$U(1)$ gauge field localization on a Bloch brane with Chumbes-Holf da Silva-Hott mechanism

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(Received 9 March 2014; published 25 August 2014)

We follow the Chumbes–Holf da Silva–Hott mechanism to study the (quasi)localization of the $U(1)$ gauge field on the Bloch brane. The localization and resonances of the $U(1)$ gauge field are discussed for four kinds of Bloch brane solutions: the original and generalized Bloch brane solutions, as well as the degenerate Bloch brane solutions I and II. With the Chumbes–Holf da Silva–Hott mechanism, we find that the mass spectrum of the gauge field Kaluza-Klein modes is continuous and there is no tachyonic mode. The zero mode is localized on all the branes and there are resonant Kaluza-Klein modes on the degenerate Bloch branes.

DOI: 10.1103/PhysRevD.90.045031 PACS numbers: 11.10.Kk, 11.27.+d

I. INTRODUCTION

In braneworld theory, gravitons can be localized on the Randall-Sundrum (RS) thin brane [1,2] and on the thick brane [3], naturally. In addition to graviton localization, the localization of Standard Model particles is also an important issue for any braneworld model.

The localization of fermions on the brane can be realized by introducing the usual Yukawa coupling between the background scalar field and the fermion field [4–10] when the background scalar field has a kinklike configuration interpolating between the two different vacua at the two sides of the brane. However, when the scalar field is an even function of the extra dimension, one needs to introduce the new localization mechanism presented in Ref. [11]. Real scalar fields can be localized on a brane as long as the graviton is localizable [5].

For the $U(1)$ gauge field, however, localization is more complex than for the fermion and scalar fields. In the RS thin brane scenario, the $U(1)$ gauge field with the following standard five-dimensional action,

$$S \sim \int d^5x\sqrt{-g}F_{MN}F^{MN}, \tag{1}$$

cannot be localized [12]. Here, $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength of the $U(1)$ gauge field. In order to localize the $U(1)$ gauge field on the RS brane, many ideas were proposed [13–21].

In the thick brane scenario, the $U(1)$ gauge field with the action (1) can be localized on some thick branes. For example, it can be localized on the thick de Sitter (dS) brane [22–24], the Weyl thick brane [25], and the brane with finite extra dimension [26]. In Ref. [23], especially, the potentials in the corresponding Schrodinger equations for the Kaluza-Klein (KK) modes of the vector field are modified Poschl-Teller potentials, which lead to the localization of the vector zero mode on the brane as well as to mass gaps in the mass spectra, but it cannot be localized on thick brane models that are asymptotically RS.

In order to localize the gauge field on the thick brane, Kehagias and Tamvakis (KT) proposed a general mechanism, in which a coupling between the gauge field and an extra dilaton field is introduced [27]. The Kehagias and Tamvakis mechanism has been applied in many different braneworld scenarios to localize the vector [28–31] and Kalb-Ramond fields [31–34]. Recently, Chumbes, Holf da Silva, and Hott (CHH) proposed a new mechanism to localize gauge and tensor fields on a thick brane [35]. In their method, gauge and tensor fields are directly coupled to a function of the background scalar field.

On the other hand, the thick brane is usually generated by a background scalar field. In Ref. [36], Bazeia and Gomes introduced the Bloch brane generated by two real scalar fields. This brane model was further generalized in Ref. [37] and investigated in Refs. [11,30,34,38–40]. It is known that the $U(1)$ gauge field with the action (1) cannot be localized on the Bloch brane [30]. In Ref. [30] the localization of the gauge field on the Bloch brane was discussed with the KT mechanism. In order to localize the zero mode on the Bloch brane, an extra dilaton scalar field is introduced in Ref. [30].

In this paper, we will investigate the localization of the $U(1)$ gauge field with the CHH mechanism. In this mechanism the third dilaton scalar field, which appears in Ref. [30], is not needed. The localization and

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quasilocalization of the gauge field for four kinds of Bloch brane solutions are discussed and the localized zero mode and resonant KK modes are found.

This paper is constructed as follows. In Sec. II, the Bloch brane scenario and its four kinds of solutions are reviewed briefly. The localization and quasilocalization of the $U(1)$ gauge field are discussed in Sec. III. Finally, we give our conclusions in Sec. IV.

II. REVIEW OF THE BLOCH BRANE

The action for the Bloch brane model reads [36]

$$S = \int d^4x dy \sqrt{-g}\left[\frac{1}{4}R - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{2}\partial_M\chi\partial^M\chi - V(\phi,\chi)\right], \tag{2}$$

where $g = \det(g_{MN})$, R is the scalar curvature of the five-dimensional space-time, $M, N = 0, 1, 2, 3, 4$, and ϕ, χ are two real scalar fields depending only on the extradimensional coordinate y for the static flat brane model.

The line element for the five-dimensional space-time is assumed as

$$ds^2 = g_{MN}dx^M dx^N = e^{2\alpha(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{3}$$

where $e^{2\alpha(y)}$ is the warp factor, $\alpha(y)$ is only the function of the extradimensional coordinate y, and $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. From the above action (2), one can get the equations of motion of ϕ, χ, and the Einstein equations [36],

$$\phi'' = -4\alpha'\phi' + \frac{\partial V(\chi,\phi)}{\partial\phi}, \tag{4}$$

$$\chi'' = -4\alpha'\chi' + \frac{\partial V(\chi,\phi)}{\partial\chi}, \tag{5}$$

$$\alpha'' = -\frac{2}{3}(\phi'^2 + \chi'^2), \tag{6}$$

$$\alpha'^2 = \frac{1}{6}(\phi'^2 + \chi'^2) - \frac{1}{3}V(\phi,\chi), \tag{7}$$

where the prime stands for the derivative with respect to y. By introducing a superpotential $W(\phi,\chi)$, the above equations can be reduced to the following first-order form,

$$\phi' = \frac{\partial W(\chi,\phi)}{\partial\phi}, \tag{8}$$

$$\chi' = \frac{\partial W(\chi,\phi)}{\partial\chi}, \tag{9}$$

$$\alpha' = -\frac{2}{3}W(\chi,\phi), \tag{10}$$

and the scalar potential is determined in terms of the superpotential by

$$V = \frac{1}{2}\left[\left(\frac{\partial W(\chi,\phi)}{\partial\phi}\right)^2 + \left(\frac{\partial W(\chi,\phi)}{\partial\chi}\right)^2\right] - \frac{4}{3}W^2(\chi,\phi). \tag{11}$$

