New expectations and uncertainties on neutrinoless double beta decay

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The hypothesis that the Majorana mass of ordinary neutrinos dominates the rate of neutrinoless double beta decay is investigated. Predictions from neutrino oscillations are updated. Nuclear uncertainties are discussed, evaluating the impact of the quenching of the axial vector coupling constant in the nuclear medium, recently pointed out by Iachello *et al.* [Phys. Rev. C 87, 014315 (2013)]. Also, the sensitivity of present and future experiments is assessed, and possible implications of the knowledge on neutrino masses from cosmology are studied. The predictions from neutrino oscillations are compared with the results from cosmology and from neutrinoless double beta decay searches, emphasizing the important role of the measurement errors. The obstacles to an experimental determination of the Majorana phases are pointed out.

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I. INTRODUCTION

The search for neutrinoless double beta decay ($0\nu\beta\beta$) probes lepton number conservation and allows us to investigate the nature of the neutrino mass eigenstates. In this work, we perform an updated and in-depth study of the conservative assumption that this transition is due to the exchange of the three known neutrinos, endowed with Majorana mass. We emphasize the role of the uncertainties due to the quenching of the axial vector coupling constant in the nuclear medium, recently pointed out by Iachello *et al.* [1].

First, we update the predictions from oscillations (Sec. II). We compare these predictions with the recent experimental results on $0\nu\beta\beta$ in Sec. III. The sensitivity of future experiments, defined by taking into account the uncertainties, is discussed in Sec. IV. In Sec. V, we analyze the implication of recent bounds and hints for the neutrino mass obtained in cosmology. Finally, in Sec. VI, we discuss whether it could be possible to measure the *Majorana phases* and/or discriminate the two neutrino mass hierarchies while quantifying the role of the errors of measurement.

II. UPDATED PREDICTIONS FROM OSCILLATIONS

Assuming that the $0\nu\beta\beta$ transition is caused by the exchange of *ordinary* neutrinos, the key parameter that regulates its rate is the *Majorana effective mass*, namely,

$$m_{\beta\beta} \equiv |e^{i\alpha_1}|U_{ei}^2|m_1 + e^{i\alpha_2}|U_{e2}^2|m_2 + |U_{e3}^2|m_3|. \tag{1}$$

It represents the absolute value of the ee entry of the neutrino mass matrix. Here, m_i are the masses of the individual neutrinos ν_i, $\alpha_{1,2}$ are the Majorana phases, and U_{ei} are the elements of the mixing matrix that define the composition of the electron neutrino: $|\nu_e\rangle = \sum_{i=1}^{3} U_{ei}^*|\nu_i\rangle$.

The present information on three-flavor neutrino oscillations is compatible with two different neutrino mass spectra: *normal hierarchy* ($\mathcal{NH}$) and *inverted hierarchy* ($\mathcal{IH}$). In the former case the mass-squared difference between the two heavier states is much larger than the one between the two lighter states. In the latter case, the opposite is true.

Thanks to the knowledge of the oscillation parameters, it is possible to constrain the parameter $m_{\beta\beta}$. However, since the complex phases $\alpha_{1,2}$ in Eq. (1) cannot be probed by oscillations and are unknown, the allowed region for $m_{\beta\beta}$ is obtained by letting them vary freely. The expressions for the resulting extremes are [2]

$$m_{\beta\beta}^{\max} = \sum_{i=1}^{3} |U_{ei}^2| m_i \tag{2}$$

$$m_{\beta\beta}^{\min} = \max\{2|U_{ei}^2|m_i - m_{\beta\beta}^{\max}, 0\} \quad i = 1, 2, 3. \tag{3}$$

We adopt the graphical representation of $m_{\beta\beta}$ introduced in [2] and refined in [3,4]. It consists in plotting $m_{\beta\beta}$ in bilogarithmic scale as a function of the mass of the lightest neutrino, both for the cases of $\mathcal{NH}$ and of $\mathcal{IH}$. The resulting plot, according to the new values of the oscillation parameters in [5], is shown in the left panel of Fig. 1. The uncertainties on the various parameters entering Eqs. (2) and (3) are propagated using the procedures described in the Appendix [Eq. (A3)]. This results in a wider allowed region, which corresponds to the shaded parts in the picture.

