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Renormalizable toy model of massive spin-two field and new bigravity

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In this paper, we propose a toy model of the renormalizable theory describing a massive spin-two field. Although the model is renormalizable, we show that the model contains ghosts. The coupling of the theory with gravity can be regarded as a new kind of bimetric gravity or bigravity. We show that the massive spin-two field plays the role of the cosmological constant.

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I. INTRODUCTION

After the establishment of the free theory of massive gravity by Fierz and Pauli [1] (for a recent review, see Ref. [2]), a consistent interacting theory has not been found during the past three quarters of a century. One of the reasons is the appearance of the Boulware-Deser ghost [3,4] in general, and another is the appearance of the van Dam–Veltman–Zakharov (vDVZ) discontinuity [5] in the massless limit $m \to 0$, although the discontinuity can be screened by the Vainstein mechanism [6] (see, for example, Ref. [7]).

After the elapse of seventy-five years from the work by Fierz and Pauli [1], there has been remarkable progress in the study of nonlinear massive gravity, and the ghost-free models, which are called the de Rham, Gabadadze, and Tolley (dRGT) models, have been found in Refs. [8–10]. These models have a nondynamical background metric, but they have been extended to the models with a dynamical metric [11–13], which are called bigravity models. After that, cosmology was studied in the massive gravity models [14] in the decoupling limit, where the models reduce to scalar-tensor theories, and several activities in the massive gravity models [15–18] and in the bimetric gravity models [19–26] have followed after that.

The absence of ghosts was shown in the Hamiltonian analysis [13] by using the Arnowitt-Deser-Misner (ADM) formalism, where the metric is assumed to be

$$g_{00} = -N^2 + g^{ij}N_iN_j, \qquad g_{0i} = N_i, \qquad g_{ij} = g_{ij}. \quad (1)$$

Here $i, j = 1, 2, 3$; N is called the lapse function; and N_i is called the shift function. We denote the inverse of g_{ij} by g^{ij}. In the dRGT models, after the redefinition of the shift function N_i, the Hamiltonian becomes linear to the lapse function N, and in the expression of the new shift function given by solving the equation obtained from the variation of the new shift function, the lapse function N does not appear. Therefore, the variation of N gives a constraint on g_{ij} and their conjugate momenta. By combining this constraint with the secondary constraint derived from the constraint, an extra degree of freedom corresponding to the ghost is eliminated. Because the existence of the Boulware-Deser ghost may depend on initial conditions, the Boulware-Deser ghost in three-dimensional bigravity model was studied in Ref. [27] by using the Hamiltonian analysis.

Recently in Ref. [28], possibilities have been proposed of new nonlinear ghost-free derivative interactions in massive gravity. After that, however, in Ref. [29], it has been shown that a class of the derivative interactions includes ghosts, and a kind of no-go theorem prohibiting the derivative interactions has been claimed. In this paper, we show the existence of the nonlinear derivative interactions which are not included in Ref. [29], although such derivative interactions generate ghosts, unfortunately.

Motivated by such analyses, we propose a power-counting renormalizable model describing the massive spin-two field. The model could not be, however, really renormalizable, because the projection operators included in the propagator generate nonrenormalizble divergences. This problem is, however, solved by adding a term where a vector field couples with the massive spin-two field. Although the model could be renormalizable by investigating the spectrum of this model, we find that ghosts could appear, and therefore the model cannot be a realistic one but rather a kind of toy model. Because the gravity is not renormalizable, we may consider the coupling of the power-counting renormalizable model, which could not be really renormalizable, with gravity. The model can be regarded as a new kind of bigravity.

II. STILL NEW DERIVATIVE INTERACTION IN MASSIVE GRAVITY?

In Ref. [28], by using the perturbation $h_{\mu\nu}$ from the flat metric

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \quad (2)$$

as a dynamical variable, new ghost-free interactions were proposed. The interaction terms have the following form:

$$\mathcal{L}_{d,0} \sim \eta^{\mu_1\nu_1\cdots\mu_n\nu_n} h_{\mu_1\nu_1}\dots h_{\mu_n\nu_n}. \tag{3}$$

Or, in terms including d derivatives, which are called pseudolinear terms (see also Ref. [30]):

$$\mathcal{L}_{d,n} \sim \eta^{\mu_1\nu_1\cdots\mu_n\nu_n} \partial_{\mu_1}\partial_{\nu_1} h_{\mu_2\nu_2}\dots\partial_{\mu_{d-1}}\partial_{\nu_{d-1}} h_{\mu_d\nu_d} h_{\mu_{d+1}\nu_{d+1}}\dots h_{\mu_{n+d/2}\nu_{n+d/2}}. \tag{4}$$

Here $\eta^{\mu_1\nu_1\cdots\mu_n\nu_n}$ is given by the product of n $\eta_{\mu\nu}$ and antisymmetrizing the indexes $\nu_1, \nu_2, \dots,$ and ν_n, for example:

$$\eta^{\mu_1\nu_1\mu_2\nu_2} \equiv \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2} - \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1},$$
$$\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \equiv \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3} - \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_2} + \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1} - \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_3} + \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_2} - \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_1}. \tag{5}$$

It is evident that these terms are linear with respect to h_{00}, which could be a perturbation of the lapse function N in the Hamiltonian, and there do not appear terms which include both h_{00} and h_{0i}. Therefore, the variation of h_{00} gives a constraint for h_{ij} and their conjugate momenta π_{ij}, and therefore the ghost could be eliminated, although we may need more careful Hamiltonian analysis.

