

Reviving quark nuggets as a candidate for dark matterAbhishek Atreya,^{1,*} Anjishnu Sarkar,^{2,‡} and Ajit M. Srivastava^{1,†}¹*Institute of Physics, Bhubaneswar 751005, India*²*LNM Institute of Information Technology, Jaipur, Rajasthan 302031, India*

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We discuss a novel mechanism for segregation of baryons and antibaryons in the quark-gluon plasma phase which can lead to formation of quark and antiquark nuggets in the early universe, irrespective of the order of the quark-hadron phase transition. This happens due to CP violating scattering of quarks and antiquarks from moving $Z(3)$ domain walls. CP violation here is spontaneous in nature and arises from the nontrivial profile of the background gauge field (A_0) between different $Z(3)$ vacua. We study the effect of this spontaneous CP violation on the baryon transport across the collapsing large $Z(3)$ domain walls (which can arise in the context of certain low energy scale inflationary models). Our results show that this CP violation can lead to large concentrations of baryons and antibaryons in the early universe. The quark and antiquark nuggets, formed by this alternate mechanism, can provide a viable dark matter candidate within standard model without violating any observational constraints.

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I. INTRODUCTION

One of the main unsolved problems of the modern physics is the existence of dark matter in the Universe. It is usually stated that the data on nucleosynthesis and cosmic microwave background radiation (CMBR) does not allow baryonic dark matter. This indeed holds true for baryons in the form of gas (e.g. hydrogen, helium). Observational constraints from nucleosynthesis and CMBR are very strong on such forms of baryonic matter and restrict it to less than 20% of all matter/radiation in the Universe (excluding the dark energy). However, it is important to note that these constraints do not apply if baryons are in the form of heavy bodies, such as quark nuggets, MACHOS, etc., provided that such objects form before nucleosynthesis. There are separate strong observational constraints on MACHOS from gravitational microlensing observations. In any case, it is hard to come up with scenarios where such heavy objects could form before nucleosynthesis. On the other hand, quark nuggets pass through all the observational constraints, and indeed, these were considered promising dark matter candidates after the pioneering work of Witten [1] showing the possibility of formation of such objects in a strong first order quark-hadron transition in the Universe. There were many investigations discussing the issues of stability of such objects [2–4]. It was generally considered that quark nuggets (strangelets) having density above nuclear density, with baryon number ranging from few thousand to $\sim 10^{50}$ (sizes varying from fm to meters), can provide required dark matter. Such a candidate for dark

matter will be extremely appealing as it does not require any physics beyond standard model.

The interest in quark nuggets declined with results from lattice gauge theory showing that a first order quark-hadron transition is very unlikely. The transition, for the range of chemical potentials relevant for the early universe, is most likely a crossover. Witten's scenario of formation of quark nuggets does not work in such a case. However, with most attempts of explaining the dark matter not meeting any success (such as supersymmetric dark matter candidates in view of LHC results), it is important to appreciate following points about quark nuggets as dark matter candidates. As we mentioned above, here one does not need any new species of particles, quarks do the job. Secondly, any scenario of forming quark nuggets will most naturally fit in the quark-gluon plasma (QGP) phase of the Universe, well above radiation decoupling and nucleosynthesis stages. Those baryons (quarks) which form (heavy) quark nuggets completely decouple from the processes happening at nucleosynthesis stage, and later on at the radiation decoupling stage. Thus, nucleosynthesis and CMBR constraints do not apply to the fraction of baryons in quark nuggets. Further, stability of these quark nuggets, especially strangelets, has been extensively discussed and it has been argued that strangelets with baryon number of several hundred to general quark nuggets with baryon number of order up to 10^{50} may be stable up to the present stage [2–4]. The only issue then remains is how to form these objects when quark-hadron transition is a crossover. We address this issue in this paper, extending our earlier analysis of an alternate scenario of formation of quark nuggets without requiring any first order quark-hadron phase transition.

We would like to emphasize that even in the absence of a mechanism for the formation of quark nuggets, it is important to recognize that quark nuggets provide a viable

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dark matter candidate entirely within the standard model. It then provides a strong motivation to search for mechanisms which can lead to formation of such objects in the early stages of the Universe. Indeed, these exciting objects have fascinated cosmologists and even now there are attempts to detect these objects [5,6].

We briefly recall the essential physics of Witten's proposal [1] for the formation of quark nuggets. Witten proposed that if the Universe underwent a (strong) first order QCD phase transition, then localized regions of high temperature phase, trapped between expanding hadronic bubbles, will shrink, in the process trapping the baryons inside them. He also argued that resulting quark nuggets can be stable and survive up to the present epoch. In Witten's scenario, the importance of first order phase transition was due to the fact that it provides us with an interface between two regions of the Universe in different phases. The baryon transport across the phase boundary then leads to the buildup of baryon excess in the collapsing domains. Such an interface does not exist in a crossover or in a second order phase transition. Hence, with lattice QCD calculations ruling out the first order phase transition, the mechanism of formation of quark nuggets as proposed by Witten becomes inapplicable.

Our proposal for an alternate mechanism for the formation of quark nuggets is based on this crucial ingredient of Witten's scenario, that is the existence of an interface leading to quark/antiquark reflection. Quark-hadron phase boundary (for a first order transition) is one such possibility for the interface. However, in addition to this bubble wall interface between different QCD phases, there are other possibilities of extended topological objects in the QGP phase and these have been extensively discussed in the literature [7–9]. These are domain wall defects and they arise from the spontaneous breaking of $Z(3)$ symmetry in the high temperature phase (QGP phase) of QCD, with the expectation value of the Polyakov loop $L(x)$ being the order parameter for confinement-deconfinement transition. It has been pointed out that there are also topological string defects in QGP forming at the junctions of $Z(3)$ walls [10]. The existence of these defects can be probed in the ongoing relativistic heavy-ion collision experiments at Brookhaven National Laboratory and at LHC-CERN. These are the only topological defects in a relativistic quantum field theory which can be probed in lab conditions with the present day accelerators. Detailed simulations have been performed to study the formation and evolution of these objects in these experiments [11,12].

