would allow us to determine them independently of the behavior of the other TMDs.

V. THE DIFFERENCE CROSS SECTIONS WITH $K^{\pm}$

If we consider only charged kaons, we cannot use SU(2) invariance as it relates neutral to charged kaons. However, in order to simplify analysis, the assumption made in all analysis of kaon production is that the unfavored FFs of d and $\bar{d}$ quarks into K^+ are the same:

$$D^{K^+}_{dV} = H^{\perp K^+}_{dV} = 0. \tag{18}$$

Below we present the functions $F_{UU}^{K^+-K^-}$ that enter $d\sigma^{K^+-K^-}$, for proton and deuterium targets separately, using this assumption.

A. On proton target

From (9) for the terms $F_{UU}^{K^+-K^-}$ in $d^5\sigma_p^{K^+-K^-}$ we obtain

$$\begin{aligned} F_{UU}^{K^+-K^-} &= e_u^2 u_{1,V}\otimes D_{1,uV}^{K^+},\\ F_{UU}^{\cos 2\phi_h,K^+-K^-} &= e_u^2\left\{h_{1,uV}^{\perp}\otimes H_{1,uV}^{\perp K^+}\otimes w_2^{\perp}+\frac{2}{Q^2}u_{1V}\otimes D_{1,uV}^{K^+}\otimes w_2\right\},\\ F_{UU}^{\cos\phi_h,K^+-K^-} &= -\frac{2}{Q}e_u^2\{h_{1,uV}^{\perp}\otimes H_{1,uV}^{\perp K^+}\otimes w_1^{\perp}+u_{1,V}\otimes D_{1,uV}^{K^+}\otimes w_1\}. \end{aligned} \tag{19}$$

B. On deuterium target

From (13) for $d\sigma_d^{K^+-K^-}$ we obtain

$$\begin{aligned} F_{UU}^{K^+-K^-} &= e_u^2(u_{1,V}+d_{1,V})\otimes D_{1,uV}^{K^+},\\ F_{UU}^{\cos 2\phi_h,K^+-K^-} &= e_u^2\left\{(h_{1,uV}^{\perp}+h_{1,dV}^{\perp})\otimes H_{1,uV}^{\perp K^+}\otimes w_2^{\perp}+\frac{2}{Q^2}(u_{1,V}+d_{1,V})\otimes D_{1,uV}^{K^+}\otimes w_2\right\},\\ F_{UU}^{\cos\phi_h,K^+-K^-} &= -\frac{2}{Q}e_u^2\{(h_{1,uV}^{\perp}+h_{1,dV}^{\perp})\otimes H_{1,uV}^{\perp K^+}\otimes w_1^{\perp}+(u_{1,V}+d_{1,V})\otimes D_{1,uV}^{K^+}\otimes w_1\}. \end{aligned} \tag{20}$$

Here we summarize some common features of the considered difference cross sections.

(1) On a deuterium target, both for $h-\bar{h}$, $\pi^+-\pi^-$ and K^+-K^-, always the same combinations of TMD parton densities are measured: $(u_{1,V}+d_{1,V})$ and $(h_{1,uV}^{\perp}+h_{1,dV}^{\perp})$.

(2) On a deuterium target, it is always one combination of unpolarized and one of polarized quark TMD FFs that enter. This combination depends on the final hadron—for charged hadrons it is $(e_u^2 D_{1,uV}^h+e_d^2 D_{1,dV}^h)$ and $(e_u^2 H_{1,uV}^{h\perp}+e_d^2 H_{1,dV}^{h\perp})$, for $\pi^{\pm}$ it is $D_{1,uV}^{\pi^+}$ and $H_{1,uV}^{\perp\pi^+}$, and for $K^{\pm}$ it is $D_{1,uV}^{K^+}$ and $H_{1,uV}^{\perp K^+}$. However, the key point, that it is always only one quantity, remains which allows us to extract it irrespectively from the other TMD FFs.

(3) Both on proton and deuterium targets, only the valence-quark TMD functions enter all difference cross sections.

VI. THE CROSS SECTION FOR $\mathcal{K}=K^++K^--2K_s^0$

Up to now we considered production of any charged hadrons, $h-\bar{h}$ and $h=\pi^{\pm},K^{\pm}$. Now we consider production of kaons only.