Then for the superpotential,

$$W(\phi,\chi) = \phi\left[\left(1 - \frac{1}{3}\phi^2\right) - b\chi^2\right], \tag{12}$$

where b is a real parameter, the solution of Eqs. (8)–(10) is given by [36]

$$\phi(y) = \tanh(2by), \tag{13a}$$

$$\chi(y) = \sqrt{\frac{1}{b} - 2}\,\mathrm{sech}(2by), \tag{13b}$$

$$\alpha(y) = \frac{1}{9b}[(1-3b)\tanh^2(2by) - 2\ln\cosh(2by)], \tag{13c}$$

where the parameter b satisfies the constraint $0 < b < 1/2$. The above two-field solution represents a Bloch wall. When $b \to 1/2$, one will get the Ising wall of the one-field solution [36].

In addition to the above original Bloch brane solution, the generalized Bloch brane solution was found in Ref. [37] by using the following generalized superpotential,

$$W(\phi,\chi) = \phi\left[a\left(v^2 - \frac{1}{3}\phi^2\right) - b\chi^2\right], \tag{14}$$

where a, b, and v are positive constants. It reads [37]

$$\phi(y) = v\tanh(2bvy), \tag{15a}$$

$$\chi(y) = v\sqrt{\frac{a-2b}{b}}\,\mathrm{sech}(2bvy), \tag{15b}$$

$$\alpha(y) = \frac{v^2}{9b}[(a-3b)\tanh^2(2bvy) - 2a\ln\cosh(2bvy)], \tag{15c}$$

where $a > 2b > 0$.

Other solutions of the Bloch brane were also found in Ref. [37] for the same superpotential (14) with $a = b$ and $a = 4b$, namely, the degenerated Bloch brane solutions. They are

$$\phi(y) = \frac{\sqrt{c_0^2 - 4}\,v\sinh(2bvy)}{\sqrt{c_0^2 - 4}\cosh(2bvy) - c_0}, \tag{16a}$$

$$\chi(y) = \frac{2v}{\sqrt{c_0^2-4}\cosh(2bvy)-c_0}, \tag{16b}$$

$$\alpha(y) = \frac{1}{2}\left[\frac{4v^2(-\sqrt{c_0^2-4}c_0\cosh(2bvy)+c_0^2-4)}{9(\sqrt{c_0^2-4}\cosh(2bvy)-c_0)^2} - \frac{4(c_0^2-\sqrt{c_0^2-4}c_0-4)v^2}{9(\sqrt{c_0^2-4}-c_0)^2}\right] + \frac{1}{2}\log\left(\frac{\sqrt{c_0^2-4}-c_0}{\sqrt{c_0^2-4}\cosh(2bvy)-c_0}\right)^{\frac{4v^2}{9}}, \tag{16c}$$

for $c_0 < -2$ and $a = b$, and

$$\phi(y) = \frac{\sqrt{1-16c_0}v\sinh(4bvy)}{\sqrt{1-16c_0}\cosh(4bvy)+1}, \tag{17a}$$

$$\chi(y) = \frac{2v}{\sqrt{\sqrt{1-16c_0}\cosh(4bvy)+1}}, \tag{17b}$$

$$\alpha(y) = \frac{1}{2}\left[\frac{4(8c_0+\sqrt{1-16c_0}+1)v^2}{9(\sqrt{1-16c_0}+1)^2} - \frac{4v^2(\sqrt{1-16c_0}\cosh(4bvy)+8c_0+1)}{9(\sqrt{1-16c_0}\cosh(4bvy)+1)^2}\right] + \frac{1}{2}\log\left(\frac{\sqrt{1-16c_0}+1}{\sqrt{1-16c_0}\cosh(4bvy)+1}\right)^{\frac{8v^2}{9}}, \tag{17c}$$

for $c_0 < 1/16$ and $a = 4b$.

In this paper we will call solutions (13), (15), (16), and (17) the original, generalized, degenerate I and degenerate II Bloch brane solutions, respectively.

From the above solutions one can see that the Bloch brane has a rich inner structure. The details of the above solutions can be found in Refs. [36,37].

III. LOCALIZATION AND QUASILOCALIZATION OF THE GAUGE FIELD

As was analyzed in Ref. [30], for a gauge field with the following standard five-dimensional action,

$$S \sim \int d^5x\sqrt{-g}F_{MN}F^{MN}, \tag{18}$$

the corresponding zero mode cannot be localized on the Bloch brane. In order to localize the gauge field on the Bloch brane, the authors of Ref. [30] extended the Bloch brane scenario to the so-called dilatonic Bloch brane model, which is described by the following action,

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{4}R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}(\partial\pi)^2 - V(\phi,\chi,\pi)\right], \tag{19}$$

where the scalar fields ϕ and χ generate the brane, and the dilaton scalar field π is used to localize the gauge field on the brane. The action of the gauge field is assumed to be

$$S \sim \int \sqrt{-g}d^5x\mathrm{e}^{-2\lambda\pi\sqrt{2/3}}F_{MN}F^{MN}, \tag{20}$$

where the coupling between the dilation field π and the gauge field is introduced. With the action (20), the zero mass mode of the gauge field was found to be localized on the brane, and some massive resonant modes were also found [30].

