

# Tests of *CPT* and Lorentz symmetry from muon anomalous magnetic dipole moment

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We derive the relativistic factor for splitting of the  $g$ -factors of a fermion and its antifermion partner, which is important for placing constraints on dimension-five, *CPT*-odd and Lorentz-invariance-violating interactions from experiments performed in a cyclotron. From existing data, we extract limits ( $1\sigma$ ) on the coupling strengths of the temporal component,  $f^0$ , of a background field (including the field amplitude), which is responsible for such  $g$ -factor splitting, with an electron, proton, and muon:  $|f_e^0| < 2.3 \times 10^{-12} \mu_B$ ,  $|f_p^0| < 4 \times 10^{-9} \mu_B$ , and  $|f_\mu^0| < 8 \times 10^{-11} \mu_B$ , respectively, in the laboratory frame ( $\mu_B$  is the Bohr magneton). From existing data, we also extract limits on the coupling strengths of the spatial components,  $d^\perp$ , of related dimension-five interactions of a background field with an electron, proton, neutron, and muon:  $|d_e^\perp| \lesssim 10^{-9} \mu_B$ ,  $|d_p^\perp| \lesssim 10^{-9} \mu_B$ ,  $|d_n^\perp| \lesssim 10^{-10} \mu_B$ , and  $|d_\mu^\perp| \lesssim 10^{-9} \mu_B$ , respectively, in the laboratory frame.

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## I. INTRODUCTION

The violation of the fundamental symmetries of nature is an area of substantial interest, both experimentally and theoretically. Field theories, which are constructed from the principles of locality, spin statistics and Lorentz invariance, conserve the combined *CPT* symmetry. The violation of one or more of these three principles, presumably from some form of ultra short-distance scale physics, opens the door for the possibility of *CPT*-odd physics. *CPT*-odd and Lorentz-invariance-violating physics has been sought for experimentally in the form of the coupling

$$\hat{H}_{\text{int}} = \mathbf{b} \cdot \boldsymbol{\sigma} \quad (1)$$

between a background cosmic field,  $\mathbf{b}$ , and the spin of an electron, proton, neutron and muon,  $\boldsymbol{\sigma}$ , and constraints on the strengths of such interactions have been obtained [1–12]. For further details on the broad range of experiments performed in this field and a brief history of the improvements in these limits, we refer the reader to the reviews of Refs. [13,14] and the references therein. Electric dipole moment (EDM) measurements have also been proposed as sensitive probes of *CPT*-odd physics [15]. Limits on *P*-odd fermion effects induced by *CPT*-odd, Lorentz-invariance-violating couplings have been extracted from existing parity nonconservation (PNC) and anapole moment data [16]. Atomic dysprosium has been proposed for odd-parity tests of Lorentz symmetry [17] and has been used to place limits on local Lorentz invariance [18]. Pion and kaon systems have also been suggested for tests of Lorentz invariance [19].

Dirac theory predicts that all elementary standard model (SM) fermions should have the gyromagnetic ratio  $g^{\text{Dirac}} = 2$ . Quantum field theory corrections result in deviations from  $g^{\text{Dirac}} = 2$ , which can be quantified by the anomalous magnetic dipole moment (MDM) parameter

$$a = \frac{g - 2}{2}. \quad (2)$$

Consider, for instance, the particularly interesting case of the muon. The current SM prediction for the anomalous MDM of the muon consists of quantum electrodynamic, weak and hadronic contributions [20] (see also the multitude of references therein for more details of some of the pioneering theory and experiments, which led to the current refined prediction of  $a_\mu^{\text{SM}}$ ):

$$a_\mu^{\text{SM}} = 116591803(1)(42)(26) \times 10^{-11}, \quad (3)$$

where the uncertainties are due to the electroweak, lowest-order hadronic and higher-order hadronic contributions, respectively. The most accurate measurement to date for the anomalous MDMs of the muon and antimuon are [21,22]

