

**Dynamics of electroweak gauge fields during and after Higgs inflation**Ippei Obata,<sup>1,2,\*</sup> Takashi Miura,<sup>2,†</sup> and Jiro Soda<sup>2,‡</sup><sup>1</sup>*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*<sup>2</sup>*Department of Physics, Kobe University, Kobe 657-8501, Japan*

(Received 20 May 2014; published 1 August 2014)

We study the dynamics of  $SU(2)_L \times U(1)_Y$  electroweak gauge fields during and after Higgs inflation. In particular, we investigate configurations of the gauge fields during inflation and find the gauge fields remain topologically nontrivial. We also find that the gauge fields grow due to parametric resonances caused by oscillations of a Higgs field after inflation. We show that the Chern-Simons number also grows significantly. Interestingly, the parametric amplification gives rise to sizable magnetic fields after the inflation whose final amplitudes depend on the anisotropy survived during inflation.

DOI: [10.1103/PhysRevD.90.045005](https://doi.org/10.1103/PhysRevD.90.045005)

PACS numbers: 14.80.Bn, 98.80.Cq, 98.80.-k

**I. INTRODUCTION**

It is well known that an inflationary scenario has succeeded in solving various cosmological problems of the standard big bang model and predicting scale-invariant fluctuations observed in the large scale structure of the Universe and the cosmic microwave background radiation (CMB). In this scenario, a scalar field called inflaton is supposed to be a source of quasiexponential expansion of the Universe. Thus, it is important to identify the inflaton in a model of particle physics. An attractive possibility is that the inflaton is one of scalar fields in a supersymmetric extension of the standard model of particle physics. So far, however, there exists no signal of supersymmetry in experiments at the LHC, which has been supposed to discover supersymmetric particles. This is one of the reasons people tend to prefer Higgs inflation proposed in [1], where the Higgs field plays a role of the inflaton. It should be emphasized that inflation can be realized in the electroweak sector of the standard model of particle physics. Remarkably, new data released by Planck strongly support Higgs inflation [2] although recent BICEP2 results raised a challenge to Higgs inflation [3].

Needless to say, gauge fields are essential ingredients of electroweak theory. Thus, it is intriguing to explore the cosmological dynamics of the electroweak gauge fields in an inflationary scenario. Indeed, in view of recent findings concerning roles of gauge fields in inflation [4], it is worth investigating a possible role of the electroweak gauge fields in Higgs inflation. It should be noted that the dynamics of electroweak theory coupled with Einstein gravity has been already studied in [5]. There, it is found that a nontrivial local minimum appears in the effective potential of the gauge fields. In their paper, however, the presence of a cosmological constant is assumed to realize inflation.

In fact, it is inevitable to consider a concrete inflation model for making predictions on observables at present. Fortunately, as mentioned above, electroweak theory itself prepares Higgs inflation. Thus, it is worth redoing their analysis in the context of Higgs inflation.

The other motivation stemmed from primordial magnetic fields [6,7]. It is well known that there exist magnetic fields in galaxies. Moreover, there are several observational evidences [8–11] for the presence of magnetic fields in intergalaxies or interclusters of galaxies, which seems difficult to explain by astrophysical processes. One attractive mechanism for producing magnetic fields with such a long correlation length is the generation of magnetic fields in inflation. Indeed, the generation of primordial magnetic fields during inflation has been discussed in many papers [12–18]. In the course of these studies, strong coupling and backreaction problems are recognized [19]. Although a partial answer to the backreaction problem is given in [20] and there is a proposal for resolving the strong coupling problem [21], it is fair to say that further works including theoretical constraints on the magnitude of magnetic fields from observations [22,23] are necessary. At this point, it should be noticed that all of these works have been done in Einstein-Maxwell theory. However, inflation occurs at the energy scale above the electroweak phase transition. Hence, it is natural to study generation of primordial magnetic fields in the context of Einstein-electroweak theory. It is legitimate to expect a simple solution to the above difficulties in this natural setup.

In this paper, therefore, we focus on the dynamics of the  $SU(2)_L \times U(1)_Y$  gauge fields during and after Higgs inflation aiming at clarifying the evolution of gauge field configurations and seeking a novel mechanism for generating primordial magnetic fields. We show that there appear topologically nontrivial configurations of the gauge fields during inflation. Moreover, the gauge fields require an anisotropic universe as a framework. Interestingly, it turns out that the initial anisotropy survives during inflation. Of course, the expansion rate is isotropic in

\*obata@tap.scphys.kyoto-u.ac.jp

†takashi.miura@people.kobe-u.ac.jp

‡jiro@phys.sci.kobe-u.ac.jp

accordance with the cosmic no-hair conjecture. We also observe that amplification of the gauge fields after inflation occurs due to parametric resonances induced by oscillations of the Higgs field. We evaluate magnetic fields generated by the processes and find that sizable magnetic fields are generated. Remarkably, the anisotropy survived until the end of inflation affects the final amplitude of magnetic fields.

The paper is organized as follows. In Sec. II, we provide the action of Einstein-electroweak theory with a non-minimal coupling and derive basic equations for analyzing cosmological dynamics of the gauge fields. In Sec. III, we do the numerical calculation and find parametric resonances. We also evaluate the change of the Chern-Simons number due to parametric resonances. In Sec. IV, we show that sizable magnetic fields are generated by parametric resonances. We conclude the paper with summary and outlook in Sec. V.

