Three sterile neutrinos in E_6

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Candidates for unification of the electroweak and strong interactions include the grand unified groups SU(5), SO(10), and the exceptional group E_6. The 27-dimensional fundamental representation of E_6 contains exotic fermions, including weak isosinglet quarks of charge $-1/3$, vectorlike weak isodoublet leptons, and neutral leptons which are singlets under both left-handed and right-handed SU(2). These last are candidates for light "sterile" neutrinos, hinted at by some recent short-baseline neutrino experiments. In order to accommodate three families of quarks and charged leptons, an E_6 model must contain three 27-plets, each of which contains a sterile neutrino candidate n. The mixing pattern within a 27-plet is described, and experimental consequences are discussed.

I. INTRODUCTION

The standard model group $SU(3)_{color} \times SU(2)_L \times U(1)$ can be incorporated into a grand unified group. Candidates include SU(5), SO(10), and E_6 [1]. Each quark and lepton family consists of a $5^* + 10$ representation of SU(5). Adding a right-handed neutrino N [an SU(5) singlet] to each such hypermultiplet, one gets a spinor 16-plet of SO(10). A right-handed neutrino can pair with a left-handed one to generate a Dirac mass m_D as occurs for charged leptons and quarks. However, the neutrality of the right-handed neutrino under the standard model group allows it to have a large Majorana mass M, leading via the seesaw mechanism [2] to light-neutrino masses $m_\nu = m_D^2/M$. At this stage there are three light neutrinos (mostly electroweak doublets) and three heavy ones (mostly electroweak singlets).

In addition to the "active" light neutrinos ν_e, ν_μ, ν_τ, some short-baseline neutrino experiments [3–12] have hinted at the existence of one or more light "sterile" neutrinos, participating in the weak interactions only via mixing with the active ones. For schemes in which one or more of the right-hand neutrinos plays the role of a light sterile neutrino, see [13,14]. These typically involve constraints in comparison with scenarios in which all three right-handed neutrinos are heavy.

In this paper we wish to investigate a different scenario for sterile neutrinos, based on E_6. This would be especially timely if the exotic states predicted in E_6 were to show up in forthcoming experiments at the CERN Large Hadron Collider. A 10-plet of SO(10) [a $5 + 5^*$ of SU(5)] can be added to each quark and lepton family. It consists of quarks $h + h^c$ which are singlets under $SU(2)_L$ and $SU(2)_R$, and leptons $E^\pm$ and their neutral counterparts which are doublets under both. The smallest E_6 representation, a 27-plet, is formed by adding another singlet n of SO(10). The n has neither L nor R isospin. We shall explore

a scenario in which the three n states are candidates for light sterile neutrinos, leaving all three right-handed neutrinos unconstrained and potentially very heavy.

Fits to short-baseline neutrino anomalies include ones in Refs. [4–6]. There is general agreement that at least two sterile neutrinos are needed to account for these anomalies. The possibility thus remains open that the third could be a keV-scale candidate for dark matter [15–18]. For the requirements placed by experiment on such a state, see Refs. [13,14]. Sterile neutrinos in E_6 have been considered some time ago (see, e.g., [19–27]), but we discuss them now in the context of present data (see also [28–30]). A "minimal extended seesaw model" [31] has one light sterile neutrino, rather than three, coexisting with three active neutrinos and their heavy right-handed counterparts.

Fermion masses in E_6 were analyzed in Ref. [19]. (See Ref. [24] for a review.) The consequences were examined of assuming that all masses of fermions in a 27-plet were due to Higgs bosons in a 27^*-plet: $27 \otimes 27 = 27^* + \cdots$. A key shortcoming of this analysis was the lack of a source for large Majorana masses of right-handed neutrinos. Solutions proposed to this problem included introduction of discrete symmetries, higher-dimension operators, and additional fermions. For an overview, including extensive references, see [28,29]. Our approach is closest to that involving higher-dimension operators. E. Ma [26] showed that a large Majorana mass for right-handed neutrinos was permitted by a specific scheme of E_6 breaking which received much subsequent attention from S. F. King and collaborators [32]. (For a recent reference, see [33].) In the present article we update the analysis of Ref. [19] and discuss some possibilities for accommodating the recent suggestions of sterile neutrinos. We neglect mixing among fermion families, leaving that topic for further study. (For one review, see Ref. [34].)

