

Implications of an axino LSP for naturalness

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Both the naturalness of the electroweak symmetry breaking and the resolution of the strong CP problem may require a small Higgsino mass μ generated by a realization of the DFSZ axion model. Assuming the axino is the lightest supersymmetric particle, we study its implications on μ and the axion scale. Copiously produced light Higgsinos at collider (effectively only neutral next-to-lightest superparticles pairs) eventually decay to axinos leaving prompt multileptons or displaced vertices which are being looked for at the LHC. We use latest LHC7 + 8 results to derive current limits on μ and the axion scale. Various Higgsino-axino phenomenology is illustrated by comparing with a standard case without lightest axinos as well as with a more general case with additional light gauginos in the spectrum.

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I. INTRODUCTION

The strong CP problem is elegantly resolved by introducing a Peccei-Quinn (PQ) symmetry [1] and its spontaneous breaking resulting in a dynamical field called the axion [2]. In this mechanism, the CP-violating QCD θ term is determined by a vacuum expectation value of the axion that dynamically cancels out the nonzero QCD θ term. The PQ symmetry can be realized either by introducing heavy quarks (KSVZ) [3] or by extending the Higgs sector (DFSZ) [4] and its breaking scale v_{PQ} is related with the axion coupling constant as $f_a \equiv \sqrt{2}v_{\text{PQ}}/N_{\text{DW}}$ with N_{DW} being the domain wall number counting the QCD anomaly.¹ The conventionally allowed window of the axion coupling constant is $10^9 \lesssim f_a/\text{GeV} \lesssim 10^{12}$ (For a review, see [5]). The upper bound comes from the axion cold dark matter contribution which is cosmological model dependent. A recent simulation of axionic topological defect contributions provides a stringent upper bound $f_a/\text{GeV} \lesssim \text{a few} \times 10^{10}$ if PQ symmetry were broken after inflation [6]. The window can be widen if PQ-symmetry were broken before or during inflation in certain class of PQ symmetry breaking models avoiding too large axionic isocurvature perturbations [7]. The existence of such a high scale causes quadratic divergences to the Higgs boson mass and thus requires a huge fine-tuning to keep stable two scales, the electroweak scale and the PQ scale (or a generic UV scale).

Supersymmetry (SUSY) would be the best-known framework to avoid such a hierarchy problem. However, the electroweak symmetry breaking in SUSY suffers from a certain degree of fine-tuning to maintain a desirable potential minimization condition:

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \quad (1)$$

where $m_{H_{u,d}}$ are the soft masses of the two Higgs doublets, $\tan \beta \equiv v_u/v_d$ is the ratio of their vacuum expectation values, and μ is the Higgs bilinear parameter in the superpotential. As LHC finds no hint of SUSY, it pushes up the soft mass scale above TeV range, the minimization condition (1) requires a fine cancellation among different terms. Barring too huge cancellation, one may arrange $m_{H_{u,d}}$ and μ not too larger than m_Z . This has been advocated as “natural SUSY” [8] implying stops/sbottoms at sub-TeV and light Higgsinos with

$$\mu \lesssim 200 \text{ GeV}. \quad (2)$$

Such a spectrum can also be obtained radiatively with multi-TeV soft masses at a UV scale [9].

An electroweak μ may be related to the PQ symmetry in the manner of DFSZ [10], which introduces a nonrenormalizable superpotential in the Higgs sector:

$$W = \lambda_\mu \frac{P^2}{M_P} H_u H_d \quad (3)$$

where P and thus $H_u H_d$ carries a nontrivial PQ charge and M_P is the reduced Planck mass. Upon the PQ symmetry breaking $v_{\text{PQ}} \sim \langle P \rangle$, a μ term is generated by $\mu = \lambda_\mu \langle P \rangle^2 / M_P$. Once PQ-symmetry is broken, there appear the axion a , its scalar partner, the saxion s , and the fermion super-partner,

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¹The standard DFSZ model has $N_{\text{DW}} = 6$, but a certain variations can allow $N_{\text{DW}} = 1$ to avoid the domain wall problem.

the axino \tilde{a} . Forming an axion superfield $A = (s + ia, \tilde{a})$, one can schematically write down the effective μ -term superpotential,

$$W = \mu H_u H_d + c_H \frac{\mu}{v_{\text{PQ}}} A H_u H_d, \quad (4)$$

where c_H is a parameter depending on the PQ symmetry breaking sector; we use $c_H = 2$ in this paper. In the context of the natural SUSY having a small μ parameter, a neutral Higgsino tends to be the lightest supersymmetric particle (LSP) and thus is a dark matter candidate assuming R -parity. In this case, a heavy axino decay to the LSP can change the standard thermal Higgsino dark matter density resulting in different mixtures of the axion and Higgsino dark matter components depending on the PQ scale [11].

In this paper, we investigate implications of the axino LSP in the framework of “the natural SUSY DFSZ model”. Naively speaking, the axino mass is expected to be of order of the soft SUSY breaking scale, but it is in general model dependent [12,13]. As a dark matter, the abundance of axino depends on the history of the universe involving either the condensation of saxion or the reheating temperature of the primordial inflation. Axinos can be produced abundantly either by saxion decay [14,15] or by interactions with thermal particles [16–19].² To avoid axino over-production, we assume the axino is very light or the reheat temperature is low enough to suppress the thermal production in this paper.

Since Higgsinos are predicted to be light in the natural SUSY scenario, they can be copiously produced at the LHC and decay to axino plus the Higgs boson h or Z boson through the coupling in Eq. (4). This leads to interesting signatures of multileptons/jets and missing transverse energy (MET) which can be prompt or displaced depending on the PQ scale. Notice that the standard Higgsino LSP scenario is hard to probe as heavier Higgsino decays produce unobservably soft leptons or pions due to a small mass gap between a heavier Higgsino and the Higgsino LSP. Currently, the ATLAS and CMS collaborations look for prompt multilepton plus MET and displaced di-jet/lepton signatures. Applying the current search results to the Higgsino-axino system, we obtain various limits on the μ parameter as well as the PQ scale. We assume that sleptons, squarks, and gluinos are heavy, but see Refs. [21] for earlier collider studies in the presence of light sleptons.

In Sec. II, we first translate the current multilepton + MET search results to the Higgsino-bino system where the Higgsino and bino are taken to be the next-to-LSP (NLSP) and the LSP, respectively, and thus the NLSP decay to the LSP plus h or Z can lead to prompt multilepton signatures. In Sec. III, we turn into a case of the Higgsino NLSP and

the axino LSP which can lead to displaced vertices from the NLSP decay. Then, we extend our analysis to the case of the Higgsino NNLSP and the bino NLSP with the axino LSP in Sec. IV. LHC14 projections of displaced vertex searches are estimated in Sec. V to see how far the axion scale can be probed. Finally, we conclude in Sec. VI.

II. CURRENT LIMITS ON (N)LSP HIGGSINOS WITHOUT AXINOS

Before considering the axino LSP, let us first deduce and summarize the current exclusion bounds in the case of (1) the standard Higgsino-like NLSP and binolike LSP as well as in the case of (2) Higgsino-like LSP. The results will be later compared with those with axino LSPs.