## II. COSMOLOGY IN EINSTEIN-ELECTROWEAK THEORY

In this section, we present basic formulas for studying cosmology in nonminimally coupled Einstein-electroweak theory where a Higgs field  $\Phi$  plays a role of an inflaton. The action reads

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R + \xi \Phi^\dagger \Phi R - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \left( \Phi^\dagger \Phi - \frac{v_0^2}{2} \right)^2 \right], \quad (1)$$

where  $M_{\text{pl}}$  is the reduced Planck mass,  $g$  is a determinant of a metric  $g_{\mu\nu}$ , and  $\xi$ ,  $\lambda$  and  $v_0$  are parameters. Here, a gauge covariant derivative is defined as

$$D_\mu \Phi = \left( \partial_\mu - i \frac{g}{2} \tau^a A_\mu^a - i \frac{g'}{2} B_\mu \right) \Phi, \quad (2)$$

where  $g$  and  $g'$  are gauge coupling constants,  $A_\mu^a$  and  $B_\mu$  are  $SU(2)_L \times U(1)_Y$  gauge fields, and  $\tau^a$  are Pauli matrices. Note that the coupling constant  $g$  is not related to the metric. We also defined the  $SU(2)_L$  field strength  $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$  with structure constants  $f^{abc}$  and the  $U(1)_Y$  field strength  $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ . Note that, for the parameter region  $1 \ll \sqrt{\xi} \ll 10^{17}$ , inflation can be realized in the context of electroweak theory [1].

Hereafter, we assume a homogeneous universe with a spatial topology of  $S^3$ . In this case, we can use the Maurer-Cartan invariant one-form basis  $\sigma^j (j = 1, 2, 3)$  which satisfy

$$d\sigma^i = \epsilon^{ijk} \sigma^j \wedge \sigma^k, \quad (3)$$

where  $\epsilon^{ijk}$  is the Levi-Civita symbol. Using these one-forms, the metric can be written as

$$ds^2 = -N^2 dt^2 + a_1^2 ((\sigma^1)^2 + (\sigma^2)^2) + a_3^2 (\sigma^3)^2 \quad (4)$$

$$\equiv - (e^0)^2 + (e^1)^2 + (e^2)^2 + (e^3)^2, \quad (5)$$

where  $N$  is the lapse function and  $e^\mu (\mu = 0, 1, 2, 3)$  is the orthonormal basis. Here, scale factors  $a_1(t)$  and  $a_3(t)$  are different in general because of the presence of the  $U(1)_Y$  gauge field which is supposed to have a specific direction

$$\mathbf{B} = h(t) \sigma^3. \quad (6)$$

This anisotropic ansatz is inevitable when we consider cosmology in electroweak theory. It is also necessary when a nontrivial coupling between an inflaton and gauge fields exists as in electroweak theory, because there is a possibility to have anisotropic inflation [24]. Indeed, we will see that Higgs inflation can be regarded as a kind of anisotropic inflation in the sense that the survived anisotropy is relevant to the gauge field dynamics. Next, using the gauge degrees of freedom, we transform the Higgs field

$$\Phi = \frac{v}{\sqrt{2}} \begin{pmatrix} x^2 + ix^1 \\ x^4 - ix^3 \end{pmatrix} \quad (7)$$

to

$$\Phi \rightarrow H^{-1} \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (8)$$

where  $H = x^4 + ix^i \tau^i (i = 1, 2, 3)$ , with a relation  $(x^i)^2 + (x^4)^2 = 1$ , is an element of  $SU(2)_L$  group. Here,  $\tau^i$  are Pauli matrices. Under this transformation, the trivial gauge field becomes

$$-iH^{-1} dH = \sigma^j \tau^j. \quad (9)$$

Thus, we use the following ansatz of the Higgs fields

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(t) \end{pmatrix}. \quad (10)$$

Finally, we can assume the gauge fields as follows

$$\mathbf{A} = \frac{1}{2g} [f_1(t)(\sigma^1 \tau^1 + \sigma^2 \tau^2) + f_3(t)\sigma^3 \tau^3], \quad (11)$$

where we have taken into account anisotropy due to the  $U(1)$  gauge field  $B_\mu$ . Substituting the ansatz (11) into the field strength of the  $SU(2)_L$  gauge field,  $\mathbf{F} = d\mathbf{A} - ig\mathbf{A} \wedge \mathbf{A}$ , we obtain

$$\begin{aligned} \mathbf{F} = & \frac{\dot{f}_1}{ga_1 N} e^0 \wedge \left( e^1 \frac{\tau^1}{2} + e^2 \frac{\tau^2}{2} \right) + \frac{\dot{f}_3}{ga_3 N} e^0 \wedge e^3 \frac{\tau^3}{2} \\ & + \frac{f_1(f_3+2)}{ga_1 a_3} \left( e^2 \wedge e^3 \frac{\tau^1}{2} + e^3 \wedge e^1 \frac{\tau^2}{2} \right) \\ & + \frac{f_1^2 + 2f_3}{ga_1^2} e^1 \wedge e^2 \frac{\tau^3}{2}. \end{aligned} \quad (12)$$

It is easy to see a trivial configuration  $f_1 = f_3 = 0$  has the field strength  $\mathbf{F} = 0$ . Moreover, the configuration with  $f_1 = f_3 = -2$  also gives rise to  $\mathbf{F} = 0$ . Indeed, this is a case of pure gauge

$$\mathbf{A} = -\frac{1}{g} \sigma^j \tau^j = \frac{i}{g} H^{-1} d\mathcal{H}. \quad (13)$$

The field strength of the  $U(1)_Y$  gauge  $\mathbf{G} = d\mathbf{B}$  is given by

$$\mathbf{G} = \frac{\dot{h}}{Na_3} e^0 \wedge e^3 + \frac{2h}{a_1^2} e^1 \wedge e^2. \quad (14)$$

Note that  $h = 0$  yields  $\mathbf{G} = 0$ .