If in addition to the charged $K^{\pm}$ also neutral kaons $K_s^0=(K^0+\bar{K}^0)/\sqrt{2}$ are measured, SU(2) invariance of the strong interactions implies that no new FFs are introduced into the cross sections. We have

$$\begin{aligned} D_{1u}^{K^++K^--2K_s^0} &= -D_{1d}^{K^++K^--2K_s^0}=(D_{1u}-D_{1d})^{K^++K^-},\\ D_{1s}^{K^++K^--2K_s^0} &= D_{1c}^{K^++K^--2K_s^0}=D_{1b}^{K^++K^--2K_s^0}=0, \end{aligned} \tag{21}$$

and similarly for $H_{1q}^{\perp,h}$.

We show that, in the difference of charged and neutral kaon production in SIDIS, $d\sigma^{\mathcal{K}}$:

$$d\sigma^{\mathcal{K}}=d\sigma^{K^++K^--2K_s^0}\equiv d\sigma^{K^+}+d\sigma^{K^-}-2d\sigma^{K_s^0}, \tag{22}$$

only one combination of unpolarized TMD FFs $(D_{1u}-D_{1d})^{K^++K^-}$ and one combination of Collins functions $(H_{1u}-H_{1d})^{\perp,K^++K^-}$ enter, both for proton and deuterium targets. This result is obtained under the only assumption of SU(2) invariance. We give the expressions for $d\sigma^{\mathcal{K}}$ on proton and deuterium targets.

A. On proton target

Using (21) for $d\sigma_p^{\mathcal{K}}$ we obtain

$$\begin{aligned} F_{UU}^{\mathcal{K}} &= [e_u^2(u_1+\bar{u}_1)-e_d^2(d_1+\bar{d}_1)]\otimes D_{1,u-d}^{K^++K^-},\\ F_{UU}^{\cos2\phi_h,\mathcal{K}} &= [e_u^2(h_{1,u}^{\perp}+h_{1,\bar{u}}^{\perp})-e_d^2(h_d^{\perp}+h_{\bar{d}}^{\perp})]\otimes H_{1,u-d}^{\perp,K^++K^-}\otimes w_2^{\perp}\\ &\quad+\frac{2}{Q^2}[e_u^2(u_1+\bar{u}_1)-e_d^2(d_1+\bar{d}_1)]\otimes D_{1,u-d}^{K^++K^-}\otimes w_2,\\ F_{UU}^{\cos\phi_h,\mathcal{K}} &= -\frac{2}{Q}\{[e_u^2(h_{1,u}^{\perp}+h_{1,\bar{u}}^{\perp})-e_d^2(h_{1,d}^{\perp}+h_{1,\bar{d}}^{\perp})]\\ &\quad\otimes H_{1,u-d}^{\perp,K^++K^-}\otimes w_1^{\perp}+[e_u^2(u_1+\bar{u}_1)\\ &\quad-e_d^2(d_1+\bar{d}_1)]\otimes D_{1,u-d}^{K^++K^-}\otimes w_1\}. \end{aligned} \tag{23}$$

Here we have used the brief notation:

$$\begin{aligned} D_{1,u-d}^{K^++K^-} &= (D_{1u}-D_{1d})^{K^++K^-},\\ H_{1,u-d}^{\perp,K^++K^-} &= (H_{1u}-H_{1d})^{\perp,K^++K^-}. \end{aligned} \tag{24}$$

B. On deuterium target

Using (21) for $d\sigma_d^{\mathcal{K}}$ we obtain

$$F_{UU}^{\mathcal{K}} = (e_u^2 - e_d^2)(u_1 + \bar{u}_1 + d_1 + \bar{d}_1) \otimes D_{1,u-d}^{K^+ + K^-},$$
$$F_{UU}^{\cos 2\phi_h,\mathcal{K}} = (e_u^2 - e_d^2)\left\{(h_{1,u}^\perp + h_{1,\bar{u}}^\perp + h_{1,d}^\perp + h_{1,\bar{d}}^\perp) \otimes H_{1,u-d}^{\perp,K^+ + K^-} \otimes w_2^\perp + \frac{2}{Q^2}(u_1 + \bar{u}_1 + d_1 + \bar{d}_1) \otimes D_{1,u-d}^{K^+ + K^-} \otimes w_2\right\},$$
$$F_{UU}^{\cos \phi_h,\mathcal{K}} = -\frac{2}{Q}(e_u^2 - e_d^2)\{(h_{1,u}^\perp + h_{1,\bar{u}}^\perp + h_{1,d}^\perp + h_{1,\bar{d}}^\perp) \otimes H_{1,u-d}^{\perp,K^+ + K^-} \otimes w_1^\perp + (u_1 + \bar{u}_1 + d_1 + \bar{d}_1) \otimes D_{1,u-d}^{K^+ + K^-} \otimes w_1\}. \quad (25)$$

