Constraining extended gravity models by S2 star orbits around the Galactic Centre

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We investigate the possibility of explaining theoretically the observed deviations of S2 star orbit around the Galactic Centre using gravitational potentials derived from modified gravity models in the absence of dark matter. To this aim, an analytic fourth-order theory of gravity, nonminimally coupled with a massive scalar field, is considered. Specifically, the interaction term is given by the analytic functions $f(R)$ and $f(R,\phi)$ where R is the Ricci scalar and ϕ is a scalar field whose meaning can be related to further gravitational degrees of freedom. We simulate the orbit of the S2 star around the Galactic Centre in $f(R)$ (Yukawa-like) and $f(R,\phi)$ (Sanders-like) gravity potentials and compare it with New Technology Telescope/Very Large Telescope observations. Our simulations result in strong constraints on the range of gravity interaction. In the case of analytic functions $f(R)$, we are not able to obtain reliable constraints on the derivative constants f_1 and f_2, because the current observations of the S2 star indicated that they may be highly mutually correlated. In the case of analytic functions $f(R,\phi)$, we are able to obtain reliable constraints on the derivative constants f_0, f_R, f_{RR}, f_ϕ, $f_{\phi\phi}$, and $f_{\phi R}$. The approach we are proposing seems to be sufficiently reliable to constrain the modified gravity models from stellar orbits around the Galactic Centre.

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I. INTRODUCTION

Extended theories of gravity [1] are alternative theories of gravitational interaction developed from the exact starting points investigated first by Einstein and Hilbert and aimed from one side to extend the positive results of general relativity and, on the other hand, to cure its shortcomings. Besides other fundamental issues, like dark energy and quantum gravity, these theories have been proposed like alternative approaches to Newtonian gravity in order to explain galactic and extragalactic dynamics without introducing dark matter [2,3]. In particular, the search for non-Newtonian gravity is part of the quest for non-Einsteinian physics, which consists of searching for deviations from special and general relativity [4–6]. They are aimed at addressing conceptual and experimental problems that have recently emerged in astrophysics and cosmology from the observations of the Solar System, binary pulsars, spiral galaxies, clusters of galaxies, and the large-scale structure of the Universe [7–11]. In general, these theories describe gravity as a metric theory with a linear connection but there are also affine, or metric-affine, formulations of the extended theories of gravity [1]. Essentially, they are based on straightforward generalizations of the Einstein theory where the gravitational action (the Hilbert-Einstein action) is assumed to be linear in the Ricci curvature scalar R. $f(R)$ gravity is a type of modified gravity which generalizes Einstein's general relativity and it was first proposed in 1970 by Buchdahl [12]. It is actually a family of models, each one defined by a different function of the Ricci scalar. The simplest case just involves general relativity. In the case of $f(R)$ gravity, one assumes a generic function f of the Ricci scalar R (in particular, analytic functions) and searches for a theory of gravity having suitable behavior at small and large scale lengths. As a consequence of introducing an arbitrary function, there may be freedom to explain the accelerated expansion and structure formation of the Universe without adding unknown forms of dark energy or dark matter. One type of the extended theories of gravity is characterized by power-law Lagrangians [13,14]. Alternative approaches to Newtonian gravity in the framework of the weak field limit of fourth-order gravity theory have been proposed and constraints on these theories have been discussed [15–22].

Yukawa-like corrections have been obtained in the framework of $f(R)$ gravity as a general feature of these

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theories [23–26]. It is important to stress that they emerge as exact solutions in the context of extended gravity and are not just put in by hand as phenomenological terms. The physical meaning of such corrections needs to be confirmed at different scales: for short distances, the Solar System, spiral galaxies, and galaxy clusters. A compilation of experimental, geophysical, and astronomical constraints on Yukawa violations of the gravitational inverse square law are given in Figs. 9 and 10 from [27] for different ranges. These results show that the Yukawa term is relatively well constrained for the short ranges. For longer distances Yukawa corrections have been successfully applied to clusters of galaxies [23,28,29]. Lucchesi and Peron [30] analyzed pericenter general relativistic precession and gave constraints on exponential potential to Solar System measurements. However, further tests are needed in order to set robust constraints on Yukawa corrections. Galactic stellar dynamics could be of great aid in this program.

S stars are mainly young early-type stars that closely orbit the massive compact object at the center of the Milky Way, named Sgr A* [31–36]. These stars, together with a recently discovered dense gas cloud falling towards the Galactic Centre [37], indicate that the massive central object is a black hole. In our simulation we will treat the central object like a "massive compact object" since our goal was only to study orbits of stars around the Galactic Centre, no matter what the nature of the object is (black hole or not). For at least one of them, called S2, there are some observational indications that its orbit may deviate from the Keplerian case due to relativistic precession [33,38].

However, we have to point out that the present astrometric limit is still not sufficient to definitely confirm such a claim. On the other hand, the astrometric accuracy is constantly improving from around 10 mas during the first part of the observational period, currently reaching less than 1 mas (0.3 mas); see [39]. Furthermore, some recent studies provide more and more evidence that the orbit of the S2 star is not closing (see, e.g., Fig. 2 in [38]). Here, we fitted the NTT/VLT astrometric observations of the S2 star, which contain a possible indication of orbital precession around the massive compact object at the Galactic Centre, in order to constrain the parameters of Sanders-like gravity potential since this kind of potential has not been tested at these scales yet. We obtained much larger orbital precession of the S2 star in Sanders-like gravity than the corresponding value predicted by general relativity. In the paper [33] (page 1092, Fig. 13), authors presented the Keplerian orbit but they have to move the position of central point mass to explain orbital precession. In our orbit, calculated by Sanders-like potential for best fitting parameters, we also obtained precession, but with a fixed position of the central point mass. In other words, we do not need to move central point mass in order to get the fit.