Using the ansatz (10), the term proportional to the curvature becomes

$$\frac{M_{\text{pl}}^2}{2} \left( 1 + \frac{\xi v^2}{M_{\text{pl}}^2} \right) R. \quad (15)$$

In order to transform the metric into the Einstein frame, we need the conformal transformation

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega \equiv \sqrt{1 + \frac{\xi v^2}{M_{\text{pl}}^2}}. \quad (16)$$

Substituting these ansatzes into the action after the conformal transformation, we get the following action in the Einstein frame

$$\begin{aligned} S = & 2\pi^2 \int dt a_1^2 a_3 \frac{M_{\text{pl}}^2}{2} \left[ \frac{4}{a_1} \frac{d}{dt} \left( \frac{\dot{a}_1}{N} \right) + \frac{2}{a_3} \frac{d}{dt} \left( \frac{\dot{a}_3}{N} \right) + \frac{2}{N} \left( \frac{\dot{a}_1}{a_1} \right)^2 + \frac{4}{N} \left( \frac{\dot{a}_1}{a_1} \right) \left( \frac{\dot{a}_3}{a_3} \right) + N \left( \frac{8}{a_1^2} - \frac{2a_3^2}{a_1^4} \right) \right] \\ & + 2\pi^2 \int dt N a_1^2 a_3 \left[ \frac{1}{2g^2 N^2} \left( \frac{2\dot{f}_1^2}{a_1^2} + \frac{\dot{f}_3^2}{a_3^2} \right) + \frac{\dot{h}^2}{2N^2 a_3^2} + \frac{1}{2N^2} \left( 1 + \frac{6\xi^2 v^2}{M_{\text{pl}}^2 \Omega^2} \right) \frac{\dot{v}^2}{\Omega^2} \right] \\ & - 2\pi^2 \int dt N a_1^2 a_3 V(f_1, f_3, h, v, a_1, a_3), \end{aligned} \quad (17)$$

where the potential function reads

$$\begin{aligned} V(f_1, f_3, h, v, a_1, a_3) = & \frac{\lambda}{4\Omega^4} (v^2 - v_0^2)^2 + \frac{v^2}{8\Omega^2} \left( \frac{2f_1^2}{a_1^2} + \frac{(f_3 - g'h)^2}{a_3^2} \right) \\ & + \frac{1}{2g^2} \left( \frac{2f_1^2(f_3+2)^2}{a_1^2 a_3^2} + \frac{(f_1^2 + 2f_3)^2}{a_1^4} \right) + \frac{2h^2}{a_1^4}. \end{aligned} \quad (18)$$

We can envisage the dynamics of gauge fields based on the potential (18). It consists of three parts: the first term coming from the Higgs potential is independent of the scale factors, the second term is proportional to the inverse of quadratic of scale factors, and the third and fourth terms are proportional to the inverse of the quartic of the scale factors. In the course of expansion of the universe, the terms depending on  $a_1, a_3$  become negligible

compared to the first term. However, in the early stage of the universe, since the second term contains the factor  $(f_3 - g'h)$ , there exists a flat direction of the potential. So, the gauge fields first roll down to the bottom of the flat direction where the topology of the gauge fields is nontrivial. This makes the difference as we will see in the following sections.

Now, we can derive basic equations. It is convenient to define new variables as

$$a_1 \equiv e^{\alpha+\beta}, \quad a_3 \equiv e^{\alpha-2\beta}, \quad (19)$$

where  $\alpha$  and  $\beta$  describe the average and the anisotropic expansion of the universe, respectively. In this paper, we use a length unit normalized by  $\text{GeV}^{-1}$ . Taking the variation with respect to the lapse function  $N$ , we obtain the Hamiltonian constraint

$$\begin{aligned}
 3(\dot{\alpha}^2 - \dot{\beta}^2) + e^{-2(\alpha+\beta)}(4 - e^{-6\beta}) &= \frac{1}{M_{\text{pl}}^2} \left[ \frac{1}{2g^2} (2\dot{f}_1^2 e^{-2(\alpha+\beta)} + \dot{f}_3^2 e^{-2(\alpha-2\beta)}) + \frac{\dot{h}^2}{2} e^{-2(\alpha-2\beta)} \right. \\
 &+ \frac{1}{2} \left( 1 + \frac{6\xi^2 v^2}{M_{\text{pl}}^2 \Omega^2} \right) \frac{\dot{v}^2}{\Omega^2} + \frac{\lambda}{4\Omega^4} (v^2 - v_0^2)^2 + \frac{v^2}{8\Omega^2} (2f_1^2 e^{-2(\alpha+\beta)} + (f_3 - gh)^2 e^{-2(\alpha-2\beta)}) \\
 &\left. + \frac{1}{2g^2} (2f_1^2 (f_3 + 2)^2 e^{-2(2\alpha-\beta)} + (f_1^2 + 2f_3)^2 e^{-4(\alpha+\beta)}) + 2h^2 e^{-4(\alpha+\beta)} \right]. \quad (20)
 \end{aligned}$$

Hereafter, we can set  $N = 1$ . Other components of Einstein equations can be obtained by taking the variations of the action with respect to  $\alpha$  and  $\beta$ . The equation for  $\alpha$  reads

$$\begin{aligned}
 6\ddot{\alpha} + 9(\dot{\alpha}^2 + \dot{\beta}^2) + (4 - e^{-6\beta})e^{-2(\alpha+\beta)} &= -\frac{1}{M_{\text{pl}}^2} \left[ \frac{e^{-2\alpha}}{2g^2} (2\dot{f}_1^2 e^{-2\beta} + \dot{f}_3^2 e^{4\beta}) + \frac{\dot{h}^2 e^{-2(\alpha-2\beta)}}{2} + \frac{3}{2} \left( 1 + \frac{6\xi^2 v^2}{M_{\text{pl}}^2 \Omega^2} \right) \frac{\dot{v}^2}{\Omega^2} \right. \\
 &- \frac{3\lambda}{4\Omega^4} (v^2 - v_0^2)^2 - \frac{v^2 e^{-2\alpha}}{8\Omega^2} (2f_1^2 e^{-2\beta} + (f_3 - gh)^2 e^{4\beta}) \\
 &\left. + \frac{e^{-4\alpha}}{2g^2} (2f_1^2 (f_3 + 2)^2 e^{2\beta} + (f_1^2 + 2f_3)^2 e^{-4\beta}) + 2h^2 e^{-4(\alpha+\beta)} \right]. \quad (21)
 \end{aligned}$$