In the presence of quarks, questions have been raised on the existence of these objects [13,14]. However, lattice studies by Deka *et al.* [15] of QCD with quarks show strong possibility of the existence of nontrivial, metastable, $Z(3)$ vacua for high temperatures of order 700 MeV. These high temperatures occur naturally in the early universe and may be possible to reach at LHC. Hence, it seems plausible that

these defects will be naturally formed in any realistic phase transition from the confining phase to the QGP phase.

The baryon inhomogeneity generation due to the reflection of quarks/antiquarks from $Z(3)$ walls was first studied by some of us in the context of relativistic heavy-ion collision experiments (RHICE) and in the Universe, in Ref. [16]. For the case of the Universe, it was argued in [16] that these collapsing domains can concentrate enough baryon number (in certain late time inflationary models) to form quark nuggets thus providing us with an alternate scenario of quark nuggets formation in early universe, which is independent of the order of phase transition. In these works, the scattering of quarks from $Z(3)$ walls was calculated by modeling the dependence of effective quark mass on the magnitude of the Polyakov loop order parameter $L(x)$ which did not distinguish between quarks and antiquarks.

In this paper we will extend the earlier analysis [16] by incorporating an interesting possibility arising from the spontaneous CP violation from $Z(3)$ interfaces. This was first discussed by Altes *et al.* [17], who showed that spontaneous CP violation can arise from $Z(N)$ structures due to the nontrivial background gauge field configuration associated with the Polyakov loop. They showed that it can lead to the localization of either quarks or antiquarks on the domain wall. It was also argued that it can lead to baryogenesis via sphaleron transition in certain extensions of the standard model. Same possibility of spontaneous CP violation for the case of QCD was also discussed in [18]. Though, in these works, the CP violating effects were discussed primarily qualitatively, and the exact profiles of $L(x)$ or the associated A_0 profiles were not calculated.

In an earlier work [16] we have incorporated this spontaneous CP violation in the propagation of quarks and antiquarks across the $Z(3)$ walls. We use the profile of Polyakov loop $L(\vec{x})$ between different $Z(3)$ vacua [which was obtained by using specific effective potential for $L(x)$ as discussed in [19]] to obtain the profile of the background gauge field A_0 . This background A_0 configuration acts as a potential for quarks and antiquarks causing nontrivial reflection of quarks from the wall. Spontaneous CP violation arising from the background A_0 configuration leads to different reflection coefficients for quarks and antiquarks. In the present work we study the effect of this difference in the scattering of quarks and antiquarks from $Z(3)$ walls on baryon transport across the collapsing $Z(3)$ domain walls in the early universe. We calculate the transmission coefficients of quarks and antiquarks from the background A_0 profile and use those in the baryon transport equations. We show that it can lead to the generation of baryon density inhomogeneities, by segregating baryons and antibaryons in different regions of the Universe near QCD phase transition epoch. [Since the background field is a color field, not only do we get the quarks and antiquark segregation, we also find that the

segregation of the baryons/antibaryons depends on the color configuration of the specific $Z(3)$ wall. This can have important implications in the context of the early universe and heavy-ion experiments that could be worth pursuing.]

Here it should be mentioned that in the present work we use $Z(3)$ wall profile of pure $SU(3)$ gauge theory, without dynamical quarks. The quark effects may not be important in the context of heavy-ion experiments due to small length and time scales involved, but for the case of Universe these effects will be of crucial importance. We will discuss this further below and argue that in case of certain inflationary models we can work with the domain wall profile corresponding to pure $SU(3)$ gauge theory.

The organization of the paper is as follows. In Sec. II we start by discussing the effective potential for the Polyakov loop and calculate the profile of the background gauge field A_0 from the profile of the order parameter $L(\vec{x})$ between different $Z(3)$ vacua [10]. In Sec. III we discuss the formation of $Z(3)$ structures in the early universe. There we discuss in detail the effects of quarks in the context of inflationary cosmology and how in certain low energy inflationary models, these $Z(3)$ domains can survive long enough to have interesting cosmological implications. The formation of baryon inhomogeneities due to baryon transport across the $Z(3)$ walls is discussed in Sec. IV. We present our results in Sec. V. Section VI presents discussions and conclusions.

II. $Z(3)$ SYMMETRY AND SPONTANEOUS CP VIOLATION

In this section we discuss the effective potential used to study the confinement-deconfinement phase transition in QCD, and the basic physics of spontaneous CP violation from the $Z(3)$ structure. Initially we restrict our discussion to pure $SU(N)$ gauge theory. In pure gauge $SU(N)$ system, in thermal equilibrium at temperature T , Polyakov loop [20–22] is defined as

$$L(x) = \frac{1}{N} \text{Tr} \left[\mathbf{P} \exp \left(ig \int_0^\beta A_0(\vec{x}, \tau) d\tau \right) \right], \quad (1)$$

where, $\beta = T^{-1}$ and $A_0(\vec{x}, \tau) = A_0^a(\vec{x}, \tau) T^a$, ($a = 1, \dots, N$) are the $SU(N)$ gauge fields satisfying the periodic boundary conditions in the Euclidean time direction τ , viz $A_0(\vec{x}, 0) = A_0(\vec{x}, \beta)$. T^a are the generators of $SU(N)$ in the fundamental representation. \mathbf{P} denotes the path ordering in the Euclidean time τ , and g is the gauge coupling. Thermal average of the Polyakov loop, $\langle L(\vec{x}) \rangle$, acts as the order parameter for the confinement-deconfinement phase transition. For brevity, we will use $l(x)$ to denote $\langle L(\vec{x}) \rangle$ from now on. It is related to the free energy of a test quark in a pure gluonic medium, $l(x) \propto e^{-\beta F}$. In confined phase, the free energy of a test quark is infinite hence $l(x) = 0$ (i.e. system is below T_c). While $l(x) \neq 0$ in deconfined phase, because in the deconfined phase a test quark has finite free

energy (in other words, the system is above the critical temperature T_c). Under $Z(N)$ [which is the center of $SU(N)$] transformation, the Polyakov loop transforms as

$$L(x) \longrightarrow Z \times L(x), \quad \text{where } Z = e^{i\phi}, \quad (2)$$