The method used in Ref. [30] was first proposed by Kehagias and Tamvakis (KT) in Ref. [27]. In this paper we will follow another mechanism proposed by CHH in Ref. [35] and study the localization and quasilocalization of the gauge field. Compared with the KT mechanism, the dilation scalar field is not needed in the CHH mechanism. We will show that the $U(1)$ gauge field can be localized on the standard Bloch brane by introducing a coupling between the gauge field and the background scalar field χ. The action of the five-dimensional $U(1)$ gauge field reads

$$S = -\frac{1}{4}\int d^5x\sqrt{-g}\chi(y)F_{MN}F^{MN}. \tag{21}$$

By means of the decomposition of $A_\mu = \sum_n a_\mu(x)\rho_n(y)$ and the gauge $\partial_\mu A^\mu = 0$ and $A_4 = 0$, the above action (21) can be reduced to

$$S = -\frac{1}{4}\int dy\chi(y)\rho_n(y)^2\int d^4x(f_{\mu\nu}f^{\mu\nu} - 2m_n^2a_\mu a^\mu), \tag{22}$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the four-dimensional gauge field strength tensor, and $\rho_n(y)$ should satisfy the equation

$$\rho_n'' + \left(\frac{\chi'}{\chi} + 2\alpha'\right)\rho_n' = -m_n^2\rho_n e^{-2\alpha}. \tag{23}$$

The localization of the gauge field requires

$$I \equiv \int_{-\infty}^{+\infty} dy\chi(y)\rho_n^2(y) < \infty. \tag{24}$$

A. Zero mode

First we discuss the localization of the zero mode of the gauge field. Let $m_0 = 0$, then Eq. (23) reads

$$\rho_0'' + \left(\frac{\chi'}{\chi} + 2\alpha'\right)\rho_0' = 0. \tag{25}$$

By introducing the filed transformation [35]

$$\rho_0 = e^{-\gamma(y)}\hat{\rho}_0(y) \tag{26}$$

with $\gamma(y)$ satisfies $2\gamma' = 2\alpha + \chi'/\chi$, Eq. (25) can be reduced to

$$-\hat{\rho}_0'' + (\gamma'' + \gamma'^2)\hat{\rho}_0 = 0 \tag{27}$$

or

$$\left(\frac{d}{dy} + \gamma'\right)\left(-\frac{d}{dy} + \gamma'\right)\hat{\rho}_0 = 0. \tag{28}$$

The solution of the above equation is $\hat{\rho}_0 = C_1 e^{\gamma(y)}$, where C_1 is a constant. So the zero mode solution is $\rho_0 = C_1$. The localization of the zero mode needs

$$\begin{aligned} I &= \int_{-\infty}^{+\infty} dy \chi(y)\rho_0^2(y) \\ &= C_1^2 \int_{-\infty}^{+\infty} \chi(y) dy < \infty. \end{aligned} \tag{29}$$

Because the function $\chi(y)$ is continuous, the convergence of the above integration is decided by the asymptotic behavior of $\chi(y)$ at the infinity of extra dimension.

In addition to the above four kinds of analytic solutions, there may exist some other solutions of the Bloch brane. And to check the localization condition (29) for all of these brane solutions one by one is not efficient. Therefore, in the following we will try to find out the general asymptotic solution of $\chi(y)$ at the infinity and give a general conclusion.

In the Bloch brane scenario, the configuration of the scalar $\phi(y)$ is a kink, and its asymptotic solution is

$$\phi(y \to \pm\infty) \to \pm v, \tag{30}$$

where v is the vacuum expectation value of ϕ. Substituting the general superpotential (14) into Eq. (9) yields

$$\chi' = -2b\phi\chi. \tag{31}$$

When $y \to \pm\infty$, we have

$$\chi'(y \to \pm\infty) \to \mp 2bv\chi(y \to \pm\infty), \tag{32}$$

from which we can obtain the asymptotic solution of χ:

$$\chi(y \to \pm\infty) \to e^{-2bv|y|}. \tag{33}$$

With the above asymptotic solution, we know that the integration in Eq. (29) is convergent because the constants b and v are positive. So the zero mode of the vector field can be localized on the general Bloch brane.

Now we will calculate explicitly the normalization constant C_1 for the four kinds of Bloch brane solutions.

For the original and generalized Bloch brane solutions (13) and (15), the normalization constants are, respectively, given by

$$C_1 = \sqrt{\frac{2b}{\pi}\sqrt{\frac{b}{1-2b}}}, \quad (0 < b < 1/2) \tag{34}$$

$$C_1 = \sqrt{\frac{2b}{\pi}\sqrt{\frac{b}{a-2b}}}, \quad (0 < b < a/2), \tag{35}$$

which are finite.

For the degenerate Bloch solutions I (16) and II (17), the results are, respectively,

$$C_1 = \left[-\frac{2}{b}\operatorname{arctanh}\left(\frac{1}{2}\left(\sqrt{c_0^2 - 4} + c_0\right)\right)\right]^{-\frac{1}{2}}, \quad (c_0 < -2) \tag{36}$$

$$C_1 = \left[\frac{\sqrt{\sqrt{1-16c_0}+1}}{4K\left(1 - \frac{2\sqrt{1-16c_0}}{\sqrt{1-16c_0}+1}\right)}\right]^{\frac{1}{2}}, \quad (c_0 < 1/16), \tag{37}$$

where the function $K(x)$ [41] gives the complete elliptic integral of the first kind.

To sum up, with the CHH mechanism, the zero mode of the $U(1)$ gauge field can be localized on the Bloch brane.