Similarly, the equation for  $\beta$  is given by

$$\begin{aligned}
 3\ddot{\beta} + 9\dot{\alpha}\dot{\beta} + 4(1 - e^{-6\beta})e^{-2(\alpha+\beta)} &= -\frac{1}{M_{\text{pl}}^2} \left[ \frac{e^{-2\alpha}}{g^2} (\dot{f}_1^2 e^{-2\beta} - \dot{f}_3^2 e^{4\beta}) - \dot{h}^2 e^{-2(\alpha-2\beta)} - \frac{v^2 e^{-2\alpha}}{4\Omega^2} (f_1^2 e^{-2\beta} - (f_3 - gh)^2 e^{4\beta}) \right. \\
 &\left. + \frac{e^{-4\alpha}}{g^2} (f_1^2 (f_3 + 2)^2 e^{2\beta} - (f_1^2 + 2f_3)^2 e^{-4\beta}) - 4h^2 e^{-4(\alpha+\beta)} \right]. \quad (22)
 \end{aligned}$$

The equation for the Higgs field can be deduced as

$$\begin{aligned}
 \ddot{v} + 3\dot{\alpha}\dot{v} - \frac{\xi v \dot{v}^2}{M_{\text{pl}}^2 \Omega^2} &= -\frac{v}{1 + \frac{6\xi^2 v^2}{M_{\text{pl}}^2 \Omega^2}} \left[ \left( \frac{1}{4} (2f_1^2 e^{-2(\alpha+\beta)} + (f_3 - gh)^2 e^{-2(\alpha-2\beta)}) + \frac{6\xi^2 v^2}{M_{\text{pl}}^2 \Omega^2} \right) \left( 1 - \frac{\xi v^2}{M_{\text{pl}}^2 \Omega^2} \right) \right. \\
 &\left. + \frac{\lambda(v^2 - v_0^2)}{\Omega^2} \left( 1 - \frac{\xi}{M_{\text{pl}}^2 \Omega^2} (v^2 - v_0^2) \right) \right]. \quad (23)
 \end{aligned}$$

Moreover, we need the equations for  $SU(2)$  gauge fields

$$\dot{f}_1 + (\dot{\alpha} - 2\dot{\beta})f_1 = -f_1 \left( e^{-2(\alpha-2\beta)}(2 + f_3)^2 + e^{-2(\alpha+\beta)}(f_1^2 + 2f_3) + \frac{g^2 v^2}{4\Omega^2} \right) \quad (24)$$

and

$$\dot{f}_3 + (\dot{\alpha} + 4\dot{\beta})f_3 = -2e^{-2(\alpha+4\beta)}(f_1^2 + 2f_3) - 2e^{-2(\alpha+\beta)}f_1^2(f_3 + 2) - \frac{g^2 v^2}{4\Omega^2}(f_3 - gh). \quad (25)$$

Finally, the equation for the  $U(1)_Y$  gauge field reads

$$\dot{h} + (\dot{\alpha} + 4\dot{\beta})h = -4e^{-2(\alpha+4\beta)}h + \frac{g'}{4\Omega^2}v^2(f_3 - gh). \quad (26)$$

Now that we have obtained basic equations, we can move on to the analysis of cosmological dynamics of gauge fields.

### III. COSMOLOGICAL DYNAMICS OF GAUGE FIELDS

In this section, we focus on the dynamics of the gauge fields. We will see gauge fields remain nontrivial during inflation although the energy density of them rapidly decays during inflation in agreement with the cosmic no-hair conjecture. However, after inflation, the gauge fields show the parametric resonances due to oscillations of the Higgs field. Furthermore, we explicitly evaluate the Chern-Simon number to characterize the topology of the gauge fields.

In the subsequent calculations, we will use the following numerical values<sup>1</sup> adopted from experiments

$$g = 0.653, \quad g' = 0.358, \quad v_0 = 246 \text{ [GeV]},$$

$$m_H = 126 \text{ [GeV]}, \quad M_{\text{pl}} = 2 \times 10^{18} \text{ [GeV]}. \quad (27)$$

We also set initial values of time derivative of the following variables to be zero for convenience:

$$\dot{f}_{1i} = \dot{f}_{3i} = \dot{h}_i = \dot{v}_i = \dot{\beta}_i = 0, \quad (28)$$

where the index  $i$  represents the value at the initial time  $t = 0$ . According to [1], we need to take  $\xi \sim \sqrt{\lambda/3} N_e / (0.027)^2$  for inflation to be realized, where  $N_e$  is the total  $e$ -folding number. Taking into account the relation  $\lambda = m_H^2 / 2v_0^2$ , we have chosen<sup>2</sup>  $\xi = 2 \times 10^4$ . We also fixed the initial value of the Higgs field as  $v_i = 10^{17}$  [GeV]. As to the metric variables  $\alpha$  and  $\beta$ , we left them arbitrary parameters as long as the positivity of the Hubble squared is guaranteed in Eq. (20).

In Fig. 1, we plotted the time evolution of the gauge fields during inflation. For this calculation, we started from the pure gauge configuration and did not consider the initial anisotropy:

$$f_{1i} = -2, \quad f_{3i} = -2, \quad h_i = 0, \quad \beta_i = 0. \quad (29)$$

We chose the initial value  $\alpha_i = -20$  and determined  $\dot{\alpha}_i$  from the constraint (20). We can see that the gauge fields oscillate initially and converge into some values which do not correspond to the pure gauge ones. The similar behavior is also observed in Ref. [5].

In Fig. 2, we plotted  $\beta$  for several initial values  $\beta_i$ . It turns out the anisotropy remains during inflation although the expansion rate is isotropic. This anisotropy has an impact on the evolution of gauge field configurations after inflation because Eqs. (24), (25), and (26) explicitly depend on the

<sup>1</sup>For more detailed discussion of parameters, see for example [25,26]

<sup>2</sup>This choice is based on [1], which is in between the minimal coupling  $\xi = 0$  and that in ‘‘induced gravity’’ model  $\sqrt{\xi} \sim 10^{17}$  [27–29].