As a general remark, the orbit of S2 will give astronomers the opportunity to test for various effects predicted by general relativity. The orbital precession can occur due to relativistic effects, resulting in a prograde pericenter shift or due to a possible extended mass distribution, producing a retrograde shift [40]. Both prograde relativistic and retrograde Newtonian pericenter shifts will result in rosette shaped orbits [41]. We have to stress that the current astrometric limit is not sufficient to unambiguously confirm such a claim. Weinberg *et al.* [42] discussed physical experiments achievable via the monitoring of stellar dynamics near the Galactic Centre with a diffraction-limited, next-generation extremely large telescope.

The aim of this paper is to give the astronomical constraints on extended theories of gravity by using the peculiar dynamics of the S2 star. In particular, we want to fix the ranges of Yukawa-like correction parameters by adopting the NTT/VLT observations. The paper is organized as follows. Section II is devoted to a short summary of extended gravity in view of the Newtonian limit where Yukawa-like corrections emerge. The simulated orbits of the S2 star in modified potential are considered in Sec. III. In particular, we set the problem of how to constrain the Yukawa-potential parameters. Section IV is devoted to the results of the simulation. Conclusions are drawn in Sec. V.

II. EXTENDED THEORIES OF GRAVITY

Examples of extended theories of gravity are higher-order, scalar-tensor gravities; see, for example, [1,3,10,43–49]. These theories can be characterized by two main features: (i) the geometry can nonminimally couple to some scalar field and (ii) higher-order curvature invariants can appear in the action. In the first case, we are dealing with scalar-tensor gravity, and in the second case we have higher-order gravity. Combinations of nonminimally coupled and higher-order terms can also emerge in an effective Lagrangian, producing mixed higher-order/scalar-tensor gravity. A general class of higher-order-scalar-tensor theories in four dimensions is given by the effective action [1,50]:

$$\mathcal{S} = \int d^4x\sqrt{-g}[f(R, \Box R, \Box^2 R, \ldots, \Box^k R, \phi) + \omega(\phi)\phi_{;\alpha}\phi^{;\alpha} + \mathcal{X}\mathcal{L}_m], \tag{1}$$

where f is an unspecified function of curvature invariants and the scalar field ϕ and $\mathcal{X} = 8\pi G$. Here we use the convention $c = 1$.

The simplest extension of general relativity is achieved assuming

$$R \to f(R), \qquad \omega(\phi) = 0, \tag{2}$$

where the action (1) becomes [50]

$$\mathcal{S} = \int d^4x\sqrt{-g}[f(R) + \mathcal{X}\mathcal{L}_m]. \tag{3}$$

A general gravitational potential, with a Yukawa correction, can be obtained in the Newtonian limit of any analytic $f(R)$-gravity model. From a phenomenological point of view, this correction allows us to consider as viable this kind of model even at small distances, provided that the Yukawa correction turns out not to be relevant in this approximation, as in the so-called chameleon mechanism [49].

One can assume, however, analytic Taylor expandable $f(R)$ functions with respect to the value $R = 0$ that is the Minkowskian background [50]:

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} R^n = f_0 + f_1 R + \frac{f_2}{2} R^2 + \cdots \quad (4)$$

It is worth noting that, at the order $\mathcal{O}(0)$, the field equations give the condition $f_0 = 0$ and the solutions at further orders do not depend on this parameter. On the other hand, considering the first term in R, f_0 has the meaning of a cosmological constant.

A further step is to analyze the Newtonian limit starting from the action (1) and considering a generic function of the Ricci scalar and the scalar field. Then the action becomes [50]

$$\mathcal{A} = \int d^4x \sqrt{-g}[f(R,\phi) + \omega(\phi)\phi_{;\alpha}\phi^{;\alpha} + \mathcal{X}\mathcal{L}_m]. \quad (5)$$

The scalar field ϕ can be approximated as the Ricci scalar. In particular we get $\phi = \phi^{(0)} + \phi^{(1)} + \phi^{(2)} + \cdots$ and the function $f(R,\phi)$ with its partial derivatives (f_R, f_{RR}, f_ϕ, $f_{\phi\phi}$, and $f_{\phi R}$) and $\omega(\phi)$ can be substituted by their corresponding Taylor series. In the case of $f(R,\phi)$, we have [50]

$$f(R,\phi) \sim f(0,\phi^{(0)}) + f_R(0,\phi^{(0)})R^{(1)} + f_\phi(0,\phi^{(0)})\phi^{(1)} \ldots, \quad (6)$$

and analogous relations for the derivatives are obtained. From the lowest order of field equations we have [50]

$$f(0,\phi^{(0)}) = 0, \qquad f_\phi(0,\phi^{(0)}) = 0, \quad (7)$$

and also in this modified fourth-order gravity a missing cosmological component in the action (1) implies that the space-time is asymptotically Minkowskian (the same outcome as above). Moreover the ground value of scalar field ϕ must be a stationary point of the potential.

An important remark is due at this point. As discussed in detail in [50], a theory like $f(R,\phi)$ is dynamically equivalent to $f(R,\Box R)$ and the meaning of the scalar field results is also clearly related to the further gravitational degrees of freedom that come out in extended gravity [51,52].

