Thermodynamic interpretation of the generalized gravity models with geometry-matter coupling

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Modified gravity theories with geometry-matter coupling, in which the action is an arbitrary function of the Ricci scalar and the matter Lagrangian [$f(R, L_m)$ gravity], and of the Ricci scalar and of the trace of the matter energy-momentum tensor [$f(R, T)$ gravity], respectively, have the intriguing property that the divergence of the matter energy-momentum tensor is nonzero. In the present paper, by using the formalism of open thermodynamic systems, we interpret the generalized conservation equations in these gravitational theories from a thermodynamic point of view as describing irreversible matter creation processes, *which could be validated by fundamental particle physics*. Thus particle creation corresponds to an irreversible energy flow from the gravitational field to the created matter constituents, with the second law of thermodynamics requiring that space-time transforms into matter. The equivalent particle number creation rates, the creation pressure and the entropy production rates are obtained for both $f(R, L_m)$ and $f(R, T)$ gravity theories. The temperature evolution laws of the newly created particles are also obtained. In the case of the $f(R, T)$ gravity theory the open irreversible thermodynamic interpretation of a simple cosmological model is presented in detail. It is also shown that due to the geometry-matter coupling, during the cosmological evolution a large amount of comoving entropy could be produced.

I. INTRODUCTION

The recently released Planck satellite data of the 2.7 deg cosmic microwave background (CMB) full sky survey [1], as well as the measurement of the tensor modes from large angle CMB *B*-mode polarization by BICEP2 [2] have generally confirmed the standard Λ cold dark matter cosmological model. The BICEP2 data, pointing toward a tensor-to-scalar ratio $r = 0.2^{+0.07}_{-0.05}$, have provided very convincing evidence for the standard inflationary scenario, which predicts the generation of gravitational waves during the de Sitter exponential expansion. However, there are some tensions between the BICEP2 results and the Planck limits on standard inflationary models [3]. On the other hand, observations of the type Ia supernovae have convincingly shown that our universe presently experiences an accelerating evolution, for which the most natural explanation would be the presence of the dark energy, having an equation of state parameter $w = -1.018 \pm 0.057$ for a flat universe [4].

From a theoretical point of view, the necessity of explaining dark energy, as well as the second dominant component of the Universe, dark matter [5], raises the fundamental question if the standard Einstein-Hilbert action $S = \int (R/2 + L_m)\sqrt{-g}d^4x$, where R is the scalar curvature and L_m is the matter Lagrangian density, and in which matter is minimally coupled to the geometry, can give an appropriate quantitative description of the Universe

on all scales, going from the boundary of the Solar System to the edge of the Universe.

A theory with an explicit coupling between an arbitrary function of the scalar curvature and the Lagrangian density of matter was proposed in [6]. The gravitational action of this gravity theory is of the form $S = \int \{f_1(R) + [1 + \lambda f_2(R)]L_m\}\sqrt{-g}d^4x$. In these models an extra force acting on massive test particles arises, and the motion is no longer geodesic. Moreover, in this framework, one can also explain dark matter [6]. The initial "linear" geometry-matter coupling introduced in [6] was extended in [7], and a maximal extension of the Einstein-Hilbert action with geometry-matter coupling, of the form $S = \int d^4x\sqrt{-g}f(R, L_m)$ was considered in [8]. The cosmological and astrophysical implications of the $f(R, L_m)$ type gravity theories were investigated in [9].

An alternative model to $f(R, L_m)$ gravity is the $f(R, T)$ gravity theory [10], where T is the trace of the matter energy-momentum tensor $T_{\mu\nu}$. The corresponding action is given by $S = \int [f(R, T)/2 + L_m]\sqrt{-g}d^4x$. The dependence of the gravitational action on T may be due to the presence of quantum effects (conformal anomaly), or of some exotic imperfect fluids. When the trace of the energy-momentum tensor T is zero, $T = 0$, which is the case of the electromagnetic radiation, the field equations of the $f(R, T)$ theory reduce to those of the $f(R)$ gravity. The cosmological and astrophysical implications of the $f(R, T)$ gravity were considered in [11], and it was shown that such gravity theories explain the accelerating evolution

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of the Universe. For a recent review of modified gravity theories with geometry-matter coupling see [12].

Both $f(R, L_m)$ and $f(R, T)$ gravity have the intriguing property that once geometry-matter coupling is introduced, the four-divergence of the energy-momentum tensor is nonzero. It is the purpose of the present paper to give a thermodynamic interpretation of the gravity theories with geometry-matter coupling in the framework of the irreversible thermodynamic of open systems. More exactly, we consider that the nonconservativity of the matter energy-momentum tensor is related to irreversible matter creation processes, in which, due to the coupling between matter and geometry, there is an energy flow between the gravitational field and matter, with particles permanently added to the space-time.

The systematic investigation of the role of irreversible matter creation processes in general relativity and cosmology did start with the papers [13] and [14]. The description of particle creation is based on the reinterpretation of the matter-energy stress tensor in Einstein's equations, by modifying the usual adiabatic energy conservation laws, and thereby including irreversible matter creation. Thus matter creation corresponds to an irreversible energy flow from the gravitational field to the created matter constituents. This point of view results from consideration of the thermodynamics of open systems in the framework of cosmology. In a gravitational system the second law of thermodynamics requires that space-time transforms into matter, while the inverse transformation is forbidden. As shown in [14], the usual initial singularity associated with the big bang is structurally unstable with respect to irreversible matter creation. The inclusion of dissipative processes into the Einstein field equations lead to the possibility of cosmological models that start from empty conditions and gradually build up matter and entropy. Gravitational entropy takes a simple meaning as associated with the entropy that is necessary to produce matter. This leads to an extension of the third law of thermodynamics, as now the zero point of entropy becomes the space-time structure out of which matter is generated [13].

The phenomenological approach for matter creation introduced in [13,14] was reformulated in a manifestly covariant way in [15]. The expressions for the entropy production rate and the temperature evolution equation were also obtained, and some of their consequences were discussed. The effects and implications of the irreversible matter creation processes on the cosmological evolution were extensively analyzed in [16–36]. In particular, a theoretical model, called creation of cold dark matter was analyzed in [26]. It can be described macroscopically by introducing a negative pressure due to matter creation. Therefore, the mechanism is capable to accelerate the Universe, without the need of an additional dark energy component. An alternative cosmological model, which assumes the existence of gravitational particle creation

was studied in [27]. The model fits well the supernova observations. In this scenario one can alleviate the cosmic coincidence problem, with dark matter and dark energy being of the same nature, but acting at different scales.