We review some properties of E_6 multiplets, their decomposition into SO(10) and SU(5) representations, and mass matrix construction in Sec. II. An E_6-invariant

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mass matrix for neutrinos is constructed with 27^*-plet Higgs fields coupling to $27 \otimes 27$, and its shortcomings pointed out, in Sec. III. The addition of a large Majorana mass for right-handed neutrinos, permitted by a specific mode of E_6 breaking, leads to a mass matrix with the potential to describe conventional very light neutrinos and light sterile neutrinos mixed with them with arbitrary strength (Sec. IV). The entries of the neutral lepton mass matrix are compared with corresponding ones for charged leptons and quarks in Sec. V. The relevance of this scheme to present results for short-baseline neutrino oscillation experiments is noted in Sec. VI. The presence of three families of quarks and leptons necessitates separate 27-plets for each of them, entailing three states n. Fits to short-baseline neutrino oscillations [4–6] require at least two sterile neutrinos, leaving the third as a potential candidate for dark matter. Some aspects of this identification are discussed in Sec. VII. We conclude in Sec. VIII.

II. REVIEW OF BASICS

A. E_6 decomposition

The decomposition of E_6 into SO(10) and SU(5) representations leads to two U(1) subgroups: $E_6 \rightarrow SO(10) \otimes U(1)_\psi$ and $SO(10) \rightarrow SU(5) \otimes U(1)_\chi$ [35,36]. The charges of these two U(1)s are denoted by Q_ψ and Q_χ, and are listed in Table I for left-handed neutral leptons in the first (e) family. In what follows we shall refer exclusively to left-handed states, with right-handed states related to them by a CP transformation. The subscript E refers to an exotic vectorlike doublet (ν_E, E^-) belonging to a 10-dimensional representation of SO(10). A distinction is made between neutrinos ν and their charge conjugates N^c. The charge Q_N, defined by

$$2\sqrt{10}Q_N = \frac{5}{4}\left(2\sqrt{6}Q_\psi\right) - \frac{1}{4}\left(2\sqrt{10}Q_\chi\right), \quad (1)$$

is defined for use in Sec. IV.

Fermion masses E_6 27-plets arise from Higgs bosons transforming as the product

$$27 \times 27 = 27^* + 351 + 351^*. \quad (2)$$

TABLE I. Neutral leptons in E_6, their SO(10) and SU(5) representations, and their U(1) charges. For completeness we also display U(1) charges for the (16,10) of (SO(10),SU(5)), although it contains no neutral member.

27 member (SO(10),SU(5))	$2\sqrt{6}Q_\psi$	$2\sqrt{10}Q_\chi$	$2\sqrt{10}Q_N$
$\nu_e(16,5^*)$	−1	3	−2
(16,10)	−1	−1	−1
$N_e^c(16,1)$	−1	−5	0
$\nu_E(10,5^*)$	2	−2	3
$N_E^c(10,5)$	2	2	2
$n(1,1)$	−4	0	−5

Early superstring-inspired models [37–40] assumed the dominant contributions to come from the 27^*-plet, which was the case explored in Ref. [19]. We review that analysis in Sec. III and expand it to allow for heavy right-handed neutrinos in Sec. IV.

B. Mass matrices

In mass matrices involving fermions f_i, pairs of left-handed states, transforming as $(1/2, 0)$ under the SU(2) $\otimes$ SU(2) of the Lorentz group, and pairs of right-handed states, transforming as $(0, 1/2)$, must be coupled to form a Lorentz invariant $(0,0)$. In a two-component notation, especially useful in the treatment of neutral particles,

$$-\mathcal{L}_{ij} = \frac{m_{ij}}{2}\epsilon^{\alpha\beta}[(f_{iL}^c)_\alpha(f_{jL})_\beta + (f_{iL})_\alpha(f_{jL}^c)_\beta] + (L \rightarrow R), \quad (3)$$

where $\alpha, \beta = 1, 2; \epsilon^{12} = -\epsilon^{21} = 1; \epsilon^{11} = \epsilon^{22} = 0$. Consider, for example, u quarks, which are represented by a single field in each E_6 multiplet. In a basis described by the fields $u_{aL} = (u_L, u_L^c)$, the mass term (3) then takes the form

$$-\mathcal{L}_m = \frac{1}{2}\epsilon^{\alpha\beta}M_{ab}[(\psi_{a\alpha L}\psi_{b\beta L}) + (L \rightarrow R)], \quad (4)$$

where

$$M_{ab} = \begin{bmatrix} 0 & m_u \\ m_u & 0 \end{bmatrix}. \quad (5)$$

Charge conservation prevents M_{ab} from having diagonal entries. Its eigenvectors and eigenvalues, corresponding to a standard Dirac mass for u, are

$$\frac{u_{L,R} + u_{L,R}^c}{\sqrt{2}} : \text{ eigenvalue } + m_u,$$

$$\frac{u_{L,R} - u_{L,R}^c}{\sqrt{2}} : \text{ eigenvalue } - m_u. \quad (6)$$

For neutral particles, additional terms in the mass matrix become possible. For example, the mass matrix M_{ab} in the basis ν_L, ν_L^c now can have diagonal entries. The left-handed charge conjugate of ν_L will be referred to as N_L^c to denote the fact that both Majorana and Dirac masses are permitted for neutral leptons.

