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Oscillating strings and non-Abelian T-dual Klebanov-Witten background

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We study oscillating string solutions in the Klebanov-Witten and its non-Abelian T-dual background dualized along an SU(2) isometry. We find the string energy as the function of the oscillation number and angular momentum. We show that for a particular set of T-dual coordinates both the backgrounds have equal string states. We also study the string states where the strings are expanding and contracting in the T-dual coordinate direction. We expect the presence of the superconformal field theory dual operators whose anomalous dimensions depend on the T-dual coordinate.

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I. INTRODUCTION

Research on string dualities has added much of our understanding to string theory. Establishing the duality between seemingly different theories has been a major research area since the inception of string theory. This has given many interesting and useful results, from which a few are mentioned below. First, it has taught us how to relate various string theories in different regimes of validity and compactifications. It also has led us to the discovery of higher dimensional objects, such as D-branes and membranes. Moreover, it has given us new insights to the nonperturbative regime of the theory. The discovery of the D-brane and its world volume gauge theory has prompted the proposal of an example of gauge or gravity duality [1–3], which relates the type IIB string theory in the $AdS_5 \times S^5$ background to the $\mathcal{N} = 4$ supersymmetric $\mathrm{SU}(N)$ Yang-Mills gauge theory in four dimensions. This duality maps the anomalous dimensions of gauge-invariant operators in the gauge theory to the energy spectrum of the string-theory states. This equivalence beyond the Bogomol'nyi-Prasad-Sommerfield limit relies on the fact that on the string-theory side the quantum corrections of strings are suppressed by the large quantum number, while on the field-theory side the anomalous dimension matrices of the dual composite operators are related to the Hamiltonian of the integrable spin chain. The idea of the operators with large quantum number was first proposed in [4] and further explored in [5], which has prompted the research on the semiclassical analysis of the rigidly rotating string and its implications in the AdS/CFT correspondence. The duality has been generalized to different models with less-supersymmetric backgrounds with or without conformal invariance [6,7]. In the semiclassical limit, both the rotating and oscillating strings have been studied in both AdS and non-AdS, supersymmetric and less-supersymmetric backgrounds [8–25]. Though the oscillating strings have more stability [26] than the nonoscillating one, they are less explored compared to rotating strings.

The T duality is a symmetry transformation that relates different string backgrounds with some isometries. The idea of generalizing T duality to include non-Abelian isometry groups has been worked out in [27]. When isometry groups are non-Abelian, we reached a non-Abelian T duality which is a proven technique to construct supergravity duals of strongly coupled field theories [28–34], and interestingly these backgrounds retain supersymmetry [34,35]. Klebanov-Witten background is a good example of it where dualizing along SU(2) isometries provides us a type IIA supergravity σ-model background. In non-Abelian transformation the isometry is partially destroyed which can be recovered as nonlocal symmetry in the σ model, and the corresponding σ models are canonically equivalent [36]. The semiclassical analysis of strings in Klebanov-Witten background has been studies in [37–42]. Our study is motivated by the recent paper [43], where the rotating string solutions have been worked out in Klebanov-Witten and its non-Abelian T-dual background. It has been shown that both backgrounds enjoy an equivalent subsector of states depending on the values of the T-dual coordinates. Here we wish to study oscillating strings in the Klebanov-Witten and its non-Abelian T-dual background and compare the results in different regimes.

The rest of the paper is organized as follows. In Sec. II, we analyze the oscillating strings in the Klebanov-Witten background when the oscillation is in AdS and in $T^{1,1}$ separately. We find the energy and oscillation number dispersion relation. In Sec. III, we analyze different classes of oscillating string configurations in non-Abelian T-dual Klebanov-Witten background. We also study the case of the oscillating string when the oscillation is in the T-dual coordinate direction. In Sec. IV, we conclude with some discussion.