Cosmological particle creation can take place from the quantum vacuum, due to external conditions, which are caused by the expansion or contraction of the Universe. This process was first discussed by Schrödinger [37], who understood that the cosmic evolution could lead to a mixing of positive and negative frequencies and that this would mean creation or destruction of matter due to the expansion of the Universe. Later on this quantum phenomenon was studied via the modern techniques of quantum field theory in curved space-times in [38–43]. In quantum field theory particle production is directly connected with the curvature of the Universe and, when the field equations are formulated in the form of harmonic oscillator equation without friction part, the effect of gravity appears in the effective mass. The quantum rate of matter creation is maximum near the initial cosmological singularity, and postinflationary reheating may be explained from gravitational particle creation.

In this work, by using the formalism of the open irreversible thermodynamic systems, we reformulate the nonzero divergence of the energy-momentum tensor of the ideal fluid in the $f(R, L_m)$ and $f(R, T)$ gravity theories as an energy balance equation in the presence of matter creation. The particle number balance equation is also obtained, as well as the particle creation rate. The particle creation rate depends on both the thermodynamic parameters of the system, as well as of the functional form of the derivatives with respect to L_m and T of the function f giving the geometry-matter coupling. When $f \equiv 0$, the system is conservative. The supplementary particle creation pressure, induced by matter creation, is also determined, and it is shown that it is proportional to the particle creation rate. With the help of the creation pressure the divergence of the energy-momentum tensor can be rewritten in the form of an effective conservation equation, with the total pressure equal to the sum of the thermodynamic and creation pressures. The explicit form of the entropy, and of the entropy creation rates, associated with the particle transfer from geometry to matter, is also determined. The time evolution of the entropy is determined by the particle creation rates. The time evolution of the temperature of general thermodynamic systems with energy and pressure depending on both particle numbers and temperature is also obtained. In the case of $f(R, L_m)$ gravity the behavior of the thermodynamic parameters describing matter creation are explicitly obtained for $L_m = -p$. The thermodynamic interpretation of a cosmological model corresponding to simple choice of the function $f(R, T)$ is also discussed in detail. Even though the approach proposed in the present paper is phenomenological, quantum particle creation processes [37–43] can give a microscopic justification to

the proposed interpretation of the modified gravity models with geometry-matter coupling.

The present paper is organized as follows. In Sec. II the field and conservation equations of the $f(R, L_m)$ and $f(R, T)$ gravity theories are briefly reviewed. The interpretation of the $f(R, L_m)$ gravity theory in the framework of the thermodynamic processes in open irreversible systems with matter creation is developed in Sec. III. The irreversible thermodynamic interpretation of the $f(R, T)$ gravity theory is presented in Sec. IV. We discuss and conclude our results in Sec. V. In the present paper we use the natural system of units with $c = 8\pi G = 1$, and the Landau-Lifshitz [44] conventions for the geometric and physical quantities.

II. GENERALIZED GRAVITY MODELS WITH GEOMETRY-MATTER COUPLING

In the present section we briefly review two generalized gravity theories, in which the four-divergence of the matter energy-momentum tensor is nonzero. We consider the $f(R, L_m)$ and the $f(R, T)$ theories, and we present the field and the conservation equations for each theory.

A. $f(R, L_m)$ gravity

The most general action A for a $f(R, L_m)$ type modified theory of gravity involving an arbitrary coupling between matter and curvature is given by [8]

$$A = \int f(R, L_m) \sqrt{-g} d^4 x, \tag{1}$$

where $f(R, L_m)$ is an arbitrary function of the Ricci scalar R, and of the Lagrangian density corresponding to matter, L_m. The only requirement for the function $f(R, L_m)$ is to be an analytical function of R and L_m, respectively; that is, it must possess a Taylor series expansion about any point. The matter energy-momentum tensor $T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}. \tag{2}$$

By assuming that the Lagrangian density L_m of the matter depends only on the metric tensor components, and not on its derivatives, we obtain $T_{\mu\nu} = L_m g_{\mu\nu} - 2\partial L_m / \partial g^{\mu\nu}$.

Varying the action with respect to the metric tensor $g_{\mu\nu}$ we obtain the field equations of the model as

$$f_R(R, L_m) R_{\mu\nu} + \hat{P}_{\mu\nu} f_R(R, L_m) - \frac{1}{2} [f(R, L_m)$$
$$- f_{L_m}(R, L_m) L_m] g_{\mu\nu} = \frac{1}{2} f_{L_m}(R, L_m) T_{\mu\nu}, \tag{3}$$

where we have denoted $f_R(R, L_m) = \partial f(R, L_m)/\partial R$ and $f_{L_m}(R, L_m) = \partial f(R, L_m)/\partial L_m$, respectively, and we have introduced the operator $\hat{P}_{\mu\nu}$, defined as

$$\hat{P}_{\mu\nu} = g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu, \tag{4}$$

with $\Box = \nabla_\mu \nabla^\mu$. The operator $\hat{P}_{\mu\nu}$ has the property $\hat{P}^\mu_\mu = 3\Box$.

By contracting the field equations Eq. (3), we obtain the scalar equation

$$3\Box f_R + f_R R - 2f = \left(\frac{1}{2} T - 2L_m\right) f_{L_m}, \tag{5}$$

where $T = T^\mu_\mu$ is the trace of the matter energy-momentum tensor. By eliminating the term $\Box f_R(R, L_m)$ between Eqs. (3) and (18), we can reformulate the field equations as

$$R_{\mu\nu} = \Lambda(R, L_m) g_{\mu\nu} + \frac{1}{f_R(R, L_m)} \nabla_\mu \nabla_\nu f_R(R, L_m)$$
$$+ \Phi(R, L_m) \left(T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu}\right), \tag{6}$$

where we have denoted

$$\Lambda(R, L_m) = \frac{2 f_R(R, L_m) R - f(R, L_m) + f_{L_m}(R, L_m) L_m}{6 f_R(R, L_m)} \tag{7}$$

and

$$\Phi(R, L_m) = \frac{f_{L_m}(R, L_m)}{f_R(R, L_m)}, \tag{8}$$

respectively.

By taking the covariant divergence of Eq. (3), with the use of the mathematical identity [45]

$$[\Box, \nabla_\nu] F = (\Box \nabla_\nu - \nabla_\nu \Box) F = R_{\mu\nu} \nabla^\mu F, \tag{9}$$

where F is any arbitrary function of the space-time coordinates x^μ, we obtain for the divergence of the energy-momentum tensor $T_{\mu\nu}$ the following relationship:

$$\nabla^\mu T_{\mu\nu} = \nabla^\mu \ln[f_{L_m}(R, L_m)](L_m g_{\mu\nu} - T_{\mu\nu})$$
$$= 2\nabla^\mu \ln[f_{L_m}(R, L_m)] \frac{\partial L_m}{\partial g^{\mu\nu}}. \tag{10}$$