B. Massive modes

Next we investigate the (quasi)localization of the massive modes of gauge field. In this part, it is more convenient to rewrite the metric (3) in a conformally flat form, namely,

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2). \tag{38}$$

With the above metric (38) and the gauge choice $A_4 = 0$, the action of the five-dimensional gauge field (21) is reduced to

$$S = -\frac{1}{4}\sum_n \int dz \tilde{\rho}_n^2(z) \int d^4x (f_{\mu\nu}^{(n)} f^{(n)\mu\nu} - 2m_n^2 a_\mu^{(n)} a^{(n)\mu}), \tag{39}$$

where $\tilde{\rho}_n = \rho_n \chi^{1/2} e^A$, and $\tilde{\rho}_n(z)$ satisfies the following Schrödinger-like equation,

$$-\tilde{\rho}_n'' + V(z)\tilde{\rho}_n = m_n^2 \tilde{\rho}_n, \tag{40}$$

where the effective potential is given by

$$V(z) = \frac{1}{2}A''(z) + \frac{1}{4}A'^2(z) + \frac{A'(z)\chi'(z)}{2\chi(z)} + \frac{\chi''(z)}{2\chi(z)} - \frac{\chi'^2(z)}{4\chi^2(z)}, \tag{41}$$

and the prime denotes the derivative with respect to z. The above equation (40) can be recast to

$$\mathcal{T}^\dagger \mathcal{T} \tilde{\rho}_n = m_n^2 \tilde{\rho}_n, \tag{42}$$

where

$$\mathcal{T}^{\dagger} = -\frac{d}{dz} + \Gamma, \qquad \mathcal{T} = \frac{d}{dz} + \Gamma,$$

and

$$\Gamma = -\frac{1}{2}\left(\frac{\chi'}{\chi} + A'\right).$$

Equation (42) means that there is no tachyonic mode with $m^2 < 0$ in the spectrum of the KK modes [42]. Note that, in order to get the effective action (39) of the four-dimensional gauge fields from the five-dimensional one (21), we have introduced the orthonormalization condition between different massive modes:

$$\int dz \tilde{\rho}_m(z)\tilde{\rho}_n(z) = 0. \quad (m \neq n). \tag{43}$$

So the localization condition for $\tilde{\rho}_n(z)$ is

$$\int dz \tilde{\rho}_n^2(z) < \infty. \tag{44}$$

The property of $\tilde{\rho}_n$ is determined by the effective potential $V(z)$ in (41). There are two methods to get the explicit expression of the effective potential. The first one is to resolve the Einstein equations and the equations of motion of the background scalar fields with the line element (38). From our knowledge, with this method, there is no analytic solution. The second one is to write the expression of $V(z(y))$ in the y coordinate by the use of the coordinate transformation $dz = \mathrm{e}^{-\alpha(y)}dy$, and the result is

$$V(z(y)) = \frac{1}{2}\mathrm{e}^{2\alpha(y)}\left(\alpha''(y) + \frac{2\alpha'(y)\chi'(y)}{\chi(y)} + \frac{3}{2}\alpha'^2(y) + \frac{\chi''(y)}{\chi(y)} - \frac{\chi'^2(y)}{2\chi^2(y)}\right). \tag{45}$$

Then, we can use the numerical relation between y and z, $y = y(z)$, to obtain $V(z)$ from (45).

For all the Bloch brane solutions, the effective potentials $V(z)$ are of the volcano type, and they tend to vanish at the infinity of extra dimension, i.e.,

$$V(z \to \pm\infty) \to 0. \tag{46}$$

So the mass spectrum of the gauge field is continued and $m \geq 0$. For the massive KK mode, the solution of $\tilde{\rho}_n(z)$ oscillates when far away from the brane along the extra dimension. And when $m^2 \gg V_{\max}$, where $V_{\max}$ is the maximum of the $V(z)$, $\tilde{\rho}_n(z)$ approaches the plane wave solution. The shapes of $\tilde{\rho}_n(z)$ are shown in Fig. 1 for a typical potential.

Since the effective potential tends to vanish at the boundary of the extra dimension, the massive KK modes cannot be normalized.

In order to investigate the structure of the mass spectrum of these nonlocalized KK modes, we use the relative probability method introduced in Ref. [43]. The relative probability function is defined as [43]

$$P(m) = \frac{\int_{-z_b}^{z_b} \tilde{\rho}^2(z)dz}{\int_{-z_c}^{z_c} \tilde{\rho}^2(z)dz}, \tag{47}$$

where $z_c > z_b$ and $2z_b$ is about the thickness of the brane. Here and after, we set $z_c = 10z_b$. If $m^2 \gg V_{\max}$ the solution of $\tilde{\rho}$ will be approximately a plane wave, so $P(m) \approx z_b/z_c = 0.1$. In order to get the solution of Eq. (40), we introduce two boundary conditions:

$$\tilde{\rho}(0) = 0, \qquad \tilde{\rho}'(0) = 1 \tag{48}$$

for the odd parity solution and

$$\tilde{\rho}(0) = 1, \qquad \tilde{\rho}'(0) = 0 \tag{49}$$

for the even one. Next, we give the definition of a resonant or quasilocalized KK mode with the $P - m$ curve defined in

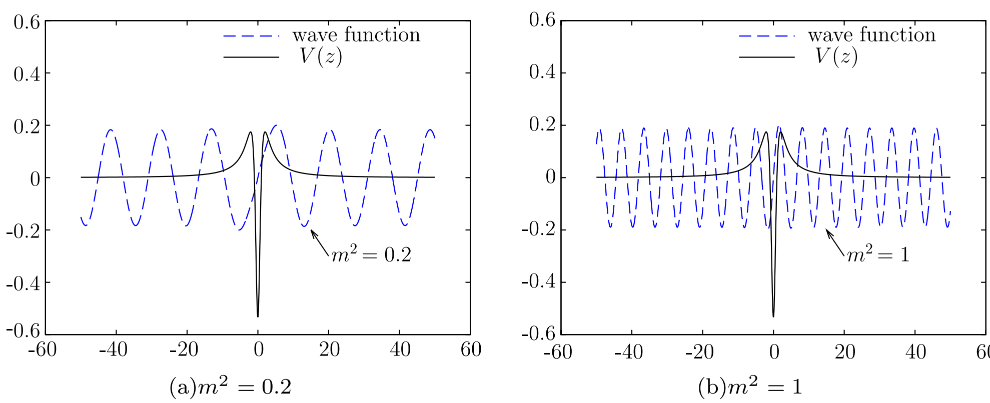


FIG. 1 (color online). The shapes of the effective potential $V(z)$ and $\tilde{\rho}_n(z)$ for the original Bloch brane solution. The parameters are set to for $b = 0.4$, $m^2 = 0.2$ (left), and $m^2 = 1$ (right).