where, $\phi = 2\pi m/N$; $m = 0, 1, \dots, (N-1)$. This leads to the spontaneous breaking of $Z(N)$ symmetry with N degenerate vacua in the deconfined phase or QGP phase. For QCD, $N = 3$ hence it has three degenerate $Z(3)$ vacua resulting from the spontaneous breaking of $Z(3)$ symmetry at $T > T_c$. This leads to the formation of interfaces between regions of different $Z(N)$ vacua. These vacua are characterized by

$$l(\vec{x}) = 1, e^{i2\pi/3}, e^{i4\pi/3}. \quad (3)$$

It has been argued that these $Z(3)$ domains do not have a physical meaning [13,14]. As dynamical quarks do not respect the $Z(N)$ symmetry, their inclusion further complicates the issue. It has been argued that the effect of addition of quarks can be interpreted as the explicit breaking of $Z(N)$ symmetry, see, for example, Refs. [19,23–25]. This leads to the lifting of degeneracy of the vacuum, with $l(\vec{x}) = 1$ as the true vacuum and $l(\vec{x}) = e^{i2\pi/3}, e^{i4\pi/3}$ as the metastable ones. We will follow this approach. This interpretation finds support in the recent lattice QCD studies with quarks [15]. These result strongly favor these metastable $Z(3)$ vacua at high temperature. These $Z(3)$ vacua can have important consequences in the case of early universe where these high temperatures occur quite naturally. However, for the time being we will consider the pure gauge case [i.e. degenerate $Z(3)$ vacua] because our emphasis here is on the interesting physics due to the spontaneous CP violation in the reflection of quarks and antiquarks from $Z(3)$ walls which leads to the segregation of baryons and antibaryons in early universe. This aspect is independent of the explicit symmetry breaking due to quark effects. We will discuss the effects of quarks again in detail when we discuss the formation of $Z(3)$ networks in the next section (Sec. III). There we will see that explicit symmetry breaking arising from the quark effects makes domain walls dynamical, with true vacuum being preferred compared to the metastable vacua in the coarsening of a domain wall network. Thus, a generic domain wall network rapidly decays, and only in certain special conditions it may survive long enough to have observational implications. We will discuss these conditions in Sec. III where we discuss formation of $Z(3)$ walls in the Universe.

An effective potential for Polyakov loop, in the spirit of Landau-Ginzberg theory of phase transitions, was proposed by Pisarski [19]. The Lagrangian density is given as

$$\mathcal{L} = \frac{N}{g^2} |\partial_\mu l|^2 T^2 - V(l), \quad (4)$$

where $N = 3$ for QCD. T^2 is multiplied with the first term to give the correct dimensions to the kinetic term. $V(l)$ is the potential term that has the form

$$V(l) = \left(-\frac{b_2}{2} |l|^2 - \frac{b_3}{6} (l^3 + (l^*)^3) + \frac{1}{4} (|l|^2)^2 \right) b_4 T^4. \quad (5)$$

When $T > T_c$ [i.e. $l(x) \neq 0$], the cubic term in the above potential gives rise to $\cos(3\theta)$ term [by writing $l(x) = |l(x)|e^{i\theta}$], that leads to three degenerate $Z(3)$ vacua. In Refs. [23–25], the coefficients b_2 , b_3 and b_4 are fixed using lattice results for the pressure and energy density for pure $SU(3)$ gauge theory [26,27]. b_2 is given by $b_2 = (1 - 1.11/x)(1 + 0.265/x)^2(1 + 0.300/x)^3 - 0.478$, where $x = T/T_c$ with $T_c \sim 182$ MeV. The other parameters are $b_3 = 2.0$ and $b_4 = 0.6061 \times 47.5/16$. The additional factor $47.5/16$ in b_4 is to account for the energy and pressure contributions from the additional quark degrees of freedom compared to pure $SU(3)$ case. With the above values, $l(x) \rightarrow y = b_3/2 + \frac{1}{2} \times \sqrt{b_3^2 + 4b_2(T = \infty)}$ as $T \rightarrow \infty$. $l(x)$ and other quantities are then normalized as follows:

$$\begin{aligned} l(x) &\rightarrow \frac{l(x)}{y}, & b_2 &\rightarrow \frac{b_2}{y^2}, \\ b_3 &\rightarrow \frac{b_3}{y}, & b_4 &\rightarrow b_4 y^4, \end{aligned} \quad (6)$$

so that $l(x) \rightarrow 1$ as $T \rightarrow \infty$. The normalized quantities are then used in Eq. (5), which is then used to calculate the $l(x)$ profile using energy minimization, see Ref. [10] for details. Figure 1 shows the plot of $|l(x)|$ for the interface between two different vacua at $T = 400$ MeV (in the absence of quarks all the three interfaces have same profile for $|l(x)|$).

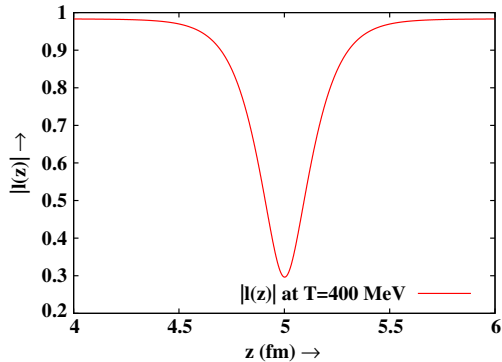


FIG. 1 (color online). Variation of $|l(x)|$ between different $Z(3)$ vacua for $T = 400$ MeV. The profile is same between any two interfaces.

We mention here that in interpreting the effective potential as in Eq. (5) one has to address the issue of convexity requirements [28]. In identifying the thermodynamic phases from the effective potential in Eq. (5), we are following the standard interpretation that the true free energy of the system (which will satisfy convexity conditions) is to be identified only with the minimum of the effective potential. The nontrivial shape of the effective potential (with barriers etc.) may imply the existence of different phases, metastable states etc. which can be interpreted in terms of nucleation of bubbles, domain walls, etc. We also mention that these different [$Z(3)$] vacua of the effective potential arise in the Euclidean formulation, and have to be interpreted as possible thermodynamic phases of the physical system which exists in the Minkowski space-time. When two such (different) phases exist in a given physical system, then the boundary between these regions must correspond to a domain wall (in the Minkowski space-time). The issue is how to get the profile of such a domain wall from the knowledge of the effective potential in the Euclidean space-time. For this we follow a simple procedure of carrying out analytic continuation from the profile in the Euclidean space-time to the Minkowski space-time, as we will discuss below in Sec. V.