Generally the matter Lagrangian L_m is a function of the matter energy density ρ, the pressure p as well as the other thermodynamic quantities, such as the specific entropy s or the baryon number n, so that $L_m = L_m(\rho, p, s, n)$. In the simple (but physically the most relevant) case in which the matter obeys a barotropic equation of state, so that the pressure is a function of the energy density of the matter only, $p = p(\rho)$, the matter Lagrangian becomes a function of the energy density only, and hence $L_m = L_m(\rho)$. Then, the matter Lagrangian is given by [46,47]

$$L_m(\rho) = \rho\left(1 + \int_0^p \frac{dp}{\rho}\right) - p(\rho), \qquad (11)$$

while the energy-momentum tensor can be written as

$$T^{\mu\nu} = [\rho + p(\rho) + \rho\tilde{\Pi}(\rho)]U^\mu U^\nu - p(\rho)g^{\mu\nu}, \qquad (12)$$

respectively, where

$$\tilde{\Pi}(\rho) = \int_0^\rho \frac{p}{\rho^2}d\rho = \int_0^p \frac{dp}{\rho} - \frac{p(\rho)}{\rho}. \qquad (13)$$

The expression $\tilde{\Pi}(\rho) + p(\rho)/\rho$ represents the specific enthalpy of the fluid. From a physical point of view $\tilde{\Pi}(\rho)$ can be interpreted as the elastic (deformation) potential energy of the body, and therefore Eq. (12) corresponds to the energy-momentum tensor of a compressible elastic isotropic system.

B. $f(R,T)$ gravity

As a second example of a modified theory of gravity with nonconserved energy-momentum tensor we consider the $f(R,T)$ gravity, with action having the following form [10]:

$$A = \frac{1}{16\pi} \int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \qquad (14)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar, R, and of the trace T of the stress-energy tensor of the matter, $T_{\mu\nu}$. L_m is the matter Lagrangian density. We define the variation of T with respect to the metric tensor as

$$\frac{\delta(g^{\alpha\beta}T_{\alpha\beta})}{\delta g^{\mu\nu}} = T_{\mu\nu} + \Theta_{\mu\nu}, \qquad (15)$$

where

$$\Theta_{\mu\nu} \equiv g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}. \qquad (16)$$

By varying the gravitational action Eq. (14), we obtain the field equations of the $f(R,T)$ gravity model as [10]

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \hat{P}_{\mu\nu}f_R(R,T)$$
$$= 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}, \qquad (17)$$

where $f_R(R,T) = \partial f(R,T)/\partial R$ and $f_T(R,T) = \partial f(R,T)/\partial T$, respectively. Note that when $f(R,T) \equiv f(R)$, from Eq. (17) we obtain the field equations of $f(R)$ gravity [48].

Contracting Eq. (17) gives the following relation between the Ricci scalar R and the trace T of the energy-momentum tensor:

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T)$$
$$= 8\pi T - f_T(R,T)T - f_T(R,T)\Theta, \qquad (18)$$

where we have denoted $\Theta = \Theta_\mu{}^\mu$.

By eliminating the term $\Box f_R(R,T)$ between Eqs. (17) and (18), the gravitational field equations can be written in the form

$$f_R(R,T)\left(R_{\mu\nu} - \frac{1}{3}Rg_{\mu\nu}\right) + \frac{1}{6}f(R,T)g_{\mu\nu}$$
$$= 8\pi\left(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}\right) - f_T(R,T)\left(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}\right)$$
$$- f_T(R,T)\left(\Theta_{\mu\nu} - \frac{1}{3}\Theta g_{\mu\nu}\right) + \nabla_\mu\nabla_\nu f_R(R,T). \quad (19)$$

By taking the covariant divergence of Eq. (17), with the use of the following mathematical identity [10,45,49]:

$$\nabla^\mu\left[f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + \hat{P}_{\mu\nu}f_R(R,T)\right]$$
$$\equiv \frac{1}{2}f_T(R,T)\nabla^\mu Tg_{\mu\nu}, \qquad (20)$$

we obtain for the divergence of the energy-momentum tensor $T_{\mu\nu}$ the equation

$$\nabla^\mu T_{\mu\nu} = \frac{f_T(R,T)}{8\pi - f_T(R,T)}\left[(T_{\mu\nu} + \Theta_{\mu\nu})\nabla^\mu \ln f_T(R,T)\right.$$
$$\left. + \nabla^\mu\Theta_{\mu\nu} - \frac{1}{2}\nabla^\mu Tg_{\mu\nu}\right]. \qquad (21)$$

For the tensor $\Theta_{\mu\nu}$ we find

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}. \qquad (22)$$

III. THERMODYNAMIC INTERPRETATION OF THE GEOMETRY-MATTER COUPLING IN $f(R,L_m)$ GRAVITY

In the present section we investigate the $f(R, L_m)$ gravity theory from the point of view of the thermodynamics of the matter creation irreversible processes. As a first step in our study we obtain the energy conservation equation, which, as compared to the standard adiabatic conservation equation, contains an extra term, which can be interpreted as a matter creation rate. Matter creation acts as an entropy source, and the entropy flux and the temperature evolution of the gravitational system with nonminimal geometry matter coupling are obtained.

In the case of a perfect cosmological fluid with energy density ϵ, thermodynamic pressure p and normalized four-velocity $U^\nu, \nu = 0, 1, 2, 3$, satisfying the condition $U_\nu U^\nu = 1$, the energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (\epsilon + p)U_\mu U_\nu - pg_{\mu\nu}. \qquad (23)$$

We also introduce the projection operator h_λ^ν, defined as $h_\lambda^\nu = \delta_\lambda^\nu - U_\lambda U^\nu$, with the property $U_\nu h_\lambda^\nu = 0$.

A. The energy conservation equation

By taking the covariant divergence of Eq. (23) we obtain

$$\nabla^\mu T_{\mu\nu} = (\nabla^\mu \epsilon + \nabla^\mu p)U_\mu U_\nu + (\epsilon + p)\nabla^\mu U_\mu U_\nu$$
$$+ (\epsilon + p)U_\mu \nabla^\mu U_\nu - \nabla^\mu p g_{\mu\nu}. \qquad (24)$$

Therefore Eq. (10) takes the form

$$(\nabla^\mu \epsilon + \nabla^\mu p)U_\mu U_\nu + (\epsilon + p)\nabla^\mu U_\mu U_\nu$$
$$+ (\epsilon + p)U_\mu \nabla^\mu U_\nu - \nabla^\mu p g_{\mu\nu} = \nabla^\mu \ln [f_{L_m}(R, L_m)]$$
$$\times [(L_m + p)g_{\mu\nu} - (\epsilon + p)U_\mu U_\nu]. \qquad (25)$$

Multiplying Eq. (25) with h_λ^ν gives the momentum balance equation for a perfect fluid in the alternative $f(R, L_m)$ gravity theory as

$$U^\mu \nabla_\mu U^\lambda = \frac{d^2 x^\lambda}{ds^2} + \Gamma_{\mu\nu}^\lambda U^\mu U^\nu$$
$$= \frac{1}{\epsilon + p}\{h^{\nu\lambda}\nabla_\nu p + h_\mu^\lambda \nabla^\mu \ln [f_{L_m}(R, L_m)](L_m + p)\}. \qquad (26)$$

By multiplying Eq. (25) with U^ν, and by taking into account the identity $U^\nu \nabla^\mu U_\nu = 0$, we finally obtain the energy balance equation in the $f(R, L_m)$ gravity theory as

$$\dot{\epsilon} + 3(\epsilon + p)H = \frac{d}{ds}\ln [f_{L_m}(R, L_m)](L_m - \epsilon), \qquad (27)$$

where we have denoted $H = (1/3)\nabla^\mu U_\mu$, and $\dot{} = U^\mu \nabla_\mu = d/d\tilde{s}$, respectively, where $d\tilde{s}$ is the line element corresponding to the metric $g_{\mu\nu}$, $d\tilde{s}^2 = g_{\mu\nu}dx^\mu dx^\nu$.