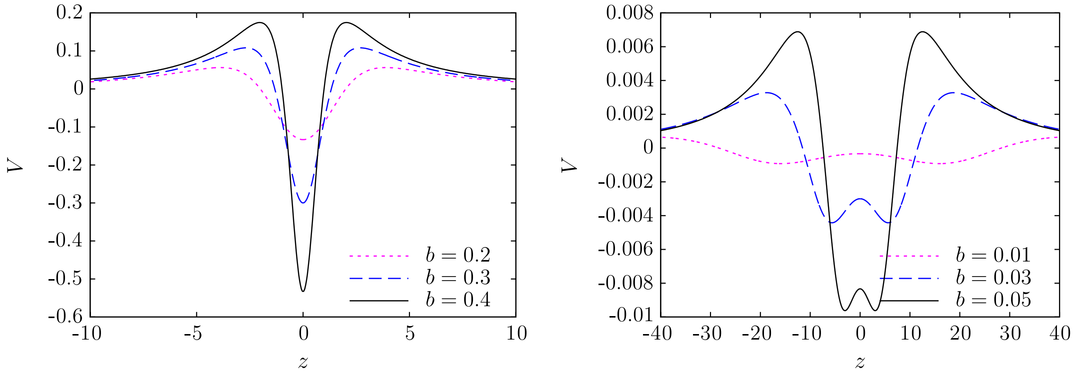


FIG. 2 (color online). The shapes of the effective potential $V(z)$ for the original Bloch brane solution with different values of the parameter b.

Eq. (47), which is usually solved by numerical method. If there is one or more peaks in the $P-m$ curve, then those peaks having full width at half maximum are called resonant peaks and the corresponding KK modes is defined as resonant KK modes. We explain this definition more explicitly. For the nth peak in the $P-m$ curve located at $m=m_n$, whose value is denoted as $P(m_n)$, there must exist two minima around the peak—one at the left-hand side of the peak (with $m=m_n^-$) and another at the right-hand side (with $m=m_n^+$), denoted as $P(m_n^-)$ and $P(m_n^+)$, respectively. If the half value of the nth peak is larger than both $P(m_n^-)$ and $P(m_n^+)$, i.e., $P(m_n)/2>P(m_n^\pm)$, then this peak has full width at half maximum and the corresponding massive KK mode with mass $m=m_n$ is a resonant KK mode. The full width at half maximum $\Gamma_n\equiv\Delta m_n$ is defined as the decay width of the nth resonant KK mode and $\tau_n\equiv 1/\Gamma_n$ is defined as its lifetime. So we can use the $P-m$ curve to check whether there are resonant modes or not in the spectrum of the vector KK modes. Note that, if a peak has no full width at half maximum, then we cannot give the lifetime for the corresponding KK mode. Such a peak is not a resonance according to our definition.

It is worth noting that in particle physics, the resonance is defined as a peak located around a certain energy found in the cross section which is a function of the total energy of colliding particles. For example, for a resonant scattering

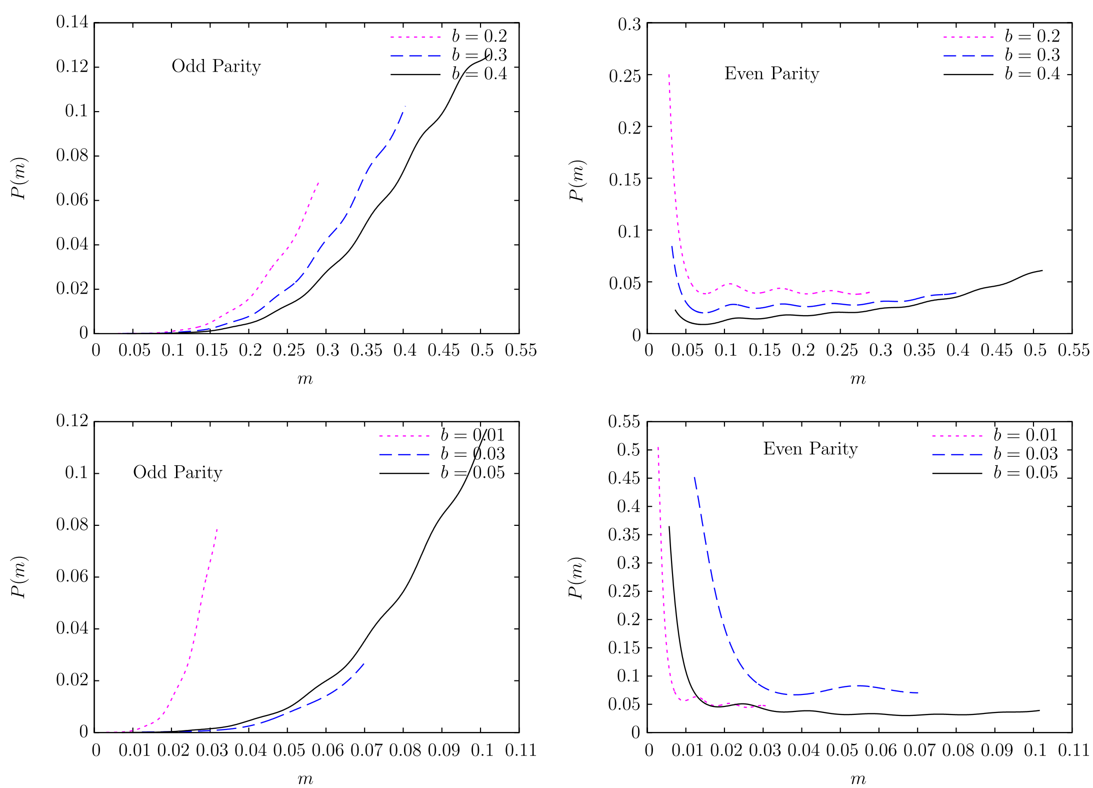
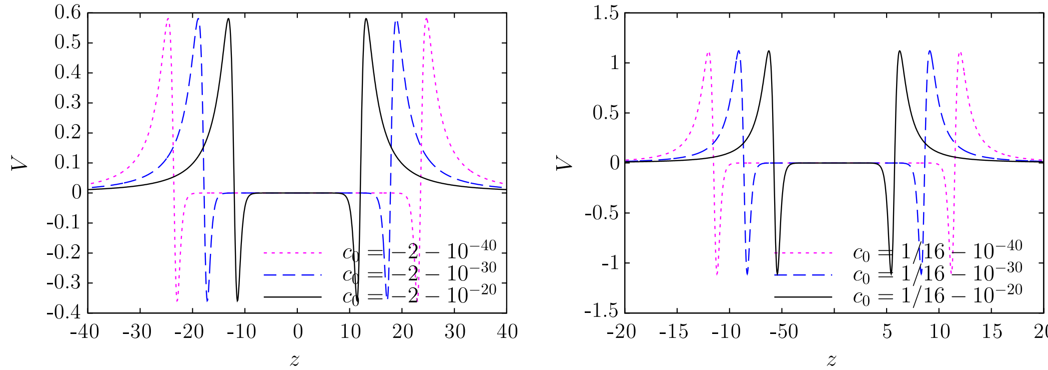