III. SIMULATED ORBITS OF THE S2 STAR AND THE YUKAWA-LIKE CORRECTIONS

In order to constrain the parameters of the $f(R)$ and $f(R,\phi)$ models, we simulate orbits of the S2 star in Yukawa-like gravity potentials and fit them to the astrometric observations obtained by the New Technology Telescope/Very Large Telescope (NTT/VLT) (see Fig. 1 in [33]), which are publicly available as the supplementary online data to the electronic version of the paper [33] at [53], and which are aimed at estimating the distance from the Galactic Centre and to map the inner region of our Galaxy.

As discussed, in $f(R)$-gravity, the scalar curvature R of the Hilbert-Einstein action is replaced by a generic function $f(R)$. As a result, in the weak field limit [54], the gravitational potential is found to be Yukawa like [10,55]:

$$\Phi(r) = -\frac{GM}{(1+\delta)r}[1 + \delta e^{-(\frac{r}{\Lambda})}], \quad (8)$$

where $\Lambda^2 = -f_1/f_2$ is an arbitrary parameter (usually referred to as the range of interaction), depending on the typical scale of the system under consideration, and $\delta = f_1 - 1$ is a universal constant. It is worth noticing that δ and Λ depend on the parameters of the given $f(R)$ gravity model. It is important to stress that a Yukawa-like correction has been invoked several times in the past [56]. Such corrections have been obtained, as a general feature, in the framework of $f(R)$ gravity [25] and successfully applied to clusters of galaxies setting [29]. In general, one can relate the length-scale Λ to the mass of the effective scalar field introduced by the extended theory of gravity and then to the mass and characteristic size of the self-gravitating system [1]. The larger the mass, the smaller Λ will be and the faster the exponential decay of the correction will be; i.e., the larger the mass, the quicker the recovery of the Newtonian dynamics. Equation (8) then gives us the opportunity to investigate in a unified way the impact of a large class of modified gravity theories, included in which are the extended theories, since other details do not have any impact on the galactic scales we are interested in.

In the $f(R,\phi)$ gravity the gravitational potential is found by setting the gravitational constant as

$$G = \left(\frac{2\omega(\phi^{(0)})\phi^{(0)} - 4}{2\omega(\phi^{(0)})\phi^{(0)} - 3}\right)\frac{G_\infty}{\phi^{(0)}}, \quad (9)$$

where G_∞ is the gravitational constant as measured at infinity and, by imposing $\alpha^{-1} = 3 - 2\omega(\phi^{(0)})\phi^{(0)}$, the gravity potential is [50]

$$\Phi_{ST}(\mathbf{x}) = -\frac{G_\infty M}{|\mathbf{x}|}\left\{1 + \alpha e^{-\sqrt{1-3\alpha}m_\phi|\mathbf{x}|}\right\} \quad (10)$$

and a Sanders-like potential is fully recovered [56]. Such a potential has often been used to obtain the rotation curves of spiral galaxies [57]. However, it is worth stressing that the only Sanders potential is unable to reproduce the rotation curves of spirals without dark matter, as pointed out in an accurate study in [58]. However, the paradigm remains valid and modifications of Newtonian potential can be investigated in view of addressing the dark matter problem.

We can set the value of the derivatives of the Taylor expansion as $f_{R\phi} = 1, f_{RR} = 0, f_R = \phi$ without losing generality.

The simulated orbits of the S2 star in these two potentials can be obtained by a numerical integration of the corresponding differential equations of motion; that is,

$$\dot{\mathbf{r}} = \mathbf{v}, \qquad \mu\ddot{\mathbf{r}} = -\nabla\Phi(\mathbf{r}), \tag{11}$$

where μ is the reduced mass in the two-body problem. We assume the following mass and distance for the Sgr A* central massive compact object around which the S2 star is orbiting: $M = 4.3 \times 10^6 M_\odot$ and $d_\star = 8.3$ kpc, respectively [33]. For simplicity reasons, we perform two-body simulations and neglect perturbations from other members of the S-star cluster, as well as from some possibly existing extended structures composed by visible or dark matter [16].

We simulate orbits of the S2 star and fit them to the NTT/VLT astrometric observations for different combinations of *a priori* given values of f_1 and f_2 in $f(R)$, and α and m_ϕ in $f(R,\phi)$ gravity potentials (below denoted as *parameter*1 and *parameter*2, respectively). Each simulated orbit is defined by the following four initial conditions: two components of initial position and two components of initial velocity in orbital plane at the epoch of the first observation. For each combination of *parameter*1 and *parameter*2, we obtain the best fit initial conditions corresponding to a simulated orbit with the lowest discrepancy in respect to the observed one. The fitting itself is performed using the LMDIF1 routine from the MINPACK-1 FORTRAN 77 library which solves the nonlinear least squares problems by a modification of the Marquardt-Levenberg algorithm [59], according to the following procedure:

(1) In the first iteration we use a guess of initial position (x_0, y_0) and velocity $(\dot{x}_0, \dot{y}_0)$ of the S2 star in the orbital plane (the true orbit) at the epoch of the first observation;
(2) the true positions (x_i, y_i) and velocities $(\dot{x}_i, \dot{y}_i)$ at all successive observed epochs are then calculated by the numerical integration of equations of motion (11), and projected to the corresponding positions (x_i^c, y_i^c) in the observed plane (the apparent orbit);
(3) discrepancy between the simulated and the observed apparent orbit is estimated by the reduced χ^2:

$$\chi^2 = \frac{1}{2N-\nu}\sum_{i=1}^{N}\left[\left(\frac{x_i^o - x_i^c}{\sigma_{xi}}\right)^2 + \left(\frac{y_i^o - y_i^c}{\sigma_{yi}}\right)^2\right], \tag{12}$$

where (x_i^o, y_i^o) and (x_i^c, y_i^c) are the corresponding observed and calculated apparent positions, N is the number of observations, ν is the number of initial conditions (in our case $\nu = 4$), and σ_{xi} and σ_{yi} are uncertainties of the observed positions;
(4) the new initial conditions are estimated by the fitting routine and steps 2 and 3 are repeated until the fit is converging, i.e., until the minimum of reduced χ^2 is achieved.