In the following we assume a homogeneous and isotropic cosmological model, with the line element given by the Friedmann-Robertson-Walker (FRW) metric,

$$d\tilde{s}^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \qquad (28)$$

where $a(t)$ is the scale factor. Moreover, we adopt a comoving coordinate system with $U^\mu = (1, 0, 0, 0)$. In the FRW geometry $H = \dot{a}/a$, $U^\mu \nabla_\mu = \dot{} = d/dt$, and Eq. (27) can be written in the equivalent form,

$$\frac{d}{dt}(\epsilon a^3) + p\frac{d}{dt}a^3 = a^3\frac{d}{dt}\ln [f_{L_m}(R, L_m)](L_m - \epsilon). \quad (29)$$

In the standard general relativistic case $f(R, L_m) = R/2 + L_m$, which gives $f_{L_m}(R, L_m) = 1$, with Eq. (29) reducing to the standard adiabatic matter conservation law

$d(\epsilon a^3) + pda^3 = 0$. Moreover, due to our choice of the geometry, all the nondiagonal components of the energy-momentum tensor are equal to zero, so that $T_{\mu\nu} = 0$, $\mu \neq \nu$. In particular this condition implies the impossibility of heat transfer in the FRW models of modified gravity with geometry-matter coupling, since the condition $T_{0i} \equiv 0$, $i = 1, 2, 3$ must always hold.

B. The matter and entropy creation rates

In its general form, for a system containing N particles in a volume $V = a^3$, the second law of thermodynamics is given by [14]

$$\frac{d}{dt}(\epsilon a^3) + p\frac{d}{dt}a^3 = \frac{dQ}{dt} + \frac{h}{n}\frac{d}{dt}(na^3), \qquad (30)$$

where dQ is the heat received by the system during time dt, $h = \epsilon + p$ is the enthalpy per unit volume, and $n = N/V$ is the particle number density. In the following we restrict our analysis to adiabatic transformations defined by the condition $dQ = 0$, that is, we ignore proper heat transfer processes in the cosmological system. For these types of thermodynamic transformations Eq. (30) represents the formulation of the second law of thermodynamics that explicitly takes into account the variation of the number of particles, described by the term $(h/n)d(na^3)/dt$. Hence in the thermodynamic approach of open systems *the "heat" (internal energy) received by the system is entirely due to the change in the number of particles.* From a cosmological perspective, *this change is due to the transfer of energy from gravitation to matter, with the creation of matter acting as a source of internal energy.* Equivalently, for adiabatic transformations $dQ/dt = 0$, Eq. (30) can be written as

$$\dot{\epsilon} + 3(\epsilon + p)H = \frac{\epsilon + p}{n}(\dot{n} + 3Hn). \qquad (31)$$

Therefore, from a thermodynamic point of view, Eq. (29), giving the energy conservation equation in the $f(R, L_m)$ gravity theory, can be interpreted as describing matter creation in a homogeneous and isotropic geometry, with the particle number time variation given by the equation

$$\dot{n} + 3nH = \frac{n}{\epsilon + p}\frac{d}{dt}\ln [f_{L_m}(R, L_m)](L_m - \epsilon) = \Gamma n, \qquad (32)$$

where the particle creation rate Γ is defined as

$$\Gamma = \frac{1}{\epsilon + p}\frac{d}{dt}\ln [f_{L_m}(R, L_m)](L_m - \epsilon). \qquad (33)$$

Therefore the energy conservation equation can be written in the $f(R, L_m)$ gravity theory as

$$\dot{\epsilon} + 3(\epsilon + p)H = (\epsilon + p)\Gamma. \tag{34}$$

For adiabatic transformations Eq. (30), describing particle creation in open systems, can be written as an effective energy conservation equation of the form

$$\frac{d}{dt}(\epsilon a^3) + (p + p_c)\frac{d}{dt}a^3 = 0, \tag{35}$$

or, equivalently,

$$\dot{\epsilon} + 3(\epsilon + p + p_c)H = 0, \tag{36}$$

where p_c, called the creation pressure, is defined as [14]

$$p_c = -\frac{\epsilon + p}{n}\frac{d(na^3)}{da^3} = -\frac{\epsilon + p}{3nH}(\dot{n} + 3nH)$$
$$= -\frac{\epsilon + p}{3}\frac{\Gamma}{H}. \tag{37}$$

Therefore in the $f(R, L_m)$ gravity theory the creation pressure is determined by the geometry-matter coupling and is given by

$$p_c = -\frac{1}{3H}\frac{d}{dt}\ln\left[f_{L_m}(R, L_m)\right](L_m - \epsilon). \tag{38}$$

The second law of thermodynamics can be written by decomposing the entropy change into an entropy flow d_eS and an entropy creation d_iS, so that the total entropy S of the system is given by [13,14]

$$dS = d_eS + d_iS, \tag{39}$$

with $d_iS > 0$. To evaluate the entropy flow and the entropy production, we start from the total differential of the entropy [14],

$$\mathcal{T}d(sa^3) = d(\epsilon a^3) + pda^3 - \mu d(na^3), \tag{40}$$

where $\mathcal{T}$ is the temperature, $s = S/a^3$, and μ is the chemical potential given by

$$\mu n = h - \mathcal{T}s. \tag{41}$$

For closed systems and adiabatic transformations $dS = 0$ and $d_iS = 0$. However, in the presence of matter creation there is a nonzero contribution to the entropy. For homogeneous systems we still have $d_eS = 0$. In contrast, matter creation contributes to the entropy production, and the entropy time variation is given by [14]

$$\mathcal{T}\frac{d_iS}{dt} = \mathcal{T}\frac{dS}{dt} = \frac{h}{n}\frac{d}{dt}(na^3) - \mu\frac{d}{dt}(na^3)$$
$$= \mathcal{T}\frac{s}{n}\frac{d}{dt}(na^3) \geq 0. \tag{42}$$