An interpolating $l(x)$ profile between different $Z(3)$ vacua essentially implies that there is a background gauge field $A_0(x)$ profile which interpolates between different $Z(3)$ vacua. (This is an important assumption for our work, and also for Refs. [17,18]). As the order parameter is the thermal expectation value of the Polyakov loop, its relation to any underlying gauge field configuration is not direct. [The assumption of a time independent background A_0 field directly determined via Eq. (7) is a simple choice, and we take it in that spirit.] This spatial variation of A_0 gives rise to a localized color electric field in the QGP medium. The quarks/antiquarks moving across the $Z(3)$ domain walls will behave differently in the presence of such (color) electric field configuration. As a result, we should have different reflection and transmission coefficient for quarks and antiquarks. This is the source of CP violation. (This CP asymmetry is spontaneous because it arises from a specific configuration of the background A_0 field, which manifests itself as a potential in the equation of motion for quarks/antiquarks.) The earlier studies [17,18] of this spontaneous CP violation arising from $Z(3)$ walls focused on the localized solution of Dirac equation (in Euclidean space), and it was shown that if a wave function for a fermion species localizes, then its CP conjugate doesn't. It was also showed in Ref. [29] that in the standard model and minimally supersymmetric extension of the standard model, this CP violation can be utilized via sphaleron processes to lead to baryogenesis in the early universe.

The background gauge potential A_0 associated with the profile of $l(x)$ was first calculated by us in Ref. [30] where we also discussed various conceptual issues related to the

ambiguities in the extraction of a colored quantity A_0 from color singlet $l(x)$. We choose Polyakov gauge (diagonal gauge) for A_0 :

$$A_0 = \frac{2\pi T}{g} (a\lambda_3 + b\lambda_8), \quad (7)$$

where, g is the coupling constant and T is the temperature, while λ_3 and λ_8 are the diagonal Gell-Mann matrices. The A_0 profile was obtained from $l(x)$ profile (Fig. 1) by inverting Eq. (1). We also calculated reflection and transmission coefficient of quarks and antiquarks and it was found that the CP violating effect was stronger for heavier quarks. For details, see Ref. [30]. We mention here that in order to address the issue of uncertainties in the determination of the A_0 profile depending on the choice of the specific form of the effective potential, we had repeated this calculation of A_0 profile, in Ref. [30], for another choice of effective potential of the Polyakov loop as provided by Fukushima [31]. It was found that even though the two effective potentials are of qualitatively different shapes, with polynomial type effective potential in Ref. [19], and logarithmic effective potential in Ref. [31], the resulting wall profile and A_0 profile were very similar. This gives us confidence that our conclusions arising from the calculations of scattering of quarks and antiquarks from $Z(3)$ walls are not crucially dependent on the specific choice of the effective potential.

III. FORMATION OF $Z(3)$ DOMAIN WALLS IN THE EARLY UNIVERSE

The possible mechanism for the formation of these $Z(3)$ domain walls in the early universe was discussed in detail in [16]. We briefly recall essential points from that discussion. One important difference for the formation of $Z(3)$ walls compared to the formation of other topological defects in the early universe arises from the fact that here symmetry is broken in the high temperature phase, and is restored as the Universe cools while expanding. Standard mechanism of formation of defects (the Kibble mechanism) leads to the formation of defects during the transition to the symmetry broken phase. What happens when the Universe was already in the symmetry broken phase from the beginning? One could use general arguments of causality etc. to get some bounds on $Z(3)$ domain walls but it is not satisfactory, especially in view of quark mass effect due to which all domain walls can disappear (in principle, in a short time). To discuss the detailed formation of $Z(3)$ structures using standard defect formation scenario, one would require a situation where the Universe undergoes the transition from the hadronic (confined/low temperature) phase to the QGP (deconfined/high temperature) phase. Kibble mechanism [32] can then be invoked to study the formation of these defects. As discussed in Ref. [16],

inflationary cosmology provides a natural resolution of this problem as we discuss below.

Before inflation, the Universe was at a very high temperature ($T \gg T_c$) and quarks and gluons were deconfined. During inflation, the temperature of the Universe decreases exponentially to zero due to the rapid expansion. As a result any previously existing $Z(3)$ interfaces disappear as the temperature drops below the critical temperature T_c (if Universe remains in quasiequilibrium during this period) or as the energy density drops below Λ_{QCD} due to expansion (in a standard out of equilibrium scenario). After inflation, the Universe starts reheating and eventually the temperature is higher than critical temperature for confinement-deconfined transition. During the stage when temperature of the Universe rises above the quark-hadron transition, $Z(3)$ symmetry will break spontaneously, and $Z(3)$ walls and associated QGP string will form via the standard Kibble mechanism. For $T \gg \Lambda_{\text{QCD}}$, the energy scale for these walls is set by the temperature of the Universe. The tension of the $Z(3)$ interface and associated string [10] is set by the QCD parameters and the temperature. As a result the dynamics, of the tension forces at the least, should be decided by the background plasma for temperatures far above the QCD scale. However, in presence of quarks, there is an explicit breaking of $Z(3)$ symmetry. Two of the vacua, with $l(x) = z, z^2$, become metastable leading to a pressure difference between the true vacuum and the metastable vacua [17,33]. This leads to a preferential shrinking of metastable vacua. As the collapse of these regions can be very fast (simulations indicate $v_w \sim 1$ [11,12]), they are unlikely to survive until late times, say until QCD scale, to play any significant role in the context of the Universe. However, there is a possibility that when effects of quarks scattering from the walls is taken into account their collapse may be slower due to the friction experienced by domain wall. For large friction, the walls may even remain almost frozen in the plasma. For example, it has been discussed in the literature that dynamics of light cosmic strings can be dominated by friction which strongly affects the coarsening of string network [34,35]. It is plausible that the dynamics of these $Z(3)$ walls is friction dominated because of the nontrivial scattering of quarks across the wall. This can lead to significant friction in wall motion.