The nonlinear counterparts for Eq. (3) are nothing but the mass terms and the interaction terms in the dRGT models:

$$\eta^{\mu_1\nu_1\cdots\mu_n\nu_n} h_{\mu_1\nu_1}\dots h_{\mu_n\nu_n} \sim \sqrt{-g} g^{\mu_1\nu_1\cdots\mu_n\nu_n} \mathcal{K}_{\mu_1\nu_1}\dots\mathcal{K}_{\mu_n\nu_n}. \tag{6}$$

Here $\mathcal{K}_{\mu\nu}$ is defined by

$$\mathcal{K}_\mu{}^\nu \equiv \delta_\mu{}^\nu - \sqrt{g^{-1}f}_\mu^\nu, \tag{7}$$

and $f_{\mu\nu}$ is the fiducial metric and often chosen to be $f_{\mu\nu} = \eta_{\mu\nu}$.

In $D = 4$ space-time dimensions, a possible nontrivial term with two derivatives is given by

$$\mathcal{L}_{2,2} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \partial_{\mu_1}\partial_{\nu_1} h_{\mu_2\nu_2} h_{\mu_2\nu_2} \tag{8}$$

and

$$\mathcal{L}_{2,3} \sim \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \partial_{\mu_1}\partial_{\nu_1} h_{\mu_2\nu_2} h_{\mu_2\nu_2} h_{\mu_3\nu_3}. \tag{9}$$

The nonlinear counterpart of Eq. (8) could be nothing but the Einstein-Hilbert term. In case of massive gravity, there is another candidate of the nonlinear counterpart for Eq. (8) [31], which is

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} R_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3}. \tag{10}$$

The nontrivial, fully nonlinear counterpart of Eq. (9) could be also given by

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} R_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3}\mathcal{K}_{\mu_4\nu_4}. \tag{11}$$

Here $g^{\mu_1\nu_1\cdots\mu_n\nu_n}$ is, as in the definition of $\eta^{\mu_1\nu_1\cdots\mu_n\nu_n}$, given by the product of n $g_{\mu\nu}$ and antisymmetrizing the indexes $\nu_1, \nu_2, \dots,$ and ν_n.

In Ref. [29], however, it has been shown that the nonlinear terms [Eqs. (10) and (11)] could generate the ghost by using the mini-superspace, where

$$N = N(t), \qquad N_i = 0, \qquad g_{ij} = a(t)^2 \eta_{ij}. \tag{12}$$

In fact, in the mini-superspace [Eq. (12)], the terms in Eqs. (10) and (11) have the following form [29]:

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} R_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3} \sim Na^3 \left[2\frac{\dot{a}^2}{a^2N^2} - \frac{\dot{a}^2}{a^3N^2} + \frac{\dot{a}^2}{a^2N^3} \right], \tag{13}$$

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} R_{\mu_1\mu_2\nu_1\nu_2} \mathcal{K}_{\mu_3\nu_3}\mathcal{K}_{\mu_4\nu_4} \sim Na^3 \left[\frac{\dot{a}^2}{a^2N^3} - \frac{\dot{a}^2}{a^3N^2} + \frac{\dot{a}^2}{a^2N^3} - \frac{\dot{a}^2}{a^3N^3} \right]. \tag{14}$$

The expressions (13) and (14) tell that in the Hamiltonian, the terms in Eqs. (10) and (11) generate the terms which are not linear with respect to the lapse function N. Therefore, the equation given by the variation of N can be solved with respect to N and does not give any constraint on g_{ij} or their conjugate momenta, which tells us that the ghost could not be eliminated.

We should note that the terms in Eqs. (10) and (11) are not unique terms reproducing Eqs. (8) and (9), respectively. Another candidate reproducing Eq. (8) is

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} (\nabla_{\mu_1}\nabla_{\nu_1}\mathcal{K}_{\mu_2\nu_2})\mathcal{K}_{\mu_3\nu_3}, \tag{15}$$

and a candidate for Eq. (9) is

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} (\nabla_{\mu_1}\nabla_{\nu_1}\mathcal{K}_{\mu_2\nu_2})\mathcal{K}_{\mu_3\nu_3}\mathcal{K}_{\mu_4\nu_4}. \tag{16}$$

In the mini-superspace [Eq. (12)], these terms can be expressed as

$$\sqrt{-g} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} (\nabla_{\mu_1}\nabla_{\nu_1}\mathcal{K}_{\mu_2\nu_2})\mathcal{K}_{\mu_3\nu_3} \sim Na^3 \left[\frac{\dot{a}^2}{a^2N^4} \right], \tag{17}$$

$$\sqrt{-g}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}(\nabla_{\mu_1}\nabla_{\nu_1}\mathcal{K}_{\mu_2\nu_2})\mathcal{K}_{\mu_3\nu_3}\mathcal{K}_{\mu_4\nu_4} \sim Na^3\left[\frac{\dot{a}^2}{a^3N^4}-\frac{\dot{a}^2}{a^2N^4}\right]. \tag{18}$$

From the above expressions (17) and (18), however, we find that the terms in Eqs. (15) and (16) could also generate the ghost. The ghost could not be eliminated even if we consider the combinations in Eqs. (13), (14), (15), (17), and (18).

We should note that there is another candidate to reproduce Eq. (8):

$$\sqrt{-g}g^{\nu\nu'\rho\rho'\sigma\sigma'}f^{\nu'\nu''}\nabla_\nu\mathcal{K}_{\rho\rho'}\nabla_{\nu''}\mathcal{K}_{\sigma\sigma'}. \tag{19}$$

Here $f^{\mu\nu}=\eta^{\mu\nu}$. In the mini-superspace, however, this term has the following form:

$$\sqrt{-g}g^{\nu\nu'\rho\rho'\sigma\sigma'}f^{\nu'\nu''}\nabla_\nu\mathcal{K}_{\rho\rho'}\nabla_{\nu''}\mathcal{K}_{\sigma\sigma'} \sim Na^3\left[-\frac{6\dot{a}^2}{a^3N}+\frac{6\dot{a}^2}{aN^3}-\frac{6\dot{a}^2}{N^4}\right], \tag{20}$$

and therefore the ghost could not be eliminated even if we consider any combination with other terms.