II. OSCILLATING STRING IN $AdS_5 \times T^{1,1}$

We start with the Klebanov-Witten background which is the infrared limit of the theory on N coincident D3 branes placed at the conical singularity of $M_4 \times C$ [44]:

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$$ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\chi^2 + \sin^2\chi d\xi_1^2 + \cos^2\chi d\xi_2^2)) + R^2(\lambda_1^2(\sigma_{\hat{1}}^2 + \sigma_{\hat{2}}^2) + \lambda_2^2(\sigma_1^2 + \sigma_2^2) + \lambda^2(\sigma_3 + \cos\theta_1 d\phi_1)^2), \quad (1)$$

where R^2 is the curvature radius of $T^{1,1}, \lambda_1^2 = \lambda_2^2 = \frac{1}{6}, \lambda^2 = \frac{1}{9}$,

$$\sigma_{\hat{1}} = \sin\theta_1 d\phi_1, \qquad \sigma_{\hat{2}} = d\theta_1, \qquad \sigma_3 = d\psi + \cos\theta_2 d\phi_2,$$
$$\sigma_1 = \cos\psi\sin\theta_2 d\phi_2 - \sin\psi d\theta_2, \qquad \sigma_2 = \sin\psi\sin\theta_2 d\phi_2 + \cos\psi d\theta_2,$$

and the chosen coordinates range as $\rho \in [0, \infty]$, $\chi, \xi_i \in [0, 2\pi], \theta_i \in [0, \pi], \phi_i \in [0, 2\pi], \psi \in [0, 4\pi]$, and $i = 1, 2$. Here $T^{1,1}$ is the homogenous space $\frac{SU(2)\times SU(2)}{U(1)}$ with the diagonal embedding of the U(1) and Einstein metric to be $R_{ij} = 4g_{ij}$. This is an $\mathcal{N} = 1$ superconformal field theory which is dual to the type IIB theory compactified on $AdS_5 \times T^{1,1}$.

A. Oscillating in AdS

Here we wish to study a class of string solutions which is oscillating in the radial ρ direction of the AdS and at the same time rotating along the ψ direction of the $T^{1,1}$ with an angular momentum. So we chose our ansatz as follows:

$$\chi = \xi_i = 0, \qquad t = t(\tau), \qquad \rho = \rho(\tau),$$
$$\theta_2 = \phi_i = 0, \qquad \psi = \psi(\tau), \qquad \theta_1 = \theta = m\sigma. \quad (2)$$

Now putting the above ansatz in Eq. (1), we get the relevant background as

$$ds^2 = R^2\left[-\cosh\rho dt^2 + d\rho^2 + \frac{1}{6}d\theta^2 + \frac{1}{9}d\psi^2\right]. \quad (3)$$

For the above background the Polyakov action is written as

$$S_p = \frac{R^2}{4\pi}\int d\sigma d\tau\left[-\cosh^2\rho\dot{t}^2 + \dot{\rho}^2 - \frac{1}{6}m^2 + \frac{1}{9}\dot{\psi}^2\right], \quad (4)$$

where the "dot" denotes the derivative with respect to τ. We can write the equations of motion for t and ρ:

$$\ddot{t} + 2\tanh\rho\dot{\rho}\,\dot{t} = 0,$$
$$\ddot{\rho} + \cosh\rho\sinh\rho\dot{t}^2 = 0. \quad (5)$$

Now from Virasoro constraint we get

$$\dot{\rho}^2 = \cosh^2\rho\dot{t}^2 - \frac{1}{6}m^2 - \frac{1}{9}\dot{\psi}^2. \quad (6)$$

From the Polyakov action we get the conserved charges

$$E = R^2\mathcal{E} = R^2\cosh^2\rho\dot{t},$$
$$J = R^2\mathcal{J} = \frac{R^2}{9}\dot{\psi}. \quad (7)$$

Now Eq. (6) changes to

$$\dot{\rho}^2 = \frac{\mathcal{E}}{\cosh^2\rho} - \kappa^2, \quad (8)$$

where $\kappa^2 = \frac{1}{6}m^2 + 9\mathcal{J}^2$. The oscillation number for the string is

$$N = R^2\mathcal{N} = \frac{R^2}{2\pi}\oint d\rho\dot{\rho}. \quad (9)$$

By putting $x = \sinh^2\rho$ in the above equation, we get

$$\mathcal{N} = \frac{1}{\pi}\int_0^a dx\frac{\sqrt{\mathcal{E}^2 - \kappa^2(1+x^2)}}{1+x^2}, \quad (10)$$