Even if the dynamics of the domain walls is not strongly friction dominated, it is still possible for these $Z(3)$ domains to survive until the QCD scale, in certain low energy inflationary models [36–38]. In these models the reheating temperature can be quite low (~ 1 TeV, or preferably, even lower). With inclusion of some friction in the dynamics of domain walls, it is then possible for the walls to survive until QCD transition. Note that the pressure difference between the true vacuum and metastable vacuum may affect the formation of these domains. For example, there may be a bias in formation of these domains as

temperature crosses T_c due to this pressure difference. Though such a bias may get washed out by the thermal fluctuations and the continued rapid reheating at the end of inflation when equilibrium concepts may not strictly apply. It is also possible that the pressure difference between the metastable $Z(3)$ vacua and the true vacuum resulting from the explicit symmetry breaking term may be small near T_c . We will assume such optimistic conditions to apply and continue to use the effective potential given in Eq. (5) for the rest of the discussion, ignoring the effects of explicit symmetry breaking due to quarks. For detailed discussion of these issues regarding formation of $Z(3)$ walls in the early universe see Ref. [16]. Certainly it is important to consider the validity of these assumptions in detail, e.g. the evolution of domain walls with due account of friction due to quark-gluon scatterings, and we hope to come back to this in future.

After formation, the domain wall network undergoes coarsening, leading to only a few domain walls within the horizon volume. Basically with our assumptions of neglect of explicit symmetry breaking due to quarks, the standard scaling distribution of domain walls will be expected, with few domain walls surviving within horizon at any stage. Detailed simulation of the formation and evolution of these $Z(3)$ walls in the context of RHICE is discussed in Ref. [11,12]. Even though the simulations are done with first order transition via bubble nucleation, resulting domain wall network is reasonably independent of that. This is because the basic physics of the Kibble mechanism only requires formation of uncorrelated domains which happens in any transition. Further, the evolution of these $Z(3)$ domain walls, once they are formed, can be understood quite well from these simulations. As we discussed previously, large friction due to quark scatterings can lead to slow dynamics of walls (with negligible wall velocities) and may help in retaining large sizes up to the stage of quark-hadron transition. (Simulation in Refs. [11,12] did not take into account of the friction due to scattering by quarks and gluons, though dissipation due to fluctuations of the Polyakov loop order parameter was automatically included.)

IV. GENERATION AND EVOLUTION OF BARYON INHOMOGENEITIES

In this section we discuss how these collapsing $Z(3)$ walls lead to the segregation of baryon number leading to the formation of quark and antiquark nuggets. After the domain walls have formed (as discussed in the previous section), the closed domains start to collapse. (Again, with neglect of explicit symmetry breaking effects, otherwise even a closed domain wall may expand depending on the pressure difference on the two sides of the wall.) As discussed in Sec. II, a nontrivial profile of $l(x)$ leads to a background A_0 profile. This A_0 will interact with quarks and antiquarks in a different manner. In other words, it will

have different reflection and transmission coefficients for the quarks and antiquarks leading to a spontaneous violation of CP symmetry. This will lead to the concentration of quarks (or antiquarks, depending on the wall) within the collapsing domain, thereby resulting in the segregation of baryons and antibaryons in the early universe. These collapsing baryon (antibaryon) rich regions can form quark (antiquark) nuggets if the baryon concentration is sufficiently high in these regions. It is important to note that these $Z(3)$ walls exist in the QGP phase as topological defects, forming irrespective of the order of the quark-hadron phase transition, even if it is a crossover. Hence, the formation of quark nuggets in our model is via a very different mechanism than the originally proposed one [1]. In context of $Z(3)$ walls, the baryon inhomogeneity generation was discussed by some of us in Ref. [16]. However there was no CP violation in that discussion as it dealt with only $l(x)$ profile and not the gauge field associated with $l(x)$.

Main aspects of calculations in Ref. [16] were along the line of Ref. [39]. We continue to follow that approach here. While studying the baryon transport across the domain wall, we assume constant temperature. A major simplification that happens due to this assumption is that one can take the height of the potential to be constant. This also makes it possible for us to ignore the effects coming from the reheating due to decreasing surface area as the wall collapses. We also assume that the thermal equilibrium is maintained as the quarks and antiquarks are reflected from the domain wall. We further assume that the collapse of the domain walls is fast. This allows us to ignore the expansion of the Universe as domain walls will then collapse in the time smaller than the Hubble time. In our calculations we take the wall velocity to be the sound velocity, $v_w = 1/\sqrt{3}$. This velocity could be larger if the friction is subdominant in comparison to the surface tension of the wall, or the velocity can be much smaller if the frictional forces are very dominant. To study the change in the number densities inside and outside the collapsing region we assume that the baryons homogenize instantaneously as the baryon transport occurs across the wall (see the discussion in Ref. [39] for the self consistency of this assumption). We can then work with only the number density inside and outside the domain wall and ignore the diffusion of baryons.

Let V be the Hubble volume at time t . In this volume suppose there are N_d number of collapsing domains. Let $V_i = 4\pi/3R(t)^3 N_d$ [$R(t)$ being the radius of domain taken to be spherical] be the volume contained within the domain walls and $V_o = V - V_i$ be the volume outside the collapsing regions. As we are ignoring the expansion of the Universe for a given domain wall, V is fixed. Note that this assumption here amounts to saying that for a reasonably large value of N_d , and with large wall velocity, the collapse of domain walls happens in a time much shorter than the Hubble time.