Note that the analogous combinations for pions $d\sigma^{\pi^+ + \pi^- - 2\pi^0}$, both for proton and deuteron targets, will be identically 0, if the usually used relation for the collinear FFs $D_q^{\pi^+ + \pi^-} = 2D_q^{\pi^0}$ that follows from the quark model holds for the TMD FFs D_{1q} and $H_{1q}^\perp$ as well.

Common for all differences is that TMD parton densities factorize from FFs.

VII. PARAMETRIZATIONS AND COMMENTS

Up to now all considerations were general, based only on C and SU(2) invariance of strong interactions, with no assumptions on the parametrizations of the TMD PDFs and the TMD FFs. Here we shall summarize the conventionally used parametrizations and suggest how they modify when applied to the considered approach.

There are four types of TMDs for each quark flavor $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$, that enter the differential cross sections $d\sigma_N^h$ of unpolarized SIDIS—the unpolarized quark densities q_1 that couple unpolarized FFs $D_{1,q}^h$, and the transversely polarized quark densities $h_{1,q}^\perp$ that couple to the transversely polarized FFs $H_{1,q}^\perp$. This makes, in total, 24 independent quantities for each type of hadrons, that have to be determined.

In the difference cross sections that we consider, the four types of TMDs are only for the two valence quarks $q_V = u_V, d_V$—the unpolarized valence-quark densities q_{1V} that couple unpolarized valence-quark FFs D_{1,q_V}^h, and the transversely polarized valence-quark densities $h_{1qV}^\perp$ that couple to the transversely polarized valence-quark FFs $H_{1,qV}^\perp$. The independent unknown quantities are reduced from 24 in total, to at most eight.

Many simplifying assumptions are made in the performed conventional analysis of $d\sigma_N^h$: the x and $k_\perp$ (z and $p_\perp$) dependence is factorized with a Gaussian dependence on the transverse momenta, no flavor, no Q^2, no x and no z dependencies in the transverse-dependent parts; the Q^2 evolution is only in the collinear PDFs and FFs according to the Dokshitzer-Gribov-Lipatov-Altarelly-Parisi (DGLAP) equations. Here we present the standard parametrizations for the TMD quark densities and FFs for all quark flavors, and discuss how they can be modified for the TMD valence quarks. We comment on the advantages of the considered approach. We consider the unpolarized and the transversely polarized quark TMD functions separately.

A. The ϕ_h-independent terms

(1) The TMD parton densities and fragmentation functions with unpolarized quarks are [12], $q = u, \bar{u}, d, \bar{d}, s, \bar{s}$:

$$q_1(x, k_\perp, Q^2) = q(x, Q^2)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle},$$
$$D_{1,q}^h(z, p_\perp, Q^2) = D_q^h(z, Q^2)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}, \quad (26)$$

where $q(x)$ and D_d^h are the collinear PDFs and FFs. The fitting parameters $\langle k_\perp^2\rangle$ and $\langle p_\perp^2\rangle$ are assumed flavor independent. This leads to a Gaussian-type dependence on P_T^2, with a z_h-dependent width $\langle P_T^2\rangle$:

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h\langle k_\perp^2\rangle. \quad (27)$$

In a very recent analysis [13], from a separate fit to multiplicities in the unpolarized SIDIS data of COMPASS (with charged unidentified hadron $h^\pm$ on a deuteron) and HERMES (with $\pi^\pm$ and $K^\pm$ on proton and deuterium targets), $\langle k_\perp^2\rangle$ and $\langle p_\perp^2\rangle$ were determined with good precision.