Then we consider the possibility of other classes of the no-ghost interactions by relaxing the assumption in Ref. [29]. In the argument so far, we have considered the terms which have invariance under the general coordinate transformation if the fiducial metric $f_{\mu\nu}$ could be a dynamical tensor. We may relax this condition and require only the Lorentz invariance. Then we may consider the term given by replacing the covariant derivatives ∇_μ in Eq. (19) with the partial derivative ∂_μ:

$$\sqrt{-g}g^{\nu\nu'\rho\rho'\sigma\sigma'}f^{\nu'\nu''}\partial_\nu\mathcal{K}_{\rho\rho'}\partial_{\nu''}\mathcal{K}_{\sigma\sigma'}. \tag{21}$$

In the mini-superspace [Eq. (12)], this term is surely linear with respect to N. Then we check if the term in Eq. (21) could give interactions without ghosts by using the full ADM formalism. Explicitly, the term in Eq. (21) has the following form:

$$\begin{aligned}
&\sqrt{-g}\delta_{\mu_1}{}^{[\nu_1}\delta_{\mu_2}{}^{\nu_2}\delta_{\mu_3}{}^{\nu_3]}\eta^{\mu_1\rho}\partial_{\nu_1}\mathcal{K}^{\mu_2}{}_{\nu_2}\partial_\rho\mathcal{K}^{\mu_3}{}_{\nu_3}\\
&=\sqrt{-g}\Big[-\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{i}\Big)^2+2\Big(\partial_i\sqrt{g^{-1}\eta}^{0}{}_{0}\Big)\Big(\partial_i\sqrt{g^{-1}\eta}^{k}{}_{k}\Big)+\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{j}\Big)\Big(\partial_i\sqrt{g^{-1}\eta}^{k}{}_{k}\Big)\\
&\quad-\Big(\partial_0\sqrt{g^{-1}\eta}^{0}{}_{i}\Big)\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{j}\Big)+\Big(\partial_i\sqrt{g^{-1}\eta}^{i}{}_{0}\Big)\Big(\partial_0\sqrt{g^{-1}\eta}^{j}{}_{j}\Big)-\Big(\partial_i\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{0}{}_{0}\Big)\\
&\quad-\Big(\partial_i\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{k}{}_{k}\Big)+\Big(\partial_0\sqrt{g^{-1}\eta}^{0}{}_{i}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)+\Big(\partial_i\sqrt{g^{-1}\eta}^{i}{}_{0}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{0}{}_{j}\Big)\\
&\quad-\Big(\partial_i\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_0\sqrt{g^{-1}\eta}^{j}{}_{0}\Big)+\Big(\partial_i\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_k\sqrt{g^{-1}\eta}^{j}{}_{k}\Big)+\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_0\sqrt{g^{-1}\eta}^{j}{}_{i}\Big)\\
&\quad-2\Big(\partial_i\sqrt{g^{-1}\eta}^{0}{}_{j}\Big)\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{0}\Big)-\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{k}\Big)\Big(\partial_i\sqrt{g^{-1}\eta}^{k}{}_{j}\Big)+\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{0}{}_{i}\Big)\\
&\quad+\Big(\partial_i\sqrt{g^{-1}\eta}^{0}{}_{j}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{i}{}_{0}\Big)-\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{0}\Big)\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)+\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{k}\Big)\Big(\partial_k\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\\
&\quad-\Big(\partial_0\sqrt{g^{-1}\eta}^{j}{}_{j}\Big)\Big(\partial_i\sqrt{g^{-1}\eta}^{0}{}_{i}\Big)+\Big(\partial_i\sqrt{g^{-1}\eta}^{j}{}_{j}\Big)\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{0}\Big)-\Big(\partial_i\sqrt{g^{-1}\eta}^{0}{}_{0}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\\
&\quad-\Big(\partial_i\sqrt{g^{-1}\eta}^{k}{}_{k}\Big)\Big(\partial_j\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big].
\end{aligned} \tag{22}$$

So that a ghost cannot appear, the term should be given in the form where the time derivative of the lapse and shift functions do not appear. This kind of form might be obtained by the cancellations between several terms after the partial integration. Because this kind of cancellation should occur between the terms including the same number of time derivatives, we now consider the following terms:

$$\sqrt{-g}\Big[-\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{i}\Big)^2+\Big(\partial_0\sqrt{g^{-1}\eta}^{i}{}_{j}\Big)\Big(\partial_0\sqrt{g^{-1}\eta}^{j}{}_{i}\Big)\Big]. \tag{23}$$

As in Ref. [10], for convenience, we use the redefined shift function n^i, which is given by

$$N^i=(\delta^i_j+ND^i{}_j)n^j. \tag{24}$$

The definition of $D^i{}_j$ is given by solving the following equation [10]:

$$(\sqrt{1-n^T\mathbf{I}n})D=\sqrt{(\gamma^{-1}-Dnn^TD^T)\mathbf{I}},\qquad \mathbf{I}=\delta_{ij},\qquad \mathbf{I}^{-1}=\delta^{ij}. \tag{25}$$

By using n^i, we rewrite $\sqrt{g^{-1}\eta}^{\mu}{}_{\nu}$ as follows:

$$\sqrt{g^{-1}\eta} = \frac{1}{N}\mathcal{A} + \mathcal{B}. \tag{26}$$

Here

$$\mathcal{A} = \frac{1}{\sqrt{1-n^T\boldsymbol{I}n}}\begin{pmatrix} 1 & n^T\boldsymbol{I} \\ -n & -nn^T\boldsymbol{I} \end{pmatrix},$$
$$\mathcal{B} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{(\gamma^{-1} - Dnn^TD^T)\boldsymbol{I}} \end{pmatrix}. \tag{27}$$