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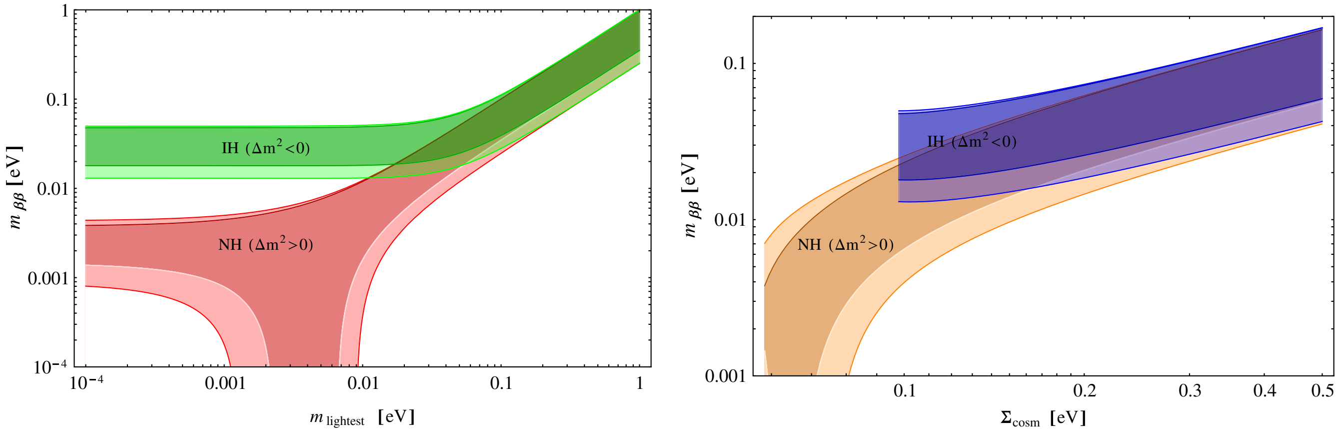

FIG. 1 (color online). Updated predictions on $m_{\beta\beta}$ from oscillations as a function of the lightest neutrino mass (left) and of the cosmological mass (right) in the two cases of $\mathcal{NH}$ and $\mathcal{IH}$. The shaded areas correspond to the 3σ regions due to error propagation of the uncertainties on the oscillation parameters.

It is also useful to express the parameter $m_{\beta\beta}$ as a function of a directly observable parameter, rather than as a function of the lightest neutrino mass. A natural choice is the cosmological mass Σ, defined as the sum of the three active neutrino masses ($\Sigma \equiv m_1 + m_2 + m_3$). The close connection between the neutrino masses' measurements obtained in the laboratory and those probed by cosmological observations was outlined long ago [6]. Furthermore, the measurements of Σ have recently reached important sensitivities, as discussed below. For these reasons, we also update the plot of the dependence of the Majorana effective mass $m_{\beta\beta}$ on the cosmological mass Σ, using the representation originally introduced in [7].

From the definition of Σ, we can write

$$\Sigma = m_l + \sqrt{m_l^2 + a^2} + \sqrt{m_l^2 + b^2}, \qquad (4)$$

where m_l is the mass of the lightest neutrino and a and b are different constants depending on the neutrino mass hierarchy. Through Eq. (4) one can establish a direct relation between Σ and m_l and thus, it is straightforward to plot $m_{\beta\beta}$ as a function of Σ. Concerning the treatment of the uncertainties, we use again the assumption of Gaussian fluctuations and the prescription reported in the Appendix. The result of the plotting in this case is shown in the right panel of Fig. 1.

III. COMPARISON WITH THE EXPERIMENTAL RESULTS

A. Experimental bounds

Recently, several experiments have obtained bounds on $t^{1/2}(\text{exp})$ above 10^{25} yr. The results are summarized in the upper part of Table I. They were achieved thanks to the study of two nuclei: ^{76}Ge and ^{136}Xe. The 90% C.L. bound from ^{76}Ge, obtained by combining GERDA-I, Heidelberg-Moscow, and IGEX via the recipe of Eq. (A1), $3.2 \cdot 10^{25}$ yr, is almost identical to the one quoted by the GERDA Collaboration, $3.0 \cdot 10^{25}$ yr [11]. By combining the first KamLAND-Zen results on $0\nu\beta\beta$ (namely, KamLAND-Zen-I [12]), and the new ones obtained after the scintillator purification (KamLAND-Zen-II [13]), the same procedure gives $2.3 \cdot 10^{25}$ yr, which differs a little bit from the combined limit quoted by the Collaboration [13], $2.6 \cdot 10^{25}$ yr. When we combine the two results of KamLAND-Zen and the one from EXO-200 using again the procedure of Eq. (A1), we get $2.6 \cdot 10^{25}$ yr, which is equal to the KamLAND-Zen limit alone. In view of the above discussion and in order to be as conservative as possible, we will adopt as combined 90% C.L. bounds the following values:

$$t_{\text{Ge}}^{1/2} > 3.0 \cdot 10^{25}\ \text{yr} \quad \text{and} \quad t_{\text{Xe}}^{1/2} > 2.6 \cdot 10^{25}\ \text{yr}. \qquad (5)$$

More experiments are also expected to produce important new results in the coming years. A few selected ones are also reported in the lower part of Table I.

B. Nuclear physics and $0\nu\beta\beta$

Assuming that the transition is dominated by the exchange of ordinary neutrinos with Majorana mass, the theoretical expression of the half-life in an ith experiment based on a certain nucleus is

$$t_i^{1/2}(\text{th}) = \frac{m_e^2}{\mathcal{G}_{0\nu,i}\mathcal{M}_i^2 m_{\beta\beta}^2}, \qquad (6)$$

where m_e is the electron mass, $\mathcal{G}_{0\nu,i}$ the phase space factor (usually given in inverse years), and $\mathcal{M}_i$ the nuclear matrix element, an adimensional quantity of enormous importance. In recent works, this last term is written emphasizing the axial coupling g_A:

$$\mathcal{M}_i = g_A^2 \cdot \mathcal{M}_{0\nu,i}. \qquad (7)$$

TABLE I. Lower bounds achievable for $m_{\beta\beta}$ by some $0\nu\beta\beta$ experiments, depending on their reached sensitivities (upper group) or sensitivity goals (lower group). The different results correspond to the different quenching of g_A, according to the definitions in Eq. (9). The 1σ uncertainties on $m_{\beta\beta}$ are calculated by assuming uncertainties both on the matrix elements and phase space factors, according to [1] and [8], respectively.