The radius of the collapsing domain, at time t , is given by the expression

$$R(t) = \frac{r_H}{N_d^{1/3}} - v_w(t - t_0), \quad (8)$$

where r_H is the horizon size at the initial time $r_H \simeq t_0 \simeq 30(\frac{150}{T(\text{MeV})})^2$ (in the units of micro seconds). If n_i and n_o are the number densities of baryons in the regions inside and outside the domain walls, then the total number of baryons in each region is $N_i = n_i V_i$ and $N_o = n_o V_o$. The equations for studying quark number density concentration inside and outside the domain wall can then be written as

$$\dot{n}_i = \left(-\frac{2}{3} v_w T_w n_i + \frac{v_o^{\text{rel}} n_o T_- - v_i^{\text{rel}} n_i T_+}{6} \right) \frac{S}{V_i} - n_i \frac{\dot{V}_i}{V_i} \quad (9)$$

$$\dot{n}_o = \left(\frac{2}{3} v_w T_w n_i - \frac{v_o^{\text{rel}} n_o T_- - v_i^{\text{rel}} n_i T_+}{6} \right) \frac{S}{V_i} + n_o \frac{\dot{V}_i}{V_o}, \quad (10)$$

where S is the surface area of the collapsing wall. T_w is the transmission coefficient for the quarks inside the domain and moving parallel to the wall. The relative velocity for such quarks with respect to the wall is v_w and they constitute 4/6 of the total number of the inside quarks. $T_-(T_+)$ is the transmission coefficient calculated for the quarks that are moving from outside (inside) of the wall towards the inside (outside) with the relative velocity $v_o^{\text{rel}}(v_i^{\text{rel}})$ with respect to the wall. Each contributes towards 1/6 of the corresponding number densities. Equations (8), (9) and (10) are then solved simultaneously to get the evolution of the baryon densities inside the collapsing domain.

As the wall collapses, it leaves behind a profile of baryon density. Consider a spherical shell of thickness dR , at a distance R from the center of the domain wall. Then if $\rho(R)$ is the baryon density, then total number of baryons in the shell is given by $dN_i = 4\pi R^2 \rho(R) dR$. Using Eq. (8) we get

$$\rho(R) = -\frac{\dot{N}_i}{4\pi v_w R^2}. \quad (11)$$

Equations (8) and (11) are solved simultaneously to get the density profile. It is important to note that during last stages of the collapse of domain wall, it is possible that the baryon concentration becomes so large that chemical potential in the region is comparable to the temperature. This will alter the transmission probability of the baryons across the domain wall. We are neglecting any such effects that may arise during the evolution.

As we discussed in Sec. II, the domain wall is selective in the transmission of baryons and antibaryons due to its

CP odd nature. This will lead to the baryon antibaryon segregation. As a result we get baryon rich and antibaryon rich regions that can form nuggets and antinuggets if there is sufficient concentration of baryons or antibaryons. In addition, the domain wall is also sensitive to the color of quark as it has different reflection and transmission coefficient for different colors. Equations (8) to (11) need to be solved for each color which will result in the color specific baryon concentration. This in itself is not surprising as in the QGP phase, the degrees of freedom are color degrees of freedom and the requirement to have colorless objects in QGP would be an artificial one.

V. RESULTS

We present a brief discussion of how to obtain A_0 profile from $l(x)$ profile. See Ref. [30] for details. Substituting Eq. (7) in Eq. (1) and comparing the real and imaginary parts, we get

$$\cos(\alpha) + \cos(\beta) + \cos(\gamma) = 3|l(x)| \cos(\theta), \quad (12a)$$

$$\sin(\alpha) + \sin(\beta) + \sin(\gamma) = 3|l(x)| \sin(\theta), \quad (12b)$$

where, $\alpha = 2\pi(\frac{a}{3} + \frac{b}{2})$, $\beta = 2\pi(\frac{a}{3} - \frac{b}{2})$ and $\gamma = 2\pi(\frac{-2a}{3})$ [(a, b) are defined in Eq. (7)]. θ is defined by $l(x) = |l(x)|e^{i\theta}$. For each of the $l = 1, z, z^2$ vacuum, the solutions are a set of ordered pairs $(a, b)_{L=1, z, z^2}$. We choose one pair $(a, b)_{L=1}$ as the initial condition. By demanding that a and b (and hence A_0) vary smoothly across the wall [as the profile of $L(x)$ changes smoothly], we approach the appropriate values of $(a, b)_{L=z}$ in $L = z, z^2$ vacuum. Once, we have a and b profiles, A_0 was calculated using Eq. (7). Figure (2) shows the background A_0 profile between $l = 1$ and $l = z^2$, calculated using the profile given in Fig. (1) for $T = 400$ MeV.

To calculate the reflection and transmission coefficient, we need the solutions of Dirac equation in the Minkowski space but the A_0 profile is calculated in Euclidean space. We start with the Dirac equation in the Euclidean space, with the spatial dependence of A_0 calculated from $Z(3)$ wall profile as mentioned above. Then we do the analytic continuation of the full equation to the Minkowski space and use the resulting equation to calculate the reflection and transmission coefficients. We first approximated domain wall by the step potential. For a general smooth potential we followed a numerical approach given by Kalotas and Lee [40]. They have discussed a numerical technique to solve Schrödinger equation with potentials having arbitrary smooth space dependence. We applied this technique of Ref. [40] for solving the Dirac equation.

We will discuss the concentration of charm quarks in the following. Their number density at $T \simeq 400$ MeV is still significant and with large reflection coefficients, they lead to large baryon/antibaryon concentrations. Up and down

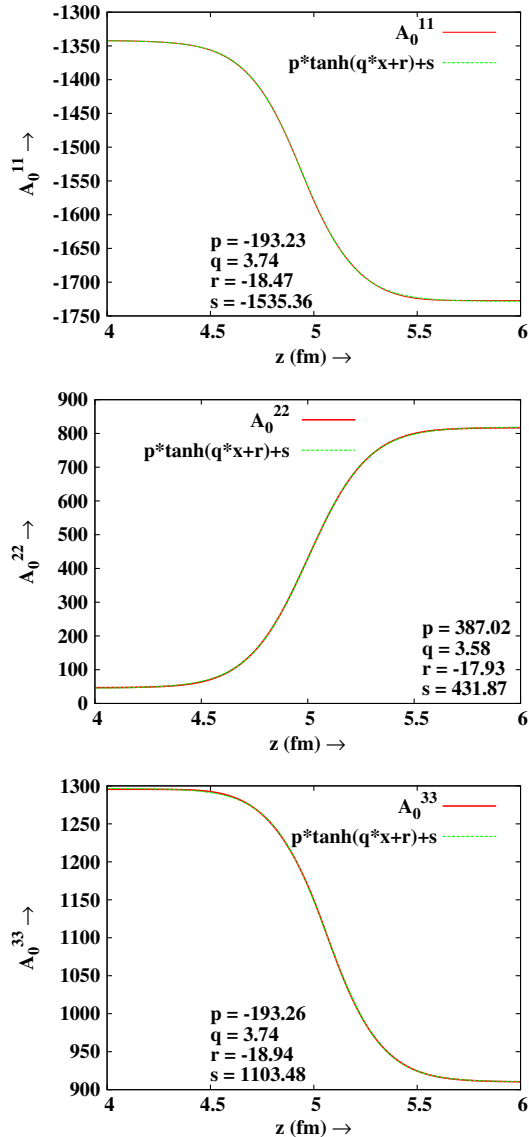


FIG. 2 (color online). The background A_0 profile calculated from the $l(x)$ profile. The profile is fitted to a tanh curve.