From Eq. (42) we obtain for the time variation of the entropy the equation

$$\frac{dS}{dt} = \frac{S}{n}(\dot{n} + 3Hn) \geq 0. \tag{43}$$

With the use of Eq. (32), giving the particle number balance in $f(R, L_m)$ gravity theory, we obtain for the entropy production the equation

$$\frac{1}{S}\frac{dS}{dt} = \frac{1}{\epsilon + p}\frac{d}{dt}\ln\left[f_{L_m}(R, L_m)\right](L_m - \epsilon) \geq 0. \tag{44}$$

The entropy flux vector S^μ is defined as [15]

$$S^\mu = n\sigma U^\mu, \tag{45}$$

where $\sigma = S/N$ is the specific entropy per particle. The entropy creation rate must satisfy the second law of thermodynamics, $\nabla_\mu S^\mu \geq 0$. Since the specific entropy obeys the Gibbs relation [15],

$$nT d\sigma = d\epsilon - \frac{h}{n}dn, \tag{46}$$

and by taking into account the definition of the chemical potential μ of the system as

$$\mu = \frac{h}{n} - \mathcal{T}\sigma, \tag{47}$$

we obtain

$$\nabla_\mu S^\mu = (\dot{n} + 3nH)\sigma + nU^\mu\nabla_\mu\sigma$$
$$= \frac{1}{\mathcal{T}}(\dot{n} + 3Hn)\left(\frac{\epsilon + p}{n} - \mu\right), \tag{48}$$

where we have taken into account the relation

$$nT\dot{\sigma} = \dot{\epsilon} - \frac{\epsilon + p}{n}\dot{n} = 0, \tag{49}$$

which follows immediately from Eq. (31). With the use of Eq. (32) we obtain for the entropy production rate associated with the particle creation processes in $f(R, L_m)$ gravity the expression

$$\nabla_\mu S^\mu = \frac{1}{\mathcal{T}} \frac{n}{\epsilon + p} \frac{d}{dt} \ln [f_{L_m}(R, L_m)]$$
$$\times (L_m - \epsilon) \left(\frac{\epsilon + p}{n} - \mu \right). \tag{50}$$

In the general case of a perfect comoving fluid with two essential thermodynamic variables, the particle number density n, and the temperatures $\mathcal{T}$, it is conventional to express ϵ and p in terms of n and $\mathcal{T}$ by means of the equilibrium equations of state,

$$\epsilon = \epsilon(n, \mathcal{T}), \qquad p = p(n, \mathcal{T}). \tag{51}$$

Then the energy conservation equation (34) takes the form

$$\frac{\partial \epsilon}{\partial n} \dot{n} + \frac{\partial \epsilon}{\partial \mathcal{T}} \dot{\mathcal{T}} + 3(\epsilon + p)H = \Gamma n. \tag{52}$$

With the help of the general thermodynamic relation [15]

$$\frac{\partial \epsilon}{\partial n} = \frac{\epsilon + p}{n} - \frac{\mathcal{T}}{n} \frac{\partial p}{\partial \mathcal{T}}, \tag{53}$$

we obtain the temperature evolution of the newly created particles in the $f(R, L_m)$ gravity theory as

$$\frac{\dot{\mathcal{T}}}{\mathcal{T}} = \frac{\dot{n}}{n} \frac{\partial p}{\partial \epsilon} = (\Gamma - 3H) \frac{\partial p}{\partial \epsilon}$$
$$= \frac{1}{\epsilon + p} \frac{\partial p}{\partial \epsilon} \left\{ \frac{d}{dt} \ln [f_{L_m}(R, L_m)](L_m - \epsilon) - 3H(\epsilon + p) \right\}. \tag{54}$$

C. The case $L_m = -p$

The exact form of the Lagrangian of the matter is one of the most intriguing theoretical problems in general relativity. One possible choice is $L_m = -p$, which was used in [50–52] to derive the equation of motion of test fluids in standard general relativity. In this case, as pointed out in [53], the extra force, given by Eq. (26), and which is one of the distinguishing features of modified gravity theories with geometry-matter coupling, identically vanishes. On the other hand, as argued in [54], other choices for the matter Lagrangian, like $L_m = \epsilon$ or $L_m = -na$, where a is the physical free energy defined as $a = \epsilon/n - \mathcal{T}s$, are also possible [52,55]. In fact, all these expressions are on shell representations of a more general Lagrangian density obtained through backsubstitution of the equations of motion into the related action [52].

In the following we investigate the consequences of the choice $L_m = -p$ for the matter Lagrangian. In this case the extra force in Eq. (26) vanishes, showing that in this model of $f(R, L_m)$ gravity the motion of the test fluids is geodesic. On the other hand, with this choice the right hand side of the energy conservation equation (27) does not vanish,

showing that the particle creation processes are present in the model. The particle number balance equation (32) becomes

$$\dot{n} + 3nH = -n \frac{d}{dt} \ln [f_{-p}(R, -p)], \tag{55}$$

where f_{-p} denotes the derivative of $f(R, -p)$ with respect to $-p$, and can be immediately integrated to give

$$na^3 = \frac{n_0 a_0^3}{f_{-p}(R, -p)}. \tag{56}$$

Therefore the variation of the total number of particles within a volume V is inversely proportional to the derivative of f with respect to the matter Lagrangian. The energy conservation equation (34) takes the form

$$\dot{\epsilon} + 3(\epsilon + p)H = -(\epsilon + p) \frac{d}{dt} \ln [f_{-p}(R, -p)]. \tag{57}$$

From Eq. (38) the creation pressure p_c is obtained as

$$p_c = \frac{\epsilon + p}{3H} \frac{d}{dt} \ln [f_{L_m}(R, L_m)]. \tag{58}$$

For a radiationlike fluid with $p = \epsilon/3$ we obtain for the variation of the energy density the expression

$$\epsilon a^4 = \frac{\epsilon_0 a_0^4}{[f_{-p}(R, -p)]^{4/3}}, \tag{59}$$

where ϵ_0 and a_0 are arbitrary constants of integration. For the total entropy variation from Eq. (44) we find

$$S = \frac{S_0}{f_{-p}(R, -p)}, \tag{60}$$

where S_0 is an arbitrary constant of integration. In a cosmological fluid where the density and pressure are functions of the temperature T only, $\epsilon = \epsilon(T)$, $p = p(T)$, the entropy is given by [56]

$$S = \frac{(\epsilon + p)V}{T}. \tag{61}$$

For a radiation fluid $\epsilon = 3p$, and it follows that the function f satisfies the following partial differential equation:

$$\frac{\partial}{\partial(-p)} f(R, -p) = -\frac{S_0 T(a)}{4(-p)a^3}. \tag{62}$$