Experiment	Isotope	$t^{1/2}(90\%\ \text{C.L.})(10^{25}\ \text{yr})$	Lower bound for $m_{\beta\beta}(\text{eV})$		
			g_{nucleon}	g_{quark}	$g_{\text{phen.}}$
IGEX [9]	^{76}Ge	1.57	0.31 ± 0.03	0.49 ± 0.05	1.44 ± 0.16
HEIDELBERG-MOSCOW [10]	^{76}Ge	1.9	0.28 ± 0.03	0.44 ± 0.05	1.31 ± 0.14
GERDA-I [11]	^{76}Ge	2.1	0.26 ± 0.03	0.42 ± 0.05	1.25 ± 0.14
KamLAND-Zen-I [12]	^{136}Xe	1.9	0.18 ± 0.02	0.29 ± 0.03	1.06 ± 0.12
KamLAND-Zen-II [13]	^{136}Xe	1.3	0.22 ± 0.02	0.35 ± 0.04	1.28 ± 0.14
EXO-200 [14]	^{136}Xe	1.1	0.24 ± 0.03	0.38 ± 0.04	1.39 ± 0.15
Combined Ge [11]	^{76}Ge	3.0	0.22 ± 0.02	0.35 ± 0.04	1.05 ± 0.11
Combined Xe	^{136}Xe	2.6	0.15 ± 0.02	0.25 ± 0.03	0.91 ± 0.10
Combined Ge + Xe	^{76}Ge/^{136}Xe		0.15 ± 0.01	0.24 ± 0.02	0.81 ± 0.07
CUORE [15]	^{130}Te	9.5	0.07 ± 0.01	0.11 ± 0.01	0.39 ± 0.04
GERDA-II [16]	^{76}Ge	15	0.10 ± 0.01	0.16 ± 0.02	0.47 ± 0.05
SuperNEMO [17]	^{82}Se	10	0.07 ± 0.01	0.12 ± 0.01	0.36 ± 0.04

$\mathcal{M}_{0\nu,i}$ depends mildly on g_A and can be evaluated by theoretically modeling the nucleus. This is independent on g_A if the same quenching is assumed both for the vector and axial coupling constants, as we do here for definiteness, following [1] (as discussed in the reference, some residual dependence upon g_A could be attributed to a different renormalization of the two coupling constants). On the contrary, $\mathcal{G}_{0\nu,i}$ is a constant parameter, independent on g_A, and it is reasonably well known. Its value can be found, e.g., in [8] for all the candidate $0\nu\beta\beta$ emitters.

As a consequence, an experimental limit on the half-life translates into a limit on the mass parameter:

$$m_{\beta\beta} \leq \frac{m_e}{\mathcal{M}_i\sqrt{\mathcal{G}_{0\nu,i}t_i^{1/2}(\text{exp.})}}. \tag{8}$$

The main sources of uncertainties in the inference are the nuclear matrix elements. The first calculations of $\mathcal{M}_{0\nu,i}$ that also estimated the errors, based on the "QRPA" description of the nucleus, assessed a relatively small intrinsic error of ∼20% [18,19]. The validity of these conclusions has been recently supported by a completely independent calculation based on the "IBM2" description of the nucleus [1,8].

However, the same papers have also emphasized a more important role of the axial coupling g_A than originally thought. In other words, the real theoretical issue concerns $\mathcal{M}_i$. Indeed, it is commonly expected that the value $g_A \simeq 1.269$ measured in the weak interactions and decays of nucleons is modified (or, *renormalized*) in the nuclear medium toward the value appropriate for quarks [18–20]; the plausibility of further modification (reduction) has been argued in [1], based on the knowledge on the double beta decay with neutrinos ($2\nu\beta\beta$). In light of this discussion, a conservative treatment of the uncertainties should consider at least three cases:

$$g_A = \begin{cases} g_{\text{nucleon}} = 1.269 \\ g_{\text{quark}} = 1 \\ g_{\text{phen}} = 1.269 \cdot A^{-0.18}. \end{cases} \tag{9}$$

We will refer to the last formula with the name *maximal quenching*. It includes phenomenologically the effect of the atomic number A. The g_{phen}. parametrization as a function of A comes directly from the comparison between the theoretical half-life for $2\nu\beta\beta$ and its observation in different nuclei [1].