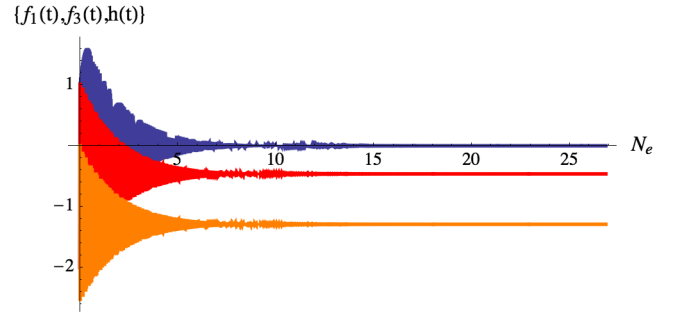


FIG. 1 (color online). The time evolution of  $f_1(t)$  (top),  $f_3(t)$  (middle), and  $h(t)$  (down) during inflation with the initial conditions  $f_{1i} = -2, f_{3i} = -2, h_i = 0, \alpha_i = -20, \beta_i = 0$ . They oscillate near the bottom of the effective potential with damping.

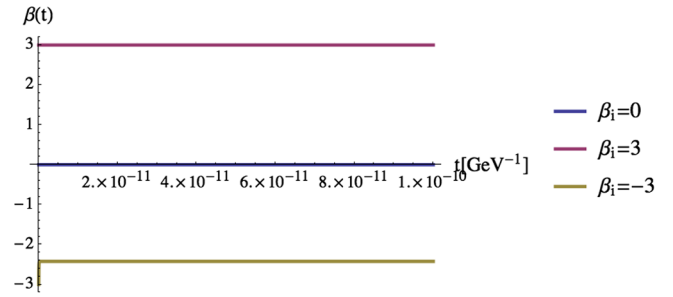


FIG. 2 (color online). The time evolution of  $\beta(t)$  for several  $\beta_i$  values with the initial conditions  $f_{1i} = -2, f_{3i} = -2, h_i = 0, \alpha_i = -20$ .

anisotropy  $\beta$ . Because of this anisotropy as well as the nontrivial gauge fields, Higgs inflation is anisotropic.

We plotted the time evolutions of the Higgs field in phase space in Fig. 3. In this case, we have about a 40  $e$ -folding number. We can see that the Higgs field shows damping oscillation after inflation. During this stage, the gauge field also shows chaotic oscillation. This kind of chaotic

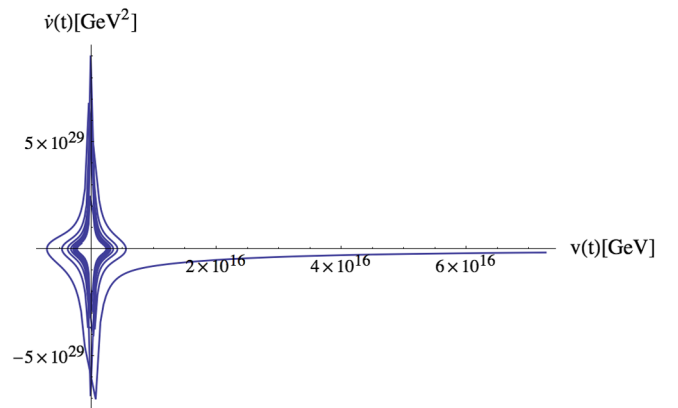


FIG. 3 (color online). The time evolution of the Higgs field in phase space with the initial conditions  $f_{1i} = -2, f_{3i} = -2, h_i = 0, \alpha_i = -20, \beta_i = 0$ .



behavior is also found in other models [30] where a gauge kinetic function keeps gauge fields nonzero. While, in this case, the gauge fields remain nonzero due to the flat direction.

In Figs. 4 and 5, we plotted the absolute value of gauge fields with different initial conditions. There, we can see rapid growth of the amplitude of the gauge fields for both  $f_1$  and  $f_3 - g'h$ . It is easy to check that the energy density of gauge fields also rapidly grows. Remarkably, the gauge fields converge to a value due to the nonlinear effect. From Fig. 5, we see even if we started with  $f_3 = h = 0$ , the final value is similar to other cases.

We can understand the behavior in Figs. 4 and 5 as follows. In fact, after inflation, we can approximate the equations for the gauge fields (24)–(26) as follows

$$\ddot{f}_1 + \frac{g^2}{4} v^2 f_1 \simeq 0 \quad (30)$$

and

$$(\ddot{f}_3 - g'\ddot{h}) + \frac{g^2 + g'^2}{4} v^2 (f_3 - g'h) \simeq 0. \quad (31)$$

In this era, the Higgs field can be also approximated as

$$v^2 \equiv V(t)^2 \cos^2 \omega t = \frac{V(t)^2}{2} (1 + \cos 2\omega t), \quad V(t) \equiv V_0 e^{-At}, \quad (32)$$

where the parameters  $\omega, V_0, A > 0$  are determined from numerical calculations. We have numerically verified the condition  $A\delta t \ll 1$  is satisfied. From Eqs. (30) and (31), we see a parametric resonance appears in a certain band around the center value where the following relationship between the frequency and amplitude holds;

$$2\omega = 2 \frac{V(t)}{\sqrt{2n}}. \quad (33)$$

Here,  $n$  is an integer and hence there are many resonance bands. It is known that, for smaller  $n$ , the resonance band becomes broad and the growth rate becomes large. It is useful to rewrite the above relation as

$$n = \frac{V_0}{\sqrt{2}\omega} e^{-At}. \quad (34)$$

Since the  $V(t)$  is decaying, this condition will be satisfied at a moment which is a center of a band and a resonance will occur for some period around the center of the resonance. As time goes by, the resonance becomes rare but the growth