In order to simplify the notation, we define the following quantities:

$$A := \frac{1}{\sqrt{1-n^T\boldsymbol{I}n}}, \quad B^l := n^l, \quad C^i{}_j := \sqrt{(\gamma^{-1} - Dnn^TD^T)\boldsymbol{I}}. \tag{28}$$

By using Eq. (28), $\sqrt{g^{-1}\eta}^{\mu}{}_{\nu}$ can be rewritten as

$$\sqrt{g^{-1}\eta}^{\mu}{}_{\nu} = \begin{pmatrix} A/N & AB^l\delta_{li}/N \\ -AB^j/N & -B^iB^k\delta_{kj}/N + C^i{}_j \end{pmatrix}, \tag{29}$$

and $\partial_0\sqrt{g^{-1}\eta}^i{}_j$ can be expressed as follows:

$$\partial_0\sqrt{g^{-1}\eta}^i{}_j = \frac{B^iB^k\delta_{kj}\dot{N}}{N^2} - \frac{\dot{B}^iB^k\delta_{kj}}{N} - \frac{B^i\dot{B}^k\delta_{kj}}{N} + \dot{C}^i{}_j. \tag{30}$$

Therefore, by using ADM variables, Eq. (23) has the following form:

$$\begin{aligned}
&\sqrt{-g}\left[-\left(\partial_0\sqrt{g^{-1}\eta}^i{}_i\right)^2 + \left(\partial_0\sqrt{g^{-1}\eta}^i{}_j\right)\left(\partial_0\sqrt{g^{-1}\eta}^j{}_i\right)\right] \\
&= N\sqrt{\gamma}\left[-\frac{2(B^i\dot{B}^k\delta_{ik})^2}{N^2} - (\dot{C}^i{}_i)^2 + \frac{4(B^i\dot{B}^k\delta_{ik})\dot{C}^j{}_j}{N} - \frac{2(B^iB^k\delta_{ik})\dot{N}\dot{C}^j{}_j}{N^2}\right. \\
&\quad \left. + \frac{2B^iB^k\delta_{kj}\dot{C}^j{}_i\dot{N}}{N^2} + \frac{2(B^l\delta_{li}B^i)(\dot{B}^l\delta_{li}\dot{B}^i)}{N^2} - \frac{2\dot{B}^iB^k\delta_{kj}\dot{C}^j{}_i}{N} - \frac{2B^i\dot{B}^k\delta_{kj}\dot{C}^j{}_i}{N} + \dot{C}^i{}_j\dot{C}^j{}_i\right].
\end{aligned} \tag{31}$$

From expression (31), we find that the time derivatives of the lapse and shift functions cannot be canceled, and therefore ghosts could appear.

III. RENORMALIZABLE MODEL OF MASSIVE SPIN-TWO FIELD

We now propose a power-counting renormalizable model of the massive spin-two field, whose Lagrangian density is given by

$$\begin{aligned}
\mathcal{L}_{h0} &= -\frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2})h_{\mu_3\nu_3} + \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2} \\
&\quad - \frac{\mu}{3!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\lambda}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4} \\
&= -\frac{1}{2}(h\Box h - h^{\mu\nu}\Box h_{\mu\nu} - h\partial^\mu\partial^\nu h_{\mu\nu} - h_{\mu\nu}\partial^\mu\partial^\nu h + 2h_\nu{}^\rho\partial^\mu\partial^\nu h_{\mu\rho}) \\
&\quad + \frac{m^2}{2}(h^2 - h_{\mu\nu}h^{\mu\nu}) - \frac{\mu}{3!}(h^3 - 3hh_{\mu\nu}h^{\mu\nu} + 2h_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\mu) \\
&\quad - \frac{\lambda}{4!}(h^4 - 6h^2h_{\mu\nu}h^{\mu\nu} + 8hh_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\mu - 6h_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\sigma h_\sigma{}^\mu + 3(h_{\mu\nu}h^{\mu\nu})^2).
\end{aligned} \tag{32}$$

Here m and μ are parameters with the dimension of mass, and λ is a dimensionless parameter. Therefore, the model given by the Lagrangian is power-counting renormalizable. The model could be also free from ghosts.

We should note, however, that the propagator is given by

$$D^m_{\alpha\beta,\rho\sigma} = -\frac{1}{2(p^2+m^2)}\left\{P^m_{\alpha\rho}P^m_{\beta\sigma} + P^m_{\alpha\sigma}P^m_{\beta\rho} - \frac{2}{3}P^m_{\alpha\beta}P^m_{\rho\sigma}\right\}, \tag{33}$$

$$P^m_{\mu\nu} \equiv \eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}. \tag{34}$$

Then when p^2 is large, the propagator behaves as $D^m_{\alpha\beta,\rho\sigma} \sim \mathcal{O}(p^2)$ due to the projection operator $P^m_{\mu\nu}$, which makes the behavior for large p^2 worse, and therefore the model should not be renormalizable.

There is a similar problem in the model of the massive vector field, whose Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}m^2 A^\mu A_\mu. \tag{35}$$

The propagator $D_{\mu\nu}$ of the massive vector is given by

$$D_{\mu\nu} = -\frac{1}{p^2 + m^2} P^m_{\mu\nu}, \tag{36}$$

which is the inverse of

$$O^{\mu\nu} \equiv -(p^2 + m^2)\eta^{\mu\nu} + p^\mu p^\nu, \tag{37}$$

that is,

$$O^{\mu\nu} D_{\nu\rho} = \delta^\mu{}_\rho. \tag{38}$$

The expression (36) tells us that for large p^2, $D_{\mu\nu}$ behaves as $\mathcal{O}(1)$, and therefore the model in Eq. (35) could not be renormalizable. If the vector field, however, couples only with the conserved current J_μ which satisfies the conservation law $\partial^\mu J_\mu = 0$, the term $\frac{p_\mu p_\nu}{m^2}$ in the projection operator $P^m_{\mu\nu}$ drops, and the propagator behaves as $D_{\mu\nu} \sim \mathcal{O}(1/p^2)$, and therefore the model may become renormalizable.