Needless to say, the validity of the assumption that the quenching is the same both for the $2\nu\beta\beta$ and the $0\nu\beta\beta$ cases is still an open issue. We stress that this is just a phenomenological description of the quenching, since the specific behavior is different in each nucleus and it somewhat differs from this parametrization [1]. Nonetheless, the assumption described in Eq. (9) seems a reasonable one and deserves discussion in the present context. In fact, the question of *which is the true value of* g_A introduces a considerable uncertainty in the inferences concerning massive neutrinos.

C. Sensitivity of present experiments

Once the experimental limits on the half-lives are known, by using the phase space of [8] and the matrix elements of [1], it is possible to find the lower bounds on $m_{\beta\beta}$ according to Eq. (8). Table I shows the results for the experiments considered in Sec. II. In order to obtain the combined bounds, the procedure shown in the Appendix was used. The different values of $m_{\beta\beta}$ correspond to the three quenching scenarios considered in Sec. III B. The 1σ errors

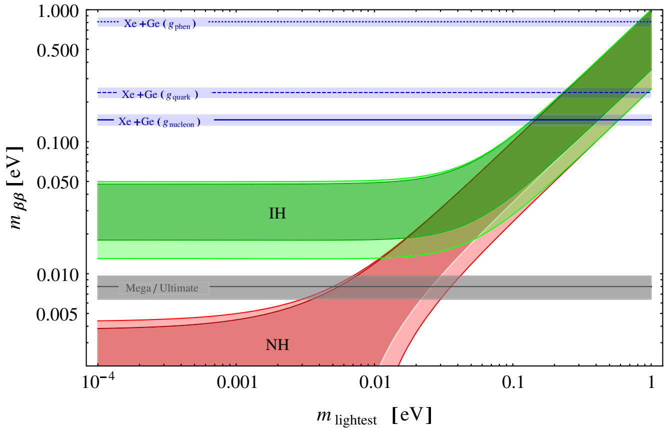

FIG. 2 (color online). Present sensitivity on $m_{\beta\beta}$, according to the Ge + Xe combined limit, in the three quenching scenarios. The upper (1σ) bands come from the uncertainties on the nuclear matrix elements. The darker band at $m_{\beta\beta} = (8 \pm 1.6)$ meV concerns the ultimate and mega experiments, discussed in the text.

of Table I were computed according to Eq. (A3), assuming both uncertainties on the matrix elements and phase space factors, as reported in [1] and [8], respectively. Nonetheless, the former error gives the main contribution.

The importance of the g_A quenching is evident from the table: the sensitivity for the same experiment in the two cases of g_{nucleon} and g_{phen}. differs by a factor ~5. This is graphically shown in Fig. 2, where the present best limit on $m_{\beta\beta}$ coming from the combination Ge + Xe (obtained as described in the Appendix) is plotted in correspondence of the considered values of g_A.

IV. SENSITIVITY OF FUTURE EXPERIMENTS

Now we consider a next generation experiment (call it a *mega* experiment) and a next-to-next generation one (an *ultimate* experiment) with enhanced sensitivity. First of all, we should clarify which is the physics goal that we would like to achieve.

Plausibly, the most honest way to talk of the sensitivity is in terms of exposure or of half-life time that can be probed. From the point of view of the physical interest, however, besides the hope of discovering the $0\nu\beta\beta$, the most exciting investigation that can be imagined at present is the exclusion of the $\mathcal{IH}$ case. This is the goal that most of the experimentalists are trying to reach with $0\nu\beta\beta$ experiments, working in the above assumptions and supposing that $0\nu\beta\beta$ will not be found. For this reason, we require a sensitivity:

$$m_{\beta\beta} = 8 \text{ meV}.$$

The *mega* experiment is the one that satisfies this requirement in the most favorable case, namely, when the quenching of g_A is absent. Instead, the *ultimate* experiment assumes that g_A is maximally quenched. We chose the 8 meV value because, even taking into account the residual uncertainties on the nuclear matrix elements, the overlap with the allowed band for $m_{\beta\beta}$ in the $\mathcal{IH}$ is excluded. In fact, the uncertainties on Ge and Xe nuclei amount to ~20%, as discussed above. Notice that we are assuming that at some point the issue of the quenching will be sorted out. Through Eq. (8), we obtain the corresponding value of $t^{1/2}$ and thus, we calculate the needed exposure to accomplish the task.

The law of radioactive decay prescribes that

$$t^{1/2} = \ln 2 \cdot T \cdot \varepsilon \cdot \frac{x \cdot \eta \cdot N_{\mathrm{A}} \cdot M}{\mathcal{M}_A \cdot N_{\mathrm{S}}}, \tag{10}$$

where T is the measuring time, ε is the detection efficiency, x is the stoichiometric multiplicity of the element containing the $\beta\beta$ candidate, η is the $\beta\beta$ candidate isotopic abundance, N_{A} is the Avogadro number, $\mathcal{M}_A$ is the compound molar mass, and N_{S} is the number of observed decays in the region of interest. Let us focus on the optimal experimental condition, when the contribution of the background counts is negligible (*zero background* condition). This means that we require

$$M \cdot T \cdot B \cdot \Delta \lesssim 1, \tag{11}$$

where M is the detector mass; B is the background level per unit mass, energy, and time; and Δ is the full width half maximum (FWHM) energy resolution. Now, if we suppose $\varepsilon \simeq 1$ (detector efficiency of 100% and no fiducial volume cuts), $x \simeq \eta \simeq 1$ (all the mass is given by the candidate nuclei), and we assume one observed event (i.e., $N_{\mathrm{S}} = 1$) in the region of interest, Eq. (10) simplifies to

$$M \cdot T = \frac{\mathcal{M}_A \cdot t^{1/2}}{\ln 2 \cdot N_{\mathrm{A}}}. \tag{12}$$