Consider first the case (1) with Higgsino NLSP and bino LSP. Being relatively light, a sizable number of charged and neutral Higgsinos, χ_1^\pm and $\chi_{2,3}^0$, can be produced electro-weakly and decay to the LSP χ_1^0 through $\chi_1^\pm \rightarrow \chi_1^0 W^\pm$ and $\chi_{2,3}^0 \rightarrow \chi_1^0 + h, Z$. The neutral Higgsino decays to the Z boson are relevant to the multilepton searches,³ and its branching ratio (BR) is a function of t_β and the sign of μ . In Fig. 1, we show current bounds on the Higgsino NLSP overlapping the officially reported bound on the wino NLSPs from the $3\ell + \text{MET}$ search [27] for reference. The associate production of charged and neutral Higgsinos is the largest and is constrained from the $3\ell + \text{MET}$ search: $\chi_1^\pm \chi_2^0 \rightarrow \chi_1^0 \chi_1^0 W Z \rightarrow \chi_1^0 \chi_1^0 + 3\ell\nu$. Here, the dependence on the underlying model parameters such as t_β and the sign of μ is weak as demonstrated in the left panel of Fig. 2—the relevant BR of Higgsino pairs is in general close to a half [24]. So we use a positive μ to draw the bound in Fig. 1. The bound on Higgsinos is weaker than the official bound on winos due to two modifications: (i) the total production cross section of Higgsino pair $\chi_2^0 \chi_1^\pm + \chi_3^0 \chi_1^\pm$ is smaller than that of wino pairs $\chi_2^0 \chi_1^\pm$ (by about a factor 2 for $\mathcal{O}(100)$ GeV NLSPs), and (ii) the BR for $\chi_{2,3}^0 \rightarrow \chi_1^0 + Z$ is smaller than 1. The actual bound on winos will also be weaker than the officially reported one according to a smaller BR. On the other hand, other multilepton searches contributed mainly from other pair productions of Higgsinos currently lead to weaker or null bounds; for example, the associate production of two neutral Higgsinos is only weakly constrained from the $4\ell + \text{MET}$ search [28] via $\chi_2^0 \chi_3^0 \rightarrow \chi_1^0 \chi_1^0 Z Z \rightarrow \chi_1^0 \chi_1^0 4\ell$. In all, the NLSP Higgsino mass exclusion currently reaches up to about 250 GeV, while the LHC sensitivity drops quickly as the mass gap between the NLSP and LSP becomes smaller.

²For the axino dark matter property in the KSVZ model, see Ref. [20].

³Contributions from intermediate Higgs bosons are generally small because of the small leptonic BR via $h \rightarrow WW^*, ZZ^*$ although Higgs decay products can certainly be useful when Higgsinos are heavy [22–25]; see also [26]. Considering light Higgsinos, we ignore Higgs contributions in this work.

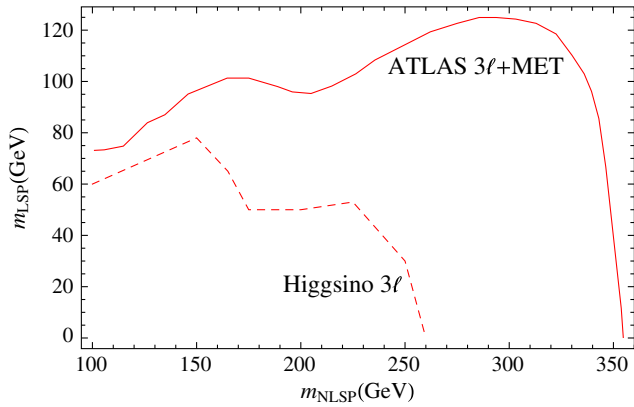


FIG. 1 (color online). The relevant BR is taken into account with $\mu > 0$, and the 3ℓ search is not sensitive to the sign of μ as depicted in Fig. 2. We assume $M_2 = 2$ TeV and $t_\beta = 3$. More in Sec. II.

There exist other experimental results on $3\ell + \text{MET}$ [28] and $4\ell + \text{MET}$ [29]. But they are not essentially different from the ones used above. The $2\ell + 2j$ [28,30] and the same-sign dilepton [28] searches do not give a much stronger bound for such a light μ . The $2\ell + 0j + 0Z$ search for the WW is also potentially useful [23,30]. In any case, our interpretation of a few standard searches in Fig. 1 (and similar figures throughout in this paper) is a reasonable and useful estimation of current Higgsino exclusion limits. See Appendix B for more details on how we obtain the bounds.

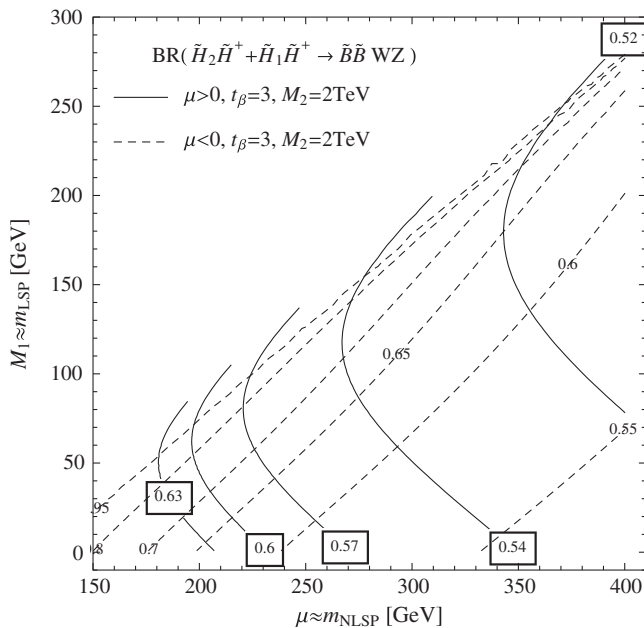


FIG. 2. The branching ratios of NLSP Higgsino pairs to LSP bins that are relevant to the $3\ell + \text{MET}$ search. Both $\mu > 0$ (solid) or $\mu < 0$ (dashed) are shown. The relevant BR shown does not vary much in most of the parameter space with both signs of μ [24]. This result is used in Fig. 1.

The Higgsino can also be the LSP (the case (2) above). If other gauginos are far away in mass, all three Higgsino states—one charged and two neutral—are nearly degenerate. Even though light Higgsinos are abundantly produced, visible decay products of decays between them are generally too soft to be observable at collider and two LSPs are produced in back-to-back directions giving a small MET. It is why the search of nearly degenerate spectrum is difficult. The squeezed spectrum is typically searched by triggering hard initial state radiations which subsequently boost the visible and invisible decay products. No dedicated LHC search is reported yet, but several theoretical studies of LHC prospects have been carried out in Refs. [31]. It is expected that the monojet + MET alone at LHC14 would be sensitive to nearly degenerate ~ 100 GeV Higgsinos only with $\mathcal{O}(1)/\text{ab}$ of data, but somewhat more optimistic approach would be to utilize soft leptons from heavier Higgsino decays when the (model-dependent) mass splitting is ~ 10 GeV or larger. Decays between Higgsinos are rather prompt [32] (even when the splitting is dominated by small loop-induced contributions), so the disappearing track searches [33] that are sensitive to the degenerate wino LSPs are not so useful for Higgsino LSPs; see Appendix A.

III. HIGGSINO NLSP AND AXINO LSP

In this section, we consider the situation of light (NLSP) Higgsinos and heavy gauginos with the axino LSP. In the decoupling limit of gauginos, there occurs an interesting and rich situation for the decays of heavier Higgsinos. Since the axino LSP is weakly interacting, Higgsinos can dominantly decay either to the lightest Higgsino or to the axino LSP, depending on the gaugino masses and the PQ symmetry breaking scale, v_{PQ} . In Fig. 3, we show relative decay widths of the heavier Higgsinos for massless axinos and $v_{\text{PQ}} = 10^9$ GeV. Heavier axinos (for a fixed μ) and a higher v_{PQ} scale only make the decays to the axino smaller. For $M_1 = M_2 \lesssim$ a few TeV, both charged and neutral Higgsinos decay dominantly to the lightest Higgsinos even with massless axinos and $v_{\text{PQ}} = 10^9$ GeV. For larger $M_1 = M_2$, the mass splitting between Higgsino states are too small to have quick enough decays between them. In this paper, we simply assume $M_1 = M_2 = 2$ TeV for which all heavier Higgsinos decay to the lightest Higgsinos; also as long as M_1 and M_2 are TeV scales, the mass splitting between Higgsinos are $\mathcal{O}(1)$ GeV (see Fig. 3) and soft leptons from decays between Higgsinos are too soft to be reliably measurable.