Instead of imposing the conservation law, we may add the following term to the action:

$$2\alpha\phi\partial^\mu A_\mu, \tag{39}$$

and consider the inverse of the operator

$$O_{A\phi} = \begin{pmatrix} O^{\mu\nu} & -i\alpha p^\mu \\ i\alpha p^\nu & 0 \end{pmatrix}, \tag{40}$$

which is given by

$$D_{A\phi} = \begin{pmatrix} -\frac{1}{p^2+m^2} P_{\nu\rho} & -i\frac{p_\nu}{\alpha p^2} \\ i\frac{p_\rho}{\alpha p^2} & \frac{m^2}{\alpha^2 p^2} \end{pmatrix}, \tag{41}$$

$$P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}, \tag{42}$$

$$O_{A\phi} D_{A\phi} = \begin{pmatrix} \delta^\mu{}_\rho & 0 \\ 0 & 1 \end{pmatrix}. \tag{43}$$

The projection operator $P_{\mu\nu}$ is equal to the projection operator $P^m_{\mu\nu}$ on shell, $p^2 = -m^2$, but their behavior for large p^2 becomes different from each other. As a result, the propagator between two A_μ's behaves as $\mathcal{O}(1/p^2)$, and therefore the model could become renormalizable if the interaction terms are also renormalizable. We should note that by construction, we are assuming that the interactions are given by A_μ, and the interactions do not include the scalar field ϕ. This tells us that in the internal lines of the loops in the Feynmann diagrams, the propagators of the two vector fields A appear, but the propagators between two scalars ϕ and those between the vector field A_μ and the scalar field ϕ do not appear. Therefore, although the propagator between the vector field A_μ and the scalar field ϕ behaves as $\mathcal{O}(1/p)$ instead of $\mathcal{O}(1/p^2)$, this behavior could not generate non-renormalizable divergence.

As we will see, however, the term (39) generates a ghost. The total Lagrangian density, Eq. (35) with Eq. (39), can be diagonalized by redefining the vector field A_μ by a new vector field B_μ, which is given by

$$A_\mu = B_\mu - \frac{2\alpha}{m^2}\partial_\mu\phi, \tag{44}$$

and we obtain

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) - \frac{1}{2}m^2 B^\mu B_\mu + \frac{2\alpha^2}{m^2}\partial^\mu\phi\partial_\mu\phi. \tag{45}$$

The propagator of the redefined vector field is given by Eq. (36), and therefore this propagator might appear to generate non-renormalizable divergences. We should also note that there appear non-renormalizable derivative interactions of the scalar field, which include $\partial_\mu\phi$. The non-renormalizable divergences generated by the derivative interactions should be canceled by the non-renormalizable divergences coming from the propagator [Eq. (36)] of the redefined vector field B_μ, and there could remain only renormalizable divergences. The cancellation is consistent with the renormalizability given by the propagator in Eq. (41). An important point is the following: We assume, by construction, that the interactions are not given in terms of the redefined vector field B_μ but in terms of A_μ, which is the vector field before the redefinition [Eq. (44)], and the interactions do not include the scalar field ϕ, either. Therefore, in the internal lines of the loops in the Feynmann diagrams, the propagators of the two vector fields always appear in the form of the propagators between the two vector fields A_μ in Eq. (41), and therefore there could not appear non-renormalizable divergences coming from the projection operator [Eq. (34)] in the propagator [Eq. (36)].

We should note, however, that the + sign in front of the kinetic term of the scalar field tells us that the scalar field is a ghost, which generates the negative norm states in the quantum theory, and therefore the model given here is not consistent as a quantum theory.

Anyway, we may consider deformation of the model similar to that in Eq. (39) by adding the following new term to the Lagrangian density [Eq. (32)]:

$$\mathcal{L} = \mathcal{L}_{h0} + 4\alpha A^{\mu}\partial^{\nu}h_{\mu\nu}, \tag{46}$$

and consider the following equation:

$$\begin{pmatrix} \mathcal{O}^{\mu\nu,\alpha\beta} & -i\alpha(p^{\mu}\eta^{\alpha\nu} + p^{\nu}\eta^{\alpha\mu}) \\ i\alpha(p^{\alpha}\eta^{\mu\beta} + p^{\beta}\eta^{\mu\alpha}) & 0 \end{pmatrix} \begin{pmatrix} D_{\alpha\beta,\rho\sigma} & -iE_{\sigma,\alpha\beta} \\ iE_{\alpha,\rho\sigma} & F_{\alpha\sigma} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\delta^{\mu}{}_{\rho}\delta^{\nu}{}_{\sigma} + \delta^{\mu}{}_{\alpha}\delta^{\nu}{}_{\beta}) & 0 \\ 0 & \delta^{\mu}{}_{\sigma} \end{pmatrix}. \tag{47}$$

We should note that $\mathcal{O}^{\mu\nu,\alpha\beta}$ is given by

$$\begin{aligned} \mathcal{O}^{\mu\nu,\alpha\beta} = &-\left\{\frac{1}{2}(P^{\mu\alpha}P^{\nu\beta} + P^{\mu\beta}P^{\nu\alpha}) - P^{\mu\nu}P^{\alpha\beta}\right\}(p^2+m^2) \\ &-\left\{\frac{1}{2}(p^{\alpha}p^{\mu}P^{\nu\beta} + p^{\beta}p^{\mu}P^{\nu\alpha} + p^{\alpha}p^{\nu}P^{\mu\beta} + p^{\beta}p^{\nu}P^{\mu\alpha}) - p^{\mu}p^{\nu}P^{\alpha\beta} - p^{\alpha}p^{\beta}P^{\mu\nu}\right\}\frac{m^2}{p^2}. \end{aligned} \tag{48}$$