This is the equation we used to estimate the product $M \cdot T$ (exposure), and thus to assess the sensitivity of the mega and ultimate scenarios. The key input is, of course, the theoretical expression of $t^{1/2}$. The calculated values of the exposure are shown in Table II for the three considered nuclei: ^{76}Ge, ^{130}Te, and ^{136}Xe. The last column of the table gives the maximum allowed value of the product $B \cdot \Delta$ that satisfies Eq. (11).

Finally, Fig. 2 shows the present knowledge on $0\nu\beta\beta$ according to the best combined limit of Ge + Xe of Table I compared to the mega/ultimate scenarios. We report the three possible predictions on the bounds on $m_{\beta\beta}$ according to the three quenching scenarios considered. The presence of 1σ bands instead of single lines is due to the propagation of the residual uncertainties on the nuclear matrix elements. The mega/ultimate scenarios are presented as the gray band.

TABLE II. Sensitivity and exposure necessary to discriminate between $\mathcal{NH}$ and $\mathcal{IH}$: the goal is $m_{\beta\beta} = 8$ meV. The two cases refer to the unquenched value of $g_A = g_{\mathrm{nucleon}}$ (mega) and $g_A = g_{\mathrm{phen}}$. (ultimate). The calculations are performed assuming *zero background* experiments with 100% detection efficiency and no fiducial volume cuts. The last column shows the maximum value of the product $B \cdot \Delta$ in order to actually comply with the zero background condition.

			Exposure (estimate)	
Experiment	Isotope	$t^{1/2}$(yr)	$M \cdot T$ (ton · yr)	$B \cdot \Delta_{(\mathrm{zero\ bkg})}$(counts/kg/yr)
Mega Te	^{130}Te	6.8×10^{27}	2.1	4.7×10^{-4}
Mega Ge	^{76}Ge	2.3×10^{28}	4.1	2.4×10^{-4}
Mega Xe	^{136}Xe	9.7×10^{27}	3.2	3.2×10^{-4}
Ultimate Te	^{130}Te	2.3×10^{29}	71	1.4×10^{-5}
Ultimate Ge	^{76}Ge	5.1×10^{29}	93	1.1×10^{-5}
Ultimate Xe	^{136}Xe	3.3×10^{29}	109	9.2×10^{-6}

V. IMPLICATIONS OF COSMOLOGY

Here we discuss the possibility of taking advantage of the knowledge about the neutrino cosmological mass to make inferences on some $0\nu\beta\beta$ experiment results (or expected ones). We consider only the optimistic assumption that g_A is unquenched. The changes induced by the quenching can be easily understood by considering, e.g., its impact in Fig. 2. Evidently, this weakens the reach of each experiment, rescaling the possible bounds or measurements toward larger values.

A. Information from cosmology

The new experimental limit provided by the Planck experiment on Σ is 0.23 eV at 95% C.L. [21]. Interestingly, several studies have emphasized some tension between the data of Planck and those from galaxy counts and lensing. Their combination suggests a nonzero best fit value of the mass, in the range (0.3–0.4) eV and with an error of about 30% [22,23]. Taking into account these data, we will consider two scenarios for the subsequent discussion:

$$\Sigma < 0.19 \text{ eV}(90\% \text{ C.L.}) \quad (\text{conservative}), \ [21]$$
$$\Sigma = (0.320 \pm 0.081) \text{ eV} \quad (\text{aggressive}), \ [23]. \qquad (13)$$

We consider these two cases since, at present, the evidence for nonzero neutrino masses is not strong and the possibility of unexpected systematics cannot be excluded.

The results on Σ can provide us precious information on $0\nu\beta\beta$ in the assumption that this transition is dominated by the light neutrinos exchange. For example, by looking at Fig. 1 and in the conservative limit in Eq. (13), it seems useless to look for $0\nu\beta\beta$ with an experiment with a sensitivity on $m_{\beta\beta}$ of 100 meV or more. If, instead, the claim of measurement in the equation is correct, and as soon as $0\nu\beta\beta$ experiments probe the transition rate, we will obtain information on the quenching factor. A successful measurement in the next generation of experiments, for example, would mean that the quenching is reduced or absent.