$$a_\mu^{\text{exp}} = 116592150(80)(30) \times 10^{-11}, \quad (4)$$

$$a_{\bar{\mu}}^{\text{exp}} = 116592040(60)(50) \times 10^{-11}, \quad (5)$$

respectively, and, assuming *CPT* invariance and taking into account correlations between systematic uncertainties, this gives the average

$$a_\mu^{\text{exp}} = 116592091(54)(33) \times 10^{-11}, \quad (6)$$

where the quoted uncertainties are due to statistical and systematic sources, respectively. The result (6) represents

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about an order-of-magnitude improvement in precision compared with the now classic experiment of Ref. [23]. The difference between the SM prediction and experimental value

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(80) \times 10^{-11}, \quad (7)$$

with experimental and theoretical uncertainties added in quadrature, represents a discrepancy between the two values of  $3.6\sigma$ , suggesting that the effects of new physics beyond the SM may be manifesting themselves. Some of the most promising current explanations for this discrepancy are supersymmetric models [24–36] and the dark photon, which is a massive vector boson from the dark matter sector that couples to SM particles by mixing with the ordinary photon [37–39]. See also Refs. [40–43] for some of the more recent reviews on the muon anomalous MDM puzzle. Future measurements of the anomalous MDM of the muon with increased precision are currently planned [44].

Tests of *CPT*-odd and Lorentz-invariance-violating physics from measurements of the anomalous MDMs of various particles have been proposed previously [45–49]. In the framework of the Kostelecký *et al.* standard model extension parametrization [45–47], it was found that there were no leading-order corrections to the  $g$ -factors of fermions and their respective antifermions. Instead, measurements of magnetic-field-independent splittings of the anomalous precession frequencies for a fermion and its respective antifermion were proposed for placing limits on *CPT*-odd and Lorentz-invariance-violating physics. In the dimension-five framework of Ref. [49], it was demonstrated that the  $g$ -factors of an electron and a positron may be split by a *CPT*-odd and Lorentz-invariance-violating interaction, and a limit on the relevant interaction parameter was extracted from existing data at the time.

In the present work, we consider *CPT*-odd, Lorentz-invariance-violating dimension-five couplings, which are linear in the gauge field strength. We derive the relativistic factor for splitting of the  $g$ -factors of a fermion and its antifermion partner, which is important for placing constraints on dimension-five, *CPT*-odd and Lorentz-invariance-violating interactions from experiments performed in a cyclotron. Anomalous MDMs are ideal physical quantities for tests of *CPT*-odd and Lorentz-invariance-violating physics, because of the high precision with which these quantities can be determined. We extract limits on the coupling strengths of the background field, which splits  $g$ -factors, with an electron, proton and muon. We also extract limits on the coupling strengths of related dimension-five interactions of a background field with an electron, proton, neutron and muon. For details of other recently proposed laboratory tests of *CPT*-odd and Lorentz-invariance-violating physics with muons, we refer the reader to Ref. [50].

## II. THEORY

We employ the natural units  $\hbar = c = 1$  and the metric signature  $(+ - - -)$  in this work. Various *CPT*-odd couplings of dimension-five have been classified (see e.g. Refs. [15,49,51,52]). In the present work, we consider the following *CPT*-odd, Lorentz-invariance-violating dimension-five couplings, which are linear in the gauge field strength [15] (see also Ref. [49] for a more detailed discussion of some of the relevant terms):

$$\mathcal{L} = - \sum [c^\nu \bar{\psi}_f \gamma^\lambda F_{\lambda\nu} \psi_f + d^\nu \bar{\psi}_f \gamma^\lambda \gamma^5 F_{\lambda\nu} \psi_f + f^\nu \bar{\psi}_f \gamma^\lambda \gamma^5 \tilde{F}_{\lambda\nu} \psi_f + g^\nu \bar{\psi}_f \gamma^\lambda \tilde{F}_{\lambda\nu} \psi_f], \quad (8)$$