FIG. 3 (color online). The shapes of $P(m)$ as a function of m for different values of the parameter b for the original Bloch brane solution.



(a)The degenerate Bloch brane solution I (b)The degenerate Bloch brane solution II

FIG. 4 (color online). The shapes of the effective potential $V(z)$ with different values of the parameter c_0 for the degenerate Bloch brane solutions I (16) (left) and II (17) (right). The other parameters are set to $v = 1$ and $b = 1$.

from an initial two-body state n to a final two-body state n', the corresponding cross section reads [44]

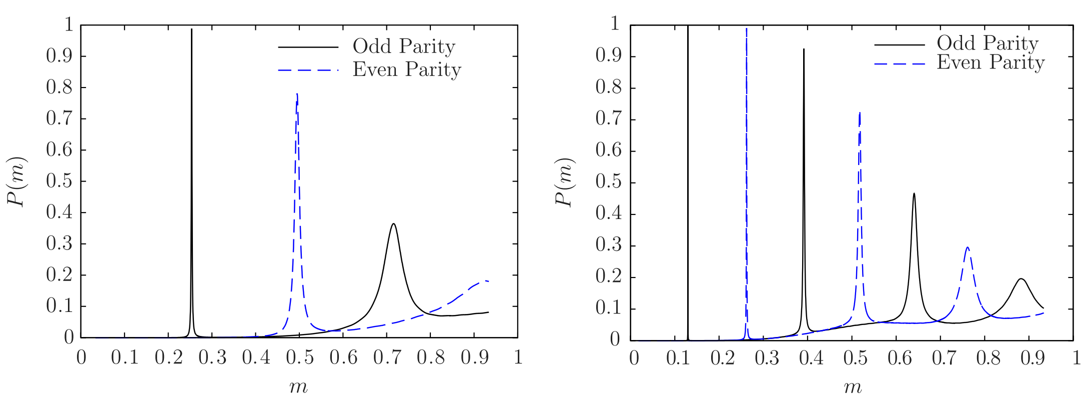
$$\sigma(n \to n', E) \propto \frac{\Gamma_n \Gamma_{n'}}{(E - E_R)^2 + \Gamma^2/4}, \tag{50}$$

where E_R is the energy of the resonance, $\Gamma_{n'}$ is the probability for the resonance decay into the final two-body state n', and Γ is the total decay rate which is the sum of all $\Gamma_{n'}$. The lifetime of a resonance is given by $\tau \equiv 1/\Gamma$ according to the uncertainty principle, where Γ is the width of the peak at the half maximum. After some time (lifetime) the resonant particle will decay into more stable particles. While in our definition, the resonances are some mass states for four-dimensional KK particles, which are governed by the Schrödinger-like equation (40). The lifetime of a resonance is the time for a four-dimensional KK particle living on branes. Because a resonant KK mode is not localizable, the corresponding KK mode particle will spread into the extra dimension after some time (lifetime). The function $P(m)$ in this paper has a similar status with the cross-section function $\sigma(E)$ in the scattering theory, and the same for m and E. Corresponding to the cross-section σ in Eq. (50), $P(m)$ does not have an analytical form in our paper, but it is effective for us to find out the resonant KK modes.

For the original Bloch brane solution, the shapes of the effective potential $V(z)$ for different values of the parameter b are shown in Fig. 2, and the corresponding $P - m$ curves are plotted in Fig. 3. The results are similar for the generalized Bloch solution and we do not show them. A large range of values of the parameters are checked for both the original and generalized Bloch solutions, but no resonant mode is found.