The popular "seesaw" mechanism [2] provides an explanation of why neutrinos are so light. Let us restrict the discussion to a single family. In the basis (ν_L, N_L^c) the mass matrix is assumed to take the form

$$\mathcal{M} = \begin{bmatrix} 0 & m \\ m & M \end{bmatrix}. \quad (7)$$

The Dirac mass terms m transform as $SU(2)_L$ doublets, while the "right-handed neutrino" Majorana mass term M transforms as an $SU(2)_L$ singlet and hence is not prevented from taking on a very large value. The (unnormalized) eigenvectors and eigenvalues of M are, approximately,

$$\nu_L - (m/M)N_L^c: \text{ eigenvalue} \simeq -m^2/M;$$
$$(m/M)\nu_L + N_L^c: \text{ eigenvalue} \simeq M. \qquad (8)$$

An equivalent description [41] is to note the possibility in the standard electroweak model of a dimension-5 operator describing neutrino mass bilinear in electroweak doublet Higgs fields H, generated by a term $-\mathcal{L}_m = (HL)^2/M$, where L is a lepton doublet. Such a term could arise, for example, if the heavy right-handed neutrino were integrated out. The generated neutrino mass would then be of the form $m_\nu = m^2/M$, where m is of the order of a Dirac mass of quarks or charged leptons, and M is sufficiently large to yield neutrino mass in the sub-eV range. Another approach is to use perturbation theory, with $\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1$,

$$\mathcal{M}_0 = \begin{bmatrix} 0 & 0 \\ 0 & M \end{bmatrix}, \qquad \mathcal{M}_1 = \begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix} \qquad (9)$$

and the unperturbed eigenvectors and eigenvectors $[1,0]^T$ (eigenvalue 0) and $[0,1]$ (eigenvalue M). This is the method we shall use to describe mixing among more than two neutral leptons when some entries in the mass matrix are much larger than others.

III. E$_6$-INVARIANT COUPLINGS

Assuming the mass terms for 27-plets of E$_6$ in Eq. (2) to transform as members of the 27*-plet, we find a mass matrix for neutral leptons with nonzero entries indicated in Table II. The corresponding mass matrix, assuming it is symmetric and real, is

$$\mathcal{M}_6 = \begin{bmatrix} 0 & m_{12} & 0 & M_{14} & 0 \\ m_{12} & 0 & 0 & m_{24} & 0 \\ 0 & 0 & 0 & M_{34} & m_{35} \\ M_{14} & m_{24} & M_{34} & 0 & m_{45} \\ 0 & 0 & m_{35} & m_{45} & 0 \end{bmatrix} \qquad (10)$$

TABLE II. Values of $(2\sqrt{6}Q_\psi, 2\sqrt{10}Q_\chi)$ in the product of two neutral lepton 27-plets represented in a Higgs boson 27*-plet.

	$\nu_e(-1,3)$	$N_e^c(-1,-5)$	$\nu_E(2,-2)$	$N_E^c(2,2)$	$n(-4,0)$
$\nu_e(-1,3)$	$\cdots$	$(-2,-2)$	$\cdots$	$(1,5)$	$\cdots$
$N_e^c(-1,-5)$	$(-2,-2)$	$\cdots$	$\cdots$	$(1,-3)$	$\cdots$
$\nu_E(2,-2)$	$\cdots$	$\cdots$	$\cdots$	$(4,0)$	$(-2,-2)$
$N_E^c(2,2)$	$(1,5)$	$(1,-3)$	$(4,0)$	$\cdots$	$(-2,2)$
$n(-4,0)$	$\cdots$	$\cdots$	$(-2,-2)$	$(-2,2)$	$\cdots$

in the basis of Table II. Here we have denoted mass terms transforming as weak isodoublets with small letters and those transforming as weak isosinglets with large letters. The latter can take on arbitrarily large values without violating weak SU(2), while the former are restricted to be less than the electroweak scale. The subscript 6 on the mass matrix refers to the rank of the group under which the couplings of $27 \otimes 27$ to a 27*-dimensional Higgs representation are invariant. Note that the seesaw term M_{22}, which would have given a large Majorana mass to the right-handed neutrino N_e, is absent. It corresponds to Q_ψ, Q_χ values not represented in a 27*-plet.

We shall now make two further assumptions about the properties of $\mathcal{M}_6$ in Eq. (10). (1) We shall assume that the exotic vectorlike neutrino ν_E pairs up with its charge conjugate to obtain a large Dirac mass, i.e., that M_{34} is very large. The absence up to the ~TeV scale of exotic weak isosinglet quarks h with charge $Q = -1/3$ or vectorlike charged leptons E with charge -1 supports this assumption. (2) We shall assume that active-sterile mixing involving ν_e is small, as supported by tests of weak universality. This corresponds to taking M_{14} relatively small despite its $\Delta I = 0$ nature.