where the sum is over all SM fermions  $f$  and SM gauge groups, with  $F_{\lambda\nu}$  and  $\tilde{F}_{\lambda\nu}$  representing the field and dual field tensor strengths, respectively. The terms  $c^\nu$ ,  $d^\nu$ ,  $f^\nu$  and  $g^\nu$  in Eq. (8) represent the amplitudes of the background cosmic field(s) with the corresponding interaction strength amalgamated into them. In the present work, we are interested in systems exposed to external magnetic and electric fields. From the Lagrangian (8), we find the following interaction Hamiltonians:

$$\hat{H}_{\text{int}}^A = f^0 \mathbf{B} \cdot \Sigma, \quad (9)$$

$$\hat{H}_{\text{int}}^B = (\mathbf{d} \times \mathbf{B}) \cdot \Sigma, \quad (10)$$

$$\hat{H}_{\text{int}}^C = -\mathbf{f} \cdot \mathbf{B} \gamma^5, \quad (11)$$

$$\hat{H}_{\text{int}}^D = (\mathbf{c} \times \mathbf{B}) \cdot \boldsymbol{\alpha}, \quad (12)$$

$$\hat{H}_{\text{int}}^E = g^0 \mathbf{B} \cdot \boldsymbol{\alpha}, \quad (13)$$

$$\hat{H}_{\text{int}}^F = -\mathbf{g} \cdot \mathbf{B}, \quad (14)$$

$$\hat{H}_{\text{int}}^G = d^0 \mathbf{E} \cdot \Sigma, \quad (15)$$

$$\hat{H}_{\text{int}}^H = -(\mathbf{f} \times \mathbf{E}) \cdot \Sigma, \quad (16)$$

$$\hat{H}_{\text{int}}^I = -\mathbf{d} \cdot \mathbf{E} \gamma^5, \quad (17)$$

$$\hat{H}_{\text{int}}^J = -(\mathbf{g} \times \mathbf{E}) \cdot \boldsymbol{\alpha}, \quad (18)$$

$$\hat{H}_{\text{int}}^K = c^0 \mathbf{E} \cdot \boldsymbol{\alpha}, \quad (19)$$

$$\hat{H}_{\text{int}}^L = -\mathbf{c} \cdot \mathbf{E}, \quad (20)$$

where  $\mathbf{B}$  is the external magnetic field strength,  $\mathbf{E}$  is the external electric field strength;  $\Sigma \equiv \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$ ,  $\boldsymbol{\alpha} \equiv \gamma^0 \boldsymbol{\gamma}$ , and  $\gamma^5 \equiv i\gamma^0 \gamma^1 \gamma^2 \gamma^3$  are Dirac matrices, and we have suppressed the possible dependence of the cosmic field parameters on the fermion species  $f$  in our notation.

We are specifically interested in interactions that can alter the spin-precession frequency of a fermion to leading order. Of the interactions (9)–(20), only Eq. (9) alters the spin-precession frequency of a fermion to leading order [in fact the interaction (9) splits the  $g$ -factors of fermions and their corresponding antifermions], since the other interactions satisfy at least one of the following three criteria:

- (I) the interaction produces no observable effect;
- (II) the interaction mixes opposite-parity states;
- (III) the interaction produces sidereal effects, which average to zero after the averaging of acquired data.

Crucial to the point (I) above are the identity [53]

$$\mathbf{E} \cdot \boldsymbol{\Sigma} = \frac{[\boldsymbol{\Sigma} \cdot \nabla, \hat{H}]}{e}, \quad (21)$$

where  $\hat{H}$  is the full Dirac Hamiltonian and  $-e$  is the electron charge, and the more general identity

$$E_j \Sigma_k = \frac{[\Sigma_k \nabla_j, \hat{H}]}{e}. \quad (22)$$

Note that none of the interactions (9)–(20) affect the cyclotron frequency.