Finally, we kept the values of *parameter*1 and *parameter*2 for which the smallest value of minimized reduced χ^2 is obtained—in other words, which results with the best fit simulated orbit of the S2 star with the lowest discrepancy with respect to the observed one.

IV. RESULTS AND DISCUSSION

Our point is that the Yukawa-like correction, coming from $f(R)$ gravity, can be used in order to fix the coefficients in the expansion (4). For the expansion up to the second order, we have two parameters to fix. Orbits of the S2 star around the Galactic Centre are, in principle, a very straightforward tool in order to test any theory of gravity. In [2], there is an overview of self-gravitating structures, at different scales, whose dynamics could be described without asking for dark matter. According to [2], the relations between f_1, f_2, and δ and the Λ parameters are $f_1 = 1 + \delta$, $f_2 = -(1+\delta)/(\Lambda^2)$.

Specifically, we have to find the minimal values of the reduced χ^2 in order to determine f_1 and f_2 assuming $f_0 = 0$. This allows us to reconstruct $f(R)$ models up to the second order.

Figure 1 presents the maps of the reduced χ^2 over the $\{f_1 - f_2\}$ parameter space for all simulated orbits of the S2 star which give at least the same or better fits to the NTT/VLT observations of the S2 star than the Keplerian orbits ($\chi^2 = 1.89$). The left panel corresponds to f_1 in the range $[-25, 0]$, and the right panel to the range [0, 25], respectively. We can see that, in a large region of the parameter space, χ^2 of the orbits in modified potential is less than the value in Newtonian potential. However, it seems that we cannot constrain both f_1 and f_2 using only the observed S2 orbits because these two parameters are strongly correlated. We can constrain only their ratio f_1/f_2. According to [2], the effective mass is $m^2 = -f_1/(3f_2)$. The solutions are valid if $m^2 > 0$, i.e., f_1 and f_2 are assumed to have different signs.

This is a degeneracy problem that has to be removed in order to obtain reliable results. Such a problem is also found in fitting the flat rotation curve of spiral galaxies. As is discussed in detail in [50], the f_1/f_2 degeneracy can be removed by using potentials coming from $f(R,\phi)$ gravity. In this model, two potentials, $\Psi(x)$ and $\Phi(x)$, result as entries of the metric in the Newtonian limit. The combination of both potentials gives rise to the effective potential (10) that affects the particle (in our case the S2 star).

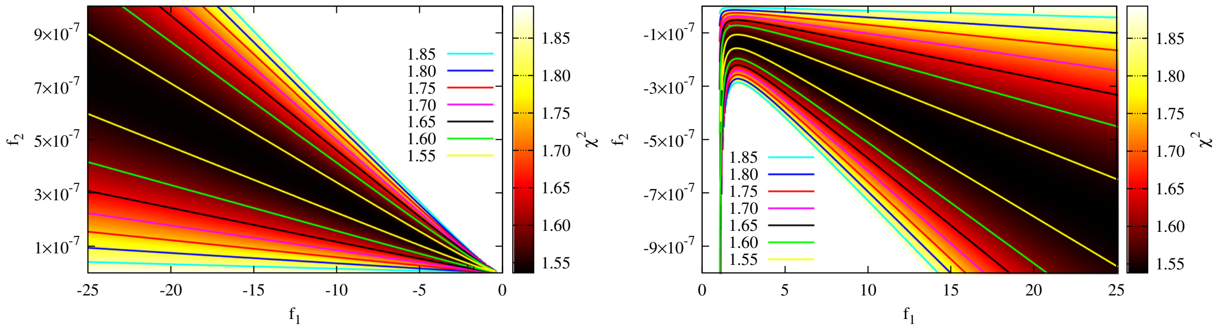

FIG. 1 (color online). The maps of reduced χ^2 over the $\{f_1 - f_2\}$ parameter space of $f(R)$ gravity in case of NTT/VLT observations of the S2 star which give at least the same ($\chi^2 = 1.89$) or better fits ($\chi^2 < 1.89$) than the Keplerian orbits. The left panel corresponds to f_1 in $[-25, 0]$, and the right panel to f_1 in [0, 25]. With a decreasing value of χ^2 (better fit) colors in gray scale are darker. A few contours are presented for specific values of reduced χ^2 given in the figure's legend.

The potential includes the gravitational constant measured at infinity G_∞. The relation between G_∞ and G, the gravitational constant measured in the laboratory, is given by the above formula (9). If we take $\omega(\phi_0) = 1/2$ and use the relation $1/\alpha = 3 - 2\omega(\phi_0)\phi_0$, we get $\phi_0 = 3 - 1/\alpha$. Combining these relations, we get $G_\infty = G/(1+\alpha)$.