quarks are ultrarelativistic and have very small reflection coefficients. The case of strange quark is an important one. We will comment on that case at the end of this section. For charm quark at $T = 400$ MeV, the thermal velocity v_p is less than the sound velocity v_s . As we are assuming the wall velocity v_w to be same as v_s , the particles moving from outside towards the wall are unable to catch up. This means that T_+ is identically zero. For the particles moving towards the wall, the energy (in the rest frame of the wall) is much larger than the potential so most of them pass through (T_- is close to unity). Only the particles moving parallel to the wall can get concentrated. The potential as seen by the incoming fermion is $V(z) = -gA_0(z)$. The value of g is chosen such that $N/g^2 = 0.8$. Since g is positive for quarks, the background A_0 profile dictates that red, green and anti-blue quarks are concentrated in the collapsing regions with

$l = z^2$. (Note, in Fig. 2, A_0^{22} has opposite sign compared to A_0^{11} and A_0^{33} . Thus, while red and green quarks experience a potential barrier leading to significant reflection, the blue quark sees a potential well. It is the blue antiquark which experiences a potential barrier and undergoes significant reflection.) Table I lists the values of T_w for charm quark for smooth profile. It clearly indicates that two color species of quarks and one color species of antiquark are not transmitted. These transmission coefficients were then used to solve Eqs. (9) and (10) simultaneously. This gives us the evolution of number densities inside and outside the domain wall for each color. Figures 3(a) and 3(b) show the evolution of number densities for charm quark and antiquark inside the collapsing domain wall at $T = 400$ MeV for the case of step potential approximation. The result is for $N_d = 10$. It is clear that the number of quarks contained in the domain wall is several orders of magnitude higher than the number of antiquarks. The number densities of quarks and antiquarks are shown in Figs. 4(a) and 4(b). Looking at Fig. 3(a) and Fig. 4(a) we note that the number densities are not much different for the smooth and step potential. This might seem surprising. However a look at Fig. 3(b) and Fig. 4(b) clearly shows that the number density of anti-red (and other corresponding) quarks, that are not getting concentrated, is much less for the smooth profile than the step potential. So, the number densities in Figs. 3(a) and 4(a) have same order of magnitude but not same numbers. Figure 5 shows the density profile of red charm quark. As the majority of antiquarks are completely transmitted, they do not leave any density profile behind.

Figures (3) and (4) give the number density of quarks in units of the background quark/antiquark number density n_0 , as a function of the size of the collapsing domain wall. At $T = 400$ MeV, $n_0 \approx O(1)/\text{fm}^3$ for each type of quark. This gives the net baryon number trapped inside the domain wall to be of order 10^{52} when domain wall collapses to a size of order one meter. This is with the optimistic assumption that all the baryons get trapped inside the wall while antiquarks leave the wall virtually unreflected. This may not be a reasonable assumption, especially in view of the assumption of thermal equilibrium and homogeneous baryon distribution inside the wall. In the most conservative scenario, the net baryon number inside the domain wall should remain trapped. Net baryon number to entropy ratio being of order 10^{-10} , it is safe to say that at

TABLE I. Table for the transmission coefficients for charm quarks and antiquarks, with thermal velocity parallel to the wall, for the $l = z^2$ wall.

	r	b	g
c	0.0	0.936623	0.0
\bar{c}	0.997471	0.0	0.99903

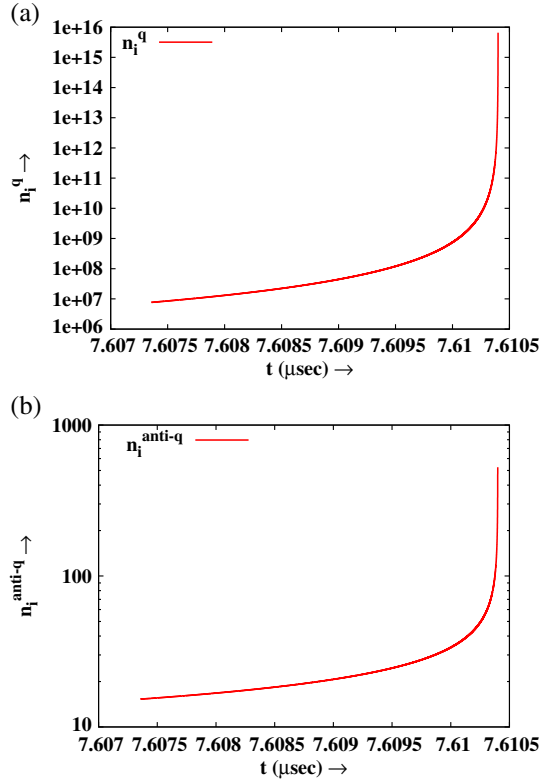


FIG. 3 (color online). Number density evolution with step function profile: (a) For red, green and anti-blue charm quark. (b) For anti-red, anti-green and blue charm quark.

least net baryon number of order 10^{42} can be trapped inside collapsing domain walls. These *quark nuggets* may then survive until present and provide dark matter. In this case [Fig. 4(a)], we had a concentration of baryons. This concentration is due to the wall between $l(x) = 1$ and $l(x) = z^2$ vacua. There would also be a wall between $l(x) = 1$ and $l(x) = z$ vacua, which will be the conjugate of the wall between $l(x) = 1$ and $l(x) = z^2$. In this domain, it will be the antibaryons which will get concentrated. As a result we will have a net segregation of baryons and antibaryons. Though, note that for the concentration of antiquarks, the above type of conservative estimate of 10^{42} baryon number may not be applicable.