Regarding possible  $P$ -odd interactions, we mention that the interactions (12) and (13) do not contribute to PNC amplitudes [53] of the form

$$E_{\text{PNC}}^{a \rightarrow b} = \sum_n \left[ \frac{\langle b | \hat{\mathbf{d}} | n \rangle \langle n | \hat{H}_{\text{int}} | a \rangle}{E_a - E_n} + \frac{\langle b | \hat{H}_{\text{int}} | n \rangle \langle n | \hat{\mathbf{d}} | a \rangle}{E_b - E_n} \right] \quad (23)$$

for transitions  $|a\rangle \rightarrow |b\rangle$ , where  $\hat{H}_{\text{int}}$  is a perturbing  $P$ -odd operator and  $\hat{\mathbf{d}}$  is the electric dipole ( $E1$ ) operator, because of the identity  $\boldsymbol{\alpha} = i[\hat{H}, \mathbf{r}]$ . Also, matrix elements of Eqs. (12) and (13) between a pair of nearly degenerate levels of opposite parity are negligibly small. The interaction (11) can give rise to sidereal nonzero PNC amplitudes and matrix elements between opposite-parity levels, which do not necessarily scale as the energy difference between the pair of opposite-parity levels. PNC amplitudes of this nature are determined entirely by relativistic effects and such matrix elements are typically dominated by relativistic effects (see e.g. Refs. [16,54]).

### III. RESULTS

#### A. Electrons and protons

We first consider the interaction with electrons and protons described by Eq. (9) in the nonrelativistic limit, which is the appropriate description of experiments for determining anomalous MDMs using a Penning trap, where the Lorentz factor is  $\gamma = (1 - v^2)^{-1/2} \approx 1$ . In an applied magnetic field, the combined potentials experienced by an electron and positron due to the interactions of

their MDMs with the magnetic field and also their interactions via Eq. (9) may be written as

$$U_e = \left( \frac{g_e \mu_B}{2} + f_e^0 \right) \boldsymbol{\sigma}_e \cdot \mathbf{B}, \quad (24)$$

$$U_{\bar{e}} = \left( -\frac{g_e \mu_B}{2} + f_e^0 \right) \boldsymbol{\sigma}_{\bar{e}} \cdot \mathbf{B}, \quad (25)$$

respectively, where  $\mu_B$  is the Bohr magneton. Likewise, for a proton and antiproton, the potentials may be written as

$$U_p = \left( -\frac{g_p \mu_N}{2} + f_p^0 \right) \boldsymbol{\sigma}_p \cdot \mathbf{B}, \quad (26)$$

$$U_{\bar{p}} = \left( \frac{g_p \mu_N}{2} + f_p^0 \right) \boldsymbol{\sigma}_{\bar{p}} \cdot \mathbf{B}, \quad (27)$$

respectively, where  $\mu_N$  is the nuclear magneton. The resulting splitting in the  $g$ -factors of an electron and a positron can thus be expressed as

$$a_e^{\text{exp}} - a_{\bar{e}}^{\text{exp}} = 2a_e^{\text{CF}}, \quad (28)$$

where  $a_e^{\text{CF}} = f_e^0 / \mu_B$ , whereas the splitting in the  $g$ -factors of a proton and antiproton can thus be expressed as

$$a_{\bar{p}}^{\text{exp}} - a_p^{\text{exp}} = 2a_p^{\text{CF}}, \quad (29)$$

where  $a_p^{\text{CF}} = f_p^0 / \mu_N$ .