Whether or not the decays of the Higgsino NLSP to the axino LSP can leave observable displaced vertices depends on the values of μ , v_{PQ} and the mass gap between the NLSP and LSP. The proper decay length of the Higgsino NLSP, \tilde{H}_1^0 , is shown in Fig. 4. The distinction between the prompt and displaced decays (also whether decaying inside or outside detector) is not determined solely by the $c\tau$ but also

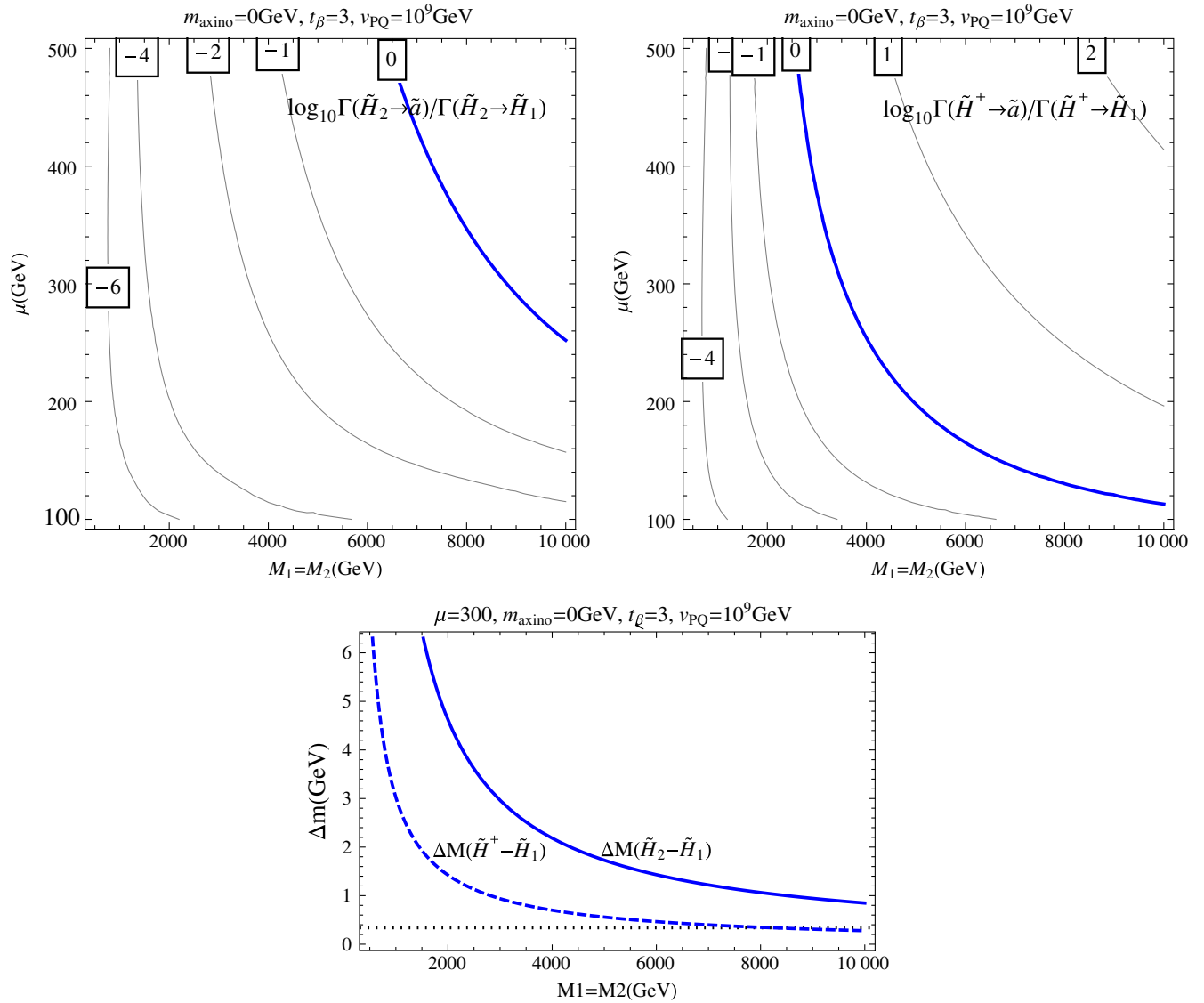


FIG. 3 (color online). The decays of heavier Higgsinos (neutral one in the left panel, and charged one in the right panel) to the lightest Higgsino vs to the axino LSP are compared in the upper panels. Massless axinos and $v_{\text{PQ}} = 10^9$ GeV are assumed here. In the lower panel, we show tree-level mass splittings between Higgsino states. The loop-induced mass splitting of the Higgsinos, $\Delta m \sim 355$ MeV [32], is marked as a horizontal dotted line.

by kinematics of decay products and the probabilistic distributions of decay lengths. But by conveniently referring to the contours of $c\tau = 200 \mu\text{m}$ (blue) and 10 m (red)—standard tight leptons are required to satisfy $d_0 \gtrsim 200 \mu\text{m}$ at LHC [34] and the size of ATLAS detector, for example, is ~ 10 m [34], we find that the decay is most likely be inside detector (and to be displaced) at collider for the favored region of parameter space with a smaller μ and $v_{\text{PQ}} \gtrsim 10^9$ GeV (unless the mass gap between Higgsino NLSPs and axino LSPs is very small). See Ref. [35] for earlier studies of displaced decays of singlinos in a most related context, Refs. [36,37] for displaced decays of standard neutralinos and Refs. [38,39] for lightest Higgsino phenomenology with gravitino LSPs.

Based on the Higgsino decay patterns discussed above, we have a simple scenario where any Higgsino pair productions would essentially be the same as the $\tilde{H}_1^0 \tilde{H}_1^0$ pair production and relevant collider signals come only from $\tilde{H}_1^0 \rightarrow \tilde{a} + h/Z$. It is useful to summarize several differences between the current situation and the standard Higgsino NLSP and bino LSP case discussed in Sec. II:

- (1) The $\tilde{H}_1^0 \tilde{H}_1^0$ production is sizable. Any pair productions of Higgsinos essentially lead to the $\tilde{H}_1^0 \tilde{H}_1^0$ and resulting total production rate (adding all) is about 8 times larger than that of the usual $\tilde{H}_1^0 \tilde{H}_2^0$ pair production as shown in Fig. 5. Note that pair productions of neutral winos or bins are highly