We may analyze the eigenvectors and eigenvalues of the matrix $\mathcal{M}_6$ by using degenerate perturbation theory, expanding around the corresponding matrix with only M_{34} nonzero. It is helpful to first diagonalize $\mathcal{M}_6$ in M_{34} by a rotation about the 3–4 axis; the result is

$$\mathcal{M}_6' = \begin{bmatrix} 0 & m_{12} & M_{14}/\sqrt{2} & M_{14}/\sqrt{2} & 0 \\ m_{12} & 0 & m_{24}/\sqrt{2} & m_{24}/\sqrt{2} & 0 \\ M_{14}/\sqrt{2} & m_{24}/\sqrt{2} & M_{34} & 0 & (m_{35}+m_{45})/\sqrt{2} \\ M_{14}/\sqrt{2} & m_{24}/\sqrt{2} & 0 & -M_{34} & (m_{45}-m_{35})/\sqrt{2} \\ 0 & 0 & (m_{35}+m_{45})/\sqrt{2} & (m_{45}-m_{35})/\sqrt{2} & 0 \end{bmatrix}. \qquad (11)$$

In the limit where all masses except M_{34} are neglected, the eigenvectors of this matrix with nonzero eigenvalues are $[0,0,1,0,0]^T$ (eigenvalue M_{34}) and $[0,0,0,1,0]^T$ (eigenvalue $-M_{34}$), the hallmarks of a four-component Dirac spinor. The three states orthogonal to these are eigenvectors of a 3×3 submatrix in the orthonormal basis $([1,0,0,0,0]^T, [0,1,0,0,0]^T, [0,0,0,0,1]^T) = (\nu_e, N_e^c, n)$. Applying second-order degenerate perturbation theory, we find

$$\mathcal{S}_3 = \begin{bmatrix} 0 & m_{12} & -M_{14}m_{35}/M_{34} \\ m_{12} & 0 & -m_{24}m_{35}/M_{34} \\ -M_{14}m_{35}/M_{34} & -m_{24}m_{35}/M_{34} & -2m_{35}m_{45}/M_{34} \end{bmatrix}. \tag{12}$$

Under our assumptions, the dominant terms in $\mathcal{S}_3$ are the two off-diagonal masses m_{12}, leading to eigenvalues $\pm m_{12}$ and a pseudo-Dirac mass for the conventional neutrino. The remaining eigenvalue is approximately $-2m_{35}m_{45}/M_{34}$, associated with a state which is mostly the sterile neutrino n. As has been noted, this does not provide a satisfactory picture of very light-neutrino masses.

IV. ALLOWING FOR A MASSIVE RIGHT-HANDED NEUTRINO

E. Ma [26] has pointed out that the state N_e^c has zero charge under a linear combination of Q_χ and Q_ψ. In our notation, this is

$$Q_N = -\frac{1}{4}Q_\chi + \frac{\sqrt{15}}{4}Q_\psi, \tag{13}$$

equivalent to Eq. (1). Values of $2\sqrt{10}Q_N$ for neutral leptons in a 27-plet of E_6 are listed in the last column of Table I.

The values of $2\sqrt{10}Q_N$ corresponding to products of two neutral 27-plet members are shown in Table III. The zero value corresponding to the 22 entry of $\mathcal{M}_6$ allows for a higher-dimension operator which breaks $U(1)_\chi$ and $U(1)_\psi$ but preserves their linear combination Q_N. The corresponding mass matrix $\mathcal{M}_5$ (with the subscript denoting the rank of the group under which couplings to Higgs fields are invariant) is

$$\mathcal{M}_5 = \begin{bmatrix} 0 & m_{12} & 0 & M_{14} & 0 \\ m_{12} & M_{22} & 0 & m_{24} & 0 \\ 0 & 0 & 0 & M_{34} & m_{35} \\ M_{14} & m_{24} & M_{34} & 0 & m_{45} \\ 0 & 0 & m_{35} & m_{45} & 0 \end{bmatrix}. \tag{14}$$

After diagonalization with respect to M_{34}, this becomes

$$\mathcal{M}_5' = \begin{bmatrix} 0 & m_{12} & M_{14}/\sqrt{2} & M_{14}/\sqrt{2} & 0 \\ m_{12} & M_{22} & m_{24}/\sqrt{2} & m_{24}/\sqrt{2} & 0 \\ M_{14}/\sqrt{2} & m_{24}/\sqrt{2} & M_{34} & 0 & (m_{35}+m_{45})/\sqrt{2} \\ M_{14}/\sqrt{2} & m_{24}/\sqrt{2} & 0 & -M_{34} & (m_{45}-m_{35})/\sqrt{2} \\ 0 & 0 & (m_{35}+m_{45})/\sqrt{2} & (m_{45}-m_{35})/\sqrt{2} & 0 \end{bmatrix}. \tag{15}$$