In the absence of a recent experimental value for  $a_{\bar{e}}$ , we extract a limit on  $f_e^0$  from the SM prediction and experimentally measured values for  $a_e$ . This is likely to be a weaker limit than that, which may be extracted from  $a_e$  and a future value for  $a_{\bar{e}}$ , since the new physics (such as supersymmetry), which contribute equally to both  $a_e$  and  $a_{\bar{e}}$ , is likely to occur at a lower energy scale than that for  $CPT$ -violating physics, which may result in a splitting of  $a_e$  and  $a_{\bar{e}}$ . This is already borne out, for instance, in the muon anomalous MDM values in Eqs. (4), (5) and (7). Noting that the SM prediction for the anomalous MDM of the electron is [55] (see also Refs. [56–64] for some of the pioneering theory and experiments, which led to the current refined prediction of  $a_e^{\text{SM}}$ )

$$a_e^{\text{SM}} = 1159652181.82(78) \times 10^{-12}, \quad (30)$$

with associated uncertainties added in quadrature, and that the experimentally measured value for the anomalous MDM of the electron is [21,65]

$$a_e^{\text{exp}} = 1159652180.76(27) \times 10^{-12}, \quad (31)$$

we extract the limit ( $1\sigma$ ) on the coupling strength of the background field with an electron via the interaction (9) to be

$$|f_e^0| < 2.3 \times 10^{-12} \mu_B, \quad (32)$$

in the laboratory frame. Further measurements, in particular of the positron anomalous MDM, with increased precision would lead to a more stringent constraint on  $f_e^0$ .

The most accurate measurement to date for the MDM of the proton is [66]

$$\mu_p^{\text{exp}} = 2.792847350(9) \mu_N, \quad (33)$$

while for the antiproton [67]

$$\mu_{\bar{p}}^{\text{exp}} = -2.792845(12) \mu_N, \quad (34)$$

from which we extract the limit ( $1\sigma$ ) on the coupling strength of the background field with a proton via the interaction (9) to be

$$|f_p^0| < 4 \times 10^{-9} \mu_B, \quad (35)$$

in the laboratory frame.

### B. Muons

We now consider the interaction with muons described by Eq. (9) in the relativistic case, which is the appropriate description of experiments for determining  $a_\mu$  using a cyclotron, where  $\gamma \gg 1$ . In this case, Eq. (9) can be expressed in the same form for a muon and antimuon:

$$\hat{H}_{\text{int}}^A = f_\mu^0 \left[ \mathbf{B} \cdot \boldsymbol{\sigma}_\mu + \frac{2(\mathbf{p}_\mu \cdot \boldsymbol{\sigma}_\mu)(\mathbf{p}_\mu \cdot \mathbf{B}) - p_\mu^2 (\mathbf{B} \cdot \boldsymbol{\sigma}_\mu)}{(\gamma + 1)^2 m_\mu^2} \right], \quad (36)$$

where  $\mathbf{p}_\mu$  is the muon relativistic momentum and  $m_\mu$  is the muon mass. In a cyclotron,  $\mathbf{p}_\mu$  and  $\mathbf{B}$  are perpendicular and so Eq. (36) simplifies to

$$\hat{H}_{\text{int}}^A = f_\mu^0 \mathbf{B} \cdot \boldsymbol{\sigma}_\mu \left[ 1 - \frac{\gamma^2 v_\mu^2}{(\gamma + 1)^2} \right]. \quad (37)$$

The background field contribution to the observed anomalous MDM of the muon is

$$a_\mu^{\text{CF}} = \frac{2f_\mu^0 m_\mu}{e} \left[ 1 - \frac{\gamma^2 v_\mu^2}{(\gamma + 1)^2} \right]. \quad (38)$$