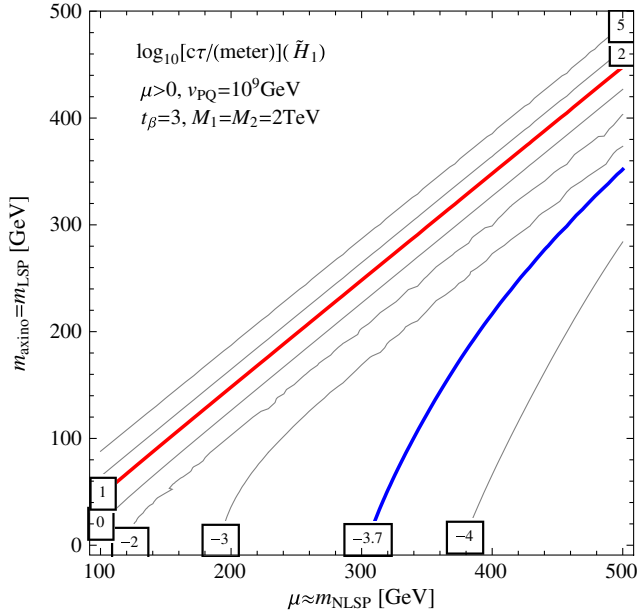


FIG. 4 (color online). The proper decay length of the lightest Higgsino NLSP, \tilde{H}_1^0 , in the presence of the axino LSP. $v_{\text{PQ}} = 10^9$ GeV here, and the lifetime scales with v_{PQ}^2 . We mark $c\tau \sim 200 \mu\text{m}$ with a blue line as a convenient reference for the displaced decay, and we mark $c\tau = 10$ m with a red line for decaying outside detector. The rapid increase of the lifetime below $m_{\text{NLSP}} - m_{\text{LSP}} \lesssim 90$ GeV is due to the closing of any two-body decay modes.

suppressed. The enhanced neutralino pair production can also be resulted in the case with the weakly interacting gravitino LSP [38].

- (2) Among standard multilepton searches, the $4\ell + \text{MET}$ search is most relevant through

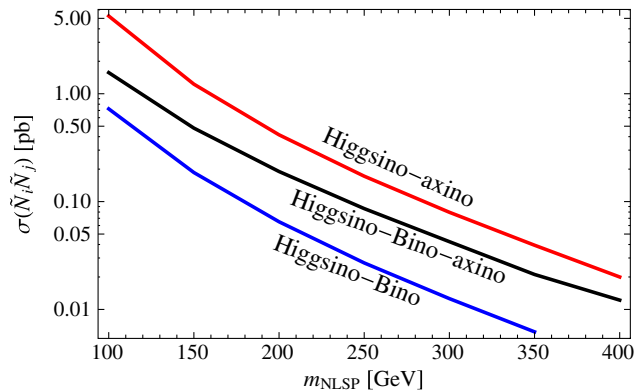


FIG. 5 (color online). The production rates of the NLSP neutralino pair effectively relevant to collider physics. For the standard Higgsino-bino case in Sec. II, $\sigma(\tilde{H}_1^0 \tilde{H}_2^0)$ is shown as blue. For the Higgsino-axino case in Sec. III, $\sigma(\tilde{H}_1^0 \tilde{H}_1^0)$ is shown as red. For the Higgsino-bino-axino case in Sec. IV, it is the $\sigma(\tilde{B} \tilde{B})$ shown as black; $\mu = M_1 + 50$ GeV is assumed. All prompt pair productions of inos effectively leading to the aforementioned production are added; see text for more discussions.

$\tilde{H}_1^0 \tilde{H}_1^0 \rightarrow \tilde{a} \tilde{a} ZZ \rightarrow \tilde{a} \tilde{a} 4\ell$. The $\tilde{H}_1^0 \tilde{H}_1^0$ can now contribute to the stringent $3\ell + \text{MET}$ ($2\ell + \text{MET}$ as well) searches only by accidentally losing one or more leptons. Thus, such multilepton searches are weakened.

- (3) Higgsino phenomenology depends only on the decay pattern of \tilde{H}_1^0 . Decays of a single neutral Higgsino, \tilde{H}_1^0 , depends sensitively on t_β and the sign of μ (as can be seen, e.g. in Fig. 8). On the other hand, in the standard case without axino LSPs, decays of all Higgsino states are indistinguishable at collider and are equally important, and summing all indistinguishable decays make some standard Higgsino phenomenology less sensitive to those parameters [24]; see one example in Fig. 2.
- (4) As discussed, the decay of \tilde{H}_1^0 is likely displaced. The displaced decay further weakens the standard multilepton SUSY searches. However, dedicated displaced vertex(DV) searches are now relevant.

In Fig. 6, we analyze the exclusion bounds on the μ - v_{PQ} parameter space with $m_{\text{axino}} = 0$ GeV. Both the $4\ell + \text{MET}$ search [28] (constraining too much prompt decays) and the CMS dijet DV search [40] (constraining a certain range of displaced decay) are relevant. Fig. 7 shows results in the more general parameter space. For high enough v_{PQ} scales, no bound exists; either the Higgsino decays still dominantly inside detector but its DV is not searched efficiently or the Higgsino dominantly decays outside detector and its phenomenology is essentially the same as that of the Higgsino-LSP case whose current null bounds are discussed in Sec. II. The bound from the DV search is sensitive to the DV reconstruction efficiency, ϵ_{DV} , which is an experimental factor capturing how much fraction of DVs are really reconstructed. For the low extreme value of $\epsilon_{\text{DV}} = 0.01$ (see Ref. [40] that $\epsilon_{\text{DV}} = 0.01 - 0.1$ is a reasonable range to consider), the bound almost disappears. The bound from the $4\ell + \text{MET}$ search is stronger for $\mu > 0$ than $\mu < 0$ because the relevant BR is larger as depicted in the right panel of Fig. 8. The total decay width of the Higgsino depends slightly on the sign of μ , thus so does Fig. 7.

It is useful to understand why the $3\ell + \text{MET}$ search is now significantly weaker than the $4\ell + \text{MET}$ search here as opposed to the results of Sec. II. The main reason why the $4\ell + \text{MET}$ search is now sensitive to this model while it is not sensitive to the standard Higgsino-bino case in Sec. II is the enhanced neutral Higgsino pair production in this model as discussed in regard of Fig. 5. Another minor reason is that the relevant BR (right panel of Fig. 8) can be somewhat larger than a half while it is typically not in the standard case (left panel of Fig. 8).⁴ On the other hand, compared to the dominant $\tilde{H}^\pm \tilde{H}_1^0$ production in the

⁴The difference is that the same Higgsino, \tilde{H}_1^0 , is pair produced here. The decays of \tilde{H}_1^0 and \tilde{H}_2^0 are typically opposite [24].