The eigenvectors corresponding to the large eigenvalues, about which we perturb, are $[0,1,0,0,0]^T$, $[0,0,1,0,0]^T$, and $[0,0,0,1,0]^T$, while we are concerned with the 2×2 submatrix $\mathcal{S}_2$ in the basis spanned by the eigenvectors $[1,0,0,0,0]^T$ and $[0,0,0,0,1]^T$. Applying second-order perturbation theory, we find

$$\mathcal{S}_2 = \begin{bmatrix} -m_{12}^2/M_{22} & -M_{14}m_{35}/M_{34} \\ -M_{14}m_{35}/M_{34} & -2m_{35}m_{45}/M_{34} \end{bmatrix}. \tag{16}$$

The matrix $\mathcal{S}_2$ describes the mixing of a conventional neutrino ν with a sterile neutrino n. The entries are independent of one another, so arbitrary mixings are possible. However, an additional constraint is that present hints of sterile neutrinos place their masses above those of the three conventional neutrinos [4–6]. So we look for solutions with small mixing but $m_n > m_\nu$.

If we denote

$$\nu = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \qquad \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}, \tag{17}$$

and $t \equiv \tan\theta$, we look for the small-t solution of the quadratic equation

TABLE III. Values of $2\sqrt{10}Q_N$ in the product of two neutral lepton 27-plets represented in a Higgs boson 27*-plet.

	$\nu_e(-2)$	$N_e^c(0)$	$\nu_E(3)$	$N_E^c(2)$	$n(-5)$
$\nu_e(-2)$	$\cdots$	-2	$\cdots$	0	$\cdots$
$N_e^c(0)$	-2	0	$\cdots$	2	$\cdots$
$\nu_E(3)$	$\cdots$	$\cdots$	$\cdots$	5	-2
$N_E^c(2)$	0	2	5	$\cdots$	-3
$n(-5)$	$\cdots$	$\cdots$	-2	-3	$\cdots$

$$t^2 + \left(\frac{m_{12}^2 M_{34}}{M_{14} m_{35} M_{22}} - \frac{2m_{45}}{M_{14}}\right) t - 1 = 0, \qquad (18)$$

in which neglecting the quadratic term gives

$$t \simeq \left(\frac{m_{12}^2 M_{34}}{M_{14} m_{35} M_{22}} - \frac{2m_{45}}{M_{14}}\right)^{-1}. \qquad (19)$$

Barring accidental cancellation of the two terms, either $|m_{12}^2 M_{34}/(M_{14} m_{35} M_{22})|$ or $|2m_{45}/M_{14}|$ must be large. If θ is small, the neutrino mass must be approximately its seesaw value $m_\nu \simeq -m_{12}^2/M_{22}$. In order that $m_n > m_\nu$ one must then have $|2m_{35} m_{45} M_{22}/(M_{34} m_{12}^2)| > 1$. But this says that the term $m_{12}^2 M_{34}/(M_{14} m_{35} M_{22})$ cannot be large unless $M_{14} \ll m_{45}$. Thus one can get $m_n > m_\nu$ with small mixing if

$$\left|\frac{m_{35} m_{45} M_{22}}{M_{34} m_{12}^2}\right| > 1, \qquad \frac{m_{45}}{M_{14}} \gg 1, \qquad (20)$$

with no accidental cancellation between the terms on the right-hand side of Eq. (19). One cannot take $m_{35} = 0$ if one wants a nonzero sterile neutrino mass. The choice of small M_{14} is demanded for self-consistency of the scheme, but remains an issue of fine-tuning. It can be forbidden in lowest order by assigning a Z_2 quantum number of -1 for SO(10) 16-plets and $+1$ for SO(10) 10-plets and singlets. In this manner both M_{14} and m_{24} are forbidden in lowest order but can be generated by small Z_2-violating vacuum expectation values of SO(10) 16-plet Higgs bosons.

If the terms in Eq. (19) do not destructively interfere with one another, one or the other will dominate, so that

$$t = \min\left(\frac{M_{14} m_{35} M_{22}}{m_{12}^2 M_{34}}, \frac{M_{14}}{2m_{45}}\right). \qquad (21)$$

But the first term is larger than the second if $m_n > m_\nu$, so $t \simeq M_{14}/(2m_{45})$.

The E$_6$ scheme should be contrasted with a "minimal extended seesaw" model [31]. That scheme introduces one sterile neutrino, rather than three, leading to a 7×7 mixing matrix when taking account of three active neutrinos and their right-handed counterparts. Under some assumptions it can generate either an eV-scale sterile neutrino to account for short-baseline anomalies, or a keV-scale neutrino as a warm dark matter candidate.