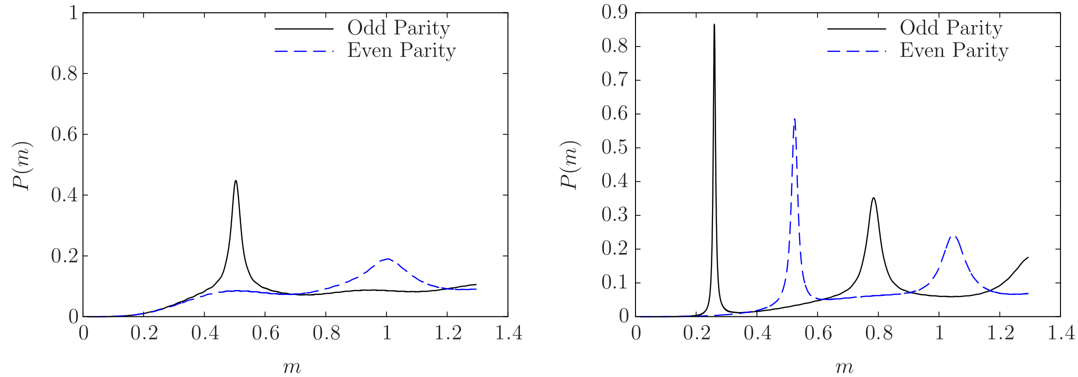
For the degenerate Bloch brane solutions I and II, the shapes of $V(z)$ are shown in Fig. 4, which shows that the width of the potential well increases with the parameter d and there are two potential wells and two barriers with vanishing potential between them when d is large enough or $c_0 \to -1$ and $c_0 \to 1/16$ for solutions I and II, respectively. Here, the parameter d is related to c_0 by $c_0 = -2 - 10^{-d}$ and $c_0 = 1/16 - 10^{-d}$ for for solutions I and II, respectively.

With the degenerate Bloch brane solutions I and II, we find resonances. In order to show the result intuitively, we



(a)$d = 10$ (b)$d = 20$

FIG. 5 (color online). The $P - m$ curves for the degenerate Bloch brane solution I (16). The parameters are set to $b = 1$, $v = 1$, $c_0 = -2 - 10^{-10}$ ($d = 10$) (Left), and $c_0 = -2 - 10^{-20}$ ($d = 20$) (Right).



(a)$d = 10$ (b)$d = 20$

FIG. 6 (color online). The $P - m$ curves for the degenerate Bloch brane solution II (17). The parameters are set to $b = 1$, $v = 1$, $c_0 = 1/16 - 10^{-10}$ ($d = 10$) (Left), and $c_0 = 1/16 - 10^{-20}$ ($d = 20$) (Right).

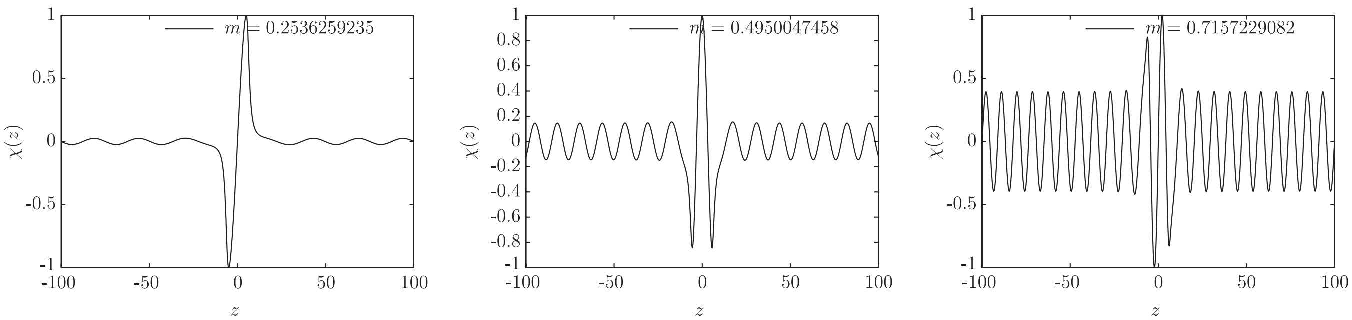


FIG. 7. The shapes of the resonant modes corresponding to the three peaks in Fig. 5(a).

plotted some curves of $P(m)$ in Figs. 5 and 6 for the degenerate Bloch brane solutions I and II, respectively. In the curves of $P(m)$, every peak corresponds to a resonant mode. And by comparing Figs. 5 and 6, we find that the number of the resonant modes increases with the width of the degenerate Bloch branes. The shapes of the resonant modes corresponding to the three peaks in Fig. 5 are shown in Fig. 7, from which it can be seen that the resonant KK modes with larger mass and shorter lifetime are nearly plane waves, while the resonances with lower mass and longer lifetime are quasibound modes.

IV. CONCLUSIONS

We have studied the localization of the U(1) gauge field on the Bloch brane with the CHH mechanism. There are two scalar fields in the Bloch brane model. In the CHH mechanism, one of the scalar fields couples directly to the U(1) gauge field. So, compared to the KT mechanism, the CHH mechanism is a simpler way to to study the localization of the $U(1)$ gauge field in the Bloch brane scenario.

In this paper, four kinds of Bloch brane solutions were discussed—the original, generalized, and degenerate I and II Bloch brane solutions, respectively. With the CHH mechanism, the Schrödinger-like equation for the vector KK modes can be recast to the supersymmetric quantum mechanics form, so the tachyonic KK modes are excluded. We found that the zero mode of the U(1) gauge field can be localized on the brane and the mass spectrum is continuous with $m^2 \geq 0$. The resonant modes in the KK spectrum were also discussed. For the original and generalized Bloch brane solutions, we did not find any resonant mode. While for the degenerate Bloch brane I and II solutions, we found some resonant modes, and the number of resonant modes is related to the inner structure of the Bloch brane and increases with the brane width.