Our aim is to determine f_0, f_R, f_{RR}, f_ϕ, $f_{\phi\phi}$, and $f_{\phi R}$. For the lowest order of the field, as we said, one can set $f_0 = 0$ and $f_\phi = 0$. We use also the further constraints given in [50] at lowest order, that is, $f_{\phi R} = 1$, $f_{RR} = 0$, and $f_R = \phi_0$. This last relation gives $f_R = \phi_0 = 3 - 1/\alpha$.

For $f(R,\phi)$ gravity, one can define a further effective mass [50], that is, $m_\phi^2 = -f_{\phi\phi}/(2\omega(\phi_0))$, and if we take $\omega(\phi_0) = 1/2$, we immediately get $f_{\phi\phi} = -m_\phi^2$.

Finally we can assume the following set of parameters: $f_0 = 0$, $f_R = 3 - 1/\alpha$, $f_\phi = 0$, $f_{RR} = 0$, $f_{\phi R} = 1$, and $f_{\phi\phi} = -m_\phi^2$. These choices are physically reliable and mean that we can assume an asymptotic Minkowski background, i.e., $f_0 = 0$, that the general relativity is recovered for $f_\phi = 0$, $f_{RR} = 0$, $f_{\phi R} = 1$, and effective massive modes (and then effective lengths) are related to $f_R = 3 - 1/\alpha$ and $f_{\phi\phi} = -m_\phi^2$. In particular, $f_0 = 0$ means that the cosmological constant can be discarded at local scales.

Figures 2 and 3 are the maps of the reduced χ^2 over the $\{\alpha - m_\phi\}$ parameter space in $f(R,\phi)$ gravity for all simulated orbits of the S2 star which give at least the same or better fits than the Keplerian orbits ($\chi^2 = 1.89$). The left panel in Fig. 2 corresponds to m_ϕ in [0, 0.06] and α in [0, 0.33], and the right panel to the zoomed range of m_ϕ in [0, 0.03] and α in [0, 0.05], respectively. For $\alpha < 0$, there is no region in the parameter space where $\chi^2 < 1.89$ (the Keplerian case). For $0 < \alpha < 1/3$ there are two regions where $\chi^2 < 1.89$ (for $m_\phi < 0$ and $m_\phi > 0$), but the absolute minimum is for $m_\phi < 0$. We obtained the absolute minimum of the reduced χ^2 for α in the interval [0.0001, 0.0004], and m_ϕ in the interval $[-0.0029, -0.0023]$ (see Fig. 3). The absolute minimum of the reduced χ^2 ($\chi^2 = 1.5011$) is obtained for $\alpha = 0.00018$ and $m_\phi = -0.0026$, respectively.

The simulated orbits of the S2 star around the Galactic Centre in Sanders gravity potential (the blue solid line) and in Newtonian gravity potential (the red dashed line) for $\alpha = 0.00018$ and $m_\phi = -0.0026$ during 10 periods are presented in Fig. 4. We can notice that the precession of the S2 star orbit has the same direction as in general relativity. The precession of the S2 star orbit in the same direction can

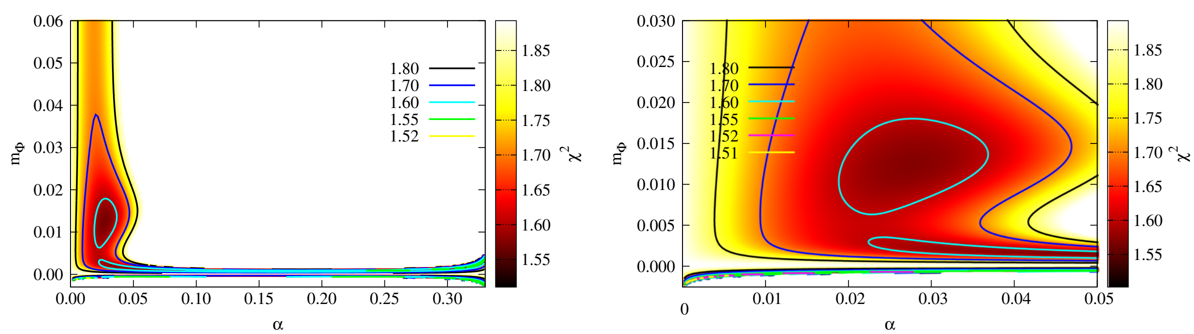

FIG. 2 (color online). The maps of reduced χ^2 over the $\{\alpha - m_\phi\}$ parameter space of $f(R,\phi)$ gravity in the case of NTT/VLT observations of the S2 star which give at least the same ($\chi^2 = 1.89$) or better fits ($\chi^2 < 1.89$) than the Keplerian orbits. The left panel corresponds to m_ϕ in [0, 0.06] and α in [0, 0.33], and the right panel to the zoomed range of m_ϕ in [0, 0.03] and α in [0, 0.05]. With a decreasing value of χ^2 (better fit) colors in gray scale are darker. A few contours are presented for specific values of reduced χ^2 given in the figure's legend.

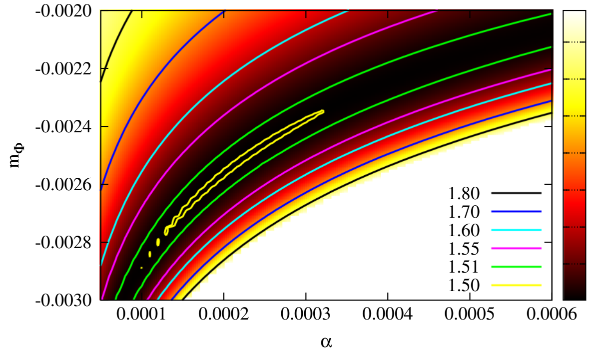

FIG. 3 (color online). The same as in Fig. 2, but for a narrow region in the $\{\alpha - m_\phi\}$ parameter space around the absolute minimum of the reduced χ^2. With a decreasing value of χ^2 (better fit) colors in gray scale are darker. A few contours are presented for specific values of reduced χ^2 given in the figure's legend.

also be obtained for some ranges of parameter δ in the general Yukawa potential (for more details see paper [21]). As can be read from Fig. 4, the best fit orbit in Sanders gravity potential precesses for about 3°.1 per orbital period.