Note that in the ultrarelativistic limit ( $\gamma \rightarrow \infty$ ), the correction to the anomalous MDM of the muon from Eq. (38) vanishes. In the experiment of Ref. [22],  $\gamma = 29.3$  and so there is only a finite suppression of the contribution to the anomalous MDM of the muon arising from Eq. (9). The anomalous precession frequency of a muon or antimuon in a cyclotron can be written, with account of both SM and cosmic field contributions, as

$$\boldsymbol{\omega}_a = \frac{e}{m_\mu} \left[ \tilde{a}_\mu \mathbf{B} - \left( \tilde{a}_\mu - \frac{1}{\gamma^2 - 1} \right) \mathbf{v}_\mu \times \mathbf{E} \right], \quad (39)$$

where  $\tilde{a}_\mu = a_\mu^{\text{SM}} + a_\mu^{\text{CF}}$  for a muon and  $\tilde{a}_\mu = a_\mu^{\text{SM}} - a_\mu^{\text{CF}}$  for an antimuon, with  $a_\mu^{\text{CF}}$  given by Eq. (38). The experimentally chosen Lorentz factor  $\gamma = 29.3$  ensures that the  $\mathbf{v}_\mu \times \mathbf{E}$  term in Eq. (39) is significantly suppressed compared to the first term. The splitting in the  $g$ -factors of a muon and an antimuon in this case can be expressed as

$$a_\mu^{\text{exp}} - a_{\bar{\mu}}^{\text{exp}} = 2a_\mu^{\text{CF}}. \quad (40)$$

From the values in Eqs. (4) and (5), we extract the limit ( $1\sigma$ ) on the coupling strength of the background field with a muon via the interaction (9) to be

$$|f_\mu^0| < 8 \times 10^{-11} \mu_B, \quad (41)$$

in the laboratory frame.

### IV. OTHER TESTS

It was pointed out in Ref. [15] that EDMs may serve as sensitive tests of  $CPT$ -odd interactions, in association with the  $d^0$  term in Eq. (8). Here we also mention a further test stemming from the interaction Hamiltonian (10), which in the nonrelativistic limit reads

$$\hat{H}_{\text{int}}^B = (\mathbf{d} \times \mathbf{B}) \cdot \boldsymbol{\sigma}. \quad (42)$$

The interaction (42), like Eq. (1), produces sidereal shifts of the energy levels in atomic and nuclear systems, but only in the presence of an external magnetic field, and can be sought for in a similar manner to the coupling (1) via sidereal modulations of transition frequencies [1–12]. Existing experiments were performed in nonzero magnetic fields and so we can extract limits on  $|d^\perp|$  from the magnetic field strengths used in these experiments and existing limits on  $|\mathbf{b}|$  (Table I). Here  $|d^\perp|$  is the magnitude of the largest component of  $\mathbf{d}$ , which is perpendicular to the applied magnetic field, at any time during the experiment.

TABLE I. Limits ( $1\sigma$ ) on the interaction strengths of a background cosmic field with an electron, proton, neutron and muon via the interaction (10). Limits are derived for  $|d^\perp|$  using existing experimental limits on  $|\mathbf{b}|$  and the magnetic field strengths employed in these experiments.  $X$  denotes fermion species.

$X$	$ \mathbf{b}_X /\text{GeV}$	Experiment $ \mathbf{B} /\text{T}$	Reference	This work $ d_X^\perp /\mu_B$
$e$	$10^{-29}$	$10^{-7}$	[5]	$10^{-9}$
$p$	$10^{-28}$	$5.5 \times 10^{-7}$	[10]	$10^{-9}$
$n$	$10^{-29}$	$5.5 \times 10^{-7}$	[10]	$10^{-10}$
$\mu$	$10^{-23}$	1.45	[6]	$10^{-9}$

Finally, we mention that the effects discussed in this work do not need to be restricted solely to static cosmic fields. Dynamic cosmic fields (one particularly important example of which is axion dark matter) are also possible and analogous effects in an oscillatory form, with oscillation frequencies determined by the mass of the field excitation, may be induced. For further details, see, e.g. Refs. [16,54,68,69]. A network of domain-wall-type structures of cosmic fields are also possible and these may induce transient effects analogous to those discussed in this work. For further details, see, e.g. Refs. [70–72].

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