V. RELATION TO CHARGED-LEPTON AND QUARK MASSES

In grand unification schemes, couplings of Higgs bosons to leptons are often related to their couplings to quarks at the unification scale. One must then apply the renormalization group to evaluate the couplings at accessible energies. Such is the case in E$_6$. At the unification scale,

TABLE IV. Charges $2\sqrt{6}Q_\psi, 2\sqrt{10}Q_\chi$, and $2\sqrt{10}Q_N$ of left-handed up-type quarks and antiquarks, and their neutral bilinears.

	$u(-1,-1,-1)$	$u^c(-1,-1,-1)$
$u(-1,-1,-1)$	$\cdots$	$(-2,-2,-2)$
$u^c(-1,-1,-1)$	$(-2,-2,-2)$	$\cdots$

we shall see that the neutral lepton parameters m_{12} and m_{35} are related to one in which quarks of charge $2/3$ acquire masses, while the parameters $m_{24}, m_{45}, M_{14},$ and M_{34} are related to ones relevant for masses of quarks of charge $-1/3$ and charged leptons (both ordinary and exotic).

A. Up-type quarks

The values of various U(1) quantum numbers for u quarks and their charge conjugates are shown in Table IV, along with charges of their neutral bilinears. Referring to Tables II and III, one sees that these charges correspond to those of the 12, 21, 35, and 53 entries in the 5×5 neutral lepton mass matrix:

$$(-2,-2,-2) \sim m_{12}, m_{35}. \qquad (22)$$

Thus, the Dirac masses of the electron neutrino and the u quark are related to one another. If this relation also holds for the second and third families, one estimates that the Dirac mass of the heaviest neutrino should be $m_{12} \simeq (m_\tau/m_b) m_t \simeq 70$ GeV (taking account of renormalization-group running). Assuming neutrino masses $m_3 \gg m_2 \gg m_1$, one would expect from $\Delta m_{32}^2 \simeq 2.5 \times 10^{-3}$ eV^2 that $m_3 \simeq 5 \times 10^{-2}$ eV and hence a seesaw scale (at least for the heaviest neutrino) of $M_{22} \simeq 10^{14}$ GeV.

B. Down-type and h quarks

The left-handed d, s, b quarks are weak isodoublets, while their charge conjugates are weak isosinglets. The left-handed h-type quarks and their charge conjugates are both weak isosinglets. In Table V we summarize various charges of states and bilinears. These bilinears have the same charges as the following entries in the neutral lepton mass matrix:

TABLE V. Charges $2\sqrt{6}Q_\psi, 2\sqrt{10}Q_\chi$, and $2\sqrt{10}Q_N$ of left-handed d- and h-type quarks and their charge conjugates, along with charges of their neutral bilinears.

	$d^c(-1,3,-2)$	$d(-1,-1,-1)$	$h^c(2,-2,3)$	$h(2,2,2)$
$d^c(-1,3,-2)$	$\cdots$	$(-2,2,-3)$	$\cdots$	$(1,5,0)$
$d(-1,-1,-1)$	$(-2,2,-3)$	$\cdots$	$(1,-3,2)$	$\cdots$
$h^c(2,-2,3)$	$\cdots$	$(1,-3,2)$	$\cdots$	$(4,0,5)$
$h(2,2,2)$	$(1,5,0)$	$\cdots$	$(4,0,5)$	$\cdots$

TABLE VI.   Charges $2\sqrt{6}Q_\psi$, $2\sqrt{10}Q_\chi$, and $2\sqrt{10}Q_N$ of left-handed charged leptons, along with charges of their neutral bilinears.

	$e^-(-1,3,-2)$	$e^+(-1,-1,-1)$	$E^-(2,-2,3)$	$E^+(2,2,2)$
$e^-(-1,3,-2)$	$\cdots$	$(-2,2,-3)$	$\cdots$	(1,5,0)
$e^+(-1,-1,-1)$	$(-2,2,-3)$	$\cdots$	$(1,-3,2)$	$\cdots$
$E^-(2,-2,3)$	$\cdots$	$(1,-3,2)$	$\cdots$	(4,0,5)
$E^+(2,2,2)$	(1,5,0)	$\cdots$	(4,0,5)	$\cdots$

$$(-2,2,-3) \sim m_{45},$$
$$(1,5,0) \sim M_{14},$$
$$(1,-3,2) \sim m_{24},$$
$$(4,0,5) \sim M_{34}. \tag{23}$$

In particular, in the absence of mixing of ordinary and exotic quarks, m_{45} is related to the Dirac mass of ordinary down-type quarks, while M_{34} is related to the Dirac mass ($> \mathcal{O}(1)$ TeV) of h-type quarks. Weak universality suggests that the mixing of weak isodoublet left-handed d-type quarks with weak left-handed isosinglet h-type quarks is small, and hence in grand unification schemes that $m_{24} \ll m_{45}$. Since M_{14} is related to a quantity which mixes weak isosinglet ordinary quarks with weak isosinglet exotic ones, there seems to be less of a constraint on that matrix element coming from quarks of charge $-1/3$ and their antiquarks. For some discussions of h-quark properties and searches, see Refs. [42–44].