By assuming a cosmological model with $f(R, -p) = f_0(R(a))g_0(-p)$, Eq. (62) gives

$$f_0(R(a)) = -\frac{S_0 T(a)}{a^3}, \qquad \frac{dg_0(-p)}{d(-p)} = \frac{1}{4(-p)}, \quad (63)$$

which fixes the matter Lagrangian dependence of the action as

$$g_0(L_m) = \frac{1}{4} \ln L_m. \tag{64}$$

IV. IRREVERSIBLE THERMODYNAMIC INTERPRETATION OF THE $f(R,T)$ GRAVITY THEORY

In the present section we consider the thermodynamic interpretation of the $f(R,T)$ gravity theory. In the present study we *assume* again that the energy-momentum tensor of the matter is given by Eq. (23). In order to obtain some explicit results we fix the matter Lagrangian from the beginning as

$$L_m = -p. \tag{65}$$

The trace of the matter energy-momentum tensor is given by $T = \epsilon - 3p$. Then, with the use of Eq. (22), we obtain for the variation $\Theta_{\mu\nu}$ of the energy-momentum tensor of a perfect fluid with respect to metric tensor $g_{\mu\nu}$ the expression [10]

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu} = -2(\epsilon + p)U_\mu U_\nu + pg_{\mu\nu}. \tag{66}$$

A. Energy and particle creation rates in $f(R,T)$ gravity

With the use of Eq. (66) the divergence of the energy-momentum tensor in $f(R,T)$ gravity theory follows immediately from Eq. (21) as

$$\nabla^\mu T_{\mu\nu} = -\frac{f_T(R,T)}{8\pi + f_T(R,T)} \left[(\epsilon + p)U_\nu U_\mu \nabla^\mu \ln f_T(R,T) \right.$$
$$\left. + \nabla_\nu \frac{\epsilon - p}{2} \right]. \tag{67}$$

Multiplying Eq. (67) with U^ν gives

$$U^\nu \nabla^\mu T_{\mu\nu} = -\frac{f_T(R,T)}{8\pi + f_T(R,T)} \left[(\epsilon + p)U_\mu \nabla^\mu \ln f_T(R,T) \right.$$
$$\left. + U^\nu \nabla_\nu \frac{\epsilon - p}{2} \right]. \tag{68}$$

From Eq. (68), with the use of the explicit form of the matter energy-momentum tensor we obtain the energy conservation equation in $f(R,T)$ gravity as

$$\dot{\epsilon} + 3(\epsilon + p)H = -\frac{f_T(R,T)}{8\pi + f_T(R,T)}$$
$$\times \left[(\epsilon + p)U_\mu \nabla^\mu \ln f_T(R,T) + U^\nu \nabla_\nu \frac{\epsilon - p}{2} \right]. \tag{69}$$

We interpret again Eq. (69) as describing adiabatic irreversible thermodynamic particle creation in a cosmological context, according to Eq. (31). Therefore the particle balance equation is given by

$$\dot{n} + 3nH = \Gamma n, \tag{70}$$

where for the case of the $f(R,T)$ gravity the particle creation rate is defined as

$$\Gamma = -\frac{f_T(R,T)}{8\pi + f_T(R,T)} \left[\frac{d}{dt} \ln f_T(R,T) + \frac{1}{2} \frac{\dot{\epsilon} - \dot{p}}{\epsilon + p} \right]. \tag{71}$$

The energy conservation equation can be formulated in an alternative way as

$$\frac{d}{dt}(\epsilon a^3) + p \frac{d}{dt} a^3 = (\epsilon + p)a^3 \Gamma. \tag{72}$$

In the $f(R,T)$ gravity theory, the creation pressure corresponding to the particle production from the gravitational field, defined in Eq. (38), is given by

$$p_c = \frac{\epsilon + p}{3H} \frac{f_T(R,T)}{8\pi + f_T(R,T)} \left[\frac{d}{dt} \ln f_T(R,T) + \frac{1}{2} \frac{\dot{\epsilon} - \dot{p}}{\epsilon + p} \right]. \tag{73}$$

If $f(R,T)$ is independent of T (the general relativistic limit), then $f_T(R,T) \equiv 0$ and both Γ and p_c vanish. Hence we reobtain the standard cosmological evolution without matter creation and with the evolution of the Universe obeying a strict conservation law of the total energy.

From Eq. (43) we obtain the time variation of the entropy, which is entirely due to the matter creation processes, as

$$S(t) = S_0 \exp \left[\int_0^t \Gamma(t')dt' \right], \tag{74}$$

where S_0 is an arbitrary constant of integration. With the use of Eq. (48), we obtain for the entropy production rate the expression

$$\nabla_\mu S^\mu = \frac{n}{\mathcal{T}} \Gamma \left(\frac{\epsilon + p}{n} - \mu \right) = -\frac{n}{\mathcal{T}} \left(\frac{\epsilon + p}{n} - \mu \right)$$
$$\times \frac{f_T(R,T)}{8\pi + f_T(R,T)} \left[\frac{d}{dt} \ln f_T(R,T) + \frac{1}{2} \frac{\dot{\epsilon} - \dot{p}}{\epsilon + p} \right]. \tag{75}$$

By assuming that the energy density and the thermodynamic pressure of the newly created particles are, according

to Eqs. (51), functions of both the particle number n and the temperature $\mathcal{T}$, the temperature time variation of the created particles in the open thermodynamic system follows from Eq. (54), and in the $f(R,T)$ gravity is given by

$$\frac{1}{c_s^2}\frac{\dot{\mathcal{T}}}{\mathcal{T}} = \Gamma - 3H = -\left\{ \frac{f_T(R,T)}{8\pi + f_T(R,T)} \right.$$
$$\left. \times \left[\frac{d}{dt}\ln f_T(R,T) + \frac{1}{2}\frac{\dot{\epsilon}-\dot{p}}{\epsilon+p} \right] + 3H \right\}, \quad (76)$$

where $c_s^2 = \partial p/\partial \epsilon$ is the speed of sound.

B. Cosmological applications

In the following we consider the thermodynamic interpretation of a simple $f(R,T)$ modified gravity theory [10]. We assume that the function $f(R,T)$ is given by

$$f(R,T) = R + 2g(T), \quad (77)$$

where $g(T)$ is an arbitrary function of the trace of the energy-momentum tensor $T = \epsilon - 3p$ of matter. The gravitational field equations immediately follow from Eq. (17) and are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} - 2g'(T)T_{\mu\nu} - 2g'(T)\Theta_{\mu\nu} + g(T)g_{\mu\nu}, \quad (78)$$

where the prime denotes a derivative with respect to the argument.