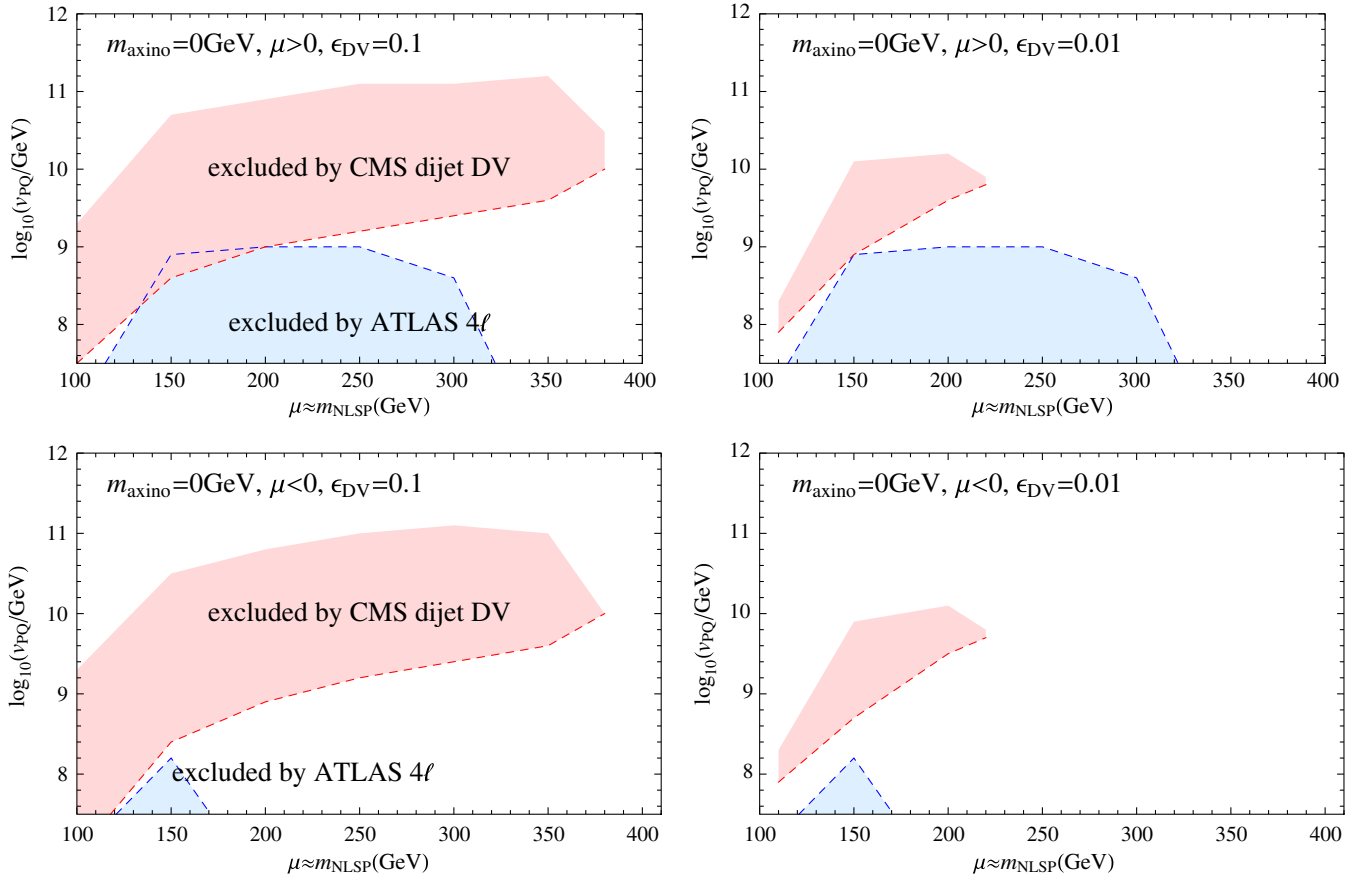


FIG. 6 (color online). The excluded parameter space of the case with the Higgsino-NLSP and axino-LSP discussed in Sec. III for $\mu > 0$ (upper) and < 0 (lower). The $4\ell + \text{MET}$ search (blue) [28] and the CMS dijet DV search (red) [40] are most relevant. We assume two extreme values of the DV reconstruction efficiencies: $\epsilon_{\text{DV}} = 0.1$ (left) and 0.01 (right). $m_{\text{axino}} = 0$ GeV is used, but see Fig. 7 for results on other values of m_{axino} . $M_1 = M_2 = 2$ TeV and $t_\beta = 3$. More in Sec. III.

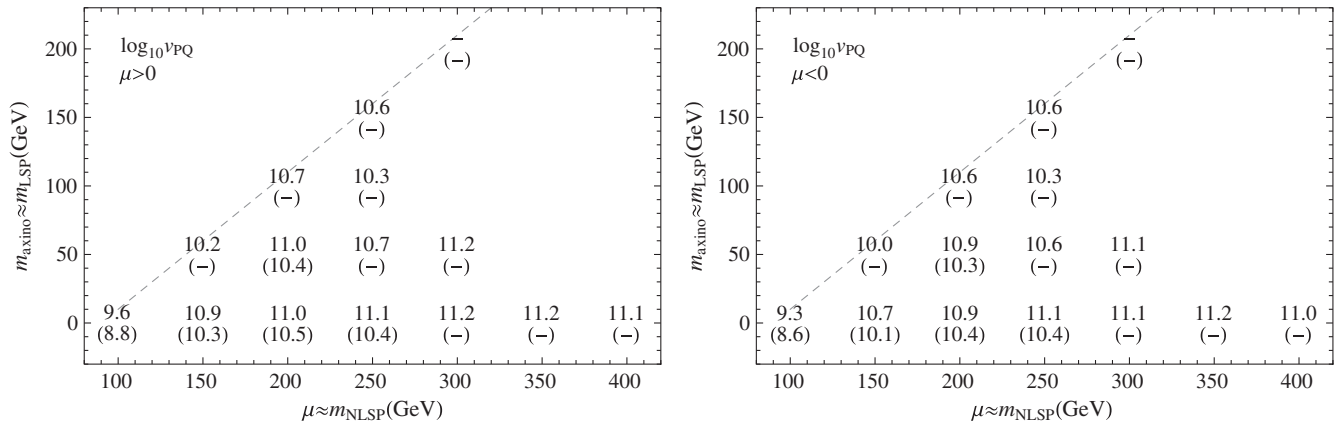


FIG. 7. The highest excluded value of $\log_{10} v_{\text{PQ}}$ for the given parameter space of Higgsino NLSP with axino LSP; for example, see Fig. 6 that the value would be ~ 11.1 for the 250-0 case with $\mu < 0$. Although there can be a smaller v_{PQ} not excluded, we conveniently choose this highest excluded value to show in these plots. CMS dijet DV and $4\ell + \text{MET}$ are used. Numbers without (with) parentheses are results with $\epsilon_{\text{DV}} = 0.1$ (0.01). The “-” implies no existing bounds. The parameter space without anything written is not simulated by ourselves. The light-gray-dashed diagonal lines imply $m_{\text{NLSP}} - m_{\text{LSP}} = 90$ GeV below which the decays to the off-shell Z boson begins to be phase-space suppressed and $v_{\text{PQ}} \gtrsim 10^8$ GeV is already high enough to make all Higgsinos decay far outer region or outside the detector—thus, no collider bounds in general.