C. Charged leptons

The pattern of mixing of charged leptons resembles that associated with quarks of charge $-1/3$ and their antiquarks. The U(1) charges of states and bilinears are summarized in Table VI. The same association of neutral lepton mass matrix entries with U(1) charges exhibited in Eq. (23) holds here as well. In the absence of ordinary-exotic mixing, m_{45} is associated with an ordinary charged-lepton Dirac mass, and M_{34} is associated with an exotic ("E-type") lepton Dirac mass. The analog of m_{24} mixes a weak isosinglet ordinary-lepton mass with a weak isodoublet exotic-lepton mass and thus must be much smaller than the analog of m_{45}, while the analog of M_{14} mixes weak isodoublets with one another and thus is not strongly constrained.

VI. RELATION TO SHORT-BASELINE OSCILLATION EXPERIMENTS

So far we have considered mixing of ν and n within a single family, finding enough freedom in E_6 to write a generic 2×2 matrix with arbitrary terms as long as we permit a large seesaw mass M_{22}. The most general mass matrix for three families will then be 6×6. As has been pointed out in Ref. [4], it is sufficient to neglect the mass differences among light neutrinos when discussing

oscillations sensitive to squared mass differences in the eV^2 range. Consequently, we may speak of squared mass differences Δm_{41}, Δm_{51}^2, and Δm_{61}^2 (in increasing order), and mixing matrices $U_{\alpha i}$, where $\alpha = (e, \mu, \tau)$ and $i = (4, 5, 6)$. There will also be CP-violating phases when there are at least two sterile neutrinos. These can be useful when accounting for differences between neutrino and antineutrino oscillations.

In Ref. [4] a model with three active and N sterile neutrinos will be referred to as a $3 + N$ model. In fits to a $3 + 1$ model, neutrino and antineutrino data favor very different oscillation parameters, as do appearance and disappearance data. The probability of compatibility among all data is rated as 0.043%. This probability rises to 13% in a $3 + 2$ model and 90% in a $3 + 3$ model. However, in $3 + 2$ and $3 + 3$ models the compatibility of appearance and disappearance data is still only about 0.008%. This is mainly due to a poor fit to the low-energy signal of electron neutrino and antineutrino appearance data in the MiniBooNE experiment [9]. It is still not settled whether those data are due to electrons (positrons for antineutrinos) or to photons, e.g., from an anomalous coherent $Z - \gamma$ interaction with the target nucleus [45]. The main improvement associated with the $3 + 3$ model appears to be increased compatibility (53%) of neutrino vs antineutrino data, compared with 5.3% for the $3 + 2$ model.

The fits assume $U_{\tau i} = 0$, allowing for maximal values of $|U_{ei}|$ and $|U_{\mu i}|$. Typical values of these parameters in both $3 + 2$ and $3 + 3$ models are about 0.15 ± 0.05. In both models Δm_{41}^2 is about 0.9 eV^2 and Δm_{51}^2 is about 17 eV^2, while an additional state with $\Delta m_{61}^2 = 22$ eV^2 is present in the $3 + 3$ model. In the $3 + 2$ model, there is an additional allowed region with $\Delta m_{51}^2 \simeq 0.9$ eV^2.

The fits of Ref. [6] also favor more than one sterile neutrino, with preference for a scheme with two extra neutrinos having $\Delta m_{14}^2 = -0.87$ eV^2 and $\Delta m_{15}^2 = 0.47$ eV^2. Mixing parameters are in the same rough range as those in Ref. [4]. Again, attention is called to the tension between neutrino appearance and disappearance data.

A cautionary note has been sounded [46] with regard to a claimed 6% deficit in the flux of reactor neutrinos [11,12]. It has been pointed out that calculations of these fluxes did not take account of the uncertainty associated with the 30% of the flux that arises from forbidden decays.

How does this relate to the $3 + 3$ scenario predicted by E_6? The freedom we have to describe a single family is an

encouraging sign that similar freedom might exist in the three-family case where mixings are governed by a 6×6 light-neutral-lepton mass matrix. However, without a basic understanding of the mixings of ν_e, ν_μ, ν_τ, it may be premature to attempt such a description. The question also is not yet settled whether a third neutrino is needed to help describe short-baseline oscillation phenomena, or is available as a dark matter candidate.