By taking the matter source as a perfect fluid, we have $\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}$, and then the field equations become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + 2g'(T)T_{\mu\nu}$$
$$+ [2pg'(T) + g(T)]g_{\mu\nu}. \quad (79)$$

By modeling the matter content of the Universe as dust with $p = 0$, the gravitational field equations can be obtained as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + 2g'(T)T_{\mu\nu} + g(T)g_{\mu\nu}. \quad (80)$$

These field equations were also proposed in [57] to solve the cosmological constant problem. The simplest cosmological model can be obtained by assuming a dust matter dominated universe ($p = 0$, $T = \rho$), and by choosing the function $g(T)$ so that $g(T) = \lambda T$, where λ is a constant, giving

$$f_T(R,T) = 2\lambda = \text{const.} \quad (81)$$

For the flat FRW metric, the gravitational field equations are given by

$$3H^2 = (8\pi + 3\lambda)\epsilon \quad (82)$$

and

$$2\dot{H} + 3H^2 = -\lambda\epsilon, \quad (83)$$

respectively. Thus this $f(R,T)$ gravity model is equivalent to a gravitational model with an effective cosmological constant $\Lambda_{\text{eff}} \propto H^2$ [10,57]. For this choice of $f(R,T)$ the gravitational coupling becomes an effective time dependent coupling, of the form $G_{\text{eff}} = G \pm 2g'(T)$. Therefore the presence of the term $2g(T)$ in the gravitational action modifies the nature of the gravitational interaction between matter and geometry, replacing the gravitational constant G by a running gravitational coupling parameter.

The field equations reduce to a single equation for H,

$$2\dot{H} + 3\frac{8\pi + 4\lambda}{8\pi + 3\lambda}H^2 = 0, \quad (84)$$

with the general solution given by

$$H(t) = \frac{2(8\pi + 3\lambda)}{3(8\pi + 4\lambda)}\frac{1}{t} = \frac{H_0}{t}, \quad (85)$$

where $H_0 = 2(8\pi + 4\lambda)/3(8\pi + 3\lambda)$. The scale factor evolves according to $a(t) = t^{H_0}$, while the matter energy density of the Universe can be obtained as

$$\epsilon(t) = \frac{\epsilon_0}{t^2}, \quad (86)$$

where $\epsilon_0 = 3H_0^2/(8\pi + 3\lambda)$. The deceleration parameter q, defined as

$$q = \frac{d}{dt}\frac{1}{H} - 1, \quad (87)$$

is obtained as

$$q = \frac{1}{H_0} - 1 = \frac{8\pi + \lambda}{2(8\pi + 4\lambda)}. \quad (88)$$

If $q < 0$, the expansion of the Universe is accelerating, while $q > 0$ corresponds to a decelerating phase.

The energy conservation equation follows from Eq. (69) and is given by

$$\dot{\epsilon} + 3\frac{8\pi + 2\lambda}{8\pi + 3\lambda}\epsilon H = 0. \quad (89)$$

The particle creation rate, due to the energy transfer from the gravitational field to the matter, is obtained as

$$\Gamma(t) = -\frac{\lambda}{8\pi + 2\lambda}\frac{\dot{\epsilon}}{\epsilon} = \frac{3\lambda}{8\pi + 3\lambda}H = \frac{\Gamma_0}{t}, \quad (90)$$

where $\Gamma_0 = (2\lambda/3)(8\pi + 4\lambda)/(8\pi + 3\lambda)^2$. The time variation of the particle number in the open matter-gravitation system is described by the equation

$$\dot{n} = \frac{\Gamma_0 - 3H_0}{t} n, \qquad (91)$$

and is given by

$$n(t) = n_0 t^{\Gamma_0 - 3H_0}, \qquad (92)$$

where n_0 is an arbitrary constant of integration. The creation pressure, obtained from Eq. (73), is determined as

$$p_c(t) = \frac{\lambda}{3(8\pi + 2\lambda)} \frac{\dot{\epsilon}}{H} = -\frac{\lambda}{8\pi + 3\lambda} \epsilon = -\frac{p_{c0}}{t^2}, \qquad (93)$$

where $p_{c0} = \lambda \epsilon_0 / (8\pi + 3\lambda)$. Finally, for the time variation of the entropy of the Universe, which is due to the creation of the particles from the gravitational field, we obtain

$$S(t) = S_0 t^{\Gamma_0}. \qquad (94)$$

Therefore we have obtained a full thermodynamic interpretation, in the framework of open irreversible thermodynamic systems, of this simple $f(R, T)$ cosmological model. The particle creation rate is proportional to the Hubble function, and it is a linearly decreasing function of time. The particle number created during this expansionary phase decreases during the cosmological evolution. The particle creation processes generate a large amount of comoving entropy, whose time dependence is given by a power law function.

V. DISCUSSIONS AND FINAL REMARKS

In the present paper we have investigated in a systematic way the possibility that modified gravity theories with geometry-matter coupling can be interpreted as providing a phenomenological description of particle production in the cosmological fluid filling the Universe. One of the common features of these classes of gravity theories is the non-conservation of the matter energy-momentum tensor. By using the mathematical formalism of the open thermodynamic systems we have provided a physical interpretation of the extra terms generated by the nonminimal geometry-matter coupling as describing particle production, with the gravitational field acting as a particle source. Both $f(R, L_m)$ and $f(R, T)$ gravity theories do admit such an interpretation, and the particle production rates, entropy, creation pressure and entropy generation rate have been explicitly obtained as functions of the coupling functions f and of their derivatives, respectively.

The phenomenological cosmological particle creation formalisms considered in the present paper have been extensively discussed in the literature within the context of standard general relativity. Hence, the geometric coupling to matter of alternative gravity theories is not strictly necessary for this type of cosmological approaches. However, there is a fundamental difference between the thermodynamic of open systems as formulated in the framework of standard general relativity, and in theories with geometry-matter coupling. While in general relativity the particle creation rates and the creation pressure either must be chosen based on some plausible physical considerations or must be inferred from some microscopic description, in both $f(R, L_m)$ and $f(R, T)$ modified gravity theories the coupling between geometry and matter completely determines the particle creation rates, the creation pressure and the entropy production, respectively. As one can see from Eqs. (33) and (38), in $f(R, L_m)$ gravity theory Γ and p_c are completely determined by the derivatives of the function $f(R, L_m)$ with respect to the matter Lagrangian L_m and by the matter Lagrangian itself. In the case of $f(R, T)$ gravity theory, the particle creation rates and the creation pressure, given by Eqs. (71) and (73), respectively, are also determined by the derivative of $f(R, T)$ with respect to T. Hence, in these classes of theories all the irreversible thermodynamic quantities are fully determined by the gravitational action. In the general relativistic limit with $f(R, L_m) = R/2 + L_m$, and $f(R, T)$ independent of T, all Γ and p_c are identically equal to zero. This result shows the essential role played by the coupling between matter and geometry in the phenomenological description of the cosmological particle production processes.

On the other hand, it has been suggested by Zeldovich [58] and later by Murphy [59] and [60] that the introduction of viscosity in the cosmological fluid is nothing but a phenomenological description of the effect of creation of particles by the nonstationary gravitational field of the expanding universe. A nonvanishing particle production rate is equivalent to a bulk viscous pressure in the cosmological fluid, or, from a quantum point of view, with a viscosity of the vacuum. This is due to the simple circumstance that any source term in the energy balance of a relativistic fluid may be formally rewritten in terms of an effective bulk viscosity.