where $a = \sqrt{\frac{\mathcal{E}^2-\kappa^2}{\kappa^2}}$ and $\mathcal{E}$ is $\mathcal{E}^2 > \kappa^2 > 0$. In order to compute the oscillation number we take the partial derivative of the above equation with respect to m as prescribed in Ref. [15]:

$$\frac{\partial\mathcal{N}}{\partial m} = -\frac{m}{6\pi}\int_0^a\frac{dx}{\sqrt{\mathcal{E}^2 - \kappa^2(1+x^2)}} = -\frac{m}{12\kappa} = -\frac{m}{12\sqrt{\frac{1}{6}m^2 + 9\mathcal{J}^2}}. \quad (11)$$

For large $\mathcal{E}$, integrating the above equation over m,

$$\mathcal{N} = \mathcal{N}_0 - \frac{1}{2}\sqrt{\frac{1}{6}m^2 + 9\mathcal{J}^2}, \quad (12)$$

where the integration constant $\mathcal{N}_0(\mathcal{E}, \mathcal{J})$ can be computed from the integral (10) for $m = 0$:

$$\mathcal{N}_0 = \frac{1}{\pi}\int_0^b dx \frac{\sqrt{\mathcal{E}^2 - \mathcal{J}^2(1+x^2)}}{1+x^2} = \frac{1}{2}(\mathcal{E} - 3\mathcal{J}), \tag{13}$$

where $b = \sqrt{\frac{\mathcal{E}^2 - 9\mathcal{J}^2}{9\mathcal{J}^2}}$. Putting the above value in Eq. (12), we get the energy as the function of the oscillation number and angular momentum as

$$\mathcal{E} = 2\mathcal{N} + 3\mathcal{J} - \sqrt{\frac{m^2}{6} + 9\mathcal{J}^2}. \tag{14}$$

The last term in the above equation can be expanded according to the angular momentum. When $m^2 < 54\mathcal{J}^2$,

$$\mathcal{E} = 2\mathcal{N} - 3\mathcal{J}\left[\frac{1}{108}\frac{m^2}{\mathcal{J}^2} - \frac{1}{23328}\frac{m^4}{\mathcal{J}^4} + \mathcal{O}\left[\frac{m^6}{\mathcal{J}^6}\right]\right]. \tag{15}$$

When $m^2 > 54\mathcal{J}^2$,

$$\mathcal{E} = 2\mathcal{N} + 3\mathcal{J} - \frac{m}{\sqrt{6}}\left[1 + 27\frac{\mathcal{J}^2}{m^2} - \frac{729}{2}\frac{\mathcal{J}^4}{m^4} + \mathcal{O}\left[\frac{\mathcal{J}^6}{m^6}\right]\right]. \tag{16}$$

B. Oscillating in $T^{1,1}$

In this subsection, we study another class of string solutions where the string is oscillating in the θ direction of the $T^{1,1}$. We chose the ansatz as

$$t = t(\tau), \quad \rho = 0, \quad \theta_2 = 0, \quad \phi_2 = 0,$$
$$\theta_1 = \theta = \theta(\tau), \quad \phi_1 = \phi = m\sigma, \quad \psi = 0. \tag{17}$$

Now the metric in Eq. (1) changes to

$$ds^2 = R^2\left[-dt^2 + \frac{1}{6}d\theta^2 + \left(\frac{1}{9} + \frac{1}{18}\sin^2\theta\right)d\phi^2\right]. \tag{18}$$

We write the Polyakov action for the fundamental string in this background:

$$S_p = \frac{R^2}{4\pi}\int d\sigma d\tau\left[-\dot{t}^2 + \frac{1}{6}\dot{\theta}^2 - \left(\frac{1}{9} + \frac{1}{18}\sin^2\theta\right)m^2\right]. \tag{19}$$