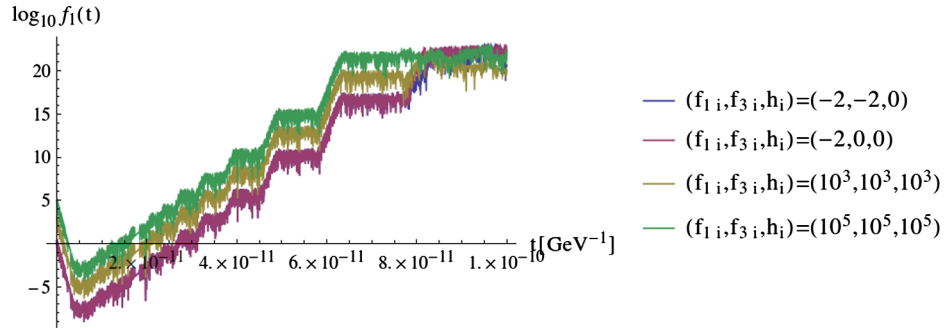


FIG. 4 (color online). The time evolution of  $f_1(t)$  after inflation for several initial sets of gauge values with the initial conditions  $\alpha_i = -20, \beta_i = 0$ . The final amplitude seems to converge to a value.

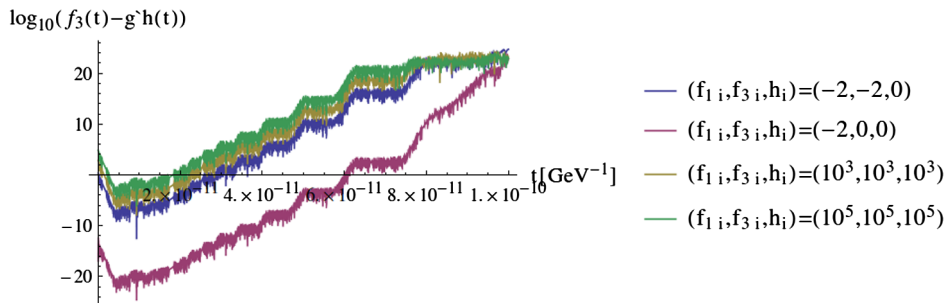


FIG. 5 (color online). The time evolution of  $f_3(t) - g'h(t)$  after inflation for several initial sets of gauge values with the initial conditions  $\alpha_i = -20, \beta_i = 0$ .

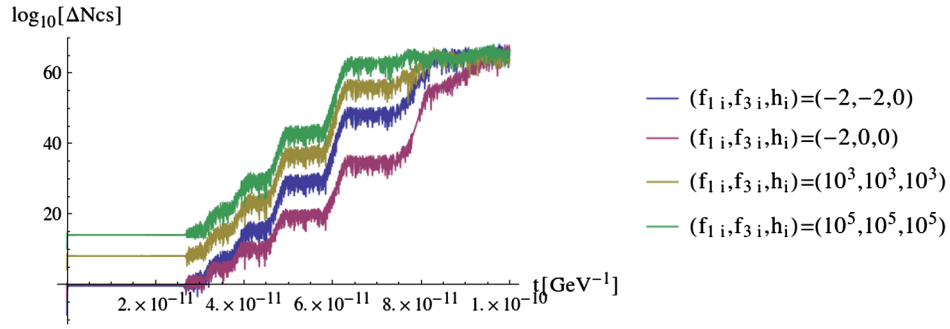


FIG. 6 (color online). The time evolution of  $\Delta N_{CS}$  after inflation for several initial sets of gauge values with the initial conditions  $\alpha_i = -20, \beta_i = 0$ .

rate of the amplitude comes to increase when a resonance happens. This is because the factor  $e^{-At}$  of  $V(t)$  in the condition (34) changes slowly and  $n$  gets small as  $t$  increases. You can see these features in Figs. 4 and 5.

Now, we consider the topological configuration of the gauge fields. As you can see from their dynamics, the gauge field configurations are quite complicated due to their interaction with the Higgs field. Here, let us characterize the gauge field configurations by the change of the Chern-Simons number:

$$\begin{aligned} \Delta N_{CS} &\equiv \frac{g^2}{16\pi^2} \int_0^t dt \sqrt{-g} \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) \\ &= -\frac{1}{8} \int_0^t \frac{d}{dt} [f_1^2 f_3 + 2f_1^2 + f_3^2] dt. \end{aligned} \quad (35)$$

We plotted its value as a function of time in Fig. 6. Apparently, the Chern-Simons number is increasing in time. Interestingly, there is a plateau for some period. Since the presence of the Chern-Simons number indicates  $CP$  violation, this result may be relevant to the origin of the baryon asymmetry.

The most important implication of our findings is that the gauge fields are nontrivial even at the background level. This would have an impact on simulations of preheating [31–34] where trivial gauge fields are assumed as initial conditions.

#### IV. GENERATION OF MAGNETIC FIELDS

In the previous section, we find the growth of gauge fields due to parametric resonances. In this section, we evaluate the magnitude of primordial magnetic fields produced in this way. Since the magnetic field is rapidly oscillating, we do not say this is directly transferred to observed magnetic fields. However, we would like to point out that it is possible to generate sizable magnetic fields with helical structure in the presence of this background because of the nontrivial Chern-Simons number.