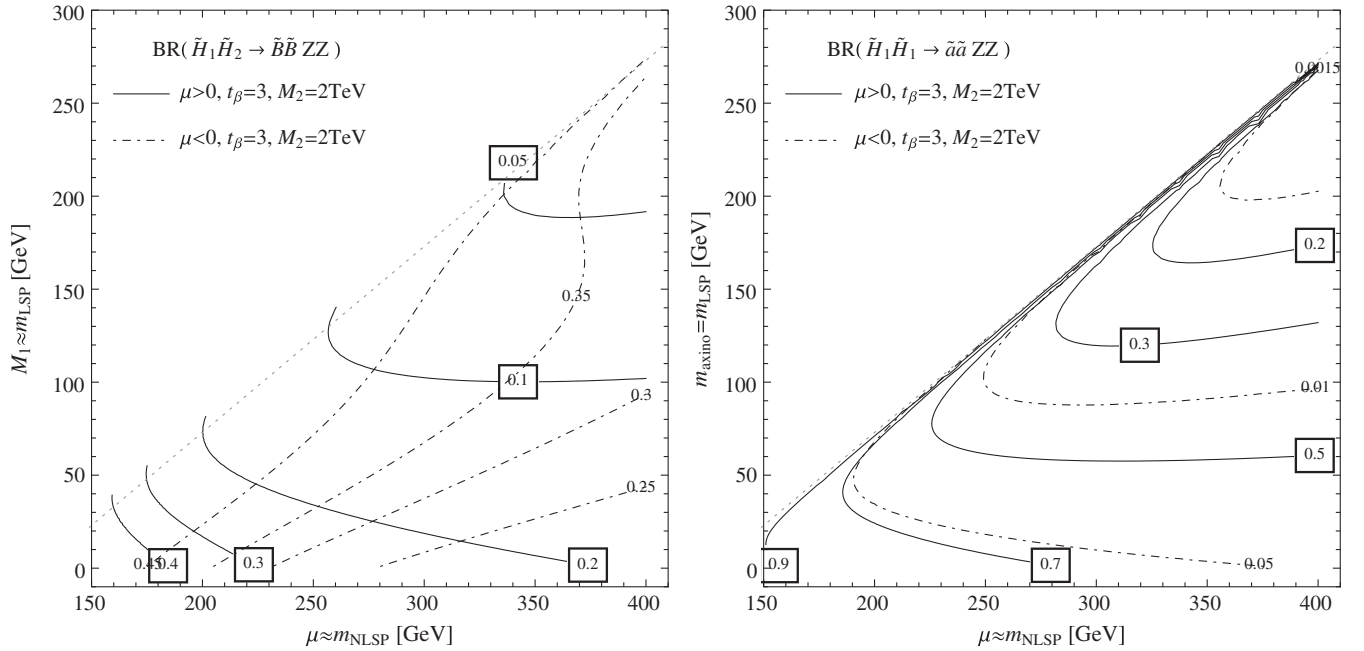


FIG. 8. The BR of the NLSP Higgsino pairs to the ZZ channel relevant to the $4\ell + \text{MET}$ search. (Left): The Higgsino-bino case in Sec. II. (Right): The Higgsino-axino case in Sec. III. Both $\mu > 0$ (solid) and $\mu < 0$ (dashed) are shown. The dotted diagonal lines are at $m_{\text{NLSP}} - m_{\text{LSP}} \approx m_h$ above which the decays to the (on- or off-shell) Z boson are almost 100%. The BR can be larger than a half in the right panel with $\mu > 0$. These results are used in obtaining the $4\ell + \text{MET}$ bounds.

Higgsino-bino case leading to the $3\ell + \text{MET}$ signal, the $\tilde{H}_1^0 \tilde{H}_1^0$ here is not much larger, thus a small selection efficiency to the $3\ell + \text{MET}$ here (needing to accidentally lose one lepton) has a big impact to decrease the exclusion reach of the $3\ell + \text{MET}$ in this model.

The CMS dilepton DV search [41] can also give a relevant bound, but this search looks for a similar range of decay length $\sim 30\text{--}60$ cm; so we conservatively use the dijet DV results to obtain bounds. Other dedicated DV searches [42] are less relevant and less stringent.

In all, by having the axino LSP, some ranges of μ and v_{PQ} can be probed at the LHC since the currently allowed range of v_{PQ} falls in the right range to allow NLSP Higgsinos to decay inside detector either promptly or with DVs. On the other hand, a higher $v_{\text{PQ}} \gtrsim 10^{10}\text{--}10^{11}$ GeV with $\mu \sim 100\text{--}400$ GeV can avoid all the current LHC searches. When the mass gap between the Higgsino and the axino is smaller than about m_Z , the Higgsino generally decays far outer region or outside the detector and no current collider searches constrain the model.

IV. HIGGSINO NNLSP, BINO NLSP, AND AXINO LSP

Having another gauginos in the light spectrum is another interesting possibility. In this section, we consider the case with the Higgsino NNLSP, bino NLSP and axino LSP. As direct bino pair production is very small, the collider phenomenology relies on all possible pair productions of

NNLSP Higgsinos and NLSP binos. We assume that $|\mu| = M_1 + 50$ GeV so that these productions are big enough for collider analysis. Due to this close-by masses and resulting mild mixing between binos and Higgsinos, the mass eigenvalue of the binolike LSP is 20 GeV lighter than the M_1 : $m_{\text{NLSP}} \approx M_1 - 20$ GeV. Similarly to the case of the Higgsino NLSP with axino LSP discussed in Sec. III, the produced Higgsinos dominantly and promptly decay to the bino NLSP. It is more obviously true here because the decays between Higgsinos and binos are not small-gap suppressed. Then, again all the Higgsino productions essentially lead to abundant bino pair productions in the collider physics point of view.

Binops can also decay to the axino LSP with substantial lifetime. In Fig. 9, we show the proper decay length of bino NLSPs. In the majority of relevant parameter space, binos likely decay inside detector either promptly or with DVs. Compared to the Higgsino NLSP's decay in Fig. 4, binos have a somewhat longer lifetime because binos couple to axinos via Higgsino mixtures in the DFSZ model. Numerically, it turns out that the bino typically has a 3–5 times longer lifetime (with the same other parameters) which implies that about 2 times lower v_{PQ} scale is needed for a similar lifetime. If Higgsinos are much heavier, the bino decays are much slower with about 10–20 times longer lifetime due to a smaller bino-Higgsino mixing.

It is useful to note several differences between this scenario and the Higgsino-axino case in Sec. III. (i) For the given NLSP mass, the effective total production of NLSP

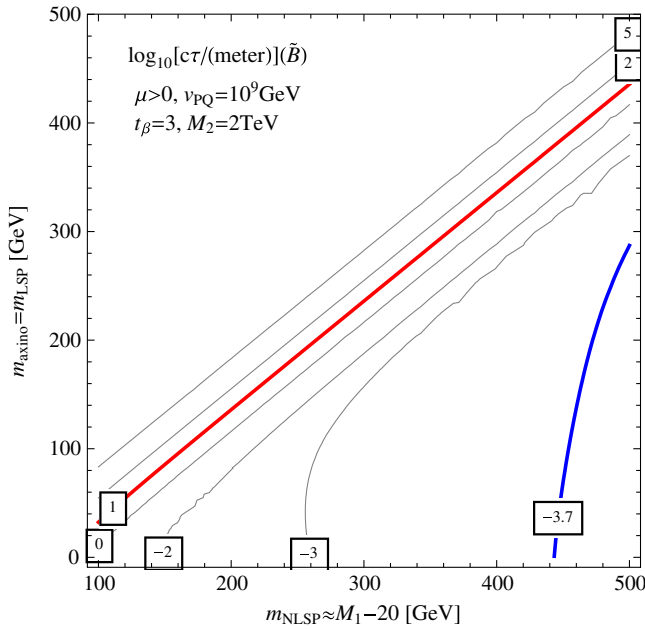


FIG. 9 (color online). The proper decay length of the bino NLSP in the presence of the axino LSP. Figure details are as in Fig. 4. The Higgsino NNLSP is assumed to be nearby with $|\mu| = M_1 + 50 > 0$. For a positive μ assumed here, the mass eigenvalue of the bino NLSP is related as $m_{\text{NLSP}} \approx M_1 - 20$ GeV due to the mild mixing between binos and Higgsinos. More in Sec. IV.

pairs is smaller here as shown in Fig. 5 because the model here relies on the (associate) productions of heavier Higgsinos. (ii) Decays of NNLSP Higgsinos to NLSP binos can produce observable particles as we assume about 50 GeV mass gap. We will explain later how we treat these visible particles in our analysis. (iii) Now the decay pattern of the bino NLSP is relevant to collider searches instead of that of the Higgsino.