The equation of motion for θ is

$$\ddot{\theta} + \frac{1}{3}m^2\sin\theta\cos\theta = 0. \tag{20}$$

But from Virasoro constraint we get

$$\dot{\theta}^2 = 6\mathcal{E}^2 - \left(\frac{2}{3} + \frac{1}{3}\sin^2\theta\right)m^2, \tag{21}$$

where $E = R^2\mathcal{E}$ is the energy. Now the oscillation number is

$$\mathcal{N} = \frac{1}{2\pi}\oint d\theta\dot{\theta} = \frac{1}{2\pi}\oint d\theta\sqrt{6\mathcal{E}^2 - \left(\frac{2}{3} + \frac{1}{3}\sin^2\theta\right)m^2}. \tag{22}$$

Putting $\sin\theta = x$ in the above equation, we get

$$\mathcal{N} = \frac{1}{2\pi}\oint\frac{dx}{1-x^2}\sqrt{6\mathcal{E}^2(1-x^2) - \left(\frac{2}{3} + \frac{1}{3}x^2\right)m^2(1-x^2)}. \tag{23}$$

Differentiating the above equation with respect to m,

$$\frac{\partial\mathcal{N}}{\partial m} = -\frac{m}{2\pi}\oint\frac{\frac{2}{3} + \frac{1}{3}x^2}{\sqrt{6\mathcal{E}^2(1-x^2) - (\frac{2}{3} + \frac{1}{3}x^2)m^2(1-x^2)}}dx. \tag{24}$$

We put $x^2 = y$ to compute the integral

$$\frac{\partial\mathcal{N}}{\partial m} = -\frac{m}{2\pi}\int_c^b\frac{\frac{2}{3} + \frac{1}{3}y}{\sqrt{6\mathcal{E}^2 y(1-y) - (\frac{2}{3} + \frac{1}{3}y)m^2 y(1-y)}}dy, \tag{25}$$

where a, b, and c are the roots of the polynomial of the denominator of the above integral (25). For large $\mathcal{E}$, we get a lower bound to energy as $\mathcal{E}$ as $6\mathcal{E}^2 > m^2$ and chose $a = \frac{18\mathcal{E}^2 - 2m^2}{m^2}$, $b = 1$, $c = 0$:

$$\frac{\partial\mathcal{N}}{\partial m} = I_1 + I_2, \tag{26}$$

where

$$I_1 = -\frac{m}{3\pi}\int_c^b\frac{dy}{\sqrt{\frac{1}{3}(y-a)(y-b)(y-c)}} = -\frac{m}{\pi}\frac{2}{\sqrt{3a}}\mathbb{K}(1/a),$$
$$I_2 = -\frac{m}{6\pi}\int_c^b\frac{y}{\sqrt{\frac{1}{3}(y-a)(y-b)(y-c)}}dy = \frac{m}{\pi}\sqrt{\frac{a}{3}}[\mathbb{E}(1/a) - \mathbb{K}(1/a)], \tag{27}$$

where $\mathbb{K}$ and $\mathbb{E}$ are the usual complete elliptic integral of the first and second kind, respectively. We can expand these elliptic integrals to get the energy as the function of oscillation number:

$$\frac{\partial \mathcal{N}}{\partial m} = \frac{m}{\pi}\sqrt{\frac{a}{3}}\left[\mathbb{E}(1/a) - \left(1+\frac{2}{a}\right)\mathbb{K}(1/a)\right], \tag{28}$$

$$\frac{\partial \mathcal{N}}{\partial m} = -\frac{5}{12\sqrt{6}}\frac{m^2}{\mathcal{E}} - \frac{17}{576\sqrt{6}}\frac{m^4}{\mathcal{E}^3} - \frac{265}{82944\sqrt{6}}\frac{m^6}{\mathcal{E}^5} + \mathcal{O}[\mathcal{E}^{-7}]. \tag{29}$$

Now, integrating the above equation over m from 0 to ∞ and setting $\mathcal{N}(\infty) = 0$, we get

$$\mathcal{N} = \frac{5}{36\sqrt{6}}\frac{m^3}{\mathcal{E}} + \frac{17}{2880\sqrt{6}}\frac{m^5}{\mathcal{E}^3} - \frac{265}{580608\sqrt{6}}\frac{m^7}{\mathcal{E}^5} + \mathcal{O}[\mathcal{E}^{-7}]. \tag{30}$$