Then we find

$$D_{\alpha\beta,\rho\sigma} = -\frac{1}{2(p^2+m^2)}\{P_{\alpha\rho}P_{\beta\sigma} + P_{\alpha\sigma}P_{\beta\rho} - P_{\alpha\beta}P_{\rho\sigma}\}, \tag{49}$$

$$E_{\alpha,\rho\sigma} = \frac{1}{2\alpha p^2}\left\{p_{\rho}P_{\alpha\sigma} + p_{\sigma}P_{\alpha\rho} - \frac{m^2 p_{\alpha}}{2(p^2+m^2)}P_{\rho\sigma} + \frac{p_{\alpha}p_{\rho}p_{\sigma}}{p^2}\right\}, \tag{50}$$

$$F_{\alpha\sigma} = \frac{m^2}{2\alpha^2 p^2}P_{\alpha\sigma} + \frac{3m^4}{8\alpha^2(p^2)^2(p^2+m^2)}p_{\alpha}p_{\sigma}. \tag{51}$$

Because the propagator between two $h_{\mu\nu}$'s behaves as $\mathcal{O}(1/p^2)$, the model could become renormalizable.

We should note that the coupling of $h_{\mu\nu}$ with the energy-momentum tensor $T_{\mu\nu}$, $\kappa^2 h^{\mu\nu}T_{\mu\nu}$, which appears in general relativity, breaks the renormalizability because κ has the dimension of length. The coupling with a scalar field ϕ or the Rarita-Schwinger field ψ_{μ} can be, however, renormalizable:

$$\phi\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}, \qquad h^{\mu\nu}\bar{\psi}_{\mu}\psi_{\nu}, \tag{52}$$

which may appear when we supersymmetrize the action of Eqs. (32) or (46).

IV. HAMILTONIAN ANALYSIS AND SPECTRUM

It is not so clear what could be physical degrees of freedom in the Lagrangian [Eq. (46)]. Then in this section, we count the physical degrees of freedom by using the Hamiltonian analysis (for example, see Ref. [2]). After that, we diagonalize the free part and find what could be the physical degrees of freedom.

The free part $\mathcal{L}_0$ of the Lagrangian [Eq. (46)] is given by

$$\mathcal{L}_0 = -\frac{1}{2}\partial_{\lambda}h_{\mu\nu}\partial^{\lambda}h^{\mu\nu} + \partial_{\mu}h_{\nu\lambda}\partial^{\nu}h^{\mu\lambda} - \partial_{\mu}h^{\mu\nu}\partial_{\nu}h + \frac{1}{2}\partial_{\lambda}h\partial^{\lambda}h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + 4\alpha A_{\mu}\partial_{\nu}h^{\mu\nu}. \tag{53}$$

We now investigate the structure of the constraints for the free Lagrangian in Eq. (53). With integration by parts, the free Lagrangian [Eq. (53)] can be rewritten as follows:

$$\mathcal{L}_0 = \mathcal{F}(h_{ij}, \dot{h}_{ij}, h_{0i}) + h_{00}\mathcal{G}(h_{ij}) + 2\alpha A_{\mu}\partial_{\nu}h^{\mu\nu} - 2\alpha\partial_{(\mu}A_{\nu)}h^{\mu\nu}, \tag{54}$$

$$\begin{aligned} \mathcal{F}(h_{ij}, \dot{h}_{ij}, h_{0i}) = &\frac{1}{2}\dot{h}_{ij}^2 - \frac{1}{2}h_{jk,i}{}^2 + 2\dot{h}_{ij,i}h_{0j} - h_{j0,i}h_{i0,j} + h_{jk,i}h_{ik,j} - 2h_{0i}\dot{h}_{kk,i} - h_{ij,i}h_{kk,j} \\ &-\frac{1}{2}\dot{h}_{ii}^2 + \frac{1}{2}h_{ii,j}{}^2 + h_{0j,i}{}^2 - \frac{m^2}{2}[-2h_{0i}{}^2 + h_{ij}{}^2 - h_{ii}{}^2], \end{aligned} \tag{55}$$

$$\mathcal{G}(h_{ij}) = -h_{ij,ij} + h_{kk,jj} - m^2 h_{ii}. \tag{56}$$

Then the conjugate momenta are given by

$$\pi_{00} = \frac{\partial\mathcal{L}_0}{\partial\dot{h}_{00}} = 2\alpha A_0, \qquad \pi_{0i} = \frac{\partial\mathcal{L}_0}{\partial\dot{h}_{0i}} = -2\alpha A_i,$$
$$\pi_{ij} = \frac{\partial\mathcal{L}_0}{\partial\dot{h}_{ij}} = \dot{h}_{ij} - \dot{h}_{kk}\delta_{ij} - \partial_i h_{j0} - \partial_j h_{i0} + 2\partial_k h_{0k}\delta_{ij},$$
$$\pi_0 = \frac{\partial\mathcal{L}_0}{\partial\dot{A}_0} = -2\alpha h_{00}, \qquad \pi_i = \frac{\partial\mathcal{L}_0}{\partial\dot{A}_i} = 2\alpha h_{0i}. \tag{57}$$

Then we find

$$\dot{h}_{ij} = \pi_{ij} - \frac{1}{2}\pi_{kk}\delta_{ij} + \partial_i h_{j0} + \partial_j h_{i0}. \tag{58}$$