In the case of Sanders potential, analytical calculation of orbital precession is very complicated to obtain, so we calculated it numerically and presented it in Figs. 5 and 6 as a function of α and m_ϕ. Assuming that a potential does not differ significantly from Newtonian potential, we derive perturbing potential from

$$V(r) = \Phi(r) - \Phi_N(r); \qquad \Phi_N(r) = -\frac{GM}{r}. \tag{13}$$

Obtained perturbing potential is of the form

$$V(r) = -\frac{GM\alpha}{r(1+\alpha)}(e^{-\sqrt{1-3\alpha}\cdot m_\phi \cdot r} - 1), \tag{14}$$

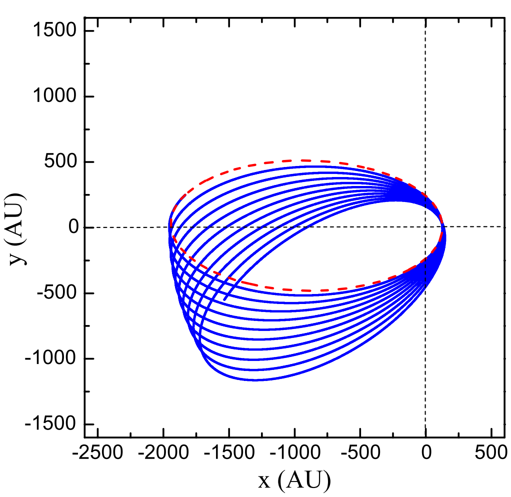

FIG. 4 (color online). Comparison between the orbit of the S2 star in Newtonian potential (the red dashed line) and Sanders-like potential for the best fit parameters (the absolute minimum of reduced $\chi^2 = 1.5011$) $\alpha = 0.00018$ and $m_\phi = -0.0026$ during ten orbital periods (the blue solid line).

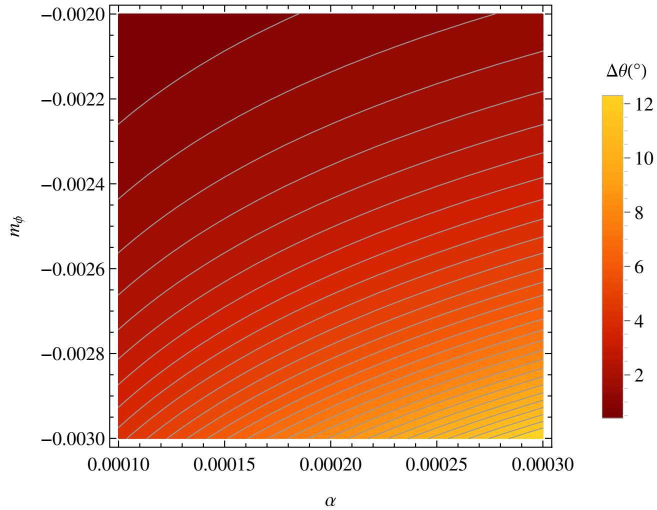

FIG. 5 (color online). Numerically calculated angle of precession per orbital period as a function of parameters α in the range [0.0001, 0.0003] and m_ϕ in the range $[-0.003, -0.002]$ in the case of Sanders-like potential. With a decreasing value of angle of precession colors are darker.

and it can be used for calculating the precession angle according to Eq. (30) from paper [41]:

$$\Delta\theta = \frac{-2L}{GMe^2}\int_{-1}^{1}\frac{z\cdot dz}{\sqrt{1-z^2}}\frac{dV(z)}{dz}, \tag{15}$$

where r is related to z via $r = \frac{L}{1+ez}$. By differentiating the perturbing potential $V(z)$ and substituting its derivative and expression for the semilatus rectum of the orbital ellipse ($L = a(1-e^2)$) in Eq. (15) above, and taking the same

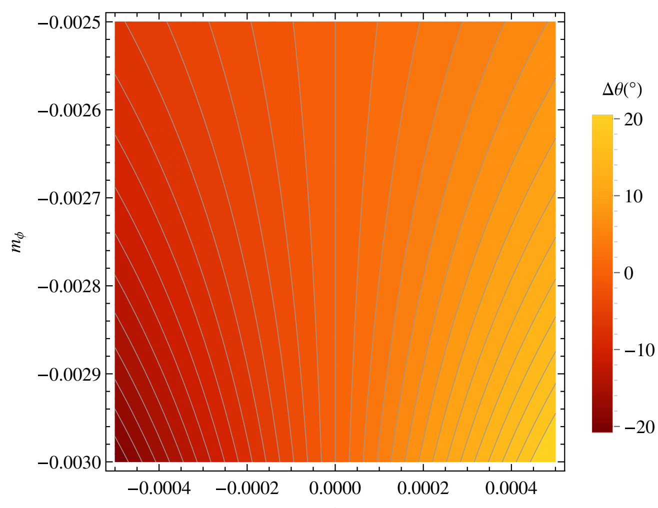

FIG. 6 (color online). The same as in Fig. 5, but for α in the range $[-0.0005, 0.0005]$ and m_ϕ in the range $[-0.003, -0.0025]$. The pericenter advance (like in GR) is obtained for positive α, and retrograde precession for negative α. With a decreasing value of angle of precession colors are darker.

values for orbital elements of the S2 star as paper [19], we obtained numerically for $\alpha = 0.00018$ and $m_\phi = -0.0026$ that precession per orbital period is $3^\circ.053$.