Inversing the series we get the energy as

$$\mathcal{E} = 0.0567\ m^3\mathcal{N}^{-1} + 0.7495\ m^{-1}\mathcal{N} - 1.789\ m^{-5}\mathcal{N}^3 + 20.67\ m^{-9}\mathcal{N}^5 + O[\mathcal{N}^7]. \tag{31}$$

III. OSCILLATING STRING IN NON-ABELIAN *T*-DUAL KLEBANOV-WITTEN BACKGROUND

Here we take the dualized metric which is presented in [34] where dualization is made with respect to the SU(2) global isometry defined by the σ_i's. As $AdS_5 \times T^{1,1}$ is a block diagonal spacetime, AdS_5 comes as a spectator space. Also two fields θ_1 and ϕ_1 of $T^{1,1}$ come as spectators as the gauge choices are taken accordingly. As the supersymmetry in the dual field theory of $AdS_5 \times T^{1,1}$ is uncharged under SU(2) flavor symmetries, it is supposed to persevere after the T dualization along this SU(2). The resulting dualized background is the σ model on a target space with the $\mathcal{N} = 1$ supersymmetric solution of type IIA:

$$ds^2_{\rm dual} = ds^2_{AdS_5} + \lambda_1^2(\sigma_{\hat 1}^2 + \sigma_{\hat 2}^2) + \frac{\lambda_2^2\lambda^2}{\Delta}x_1^2\sigma_{\hat 3}^2 + \frac{1}{\Delta}((x_1^2 + \lambda_2^2\lambda^2)dx_1^2 + (x_2^2 + \lambda_2^4)dx_2^2 + 2x_1x_2dx_1dx_2), \tag{32}$$

where $\sigma_{\hat 3} = d\psi + \cos\theta_1 d\phi_1$, $\Delta \equiv \lambda_2^2 x_1^2 + \lambda^2(x_2^2 + \lambda_2^4)$, and R is taken to be 1 for convenience and otherwise can be restored by suitable rescaling. For small values of x_1 and fixed x_2 the metric on the internal space behaves as

$$ds^2_{\rm dual} = ds^2_{AdS_5} + \lambda_1^2(\sigma_{\hat 1}^2 + \sigma_{\hat 2}^2) + \frac{\lambda_2^2}{x_2^2 + \lambda_2^4}x_1^2\sigma_{\hat 3}^2. \tag{33}$$

Though geometry is regular, the above metric has a bolt singularity which can be removed by changing the range of ψ to be 2π.

A. Oscillating in AdS

Here, we study the solution for a string moving in the background (33) which is oscillating in AdS along ρ and simultaneously rotating along ψ and localized at a fixed point in the plane (x_1, x_2). Our ansatz for this configuration is

$$t = t(\tau), \quad \rho = \rho(\tau), \quad \chi = \xi_i = \phi_1 = 0, \quad \theta_1 = m\sigma, \quad \psi = \psi(\tau), \quad x_1, x_2 = \text{fixed}. \tag{34}$$

Now Eq. (33) takes the form

$$ds^2_{\rm dual} = -\cosh^2\rho dt^2 + d\rho^2 + \lambda_1^2 d\theta_1^2 + \frac{\lambda_2^2}{x_2^2 + \lambda_2^4}x_1^2 d\psi^2. \tag{35}$$

We write the Polyakov action for the above background (35):

$$S_p = \frac{1}{4\pi}\int d\sigma d\tau\left(-\cosh^2\rho\dot{t}^2 + \dot{\rho}^2 - m^2\lambda_1^2 + \frac{\lambda_2^2}{x_2^2 + \lambda_2^4}x_1^2\dot{\psi}^2\right). \tag{36}$$

From the Virasoro constraint we get

$$\dot{\rho}^2 = \frac{E^2}{\cosh^2\rho} - m^2\lambda_1^2 - \frac{x_2^2 + \lambda_2^4}{\lambda_2^2 x_1^2}J^2, \tag{37}$$

where E and J are conserved charges and can be computed as in Sec. II A. We can see that Eqs. (8) and (37) are equivalent. So the string states in (2) and (34) are characterized by the same labels (9)–(16), if we choose appropriate values for x_1 and x_2. This is possible in the finite range of $\frac{1}{3\sqrt{6}} < x_1 < 1$ for suitable x_2 values which range as $0 < x_2 < \frac{\sqrt{53}}{6}$.