Equations in Eq. (57) give the following primary constraints:

$$\phi^1 = \pi_{00} - 2\alpha A_0, \qquad \phi_i^2 = \pi_{0i} + 2\alpha A_i,$$
$$\phi^3 = \pi_0 + 2\alpha h_{00}, \qquad \phi_i^4 = \pi_i - 2\alpha h_{i0}. \tag{59}$$

The nonvanishing components of the Poisson brackets between the constraints are given by

$$\{\phi^1(\vec{x}), \phi^3(\vec{y})\} = -4\alpha\delta(\vec{x}-\vec{y}),$$
$$\{\phi_i^2(\vec{x}), \phi_j^4(\vec{y})\} = 4\alpha\delta_{ij}\delta(\vec{x}-\vec{y}). \tag{60}$$

This tells $\det\{\phi,\phi\} \neq 0$, and we can determine the Lagrange multipliers, and we find there are no secondary constraints. Then we have a total of eight constraints in the phase space. Because the symmetric tensor has ten degrees of freedom and the vector has four, we have originally 28 degrees of freedom in the phase space. By subtracting eight degrees of freedom from the constraints, there remain 20 degrees of freedom in the phase space—that is, ten degrees of freedom in the coordinate space.

We now investigate what could be the ten physical degrees of freedom. We should also note that the free part $\mathcal{L}_0$ of the Lagrangian [Eq. (46)] can be diagonalized as in Eq. (45) by the redefinition

$$h_{\mu\nu}(x) = l_{\mu\nu}(x) + 4\alpha \int d^4y \hat{D}^m_{\mu\nu,\rho\sigma}(x-y)\partial^\rho A^\sigma(y) \tag{61}$$

as follows:

$$\mathcal{L}_0 = -\frac{1}{2}(l\Box l - l^{\mu\nu}\Box l_{\mu\nu} - l\partial^\mu\partial^\nu l_{\mu\nu} - l_{\mu\nu}\partial^\mu\partial^\nu l + 2l_\nu{}^\rho\partial^\mu\partial^\nu l_{\mu\rho})$$
$$+ \frac{m^2}{2}(l^2 - l_{\mu\nu}l^{\mu\nu}) - \frac{4\alpha^2}{m^2}\{A^\mu\Box A_\mu + (\partial^\mu A_\mu)^2\} + \frac{16\alpha^2}{3m^2}\partial^\mu A_\mu\left(1 - \frac{\Box}{m^2}\right)\partial^\nu A_\nu. \tag{62}$$

In Eq. (61), $\hat{D}^m_{\mu\nu,\rho\sigma}(x-y)$ is the propagator expressed by the coordinates x and y and defined by

$$\left(\eta^{\mu\nu}\eta^{\rho\sigma}\Box - \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})\Box - \eta^{\mu\nu}\partial^\rho\partial^\sigma - \eta^{\rho\sigma}\partial^\mu\partial^\nu l + \frac{1}{2}(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho)\right.$$
$$\left. + m^2\left(\eta^{\mu\nu}\eta^{\rho\sigma} - \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})\right)\right)\hat{D}^m_{\rho\sigma,\alpha\beta}(x-y) = \frac{1}{2}(\delta^\mu{}_\alpha\delta^\nu{}_\beta + \delta^\mu{}_\beta\delta^\nu{}_\alpha)\delta(x-y), \tag{63}$$

which is given by the Fourier transformation of $D^m_{\alpha\beta,\rho\sigma}$ in Eq. (33):

$$\hat{D}^m_{\mu\nu,\rho\sigma}(x-y) = \int \frac{d^4p}{(2\pi)^2} D^m_{\alpha\beta,\rho\sigma} e^{ip(x-y)}. \tag{64}$$

Then we find

$$h_{\mu\nu}(x) = l_{\mu\nu}(x) - \frac{2\alpha}{m^2}\left(\partial_\nu A_\mu(x) + \partial_\mu A_\nu(x) - \frac{2}{3}\eta_{\mu\nu}\partial_\rho A^\rho(x) - \frac{4}{3m^2}\partial_\mu\partial_\nu\partial_\rho A^\rho(x)\right), \tag{65}$$

which gives

$$h = h^\mu{}_\mu = l + \frac{4\alpha}{3m^4}(2\Box + m^2)\partial_\rho A^\rho. \tag{66}$$

The Lagrangian in Eq. (62) is the sum of the Lagrangian of the Fierz-Pauli massive gravity and the vector field A_μ except for the last term. The last term might be regarded to be a gauge-fixing term. The higher-derivative part can be further rewritten by using a new vector field V_μ as follows:

$$\partial^\mu A_\mu \left(1 - \frac{\Box}{m^2}\right)\partial^\nu A_\nu \sim \partial^\mu A_\mu \partial^\nu A_\nu + \partial^\mu V_\mu \partial^\nu A_\nu - \frac{m^2}{4} V^\mu V_\mu. \tag{67}$$

In fact, the variation of V_μ gives $V_\mu = -\frac{2}{m^2}\partial_\mu \partial^\nu A_\nu$. By substituting the expression of V_μ, we obtain the original expression. We now define a propagator $\Delta_{\mu\nu}$ by

$$\left(\eta^{\mu\nu}\Box + \frac{1}{3}\partial^\mu \partial^\nu\right)\Delta_{\nu\rho}(x-y) = \delta^\mu{}_\rho \delta(x-y). \tag{68}$$