Graphical presentation of the precession per orbital period for α in the range [0.0001, 0.0003] and m_ϕ in $[-0.003, -0.002]$ is given in Fig. 5, and the case for α in $[-0.0005, 0.0005]$ and m_ϕ in $[-0.003, -0.0025]$ is presented in Fig. 6. As one can see, pericenter advance (like in general relativity [GR]) is obtained for positive α, and retrograde precession for negative α. However, it should be taken into account that fits better than Keplerian are obtained only for positive α and hence for the precession in the same direction as in GR.

General relativity predicts that the pericenter of the S2 star should advance by $0^\circ.08$ per orbital revolution [34], which is much smaller than the value of precession per orbital period in Sanders gravity potential, but the direction of the precession is the same.

V. CONCLUSIONS

In this paper, we compared the observed and simulated S2 star orbits around the Galactic Centre, in order to constrain the parameters of gravitational potentials derived from modified gravity models. The obtained results are quite comfortable for the effective gravitational potential derived from $f(R,\phi)$ gravity that, essentially, reproduces Sanders-like potentials [56,57] phenomenologically adopted to explain the rotation curves of spiral galaxies. Also if these kinds of potentials are not sufficient in addressing completely the problem of dark matter in galaxies [58], they give indications that alternative theories of gravity could be viable in describing galactic dynamics.

In other words, orbital solutions derived from such a potential are in good agreement with the reduced χ^2 deduced for Keplerian orbits. This fact allows us to fix the range of variation for α and m_ϕ, the two parameters characterizing the potential (10). The precession of the S2 star orbit obtained for the best fit parameter values ($\alpha = 0.00018$ and $m_\phi = -0.0026$) has the positive direction, as in general relativity.

In particular, we fitted the NTT/VLT astrometric observations of the S2 star, which contain a possible indication for orbital precession around a massive compact object at the Galactic Centre, in order to constrain the parameters of Sanders-like gravity potential since this theory has not been tested at these scales yet. We obtained much larger orbital precession of the S2 star in Sanders-like gravity than the corresponding value predicted by general relativity. In the paper [34], the authors presented the Newtonian orbit but with a moved position of central point mass in a different way. In that way they explained observed precession. In our calculated orbit of Sanders-like potential for best fitting parameters, we also obtained precession, but with a fixed position of central point mass since we do not have to move it.

However, one should keep in mind that we considered an idealized model ignoring many uncertainty factors, such as extended mass distributions, perturbations from nonsymmetric mass distribution, etc. Therefore, future observations with advanced facilities which will enable extremely accurate measurements of the positions of stars of $\sim 10\ \mu$as [60], or the European Extra Large Telescope, with an expected accuracy of $\sim$50–100 μas [61], are needed in order to verify the claims in this paper. As a final remark, we believe that the surveys aimed at giving details on the dynamics around the Galactic Centre could be a powerful tool to test theories of gravity.

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[1] S. Capozziello and M. De Laurentis, Phys. Rep. **509**, 167 (2011).
[2] S. Capozziello and M. De Laurentis, Ann. Phys. (Amsterdam) **524**, 545 (2012).
[3] S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011).
[4] S. Kopeikin and I. Vlasov, Phys. Rep. **400**, 209 (2004).
[5] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[6] E. Fischbach and C. L. Talmadge, *The Search for Non-Newtonian Gravity* (Springer, New York, 1999).
[7] S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2002).
[8] S. Capozziello, V. F. Cardone, S. Carloni, and A. Troisi, Int. J. Mod. Phys. D **12**, 1969 (2003).
[9] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).
[10] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*, Fundamental Theories of Physics Vol. 170 (Springer, New York, 2010).
[11] G. Leon and E. N. Saridakis, Classical Quantum Gravity **28**, 065008 (2011).
[12] H. A. Buchdahl, Mon. Not. R. Astron. Soc. **150**, 1 (1970).
[13] S. Capozziello, V. F. Cardone, and A. Troisi, Phys. Rev. D **73**, 104019 (2006).
[14] S. Capozziello, V. F. Cardone, and A. Troisi, Mon. Not. R. Astron. Soc. **375**, 1423 (2007).