B. Combination of cosmology and $0\nu\beta\beta$ results

We study two different situations. In the former case (Sec. V B 1), we assume that no effect of mass is observed, and we have upper bounds both on Σ and $m_{\beta\beta}$. We use the conservative limit on Σ reported from Planck in Eq. (13). As regards $m_{\beta\beta}$, we take the current best limit coming from the Ge + Xe combination (Table I), and the one corresponding to the expected CUORE sensitivity (here, CUORE is chosen just as an example of a next generation experiment). In the latter case (Sec. V B 2), we assume that the claim for neutrino mass from cosmology is correct and that $0\nu\beta\beta$ is measured with a half-life corresponding to the lowest bound of $m_{\beta\beta}$ coming from the Ge + Xe combination or from the expected CUORE sensitivity. The values of $m_{\beta\beta}$ for these two cases are again those in Table I.

1. First scenario: Upper bounds

Let us suppose Gaussian distributions centered in zero both for Σ and $m_{\beta\beta}$, with a standard deviation coming directly from the experimental upper limit; namely, we put $\Sigma^{\mathrm{meas}} = m_{\beta\beta}^{\mathrm{meas}} = 0$ in Eq. (A5). By requiring a 90% C.L., we obtain the elliptic allowed regions in the left panel of Fig. 3. This picture shows that even in the CUORE case, there is no chance of ruling out the $\mathcal{IH}$, unless there will be a great improvement on the knowledge of Σ. However, the combination of the two parameters allows us to improve significantly the exclusion region.

2. Second scenario: Measurements

Now we assume that both Σ and $m_{\beta\beta}$ are measured with nonzero values. While the error on the former parameter comes directly from Eq. (13), the one on the latter one requires further discussion.

The error on $m_{\beta\beta}$ has at least two different contributions: one is statistical and comes from the Poisson fluctuations on the observed number of events, while the other one comes from the uncertainties on the nuclear matrix elements and the phase space factors. We will refer to this last as the *theoretical* contribution to the total uncertainty. If we assume to know exactly the detector features (i.e., the number of decaying nuclei, the efficiency, and the time of measurement), the uncertainty on $t^{1/2}$ is only due to the statistical fluctuations of the counts:

$$\frac{\delta t^{1/2}}{t^{1/2}} = \frac{\delta N_S}{N_S}. \tag{14}$$

The statistical contribution to the determination of the parameters is in general large and cannot be neglected. By emphasizing this simple but important point in the discussion, we consider a case closer to the actual experimental situation, improving on the more idealized case that has been treated by previous investigators [24].

The statistical contribution to the total error is dominant up to about 20 signal events. The theoretical error becomes the main contribution only if many events (more than a few tens) are detected. Note that a much greater contribution would come by taking into account the error on g_A. We assume here that this problem will be solved in some manner in the future, and concentrate on the discussion of the role of the statistical error.

Using the procedure described in Eq. (A3) for the Ge + Xe case, we find an uncertainty on $m_{\beta\beta}$ of about 31 meV for 5 observed events, which reduces to 24 meV for 10 events. If we neglect the statistical uncertainty, e.g., we put $N_{\mathrm{events}} > 150$, the uncertainty becomes 14 meV. This means that the Poisson fluctuations' effect is absolutely not negligible. Similarly, repeating the same calculation for CUORE, we obtain an uncertainty of 17 meV for 5 events, 13 meV for 10 events, and 8 meV for $N_{\mathrm{events}} > 150$. Therefore, referring to Table I, we consider the following cases:

$$m_{\beta\beta} = (0.15 \pm 0.01_{\mathrm{theo}} \pm 0.03(1)_{\mathrm{stat}})\ \mathrm{eV} \quad (\text{Ge-Xe})$$
$$m_{\beta\beta} = (0.07 \pm 0.01_{\mathrm{theo}} \pm 0.02(1)_{\mathrm{stat}})\ \mathrm{eV} \quad (\text{CUORE}), \tag{15}$$

where the statistical uncertainty is considered in the case of 5 (20) observed events and we computed the total error by adding the two contributions in quadrature. As for Σ, we assume the *aggressive* value of Eq. (13). The results are shown in the right panel of Fig. 3. The implication of these errors is further discussed in the next section.

VI. IS IT POSSIBLE TO PROBE MAJORANA PHASES?

Now we assume the optimistic scenario of Sec. V B 2 and consider the question of whether it is possible to measure Majorana phases. More precisely, we discuss the possibility of distinguishing the maximum and the minimum values of $m_{\beta\beta}$, Eqs. (2) and (3). In the case of quasidegenerate neutrinos that can be explored by present experiments, this possibility is closely connected with the chance to measure one Majorana phase.

Let us consider the allowed regions of parameters of the right panel of Fig. 3. This picture shows that if CUORE observes five events (larger ellipse, continuous line), we will not be able to reach any firm conclusion either on the mass hierarchy or on the Majorana phases. Interestingly, if $0\nu\beta\beta$ were instead discovered with a $m_{\beta\beta}$ a little bit below the current best limit on Ge + Xe, this could allow us to make some inference on the Majorana phases. But it is important to repeat that, in order to state anything precise about $m_{\beta\beta}$ and the Majorana phases, the present uncertainty on the quenching of the axial vector coupling constant has to be dramatically decreased.