B. Oscillating in $\hat{T}^{1,1}$

In this subsection, we study the string which moves in the non-Abelian T-dual Klebanov-Witten background which is oscillating along the θ_1 direction in $\hat{T}^{1,1}$ and localized at a fixed point in the plane (x_1, x_2). Our ansatz is

$$t = t(\tau), \quad \rho = 0, \quad \theta_1 = \theta = \theta(\tau), \quad \phi_1 = m\sigma, \quad \psi = 0, \quad x_1, x_2 = \text{fixed}. \tag{38}$$

The relevant background looks like

$$ds^2_{\rm dual} = -dt^2 + \lambda_1^2 d\theta^2 + \left[\lambda_1^2 \sin^2\theta + \frac{\lambda_2^2}{x_2^2 + \lambda_2^4} x_1^2 \cos^2\theta\right] d\phi_1^2. \tag{39}$$

From Virasoro constraint, we get

$$\dot{\theta}^2 = \frac{E^2}{\lambda_1^2} - \left[\left(1 - \frac{\lambda_2^2}{\lambda_1^2(x_2^2 + \lambda_2^4)} x_1^2\right)\sin^2\theta + \frac{\lambda_2^2}{\lambda_1^2(x_2^2 + \lambda_2^4)} x_1^2\right] m^2. \tag{40}$$

We can see that Eq. (21) is equivalent to (40) and labels (22)–(31) are the same for the string states described by (17) and (38) for the same values of x_1 and x_2 as we got in the above subsection (III A).

Now, we wish to study another class of oscillating string solution where the string is fixed at some point x_2 and oscillating in the T-dual coordinate direction x_1 from a minimum ($x_{1\,\rm min}$) to a maximum ($x_{1\,\rm max}$) value. So, for this configuration our ansatz is

$$t = t(\tau), \quad \rho = 0, \quad \theta_1 = 0, \quad \phi_1 = m\sigma, \quad \psi = 0, \quad x_1 = x_1(\tau), \quad x_2 = \text{fixed}. \tag{41}$$

As x_1 is no longer fixed, we use the background in Eq. (32), and we get

$$ds^2_{\rm dual} = -dt^2 + \frac{\lambda^2\lambda_2^2}{\Delta} x_1^2 d\phi_1^2 + \frac{1}{\Delta}(x_1^2 + \lambda^2\lambda_2^2) dx_1^2. \tag{42}$$

From the Virasoro constraint we get

$$\dot{x}_1{}^2 = \frac{\Delta E^2 - \lambda^2\lambda_2^2 m^2 x_1^2}{x_1^2 + \lambda^2\lambda_2^2}. \tag{43}$$

Now we write the oscillation number

$$N = \frac{1}{2\pi}\oint dx_1 \sqrt{\frac{\Delta E^2 - \lambda^2\lambda_2^2 m^2 x_1^2}{x_1^2 + \lambda^2\lambda_2^2}}. \tag{44}$$

Using a similar process to the previous section, we take the derivative with respect to m and put $x_1^2 = y$ to get

$$\frac{\partial N}{\partial m} = -\frac{m}{2\pi}\int_{r_2}^{r_3} dy \frac{\lambda^2\lambda_2^2 y}{\sqrt{y(y + \lambda^2\lambda_2^2)(\Delta E^2 - m^2\lambda^2\lambda_2^2 y)}}, \tag{45}$$

where r_2 and r_3 are two positive roots of the polynomial in the denominator of the above integral (45). r_1 is the other root, which is a negative one. The above integral can be written in terms of the usual complete elliptic integrals and further expanded where the roots are $-\frac{1}{54}$, 0, and $\frac{E^2(1+36x_2^2)}{6m^2-54E^2}$:

$$\begin{aligned}\frac{\partial N}{\partial m} &= \frac{m}{\pi\sqrt{54(m^2 - 9E^2)(r_3 - r_1)}}\left[(r_1 - r_3)\mathbb{E}\left(\frac{r_3 - r_2}{r_3 - r_1}\right) - r_1\mathbb{K}\left(\frac{r_3 - r_2}{r_3 - r_1}\right)\right] \\ &= -0.04167(1 + 36x_2^2)\frac{E^2}{m^2} + 0.42187(-1 - 24x_2^2 + 432x_2^4)\frac{E^4}{m^4} \\ &\quad - 0.79101(5 + 108x_2^2 - 1296x_2^4 + 46656x_2^6)\frac{E^6}{m^6} + \mathcal{O}[E^8],\end{aligned} \tag{46}$$

where $m^2 > 9E^2$, which gives the upper bound to the energy, and in the E^8 term x_2 runs up to x_2^8 and so on. Now this series can be integrated over m and inverted to get the energy

$$\begin{aligned}E = \sqrt{\frac{\tilde{m}N}{(0.04167 + 1.5x_2^2)^5}}\bigg[&1 + (-0.07031 - 1.6875x_2^2 + 30.375x_2^4)\frac{N}{\tilde{m}} \\ &+ (0.00082 - 0.11865x_2^2 - 13.526x_2^4 - 358.80x_2^6 - 2306.6x_2^8)\frac{N^2}{\tilde{m}^2} + \mathcal{O}\left(\frac{N^3}{\tilde{m}^3}\right)\bigg],\end{aligned} \tag{47}$$

where $\tilde{m} = m(0.04167 + 1.5x_2^2)^2$. This is the energy expression for the short strings which are oscillating in the T-dual coordinate direction.

IV. CONCLUSION

In this paper, we have studied various oscillating strings in the Klebanov-Witten and its non-Abelian T-dual background. First we have studied the oscillating long strings in $AdS_2 \times S^2$, where the string is expanding and contracting in the radial direction of AdS and simultaneously has an angular momentum in the $T^{1,1}$. We have found the energy as a linear function of the oscillation number. Then we have studied another class of oscillating strings which are oscillating in $\mathbb{R} \times T^{1,1}$ along the $T^{1,1}$. The energy for the long string comes to be a series as a function of the string oscillation number.

In the last section, we have studied the oscillating strings in the non-Abelian T-dual Klebanov-Witten background. We fixed the T-dual coordinates so that the remaining directions give a squashed three sphere geometry and a reduction in the range of the ψ direction in order to remove the bolt singularity. In this geometry, the T-dual coordinate $x_1 = 0$ singularity at fixed θ_i, ϕ_i is a coordinate singularity of $\mathbb{R}^2$ in polar coordinates. Here we have found both the Klebanov-Witten and its T-dual background give the same result for a range of T-dual coordinates, though their field theory duals are different in principle, because a particular sector of geometry of $T^{1,1}$ is unaffected by the non-Abelian T duality. As we see, this is not true throughout the space as it is restricted by the range of T-dual coordinates. Furthermore, we have studied some oscillating strings which are oscillating in the T-dual coordinate direction and found its energy expression for the short string configuration. We have also remarked that, though the R charge (ψ) [45] vanishes, there is a nonvanishing T-dual coordinate x_1 and a solution contrary to [43], where vanishing R charge implies the vanishing of T-dual coordinate x_1 for the rotating strings. The solutions presented here are in the $AdS_5 \times T^{1,1}$ and its T-dual background which are dual to the $\mathcal{N} = 1$ superconformal field theory with $SU(2) \times SU(2)$ flavor symmetry. The chiral operators analogue to the operators in $\mathcal{N} = 4$ supersymmetric Yang-Mills are given by $\mathrm{Tr}(AB)^k$ with R charge k and in the $(\frac{k}{2}, \frac{k}{2})$ representation of the flavor group $SU(2) \times SU(2)$. Here the two chiral multiplets A and B, which are elementary degrees of freedom, are correspondingly in the $(\mathrm{N}, \bar{N})$ and $(\bar{N}, \mathrm{N})$ representations. We can notice that the solution presented in Eq. (47) is dependent upon the T-dual coordinate x_2 which prompts us to expect the existence of a superconformal field theory dual operator to this string state whose anomalous dimension depends upon the T-dual coordinate. However, a prediction of the exact form of the operator for the solution in Eq. (47) cannot be done on the basis of the work presented here. We leave this problem for future work.

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