An important point is the choice of initial conditions for calculating A_0 . We will now discuss the effect of this choice of initial conditions on the baryon segregation. The ambiguity in the initial condition and hence in determining A_0 is reasonable as we are extracting information about a colored object (A_0) starting from a colorless variable $L(x)$. Thus there is no reason to expect unique solution for A_0 starting from a given $L(x)$ profile. This is reflected in the various sets (a, b) that are available for each of the $Z(3)$ vacua. It appears that choosing a different set (a, b) amounts to selecting domain wall profiles which carries different color information for the scattering of a fixed color (say red) quark (see Ref. [30] for a detailed discussion).

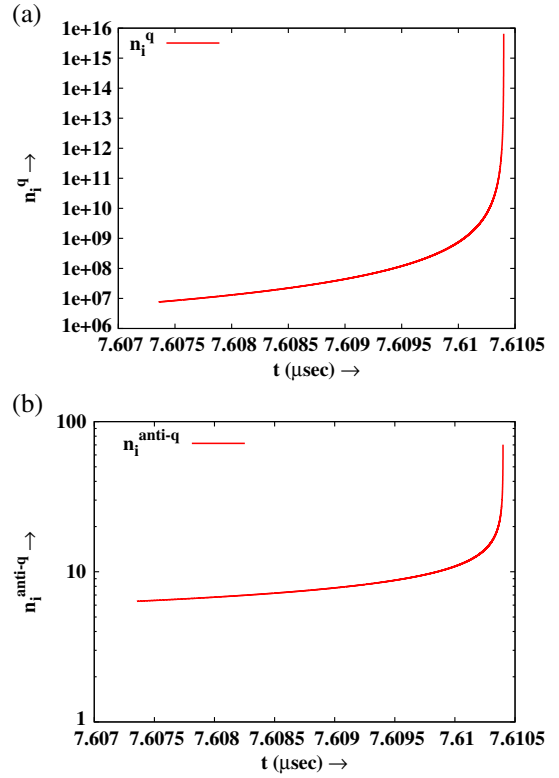


FIG. 4 (color online). Number density evolution with smooth profile: (a) For red, green and anti-blue charm quark. (b) For anti-red, anti-green and blue charm quark.

In the present context that would simply mean that if for a specific choice of (a, b) , one color (say red) is being concentrated inside the collapsing domain, another color (say blue) will be concentrated in the region for a different choice of (a, b) . Nonetheless there would be concentration of quarks (or antiquarks, as the case may be) and the number densities will also be same.

In his original proposal, Witten [1] discussed the formation of strangelets. We have not discussed concentration of strange quarks. This is due to the fact that strange quarks are in Klein regime at these temperature i.e. reflection coefficients are greater than unity. As Klein paradox is

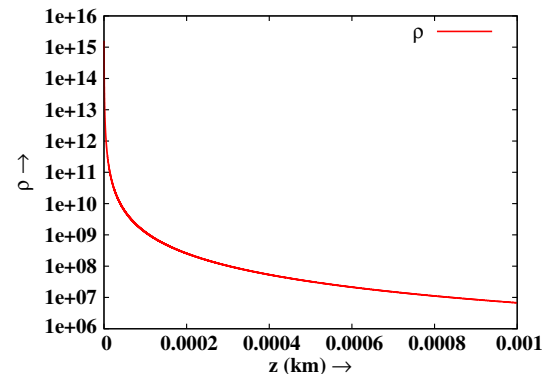


FIG. 5 (color online). Evolution of baryon density profile.

understood in terms of particle antiparticle pair production, it seems likely that we will have even larger concentration of strange quarks (or anti-strange quarks) because the pair produced species will also contribute to the number density inside the collapsing volume. However, there is a conceptual complication in doing the quantitative estimation of the number densities. In pair production, there is a back-reaction on the background field. The pair production is at the cost of the energy of the background field, which decreases as more and more particle are pair produced. This is difficult to implement in the present case as the background configuration is a topological configuration and it is not clear how to decrease the magnitude of A_0 here [affecting the magnitude of $l(x)$] while maintaining the topological property of the wall configuration. Nonetheless, it is clear that the concentration of strange quarks/antiquarks of at least same order as above will be expected in our model, naturally leading to the formation of strangelets. This is one of the strengths of our model that it can naturally lead to formation of strangeness rich quark nuggets. As we mentioned in the Introduction, stability of strangelets has been discussed extensively in the literature and for a wide range of quark numbers the strangelets could be stable. From our discussion of the formation of $Z(3)$ walls it is clear the formation of small $Z(3)$ walls is almost unavoidable in the QGP phase. Thus formation of small strangelets will happen very naturally in our model. As we have discussed above, under certain optimistic conditions, even very large strangelets are possible within our model.

VI. DISCUSSIONS AND CONCLUSIONS

We have addressed the issue of viability of quark nuggets as dark matter candidates by showing an alternate mechanism for the formation of these objects in the QGP phase of the early universe. Here the nature, or even the existence of

quark-hadron phase transition is completely irrelevant. Quarks and antiquarks are reflected by collapsing $Z(3)$ walls. This leads to concentration of baryon number in localized regions, forming quark nuggets, exactly as in the original scenario of Witten. This possibility was discussed by some of us in an earlier paper [16] where an effective constituent quark mass was introduced as a function of the Polyakov loop order parameter. Here we have extended that analysis by recognizing that the A_0 field associated with $l(x)$ leads to spontaneous CP violation leading to different scattering of quarks and antiquarks from a given $Z(3)$ wall. Thus one gets quark nuggets as well as antiquark nuggets in this scenario. Such nuggets and antinuggets have been discussed in recent publications [41] in context of a soft radio background. It would be interesting to explore if these nuggets and antinuggets discussed here can play a role in such phenomenon. Importantly, these nuggets and antinuggets provide a natural candidate, entirely within the standard model, for dark matter of the Universe. Note that as the CP violation here is resulting from a specific domain wall configuration in a given region, overall there will not be any net concentration of baryons or antibaryons. It is tempting to speculate that with the use of CP violating θ term in the QCD Lagrangian, can one get a net concentration of antibaryons over baryons? If that could be achieved then one can attempt to explain baryogenesis also in this model where excess antibaryons remain trapped in antiquark nuggets while compensating baryon number accounts for the visible matter in the Universe.

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