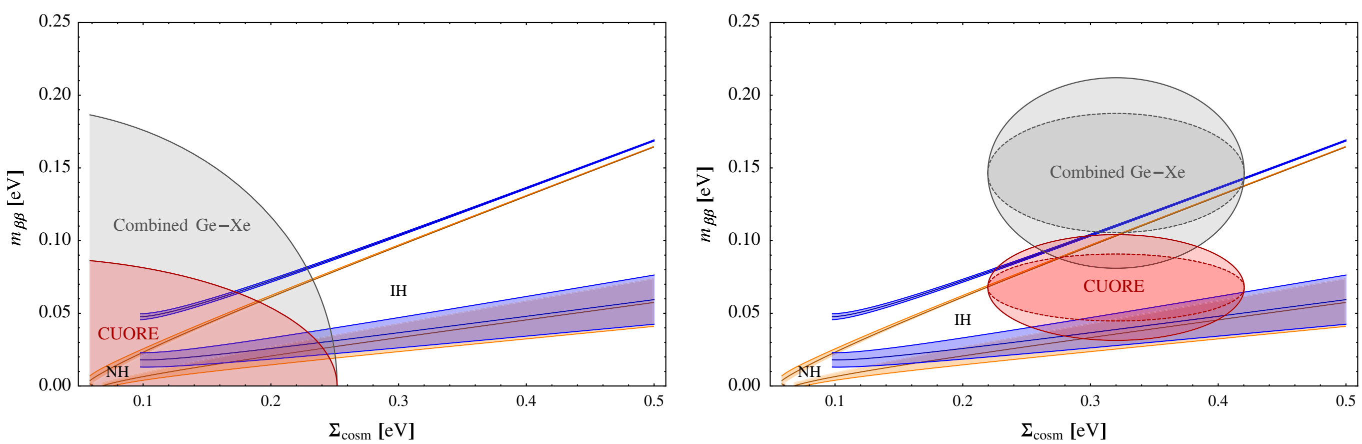

FIG. 3 (color online). Allowed regions for $m_{\beta\beta}$ as a function of the neutrino cosmological mass Σ. The colored bands correspond to the 3σ regions for the extremal values of $m_{\beta\beta}$ as a function of the neutrino cosmological mass Σ. On the left, the two ellipses represent the 90% C.L. allowed regions for the couple (Σ; $m_{\beta\beta}$) according to the experimental limits quoted in the text (Sec. V B 1). On the right, the two big (small) ellipses show the 90% C.L. regions in which a positive observation of $0\nu\beta\beta$ could be contained, according to the experimental uncertainties and 5 (20) actually observed events. In particular, they refer to two different cases: the observation of $0\nu\beta\beta$ with a $m_{\beta\beta}$ corresponding either to the Ge + Xe limit or to the CUORE expected sensitivity [Eq. (15)]. See Sec. V B 2 for a more detailed discussion.

When we repeat the same exercise assuming an observed number of 20 events, we obtain the smaller ellipses in the right plot of Fig. 3 (dashed lines). In this case, a hypothetical observation coming from the combined limit of Ge + Xe would lead to an even more precise inference on the Majorana phases whereas, in the CUORE case, we would be closer to knowing something useful on the Majorana phases, even if nothing could be said about the hierarchy.

VII. SUMMARY AND DISCUSSION

We explored the hypotheses that the ordinary neutrinos are Majorana particles and that their exchange dominates the $0\nu\beta\beta$ transition rate. In particular, we updated the predictions from neutrino oscillations, and we discussed the primary role played by considerations of nuclear physics and, more specifically, by the axial vector coupling constant of the charged-current interactions of the nucleons.

We stressed the importance of better understanding the quenching of g_A in a nuclear medium. If this turns out to be negligible, it will be possible to probe the $\mathcal{IH}$ region with the next generation experiments. Conversely, if this coupling is maximally quenched, it will be unlikely to be able to reach the minimum sensitivity required to probe the $\mathcal{IH}$ region within the next 20 years. Even in the optimistic scenario that the $0\nu\beta\beta$ will be discovered, it will be difficult to extract information on the process from the measurement, if this uncertainty persists.

We argued that a measurement or a bound from cosmology could have an important impact on the expectations on $m_{\beta\beta}$. Indeed, cosmology could be precious to understand (and possibly quantify) the actual quenching of g_A. For example, if the claim from cosmology of [23] were correct and if the future experiments measured the $0\nu\beta\beta$, we would conclude that the quenching effect is small or absent.

We critically discussed the chances of measuring the Majorana phases, by quantifying the obstacles and by assessing the role of realistic experimental uncertainties. We showed that, at present, such a measurement is really challenging, even in the most optimistic assumption on the quenching of the axial vector coupling constant.

From the above discussion, further theoretical improvements and dedicated plans of measurements seem to be necessary to clarify the expectations and decrease the uncertainties from nuclear physics.