Because of the nontrivial expectation value of the Higgs field, the  $SU(2)_L \times U(1)_Y$  gauge symmetry breaks down to

the  $U(1)_{em}$  gauge symmetry. Its gauge potential  $A_{em}$  is given by the linear combination of  $A_\mu^3$  and  $B_\mu$  with the Weinberg-angle  $\theta_W = \tan^{-1}(g'/g)$ ,

$$A_{em} = A^3 \sin \theta_W + B \cos \theta_W \quad (36)$$

$$= \frac{1}{\sqrt{g'^2 + g^2}} \left( g' \frac{f_3}{g} + gh \right) \sigma^3 \equiv h_{em} \sigma^3. \quad (37)$$

Then, its field strength can be calculated as

$$F_{em} = dA_{em} = \frac{\dot{h}_{em}}{a_3 N} e^0 \wedge e^3 + \frac{2h_{em}}{a_1^2} e^1 \wedge e^2. \quad (38)$$

So, the magnetic field points in the  $e^3$  direction:

$$B_3 = \frac{2h_{em}}{a_1^2}. \quad (39)$$

We depicted its evolution in time in Fig. 7. Similar to the results in the previous section, the strength of magnetic fields also increase drastically after inflation. Remarkably, the final amplitude of magnetic fields depends on the initial anisotropy as seen in Fig. 8. This is because the anisotropy remaining during inflation affects the dynamics of the gauge fields. We also notice there is a plateau where the growth due to parametric resonances balances with the decay due to the expansion of the universe. At some point, however, the amplitude of magnetic fields starts to grow. It is intriguing to give an analytic understanding of this feature.

It would be interesting to see if the order of magnitude of magnetic fields generated by parametric resonances is comparable with observed magnetic fields. We expect the coherent oscillation of magnetic fields found in our study will be relevant when we consider inhomogeneous fluctuations of magnetic fields and lead to the observable magnetic fields with similar amplitudes.

According to recent observations, the present magnetic field at cluster scales is expected in the range

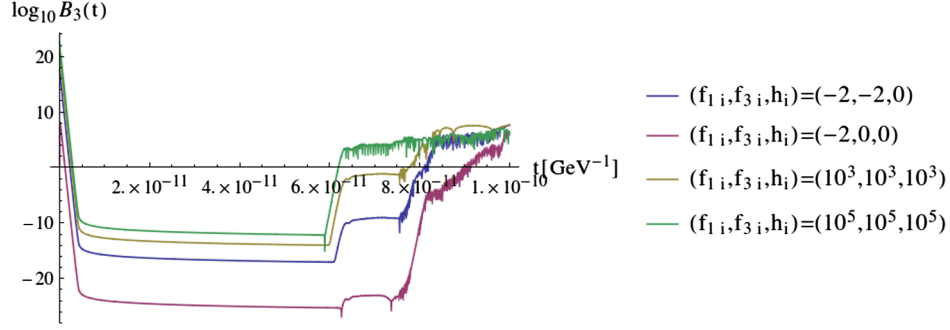


FIG. 7 (color online). The time evolution of  $B_3(t)$  after inflation for several initial sets of gauge values with the initial conditions  $\alpha_i = -20, \beta_i = 0$ .

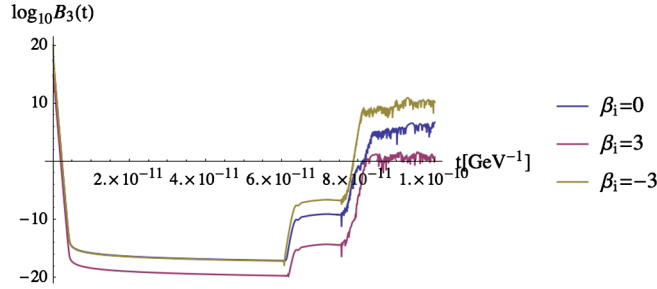


FIG. 8 (color online). The time evolution of  $B_3(t)$  after inflation for several  $\beta_i$  values with the initial conditions  $f_{1i} = -2, f_{3i} = -2, h_i = 0, \alpha_i = -20$ . Apparently, the amplitude of magnetic fields depends on the anisotropy survived during inflation.

$$10^{-15} [\text{G}] \lesssim B_{\text{obs}} \lesssim 10^{-9} [\text{G}]. \quad (40)$$

Now, we would like to translate this range into that after the reheating with a reheating temperature  $T_{\text{reh}}$ . Notice the relation  $B_{\text{reh}}/B_{\text{obs}} = (a_{\text{obs}}^2/a_{\text{reh}})^2 = (T_{\text{reh}}/T_{\text{obs}})^2$  and the present temperature  $T_{\text{obs}} \sim 10^{-4} [\text{eV}] = 10^{-13} [\text{GeV}]$  determined by observations of cosmic microwave background. Then, using the relation  $1 [\text{G}] \approx 10^{-20} [\text{GeV}^2]$ , we obtain the range

$$\begin{aligned} 10^{11} \left( \frac{T_{\text{reh}}}{10^{10} [\text{GeV}]} \right)^2 [\text{GeV}^2] &\lesssim B_{\text{reh}} \\ &\lesssim 10^{17} \left( \frac{T_{\text{reh}}}{10^{10} [\text{GeV}]} \right)^2 [\text{GeV}^2]. \end{aligned} \quad (41)$$

In the above, we used the following relationship (see for example [35])

$$N_{\text{COBE}} = 62 - \ln(10^{16} [\text{GeV}]/V_{\text{end}}^{1/4}) - \frac{1}{3} \ln(V_{\text{end}}^{1/4}/\rho_{\text{reh}}^{1/4}), \quad (42)$$

where  $V_{\text{end}}$  is the potential energy at the end of inflation,  $\rho_{\text{reh}}$  is the energy density at the reheating time, and  $N_{\text{COBE}}$

is the e-folding number corresponding to the COBE observation. Taking a look at Figs. 7 and 8, we see magnetic fields in this range can be generated depending on initial conditions of  $\beta$  or the reheating temperature  $T_{\text{reh}}$ .

## V. CONCLUSION

We studied the cosmological dynamics of the gauge fields in electroweak theory in the early universe. Since we have considered Higgs inflation, there exists nontrivial interaction between the inflaton and the gauge fields, which makes the difference from the previous work [5]. In spite of this difference, we found that there remains nontrivial configurations of the gauge fields during inflation because of the presence of the flat direction. The main finding in this paper is the growth of the gauge fields due to parametric resonances induced by oscillations of the Higgs field. To characterize the topological configuration in the gauge fields, we calculated the Chern-Simons number and found the similar growth. This implies that  $CP$  violating configuration of gauge fields exists, which would be relevant to the baryogenesis. We have also estimated the magnitude of magnetic fields and found that the order of magnitude lies in the range of observed magnetic fields at cluster scales. Remarkably, the resultant amplitude of magnetic fields depends on the anisotropy survived during inflation. This is an interesting manifestation of the anisotropy in Higgs inflation.