(2) In the discussed differences only the two valence-quark TMD parton densities $u_{1,V}$ and $d_{1,V}$, and the two valence-quark TDM FFs $D_{1,uV}^h$ and $D_{1,dV}^h$ enter. They can be parametrized analogously, $q_V = u_V, d_V$:

$$q_{1,V}(x, k_\perp, Q^2) = q_V(x, Q^2)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_{qV}}}{\pi\langle k_\perp^2\rangle_{qV}}, \quad (28)$$

$$D_{1,q_V}^h(z, p_\perp, Q^2) = D_{q_V}^h(z, Q^2)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{qV}}}{\pi\langle p_\perp^2\rangle_{qV}}, \quad (29)$$

where q_V and D^h_{qV} are the collinear valence PDFs and FFs. As multiplicities on proton and deuterium targets provide two independent measurements, one could relax the assumption of flavor independence and fit data with flavor-dependent parameters $\langle k_\perp^2\rangle_{qV}$ and $\langle p_\perp^2\rangle_{qV}$, $q_V = u_V, d_V$. This implies that the P_T^2 dependence will no longer be a simple Gaussian distribution. Recently, the first studies on flavor dependence of the partonic transverse momentum in unpolarized TMD functions were done and interesting results were obtained [14]. We hope this will help such investigations.

(3) Measurements on the deuterium target with $h-\bar{h}$, $\pi^+ - \pi^-$ and $K^+ - K^-$ final hadrons provide three independent measurements for the sum of the valence-quark TMD: $u_{1,V} + u_{1,V}$.

(4) Measurements on the deuterium target always measure only one combination of the unpolarized valence-quark TMD FFs, which allows us to determine it without additional assumptions, independently from the other TMD FFs. The combination depends on the final hadron $h-\bar{h}$, $\pi^+-\pi^-$ or K^+-K^-.

B. The ϕ_h-dependent terms

(1) Using the ansatz of Refs. [15–17], the Boer-Mulders and Collins functions, $h^\perp_{1,q}$ and $H^{\perp,h}_{1,q}$, are most generally proportional to the unpolarized TMD PDFs and FFs, respectively:

$$
\begin{aligned}
h^\perp_{1,q}(x,k_\perp,Q^2) &= \rho_q(x)\eta(k_\perp) f_{1,q}(x,k_\perp,Q^2),\\
H^{\perp,h}_{1,q}(z,p_\perp,Q^2) &= \rho^C_q(z)\eta^C(p_\perp) D^h_{1,q}(z,p_\perp,Q^2),
\end{aligned}
\tag{30}
$$

where $\rho_q(x)$ and $\rho^C_q(z)$, $\eta(k_\perp)$ and $\eta^C(p_\perp)$ are new fitting functions. Usually the transverse-dependent functions η and η^C are assumed flavor independent.

(2) Only two valence-quark Boer-Mulders densities $h_{1,uV}$ and $h_{1,dV}$, and two valence-quark Collins functions $H^h_{1,uV}$ and $H^h_{1,dV}$ enter the difference cross sections. They can be parametrized analogously:

$$
\begin{aligned}
h^\perp_{1qV}(x,k_\perp,Q^2) &= \rho_{qV}(x)\eta_{qV}(k_\perp) q_{1,V}(x,k_\perp,Q^2)\\
&= \rho_{qV}(x)\eta_{qV}(k_\perp) q_V(x,Q^2)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle},
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&H^{\perp,h}_{1qV}(z,p_\perp,Q^2)\\
&\quad= \rho^C_{qV}(z)\eta^C_{qV}(p_\perp) D^h_{1,qV}(z,p_\perp,Q^2)\\
&\quad= \rho^C_{qV}(z)\eta^C_{qV}(p_\perp) D^h_{qV}(z,Q^2)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},
\end{aligned}
\tag{32}
$$

where, given the simplicity of the approach, the TMD functions $\eta_{qV}(k_\perp)$ and $\eta^C_{qV}(p_\perp)$ can be considered flavor dependent. Measurements of the $\cos 2\phi_h$ (and $\cos\phi_h$) asymmetry on proton and deuterium targets provide two independent measurements that would allow us, in principle, to determine them separately.

(3) Measurements on the deuterium target with $h-\bar{h}$, $\pi^+ - \pi^-$ and $K^+ - K^-$ provide three independent measurements for the sum of the valence-quark Boer-Mulders functions: $h^\perp_{1,uV} + h^\perp_{1,dV}$.

(4) Measurements on the deuterium target always measure only one combination of valence-quark Collins functions. This allows us to determine it independently from the other TMD FFs. The combination depends on the final hadron $h-\bar{h}$, $\pi^+-\pi^-$ or K^+-K^-.