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APPENDIX: STATISTICAL PROCEDURES

1. Combination of measurements

Let us consider the case of different neutrinoless double beta decay experiments using the same nucleus and quoting the bounds on the half-life $t^{1/2} > t_i^{1/2}(\text{exp.})$ at the same confidence level, where $i = 1, 2, 3, \ldots$. A simple way to combine them is to suppose that the corresponding rates $\Gamma < \Gamma_i \equiv \ln 2 \cdot \hbar / t_i^{1/2}$ are Gaussian distributed in all the detectors, namely, the probability of observing a rate within the interval $[\Gamma, \Gamma + d\Gamma]$ is $dP_i \propto \exp[-\Gamma^2/2\Gamma_i^2] d\Gamma$. This is the same as saying that the number of signal events is zero up to Gaussian fluctuations. In this case, the combined Gaussian bound is $\Gamma_{\text{gaus}}^{-2} = \sum_i \Gamma_i^{-2}$. Therefore, we get the combined bound for the half-life simply as

$$t^{1/2} = \sqrt{\sum_i (t_i^{1/2}(\text{exp.}))^2}. \tag{A1}$$

The described procedure has the advantage of being simple and generally conservative, although we remind the reader that it should be validated in actual situations.

The combination of results from different nuclei is more delicate and depends on the uncertain matrix elements. An elaborate procedure is discussed in [25]. Our main goal is to outline the biggest factor of uncertainty, namely, the dependence of the results upon g_A. Thus, working in the same hypotheses mentioned above, and assuming that the (relative) matrix elements are known precisely, we immediately obtain the bound on the mass relevant to the double beta decay:

$$\frac{1}{m_{\beta\beta}} = \left[\sum_{i=\text{Ge,Te,}\ldots} \left(\frac{t_i^{1/2}(\text{exp}) \mathcal{G}_{0\nu,i} \mathcal{M}_i^2}{m_e^2} \right)^2 \right]^{1/4}. \tag{A2}$$

This is consistent with the theoretical expression of the half-life in the ith experiment, as given in Eq. (6), and coincides with Eq. (A1) for the same nuclear species.

2. Error propagation

For any choice of the Majorana phases, the massive parameter that regulates the $0\nu\beta\beta$ can be thought as $M(m, \boldsymbol{x})$. It is a function of a mass m and of certain other parameters $\boldsymbol{x}$ that are determined by oscillation experiments up to their experimental errors: $x_i \pm \Delta x_i$.

Whenever we used maximal or systematic uncertainties from the literature, we decided to interpret them as the semiwidths of flat distributions in order to propagate their effects in our calculations. Then, we considered the corresponding standard deviations as Gaussian fluctuations of the parameters around the given values.

For any fixed value of m and for the other parameters set to their best fit values x_i, we can attach the following error to M:

$$\Delta M|_m = \sqrt{\sum_i \left(\frac{\partial M}{\partial x_i}\right)^2 \Delta x_i^2}. \tag{A3}$$

If we want to consider the prediction and the error for a fixed value of another massive parameter $\Sigma(m, \boldsymbol{x})$, we have to vary also m, keeping $\delta\Sigma = \partial\Sigma/\partial m \delta m + \partial\Sigma/\partial x_i \delta x_i = 0$. Therefore, in this case we find

$$\Delta M|_\Sigma = \sqrt{\sum_i \left(\frac{\partial M}{\partial x_i} - \frac{\partial\Sigma/\partial x_i}{\partial\Sigma/\partial m}\frac{\partial M}{\partial m}\right)^2 \Delta x_i^2}. \tag{A4}$$

Of course, we calculate m by inverting the equation $\Sigma(m, \boldsymbol{x}) = \Sigma$. (Here, the symbol Σ denotes the function and also its value. However, this abuse of notation is harmless in practice.)

3. Confidence intervals

The likelihood $\mathcal{L}$ for the simultaneous observation of Σ and $m_{\beta\beta}$, Gaussian distributed variables with uncertainties $\sigma(\Sigma^{\text{meas}})$ and $\sigma(m_{\beta\beta}^{\text{meas}})$, respectively, is proportional to

$$\exp\left[-\frac{(\Sigma - \Sigma^{\text{meas}})^2}{2\sigma(\Sigma^{\text{meas}})^2}\right] \exp\left[-\frac{(m_{\beta\beta} - m_{\beta\beta}^{\text{meas}})^2}{2\sigma(m_{\beta\beta}^{\text{meas}})^2}\right]. \tag{A5}$$

This corresponds to the usual χ^2:

$$\chi^2 = \frac{(\Sigma - \Sigma^{\text{meas}})^2}{\sigma(\Sigma^{\text{meas}})^2} + \frac{(m_{\beta\beta} - m_{\beta\beta}^{\text{meas}})^2}{\sigma(m_{\beta\beta}^{\text{meas}})^2}. \tag{A6}$$

The definition of the confidence intervals has to take into account the presence of 2 degrees of freedom. Indicating with C.L. the desired confidence level, we have

$$\text{C.L.} = \iint_{\chi^2 < \chi_0^2} \mathcal{L}(\Sigma, m_{\beta\beta}) d\Sigma dm_{\beta\beta}. \tag{A7}$$

Thus, rescaling the variables of integration and integrating the angular coordinate, we have

$$\chi_0^2 = -2\ln(1 - \text{C.L.}), \tag{A8}$$

which defines the value for χ^2 corresponding to the assigned confidence level C.L.

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