In this work, we have not considered reheating process in detail. To obtain precise results on magnetic fields, we need to introduce matter fields and investigate reheating processes. Moreover, we have to study evolution of inhomogeneity during and after Higgs inflation. We leave these issues for future work.

## ACKNOWLEDGMENTS

I would like to thank Y. Fujimoto, Y. Sakakihara, and M. Watanabe for useful discussions. This work was supported in part by the Grants-in-Aid for Scientific Research (C) No. 25400251 and Grants-in-Aid for Scientific Research on Innovative Areas No. 26104708.



- [1] F. L. Bezrukov and M. Shaposhnikov, *Phys. Lett. B* **659**, 703 (2008); F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov, *J. High Energy Phys.* 01 (2011) 016.
- [2] P. A. R. Ade *et al.* (Planck Collaboration), [arXiv:1303.5082](https://arxiv.org/abs/1303.5082).
- [3] P. A. R. Ade *et al.* (BICEP2 Collaboration), *Phys. Rev. Lett.* **112**, 241101 (2014).
- [4] A. Maleknejad, M. M. Sheikh-Jabbari, and J. Soda, *Phys. Rep.* **528**, 161 (2013).
- [5] H. Emoto, Y. Hosotani, and T. Kubota, *Prog. Theor. Phys.* **108**, 157 (2002); Y. Hosotani, H. Emoto, and T. Kubota, [arXiv:hep-ph/0209112](https://arxiv.org/abs/hep-ph/0209112).
- [6] D. Grasso and H. R. Rubinstein, *Phys. Rep.* **348**, 163 (2001).
- [7] M. Giovannini, *Int. J. Mod. Phys. D* **13**, 391 (2004).
- [8] A. Bonafede, L. Feretti, M. Murgia, F. Govoni, G. Giovannini, D. Dallacasa, K. Dolag, and G. B. Taylor, [arXiv:1002.0594](https://arxiv.org/abs/1002.0594) [*Astron. Astrophys.* (to be published)].
- [9] F. Tavecchio, G. Ghisellini, L. Foschini, G. Bonnoli, G. Ghirlanda, and P. Coppi, *Mon. Not. R. Astron. Soc.* **406**, L70 (2010).
- [10] A. Neronov and I. Vovk, *Science* **328**, 73 (2010).
- [11] W. Essey, S. 'i. Ando, and A. Kusenko, *Astropart. Phys.* **35**, 135 (2011).
- [12] M. S. Turner and L. M. Widrow, *Phys. Rev. D* **37**, 2743 (1988).
- [13] B. Ratra, *Astrophys. J.* **391**, L1 (1992).
- [14] A. Dolgov, *Phys. Rev. D* **48**, 2499 (1993).
- [15] M. Giovannini and M. E. Shaposhnikov, *Phys. Rev. D* **62**, 103512 (2000).
- [16] K. Bamba and J. Yokoyama, *Phys. Rev. D* **69**, 043507 (2004).
- [17] K. Bamba and M. Sasaki, *J. Cosmol. Astropart. Phys.* 02 (2007) 030.
- [18] J. Martin and J. 'i. Yokoyama, *J. Cosmol. Astropart. Phys.* 01 (2008) 025.
- [19] V. Demozzi, V. Mukhanov, and H. Rubinstein, *J. Cosmol. Astropart. Phys.* 08 (2009) 025.
- [20] S. Kanno, J. Soda, and M.-a. Watanabe, *J. Cosmol. Astropart. Phys.* 12 (2009) 009.
- [21] R. J. Z. Ferreira, R. K. Jain, and M. S. Sloth, *J. Cosmol. Astropart. Phys.* 10 (2013) 004.
- [22] J. R. Shaw and A. Lewis, *Phys. Rev. D* **86**, 043510 (2012).
- [23] T. Fujita and S. Yokoyama, *J. Cosmol. Astropart. Phys.* 03 (2014) 013.
- [24] M.-a. Watanabe, S. Kanno, and J. Soda, *Phys. Rev. Lett.* **102**, 191302 (2009).
- [25] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, *J. High Energy Phys.* 12 (2013) 089.
- [26] A. Salvio, *Phys. Lett. B* **727**, 234 (2013).
- [27] A. Zee, *Phys. Rev. Lett.* **42**, 417 (1979).
- [28] J. L. Cervantes-Cota and H. Dehnen, *Phys. Rev. D* **51**, 395 (1995).
- [29] J. L. Cervantes-Cota and H. Dehnen, *Nucl. Phys.* **B442**, 391 (1995).
- [30] K. Murata and J. Soda, *J. Cosmol. Astropart. Phys.* 06 (2011) 037.
- [31] J. Garcia-Bellido, M. Garcia-Perez, and A. Gonzalez-Arroyo, *Phys. Rev. D* **69**, 023504 (2004).
- [32] A. Diaz-Gil, J. Garcia-Bellido, M. Garcia Perez, and A. Gonzalez-Arroyo, *Proc. Sci.*, LAT2005 (2006) 242 [[arXiv: hep-lat/0509094](https://arxiv.org/abs/hep-lat/0509094)].
- [33] J. Garcia-Bellido, D. G. Figueroa, and J. Rubio, *Phys. Rev. D* **79**, 063531 (2009).
- [34] J. T. Deskins, J. T. Giblin, and R. R. Caldwell, *Phys. Rev. D* **88**, 063530 (2013).
- [35] D. H. Lyth and A. Riotto, *Phys. Rep.* **314**, 1 (1999).