ACKNOWLEDGMENTS

We are grateful to the referee for her/his comments and suggestions, which were very useful to us in improving this manuscript. Z.-H. Z. is supported by the National Natural Science Foundation of China (Grant No. 11305095), the Natural Science Foundation of Shandong Province, China (Grant No. ZR2013AQ016), and the Scientific Research Foundation of Shandong University of Science and Technology for Recruited Talents (Grant No. 2013RCJJ026). Y.-X. L. is supported by the National Natural Science Foundation of China (Grant No. 11375075) and the Fundamental Research Funds for the Central Universities (Grant No. lzujbky-2013-18). Y. Z. is supported by the scholarship granted by the Chinese Scholarship Council (CSC).

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[3] M. Gremm, Phys. Lett. B **478**, 434 (2000).
[4] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B**, 136 (1983).
[5] B. Bajc and G. Gabadadze, Phys. Lett. B **474**, 282 (2000).
[6] S. Randjbar-Daemi and M. E. Shaposhnikov, Phys. Lett. B **492**, 361 (2000).
[7] C. Ringeval, P. Peter, and J.-P. Uzan, Phys. Rev. D **65**, 044016 (2002).
[8] R. Koley and S. Kar, Classical Quantum Gravity **22**, 753 (2005).
[9] A. Melfo, N. Pantoja, and J. D. Tempo, Phys. Rev. D **73**, 044033 (2006).
[10] Y.-X. Liu, L.-D. Zhang, L.-J. Zhang, and Y.-S. Duan, Phys. Rev. D **78**, 065025 (2008).
[11] Y.-X. Liu, Z.-G. Xu, F.-W. Chen, and S.-W. Wei, Phys. Rev. D **89**, 086001 (2014).
[12] A. Pomarol, Phys. Lett. B **486**, 153 (2000).
[13] I. Oda, arXiv:hep-th/0103052.
[14] I. Oda, Phys. Lett. B **496**, 113 (2000).
[15] G. R. Dvali, G. Gabadadze, and M. A. Shifman, Phys. Lett. B **497**, 271 (2001).
[16] P. Dimopoulos, K. Farakos, A. Kehagias, and G. Koutsoumbas, Nucl. Phys. **B617**, 237 (2001).
[17] R. Guerrero, A. Melfo, N. Pantoja, and R. O. Rodriguez, Phys. Rev. D **81**, 086004 (2010).
[18] K. Ghoroku and A. Nakamura, Phys. Rev. D **65**, 084017 (2002).
[19] M. S. Carena, E. Ponton, T. M. Tait, and C. Wagner, Phys. Rev. D **67**, 096006 (2003).
[20] H. Davoudiasl, J. Hewett, and T. Rizzo, Phys. Rev. D **68**, 045002 (2003).
[21] M. Giovannini, Phys. Rev. D **65**, 124019 (2002).
[22] Y.-X. Liu, Z.-H. Zhao, S.-W. Wei, and Y.-S. Duan, J. Cosmol. Astropart. Phys. 02 (2009) 003.
[23] H. Guo, A. Herrera-Aguilar, Y.-X. Liu, D. Malagon-Morejon, and R. R. Mora-Luna, Phys. Rev. D **87**, 095011 (2013).
[24] A. Herrera-Aguilar, A. D. Rojas, and E. Santos-Rodriguez, Eur. Phys. J. C **74**, 2770 (2014).
[25] Y.-X. Liu, L.-D. Zhang, S.-W. Wei, and Y.-S. Duan, J. High Energy Phys. 08 (2008) 041.
[26] Y.-X. Liu, C.-E. Fu, H. Guo, and H.-T. Li, Phys. Rev. D **85**, 084023 (2012).
[27] A. Kehagias and K. Tamvakis, Phys. Lett. B **504**, 38 (2001).
[28] W. Cruz, M. Tahim, and C. Almeida, Phys. Lett. B **686**, 259 (2010).
[29] G. Alencar, R. Landim, M. Tahim, C. Muniz, and R. Costa Filho, Phys. Lett. B **693**, 503 (2010).
[30] W. Cruz, A. R. Lima, and C. Almeida, Phys. Rev. D **87**, 045018 (2013).
[31] C.-E. Fu, Y.-X. Liu, and H. Guo, Phys. Rev. D **84**, 044036 (2011).
[32] W. Cruz, M. Tahim, and C. Almeida, Europhys. Lett. **88**, 41001 (2009).
[33] H. Christiansen, M. Cunha, and M. Tahim, Phys. Rev. D **82**, 085023 (2010).
[34] W. Cruz, R. Maluf, and C. Almeida, Eur. Phys. J. C **73**, 2523 (2013).
[35] A. Chumbes, J. Hoff da Silva, and M. Hott, Phys. Rev. D **85**, 085003 (2012).
[36] D. Bazeia and A. R. Gomes, J. High Energy Phys. 05 (2004) 012.
[37] A. de Souza Dutra, J. A. C. Amaro de Faria, and M. Hott, Phys. Rev. D **78**, 043526 (2008).
[38] A. R. Gomes, arXiv:hep-th/0611291.
[39] R. Correa, A. de Souza Dutra, and M. Hott, Classical Quantum Gravity **28**, 155012 (2011).
[40] Q.-Y. Xie, J. Yang, and L. Zhao, Phys. Rev. D **88**, 105014 (2013).
[41] The details of this function can be found in the following website: http://mathworld.wolfram.com/CompleteEllipticIntegraloftheFirstKind.html.
[42] D. Bazeia, C. Furtado, and A. R. Gomes, J. Cosmol. Astropart. Phys. 02 (2004) 002.
[43] Y.-X. Liu, J. Yang, Z.-H. Zhao, C.-E. Fu, and Y.-S. Duan, Phys. Rev. D **80**, 065019 (2009).
[44] S. Weinberg, *Quantum Theory of Fields*, Foundations, Vol. 1 (Cambridge University, Cambridge, England, 1995), p. 163.