[15] A. F. Zakharov, A. A. Nucita, F. De Paolis, and G. Ingrosso, Phys. Rev. D **74**, 107101 (2006).
[16] A. F. Zakharov, A. A. Nucita, F. De Paolis, and G. Ingrosso, Phys. Rev. D **76**, 062001 (2007).
[17] A. A. Nucita, F. De Paolis, G. Ingrosso, A. Qadir, and A. F. Zakharov, Publ. Astron. Soc. Pac. **119**, 349 (2007).
[18] C. Frigerio Martins and P. Salucci, Mon. Not. R. Astron. Soc. **381**, 1103 (2007).
[19] D. Borka, P. Jovanović, V. Borka Jovanović, and A. F. Zakharov, Phys. Rev. D **85**, 124004 (2012).
[20] A. F. Zakharov, D. Borka, V. Borka Jovanović, and P. Jovanović, Adv. Space Res. **54**, 1108 (2014).
[21] D. Borka, P. Jovanović, V. Borka Jovanović, and A. F. Zakharov, J. Cosmol. Astropart. Phys. 11 (2013) 050.
[22] B. Li and J. D. Barrow, Phys. Rev. D **75**, 084010 (2007).
[23] V. F. Cardone and S. Capozziello, Mon. Not. R. Astron. Soc. **414**, 1301 (2011).
[24] K. S. Stelle, Gen. Relativ. Gravit. **9**, 353 (1978).
[25] S. Capozziello, A. Stabile, and A. Troisi, Mod. Phys. Lett. A **24**, 659 (2009).
[26] L. Iorio, Mon. Not. R. Astron. Soc. **401**, 2012 (2010).
[27] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl, and S. Schlamminger, Prog. Part. Nucl. Phys. **62**, 102 (2009).
[28] S. Capozziello, A. Stabile, and A. Troisi, Phys. Rev. D **76**, 104019 (2007).
[29] S. Capozziello, E. de Filippis, and V. Salzano, Mon. Not. R. Astron. Soc. **394**, 947 (2009).
[30] D. M. Lucchesi and R. Peron, Phys. Rev. D **89**, 082002 (2014).
[31] A. M. Ghez, M. Morris, E. E. Becklin, A. Tanner, and T. Kremenek, Nature (London) **407**, 349 (2000).
[32] A. M. Ghez, S. Salim, N. N. Weinberg, J. R. Lu, T. Do, J. K. Dunn, K. Matthews, M. R. Morris, S. Yelda, E. E. Becklin, T. Kremenek, M. Milosavljević, and J. Naiman, Astrophys. J. **689**, 1044 (2008).
[33] S. Gillessen, F. Eisenhauer, T. K. Fritz, H. Bartko, K. Dodds-Eden, O. Pfuhl, T. Ott, and R. Genzel, Astrophys. J. **707**, L114 (2009).
[34] S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins, and T. Ott, Astrophys. J. **692**, 1075 (2009).
[35] R. Schödel, T. Ott, R. Genzel *et al.*, Nature (London) **419**, 694 (2002).
[36] R. Genzel, F. Eisenhauer, and S. Gillessen, Rev. Mod. Phys. **82**, 3121 (2010).
[37] S. Gillessen, R. Genzel, T. K. Fritz *et al.*, Nature (London) **481**, 51 (2012).
[38] L. Meyer, A. M. Ghez, R. Schödel, S. Yelda, A. Boehle, J. R. Lu, T. Do, M. R. Morris, E. E. Becklin, and K. Matthews, Science **338**, 84 (2012).
[39] T. Fritz, S. Gillessen, S. Trippe, T. Ott, H. Bartko, O. Pfuhl, K. Dodds-Eden, R. Davies, F. Eisenhauer, and R. Genzel, Mon. Not. R. Astron. Soc. **401**, 1177 (2010).
[40] G. F. Rubilar and A. Eckart, Astron. Astrophys. **374**, 95 (2001).
[41] G. S. Adkins and J. McDonnell, Phys. Rev. D **75**, 082001 (2007).
[42] N. N. Weinberg, M. Milosavljević, and A. M. Ghez, Astrophys. J. **622**, 878 (2005).
[43] S. Nojiri and S. D. Odintsov, Int. J. Geom. Methods Mod. Phys. **04**, 115 (2007).
[44] J. W. Moffat, J. Cosmol. Astropart. Phys. 05 (2005) 003.
[45] J. W. Moffat, J. Cosmol. Astropart. Phys. 03 (2006) 004.
[46] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012).
[47] S. Capozziello and M. Francaviglia, Gen. Relativ. Gravit. **40**, 357 (2008).
[48] S. Capozziello, M. De Laurentis, and V. Faraoni, Open Astron. J. **2**, 1874 (2009).
[49] S. Capozziello and S. Tsujikawa, Phys. Rev. D **77**, 107501 (2008).
[50] A. Stabile and S. Capozziello, Phys. Rev. D **87**, 064002 (2013).
[51] S. Gottlöber, H.-J. Schmidt, and A. A. Starobinsky, Classical Quantum Gravity **7**, 893 (1990).
[52] L. Amendola, A. Battaglia-Mayer, S. Capozziello, S. Gottlöber, V. Müller, F. Occhionero, and H.-J. Schmidt, Classical Quantum Gravity **10**, L43 (1993).
[53] See http://cdsbib.u-strasbg.fr/cgi-bin/cdsbib?2009ApJ...707L.114G.
[54] T. Clifton and J. D. Barrow, Phys. Rev. D **72**, 103005 (2005).
[55] N. R. Napolitano, S. Capozziello, A. J. Romanowsky, M. Capaccioli, and C. Tortora, Astrophys. J. **748**, 87 (2012).
[56] R. H. Sanders, Astron. Astrophys. **136**, L21 (1984).
[57] R. H. Sanders, Annu. Rev. Astron. Astrophys. **2**, 1 (1990).
[58] F. Duval *et al.*, Astron. Astrophys. **227**, 33 (1990).
[59] J. J. Moré, B. S. Garbow, and K. E. Hillstrom, User Guide for MINPACK-1, Argonne National Laboratory Report No. ANL-80-74, 1980.
[60] S. Gillessen, F. Eisenhauer, G. Perrin *et al.*, Proc. SPIE Int. Soc. Opt. Eng. **342**, 77 (2010).
[61] See http://www.eso.org/public/products/brochures/e-eltsciencecase/.