In Fig. 10, we analyze the exclusion bounds. Again, both the $4\ell + \text{MET}$ search (constraining too much prompt

decays) and the CMS dijet DV search (constraining a certain range of displaced decay) are relevant. For high enough v_{PQ} scales, no bound exists; either the bino decays still dominantly inside detector but its DV is not searched efficiently or the bino dominantly decays outside detector. When the bino decays outside detector, the visible decay products of NNLSP Higgsinos can be important in collider searches—the collider physics will then be essentially the same as that of the Higgsino NLSP and bino LSP considered in Sec. II as if axinos were absent. However, the mass-gap between the Higgsino and bino is only 50 GeV in this work, and by referring to Fig. 1 showing the current bounds on the Higgsino-bino model, we find that the visible decay products of Higgsino NNLSP with such small-gap is weakly constrained. We conservatively assume that we can ignore all (soft) leptons from Higgsino decays in our multilepton analysis, but we will include all and only leptons from bino decays to axinos in our analysis (when binos decay promptly inside detector)—the more accurate analysis will not give a much stronger bound anyway.

The Fig. 10, compared with Fig. 6 and 7, shows that the bound on this model is somewhat weaker than that of the Higgsino-axino case in Sec. III. For $\epsilon_{\text{DV}} = 0.01$, no bounds from the DV search is derived. For $\mu < 0$, no bounds from the multilepton search is derived. These weaker bounds are mainly because the effective total production of bino pairs is smaller for the given bino mass as discussed above and as shown in Fig. 5.

The results depend on the choice of $|\mu| = M_1 + 50$ GeV. The heavier Higgsinos, the smaller signal productions and the weaker collider constraints—it is thus a less interesting scenario. The lighter Higgsino closer to the bino can induce a larger mixing making binos decay more promptly (but not faster than pure Higgsinos discussed in previous section) and the excluded parameter space change slightly. If we still assume that lepton from decays between those states

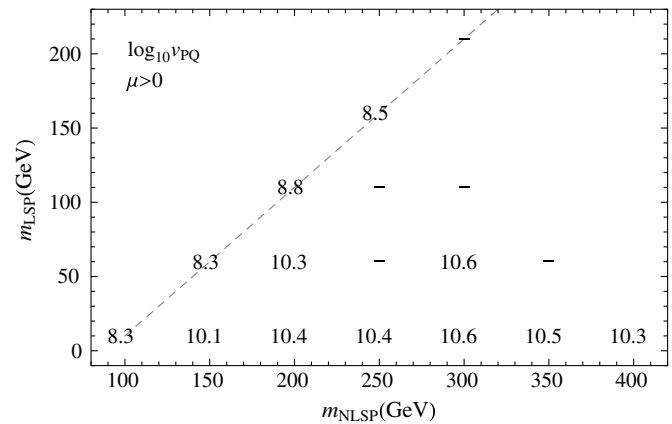
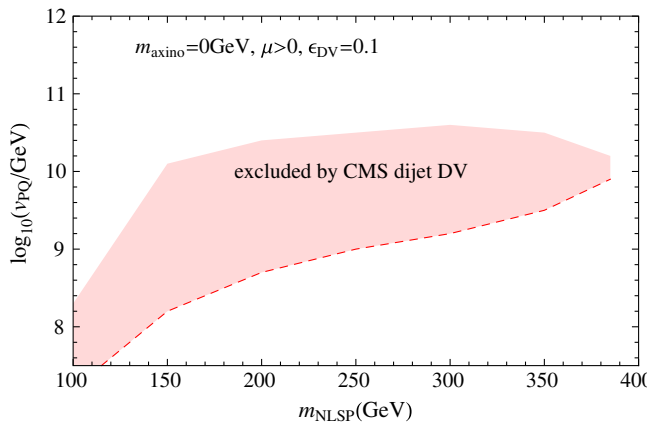


FIG. 10 (color online). The highest excluded value of $\log_{10} v_{\text{PQ}}$ for the given parameter space of the Higgsino NNLSP, bino NLSP, and axino LSP discussed in Sec. IV with $\mu > 0$. Figure details are as in Fig. 6 and 7. No bound is derived from multilepton searches in this case. For $\mu < 0$, a similar bound is obtained from the DV search. $\epsilon_{\text{DV}} = 0.1$ here, and no DV bound exists for $\epsilon_{\text{DV}} = 0.01$.

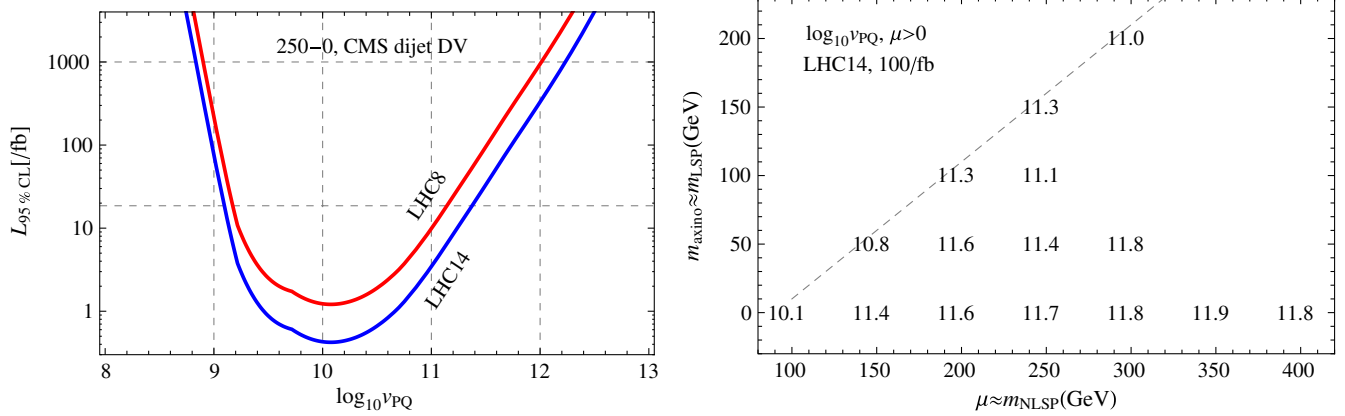


FIG. 11 (color online). LHC14 projections of the latest CMS dijet DV search. Left: the needed luminosity for 95% C.L. exclusion for the 250-0 parameter space with $\mu > 0$ and $t_\beta = 3$. The lower horizontal dashed line is at the current CMS data 18.6/fb while the upper horizontal line at 1/ab is just shown for easy reading. Right: the highest excluded $\log_{10} v_{\text{PQ}}$ value with 100/fb as in the left panel of Fig. 7 with $\epsilon_{\text{DV}} = 0.1$. Increasing to 3/ab of data roughly enhances the reach by 0.7 of $\log_{10} v_{\text{PQ}}$.

are soft enough, not much qualitative change in the collider physics would arise.

But again, in all, by having the axino LSP as well as light gauginos, some ranges of μ and v_{PQ} can be probed at the LHC since the currently allowed range of v_{PQ} falls in the right range to allow NLSP binos to decay inside detector either promptly or with DVs. On the other hand, a higher $v_{\text{PQ}} \gtrsim 10^{10}-10^{11}$ GeV with $\mu \sim 100-400$ GeV can avoid all the current LHC searches. If the NNLSP Higgsino is much heavier, the model has a looser connection with the naturalness; in any case, no any sizable production modes are available then and the search will rely on heavier particle productions.

V. LHC14 PROJECTION

As the LHC 14 TeV will start in a year, it is interesting to estimate the prospect of it. We project the current CMS dijet DV search results to study how high v_{PQ} scale can be probed at 14 TeV.