Then, redefining A_μ by

$$A_\mu = B_\mu - \frac{2}{3}\int d^4y \Delta_{\nu\rho}(x-y)\partial^\rho \partial^\sigma V_\sigma(y), \tag{69}$$

the Lagrangian density [Eq. (62)] can be rewritten as

$$\mathcal{L}_0 = -\frac{1}{2}(l\Box l - l^{\mu\nu}\Box l_{\mu\nu} - l\partial^\mu \partial^\nu l_{\mu\nu} - l_{\mu\nu}\partial^\mu \partial^\nu l + 2l_\nu{}^\rho \partial^\mu \partial^\nu l_{\mu\rho}) + \frac{m^2}{2}(l^2 - l_{\mu\nu}l^{\mu\nu}) - \frac{4\alpha^2}{m^2}\{B^\mu \Box B_\mu + (\partial^\mu B_\mu)^2\} + \frac{16\alpha^2}{3m^2}(\partial^\mu B_\mu)^2 - \frac{4\alpha^2}{3m^2}(\partial^\mu V_\mu)^2 - \frac{4\alpha^2}{3}V^\mu V_\mu. \tag{70}$$

The Lagrangian density is the sum of the Lagrangian of the Fierz-Pauli massive spin-two field $l_{\mu\nu}$ and the vector field B_μ with a gauge-fixing term and the action of an exotic vector field V_μ. V_i's are not dynamical, but V_0 is dynamical, because there is no term including the derivative of V_i's with respect to time. Therefore, V_μ contains only one degree of freedom. Because A_μ has four degrees of freedom after the gauge fixing, we have ten degrees of freedom in all, including the massive graviton $l_{\mu\nu}$, which is consistent with the previous Hamiltonian analysis.

In the Lagrangian density [Eq. (70)], the sign in front of the kinetic term of the vector field is not canonical, and therefore the vector field is a ghost. Although the model contains ghost fields, the model could be renormalizable, and therefore the model proposed in this paper might be regarded as a kind of toy model. If we could extend the model to have a local symmetry, some physical state condition may select physical states where no ghost state appears.

The bigravity model can be regarded as a model where a massive spin-two field couples with gravity. Then we may consider the model where $h_{\mu\nu}$, whose Lagrangian is given by Eq. (32), couples with gravity:

$$S = \int d^4x\sqrt{-g}\left\{-\frac{1}{2}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} + \frac{1}{2}m^2 g^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2} - \frac{\mu}{3!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\lambda}{4!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}\right\}, \tag{71}$$

which can be regarded as a new bigravity model because there appear two symmetric tensor fields $g_{\mu\nu}$ and $h_{\mu\nu}$. We should note that $h_{\mu\nu}$ is not the perturbation in $g_{\mu\nu}$, but $h_{\mu\nu}$ is a field independent of $g_{\mu\nu}$. Because the gravity is not renormalizable, we forget about the renormalizability and drop the last term in Eq. (46), where the vector field A_μ couples with $h_{\mu\nu}$.

V. SUMMARY

In summary, we considered the nonlinear derivative interactions which are not included in Ref. [29], but unfortunately we have shown that such derivative interactions could generate ghosts. We also investigated the possibility of other classes of the no-ghost interactions by only requiring the Lorentz invariance.

Motivated by the above analyses, we proposed a power-counting renormalizable model describing the massive spin-two field, which could not be really renormalizable because the projection operators included in the propagator generate non-renormalizble divergences. We solved this problem by adding a new term where a vector field A_μ couples with the massive spin-two field $h_{\mu\nu}$. By investigating the spectrum of this model, it was shown that there could appear ghosts, and therefore the model cannot be a realistic one, but we can regard this model as a kind of toy model, which may be a candidate of the renormalizable model.

Because the gravity is not renormalizable, we may consider the coupling of the power-counting renormalizable model, which could not be really renormalizable, with gravity. The model can be regarded as a new kind of bimetric gravity or bigravity. In the Appendix, we show that the field of the massive spin-two field plays the role of the cosmological constant. It is easy to see that a vacuum solution like the Schwarzschild solution or Kerr solution in the Einstein gravity becomes a solution of the new bigravity model.

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APPENDIX: COSMOLOGY BY NEW BIGRAVITY

We may consider the cosmology given by the action in Eq. (71) with the Einstein-Hilbert action:

$$S_{\rm EH} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R. \tag{A1}$$

We assume the solution of equations given by the actions (71) and (A1) is given by

$$h_{\mu\nu} = Cg_{\mu\nu}. \tag{A2}$$

Here C is a constant. We can directly check that Eq. (A2) satisfies the field equation given by the variation of $h_{\mu\nu}$ and also the Einstein equation by properly choosing C. By substituting Eq. (A2) into the action (71), we find

$$S = -\int d^4x\sqrt{-g}V(C),$$
$$V(C) \equiv -6m^2C + 4\mu C^3 + \lambda C^4. \tag{A3}$$

We should note that $\nabla_\rho g_{\mu\nu} = 0$. The constant C can be determined by the equation $V'(C) = 0$. We now parametrize m^2 and μ by

$$m^2 = -\frac{\lambda}{3}C_1C_2, \qquad \mu = -\frac{\lambda}{3}(C_1 + C_2). \tag{A4}$$

Then the solutions of $V'(C)$ are given by

$$C = 0, C_1, C_2, \tag{A5}$$

and we find

$$V(C_1) = \frac{\lambda}{3}C_1^3(-C_1 + 2C_2),$$
$$V(C_2) = \frac{\lambda}{3}C_2^3(-C_2 + 2C_1). \tag{A6}$$

Then we find that $V(C)$ plays the role of the cosmological constant. Let assume $0 < C_1 < C_2$ and $C_2 < 2C_1$. Then $V(C_1)$ is a local maximum and $V(C_2) > 0$ is a local minimum. Then $V(C_1)$ or $V(C_2)$ might generate the inflation.

It has been shown that the causality could be broken in the previous bigravity models [32] due to the existence of the superluminal mode. We should note that in the model given by the actions (71) and (A1), the superluminal mode does not appear, and therefore the causality could not be broken.

We should also note that under the assumption of Eq. (A2), we can construct black hole solutions as in the standard bigravity model (see, for example, Refs. [33,34]).

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