The energy-momentum tensor of a relativistic fluid with bulk viscosity as the only dissipative phenomena can be written as [61]

$$T_{\mu\nu} = (\epsilon + p + \Pi)U_\mu U_\nu - (p + \Pi), \qquad (95)$$

where Π is the bulk viscous pressure. The particle flow vector N^μ is given by $N^\mu = nU^\mu$. Limiting ourselves to second-order deviations from equilibrium, in the framework of causal thermodynamics the entropy flow vector S^μ takes the form [62]

$$S^\mu = sN^\mu - \frac{\tau \Pi^2}{2\xi T} U^\mu, \qquad (96)$$

where τ is the relaxation time, and ξ is the coefficient of bulk viscosity. In the case of a homogeneous and isotropic geometry, the energy conservation equation in the presence of bulk viscous dissipative processes is obtained as

$$\dot{\epsilon} + 3(\epsilon + p + \Pi)H = 0. \tag{97}$$

By comparing Eq. (97), giving the conservation equation for a bulk viscous cosmological fluid, with Eq. (36), which includes in the energy balance the creation of particles from the gravitational field, it follows that if one can take

$$p_c = \Pi, \tag{98}$$

particle creation is equivalent with the introduction of an effective bulk viscous pressure in the energy-momentum tensor of the cosmological fluid. Hence the causal bulk viscous pressure Π acts as a creation pressure. However, there is a major difference between the particle creation irreversible processes in open thermodynamic systems and bulk viscous processes, and this difference is related to the expression for the entropy production rate. While the entropy production rate associated with particle creation is given by [15]

$$\nabla_\mu S^\mu = -\frac{3Hp_c}{\mathcal{T}}\left(1 + \frac{\mu\Gamma n}{3Hp_c}\right) \geq 0, \tag{99}$$

in the presence of bulk viscous dissipative processes the entropy production rate can be obtained as [61]

$$\nabla_\mu S^\mu = -\frac{\Pi}{\mathcal{T}}\left[3H + \frac{\tau}{\xi}\dot{\Pi} + \frac{\tau}{2\xi}\Pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{\mathcal{T}}}{\mathcal{T}}\right)\right]. \tag{100}$$

In the particle creation model in open thermodynamics systems the entropy production rate is proportional to the creation pressure, while in the viscous dissipative processes thermodynamic interpretation $\nabla_\mu S^\mu$ is quadratic in the creation pressure, $\nabla_\mu S^\mu \propto p_c^2/\xi\mathcal{T}$, and, moreover, involves a new dynamical variable, the bulk viscosity coefficient.

In the present paper we have adopted a phenomenological thermodynamical approach for the description of matter creation in cosmology, with the particle creation rate being determined by the coupling between matter and geometry. Such an approach cannot give any indication on the type or nature of the created particles. However, it is natural to expect that such creation processes are consistent with the similar processes that appear in the framework of the quantum particle creation in cosmology. Generally, a static gravitational field does not produce particles. However, a time-dependent gravitational field generally produces particles. In conventional quantum mechanics, tunneling from a small scale factor a to large a would be described by the wave function which contains only

outgoing waves as $a \to \infty$ [63]. The decaying part of the wave function in the classically forbidden region can be represented as $\Psi(a) \propto \exp\{-\int\sqrt{2[V(a) - \epsilon]}da\}$, where $V(a)$ is the quantum potential, and ϵ is the conformal energy of the scalar mode. When a massive scalar field is conformally coupled to gravity, with an action of the form $S_\Phi = \int(g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi/2 - m_\Phi^2\Phi^2 + R\Phi^2/12)\sqrt{-g}d^4x$, unlike in conventional quantum mechanics, the resulting Schrödinger equation contains a negative kinetic term for the scale factor, and the potential $V(a)$ also enters with a negative sign. Therefore in quantum cosmology, and quantum particle creation processes, the excitation of the field Φ makes the tunneling processes exponentially easier, due to the increase of the effective value of ϵ [63]. Therefore, the quanta Φ of the quantum field tend to be copiously created. Therefore, based on the quantum analogy, we expect that most of the particles created due to the geometry-matter coupling may be in the form of some scalar particles (bosons) that cosmologically may represent the dark matter content of the Universe. In the simple $f(R, T)$ cosmological model analyzed in the previous section we have seen that the particle creation rate is maximum at the beginning of the cosmological expansion. Therefore we may also consider that most of the dark matter was created at the early stages of the cosmological evolution. On the other hand, self-interacting dark matter bosons may condense to form a Bose-Einstein condensate, and this hypothesis seems to be confirmed by the study of the galactic rotation curves [64,65].

One of the intriguing problems in modern cosmology is the entropy problem [66,67]. By using the open thermodynamic system interpretation, in the framework of theories with geometry-matter coupling models that start from empty conditions and gradually build up matter and entropy can easily be constructed. In these models also the gravitational entropy takes a simple meaning, being associated with the entropy that is necessary to produce matter. Matter creation process are modeled classically and on a phenomenological level by means of a creation pressure, with particles continuously added to the cosmological space-time. The dynamics and evolution of the Universe are entirely determined by particle production processes, which, in turn, essentially depend on the geometry-matter coupling, and the influence of particle production on geometry is essential. As a consequence of a large particle creation rate, inflationary behavior can also be obtained. The evolution of the Universe is determined, for a given equation of state, by the numerical values of a single parameter Γ, containing both the thermodynamics parameters of the system, as well as the geometry-matter coupling. Matter creation processes are naturally stopped after a finite interval of time, and the Universe may end in a decelerating era. A large amount of comoving entropy is produced during the evolution of the Universe.

The study of the irreversible matter-creation processes in the homogeneous and isotropic flat FRW geometry in models with geometry-matter coupling opens the possibility of considering the viability of these gravity theories in a cosmological context. However, in order to confirm the validity of the thermodynamic interpretation developed in the present paper, it is necessary to consider a wider range of cosmological and astrophysical tests of the $f(R, L_m)$ and $f(R, T)$ type theories. In particular, an essential test of these theories would be the analysis of their classical macroscopic predictions in structure formation with linear perturbations (in the Newtonian limit in small scales), and in observations fitting, including the study of the effects of matter creation on the CMB anisotropies. Essentially the models introduced in the present paper are some simple toy models, whose main interest is to begin probing and testing alternative gravity theories.

The gravitational theories considered in the present paper predict the possibility that matter creation, associated with geometry-matter coupling can also occur in the present-day universe, as initially considered by Dirac [68]. The existence of some forms of matter-geometry coupling are not in contradiction with the cosmological observations or with some astrophysical data [9,11], but observational evidence of particle creation on a cosmological scale is still missing. Presumably, the functional forms of the functions f entering in the present gravitational theories will be furnished by fundamental particle physics models of decaying vacuum energy density and gravitational coupling, thus permitting an in depth comparison of the predictions of the theory with observational data.

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