It is a technically difficult task because future detectors are different and pile-up backgrounds at higher energy collisions are larger. We, however, parameterize the DV reconstruction efficiency which will be most dependent on detector performance by an unknown ϵ_{DV} , and relatively hard cuts on jet p_T used in this analysis ($H_T > 300$ GeV and $p_T(j) > 60$ GeV which shall be scaled up at 14 TeV) will make the soft pile-up effects less influential. If we assume that cut/reconstruction efficiencies and the signal-to-background ratio after optimal cuts stay relatively constant between 8 and 14 TeV analyses, the following simple scaling rule of the statistical significance is obtained,⁵

$$\begin{aligned} (\text{significance})_i &= \frac{\sigma_{S_i} \epsilon_{S_i} \epsilon_{\text{DV}} \mathcal{P}_i}{\sqrt{\sigma_{B_i} \epsilon_{B_i}}} \sqrt{\mathcal{L}_i} \\ &= \left(\sqrt{\frac{S_i}{B_i} \epsilon_{S_i} \epsilon_{\text{DV}}} \right) \cdot \sqrt{\sigma_{S_i} \mathcal{P}_i \mathcal{L}_i}, \end{aligned} \quad (5)$$

$$\frac{(\text{significance})_i}{(\text{significance})_j} = \sqrt{\frac{\sigma_{S_i} \mathcal{P}_i \mathcal{L}_i}{\sigma_{S_j} \mathcal{P}_j \mathcal{L}_j}} = \sqrt{\frac{S_i}{S_j}}, \quad (6)$$

where each factor in the parentheses, the signal-to-background ratio $S_i/B_i = (\sigma_{S_i} \epsilon_{S_i} \epsilon_{\text{DV}} \mathcal{P}_i)/(\sigma_{B_i} \epsilon_{B_i})$, signal cut efficiency ϵ_{S_i} , and the assumed $\epsilon_{\text{DV}} = 0.1$ stays constant as discussed. σ_{S_i, B_i} are production rates of signal and background, and the probability for displaced decays to be selected by the search, \mathcal{P}_i , depends on the v_{PQ} and mass spectrum. In all, the significance simply scales with the square root of signal event counts. 8 TeV CMS dijet DV search bounds can be extrapolated to the 14 TeV bounds by finding proper v_{PQ} and mass spectrum giving the same signal event counts as the upper bound of 8 TeV results.

We show 14 TeV projected results in Fig. 11 obtained in this way. The Higgsino-axino model in Sec. III is used. For the given mass spectrum, LHC14 100(3000)/fb can probe higher v_{PQ} scale by 0.6–0.7(1.3–1.4) of $\log_{10} v_{\text{PQ}}$ as shown in the left panel for one choice of parameters. Similar size of improvement is expected for the most of light Higgsino parameter space shown in the right panel. With 3000/fb, v_{PQ} as high as 10^{12} GeV which is a general upper bound is expected to be probed with light Higgsinos. A more dedicated search will be useful in the near future.

VI. CONCLUSION

The electroweak-scale axino and Higgsino are perhaps predicted altogether by a naturalness philosophy of particle physics. The implications and the consistency of having

⁵These are often reasonable assumptions. See Ref. [43] where this scaling rule is proven for the search of gluino pairs at future high energy colliders and Ref. [44] where a public javascript code can do similar scaling for conventional searches.

both light axinos and Higgsinos are studied in the context of a few benchmark models of supersymmetry. Interestingly, for the typical range of the PQ scale, $10^9 \text{ GeV} \lesssim v_{\text{PQ}}/N_{\text{DW}} \lesssim 10^{12} \text{ GeV}$, the electroweak-scale NLSP can still decay to the axino LSP inside detector both promptly and by leaving a DV. The $4\ell + \text{MET}$ signature from the prompt decay of the NLSP is enhanced among standard SUSY searches as all heavier neutralinos and charginos decay promptly first to NLSP neutralinos so that NLSP neutralino pair productions which are relevant to the collider physics are effectively enhanced. The displaced decay of the NLSP is constrained by dedicated DV searches for a certain range of v_{PQ} typically of $10^9 \lesssim v_{\text{PQ}} \lesssim 10^{11} \text{ GeV}$ depending on the mass spectrum—searches for a wider range of decay lengths maybe possible [36,45]. A higher PQ scale of $v_{\text{PQ}} \gtrsim 10^{10}\text{--}10^{11} \text{ GeV}$ with the electroweak-scale μ or the mass spectrum with small mass gap between the NLSP and LSP is generally safe from all current collider searches. LHC14, however, is expected to probe the large part of interesting parameter space with light Higgsinos according to our naive estimation, thus a more dedicated search is motivated. We hope that we provided a basic collider physics of the natural supersymmetry with the axino LSP and light Higgsino which can also be complementary to the widely studied axino sector cosmology.

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APPENDIX A: INO DECAYS

All the relevant two- and three-body decay widths of inos are calculated and collected in Ref. [24] (see also Refs. [16,35,46] for earlier results). In this appendix, we further summarize how we calculate the two-body decays

to pions which is relevant when the mass gap is very small $\lesssim \mathcal{O}(1) \text{ GeV}$.

The two-body decay $\chi_1^+ \rightarrow \chi_1^0 \pi^+$ is calculated as [32,47]

$$\Gamma(\chi^+ \rightarrow \chi^0 \pi^+) = \Gamma(\pi^+) \cdot \frac{16\delta m^3}{m_\pi m_\mu^2} \left(1 - \frac{m_\pi^2}{\delta m^2}\right)^{1/2} \times \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^{-2}, \quad (\text{A1})$$

where the total decay width of a charged pion is $c\tau = 7.80 \text{ m}$ or $\tau = 26.03 \text{ ns}$ or $\Gamma(\pi^+) = 2.53 \times 10^{-17} \text{ GeV}$ [48]. The mass splitting between the chargino and the neutralino is denoted by δm . We use $m_\pi = 139.6 \text{ MeV}$, $m_\mu = 105.7 \text{ MeV}$ [48]. For $\delta m = 164.4(355) \text{ MeV}$ which is the one-loop asymptotic wino(Higgsino) mass splitting [32,47], the proper decay length is $c\tau = 5.9(0.34) \text{ cm}$ (equivalently, $\tau = 0.20(0.011) \text{ ns}$). The current disappearing track search [33] is sensitive to $\tau \gtrsim 0.1 \text{ ns}$, thus is currently not so sensitive to the nearly degenerate Higgsinos.

APPENDIX B: BOUND ESTIMATION

We list methods and numerical results that we used to obtain various exclusion bounds in this paper. For the $3\ell + \text{MET}$ result, we use the reported upper limits on the number of events in various $\text{SR0}\tau\text{a}$ bins of Ref. [27]. The $\text{SR0}\tau\text{a-bin16}$ is usually strongest for heavy NLSPs. For the $4\ell + \text{MET}$ result, we interpret the result in the bin of $2\text{OSSF} + 0\tau_h$ with $\text{MET} > 100 \text{ GeV}$ of Ref. [28] to the upper limit of number of events $N \lesssim 2.0$ at $1.96\sigma \approx 95\% \text{ C.L.}$ Interestingly, a very similar analysis has been carried out by ATLAS in Ref. [29], but their weaker cut on $\text{MET} > 75 \text{ GeV}$ leads to a much weaker bound. Thus, the optimization of the $4\ell + \text{MET}$ cuts in each parameter space as roughly done for the $3\ell + \text{MET}$ above will be useful. For the dijet DV result in Ref. [40], we conservatively use the result for $L_{xy} < 20 \text{ cm}$ (combined with 2 observed events) to obtain the upper limit on the new physics contribution $N \lesssim 3.1$ at $1.96\sigma \approx 95\% \text{ C.L.}$ For all results, we generate MADGRAPH [49] events with up to one additional parton and showered them by interfacing with PYTHIA [50] using the MLM [51] matching. We use FASTJET [52] for particle reconstruction.

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