(5) Following the same path of arguments, the parametrizations for $D^{K^++K^-}_{1,u-d}$ and $H^{K^++K^-}_{1,u-d}$ are

$$
\begin{aligned}
D^{K^++K^-}_{1,u-d}(z,p_\perp,Q^2) &= D^{K^++K^-}_{u-d}(z,Q^2)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_{u-d}}}{\pi\langle p_\perp^2\rangle_{u-d}},\\
H^{K^++K^-}_{1,u-d}(z,p_\perp,Q^2) &= \rho^C_{u-d}(z)\eta^C_{u-d}(p_\perp) D^{K^++K^-}_{1,u-d}\\
&\quad\times(z,p_\perp,Q^2).
\end{aligned}
\tag{33}
$$

Measurements on proton and deuterium targets could determine $\langle p_\perp^2\rangle_{u-d}$ and $\eta^C_{u-d}(p_\perp)$ independently, without any relations between them and to other TMD fragmentation functions. Note that the collinear FFs $D^{K^++K^-}_{u-d}(z)$ that enter are known solely from the inclusive e^+e^- annihilation process: $e^+e^- \to K^\pm + X$; without the assumptions of favored and unfavored FFs, they evolve in Q^2 as nonsinglets according to the DGLAP equations.

C. Common for all differences

(1) All differences rely, as known quantities, on the collinear valence-quark PDFs u_V and d_V, which are the best known parton densities (with 2%–3% accuracy at $x \lesssim 0.7$), and on the collinear valence FFs D^{h,π^+,K^+}_{uV}. Very recently, $D^{\pi^+}_{uV}$ were determined with a very good precision, directly in a model-independent analysis of the HERMES data [4].

(2) The Q^2 dependence of the nonsinglets q_V and D^h_{qV}, that enter the valence-quark parametrizations, is relatively simple. This would make it easier to investigate the Q^2 dependence in the transverse momentum-dependent part. Recently it was found [13] that a logarithmic Q^2 dependence in $\langle P_T^2\rangle$ improves description of the data.

VIII. CONCLUSIONS

We have presented an alternative, simpler approach for extracting the TMD parton densities and FFs that enter the cross section of unpolarized SIDIS.

Based only on factorization, C invariance and SU(2) invariance of strong interactions, without any assumptions about PDFs and FFs, we show that the difference cross sections of unpolarized SIDIS $d\sigma_N^{h^+} - d\sigma_N^{h^-}$, $d\sigma_N^{\pi^+} - d\sigma_N^{\pi^-}$ and $d\sigma_N^{K^+} - d\sigma_N^{K^-}$ are expressed solely in terms of the valence-quark TMD unpolarized densities $q_{1,V}$ and Boer-Mulders functions $h_{1,qV}^{\perp}$, and the valence-quark TMD unpolarized fragmentation $D_{1,qV}^h$ and Collins $H_{1,qV}^{\perp}$ functions. If measurements on proton and deuterium targets are fulfilled, model-independent information about these quantities can be obtained. Measurements on a deuterium target, both for $h^+ - h^-$, $\pi^+ - \pi^-$ and $K^+ - K^-$ production, provide information about the sum of the valence-quark TMD densities $(u_{1,V} + d_{1,V})$ and $(h_{1,uV}^{\perp} + h_{1,dV}^{\perp})$.

If in addition to charged kaons $K^{\pm}$, also the neutral K_s^0 can be measured, then SU(2) invariance implies that the difference of the produced charged and neutral kaons, $d\sigma_N^{K^+ + K^-} - d\sigma_N^{-K_s^0}$, both on proton and deuterium targets, is expressed in terms of only one combination of the TMD FFs $(D_{1,u} - D_{1,d})^{K^+ + K^-}$ and one combination of Collins functions $(H_{1,u}^{\perp} - H_{1,d}^{\perp})^{K^+ + K^-}$.

The suggested measurements of the difference cross sections provide information only about the TMD valence-quark densities and FFs, but they allow us to determine them separately, without imposing any relations among them or to other TMDs. They present a sort of sum rules, based on C and SU(2) invariance, which reduce the contribution of all TMD functions in the cross section, to a contribution only of the valence-quark TMD functions in the difference cross sections.

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