We established that typical mixings between light conventional neutrinos and sterile ones are of order $M_{14}/(2m_{45})$. In order to accommodate suggestions of $|U_{ei}|$ and $|U_{\mu i}|$ of order 0.15 ± 0.05, this ratio of mass parameters must be of the same order. As the analogue of m_{45} describes masses of down-type quarks or charged leptons, M_{14} must be a small but non-negligible fraction of this quantity. This feature is perhaps the most finely-tuned aspect of the present framework. As mentioned, assignment of a Z_2 quantum number of -1 to SO(10) 16-plets and $+1$ to 10-plets and singlets is one way to achieve this suppression.

VII. THIRD STERILE NEUTRINO AS A DARK MATTER CANDIDATE

Light sterile neutrinos with masses in the keV range have been proposed to account for some or all of the dark matter in the Universe [15–18]. (There may be more than one species [47].) Two recent claims for keV-scale dark matter are based on the observation of an x-ray line near 3.5 keV [48,49] which could arise from the decay of a 7 keV neutral lepton to a photon and a much lighter neutral lepton. (The absence of such a line in the Milky Way is a source of caution [50].)

Constraints arising from taking a keV-scale neutrino to be a source of warm dark matter, and proposals for its production, have been reviewed in Ref. [14]. Recent proposals involving a 7 keV neutrino include those in Refs. [51–55]. Values of the mixing parameter between the sterile neutrino and a light one are typically somewhat below $\sin^2 2\theta = \mathcal{O}(10^{-10})$, which is easily accommodated in an E$_6$ model. In comparison with models (see, e.g., [14]) in which a keV-scale neutrino is taken to be one of the three right-handed neutrinos, such a scenario affords greater freedom in choice of parameters.

Some remarks on the special aspects of an E$_6$ framework for keV-scale dark matter are in order. The Higgs vacuum expectation values we have introduced correspond to five neutral complex scalar bosons belonging to the 27* representation of E$_6$. The masses of these bosons are free parameters. The standard model Higgs boson *happens* to have a mass which is close to $1/\sqrt{2}$ of its vacuum expectation value, but there is no reason for this to be true in general. [In fact, two of the five neutral Higgs bosons are just those of the minimal supersymmetric Standard Model or any left-right symmetric model including SO(10).] But exchanges of these bosons can give rise to exotic processes producing the state n: e.g.,

$$d_l + h_L^c \to n_L + N_{EL}^c; \qquad e^- + E_L^+ \to n_L + N_{EL}^c. \quad (24)$$

Furthermore, if a TeV scale Z_N is produced in the early universe, it would have an appreciable branching ratio for decay into nn^c, according to the Q_N quantum numbers listed in Table I. Thus n are candidates for early overproduction unless their abundance is diluted by subsequent entropy production [56].

VIII. CONCLUSIONS

The present paper is not meant to be a definitive model for sterile neutrinos in E$_6$, but rather an answer to the question: "What does it take for E$_6$ to be a satisfactory framework for treating sterile neutrinos?" The following conditions have been found sufficient (though others may well exist):

   (i) The standard seesaw mechanism accounts for all three (light) active neutrino masses, entailing three very heavy right-handed neutrinos which are left-handed SU(2) singlets and right-handed SU(2) doublets.

  (ii) Fermion masses arise from the lowest-dimension (27*) Higgs representation in E$_6$, aside from a mechanism permitting right-handed neutrinos to acquire large Majorana masses.

 (iii) This is achieved by breaking E$_6$ down to $SU(5) \times U(1)_N$, where $U(1)_N$ is a symmetry under which right-handed neutrinos are neutral. Q_N is the corresponding charge.

  (iv) Exotic isodoublet leptons ν_E, E and isosinglet quarks h should acquire large Dirac masses and mix weakly with lighter states.

   (v) A term M_{14} in the 5×5 neutrino mass matrix is taken to be small despite carrying zero weak (left-handed) isospin. This fine-tuning, possibly achieved via a Z_2 symmetry, seems needed to describe the observed spectrum.

The grand unified group E$_6$ provides candidates for three light sterile neutrinos. At least two of these seem useful to account for present-day anomalies in short-baseline neutrino oscillation experiments. (It goes without saying that these are urgently in need of confirmation, as none rises to the level of a "discovery.") A third may improve the description of these anomalies, or could be available as a candidate for dark matter, such as suggested by recent x-ray observations. The extended Higgs and gauge structure of E$_6$ permits new mechanisms for production of these candidates in the early universe.

Conclusive evidence for three sterile neutrinos, if interpreted within the framework of E$_6$, would entail also isovector charged leptons and isosinglet quarks h with charge $-1/3$. These would then be prime targets for higher-energy searches at the CERN Large Hadron Collider.

ACKNOWLEDGMENTS

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