β-deformation on a slanted torus and deformed *pp* wave

Ariunzul Dagvadorj* and Sunyoung Shin†

Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow Region, Russia
(Received 8 May 2014; published 18 July 2014; publisher error corrected 24 July 2014)

We discuss the β-deformation of $\mathrm{AdS}_5 \times S^5$ which incorporates the $SL(2,\mathbb{R})$ symmetry of the type IIB theory. The axion-dilaton is identified with a two-torus from an 11-dimensional viewpoint. We consider the null geodesic with equal component angular momenta to take the Penrose limit of the deformed $\mathrm{AdS}_5 \times S^5$. We study the bosonic part of the string sigma model and the spectrum of the string in the pp-wave background.

DOI: 10.1103/PhysRevD.90.027704 PACS numbers: 04.65.+e

I. INTRODUCTION

The marginal deformation of $\mathcal{N}=4$ super Yang-Mills theory introduces phases in the superpotential, preserving a $U(1) \times U(1)$ non-R-symmetry. The deformation reduces the supersymmetry $\mathcal{N}=4$ to $\mathcal{N}=1$. The phases in the superpotential can be complexified[1] [1]. In the gravity side, the $U(1) \times U(1)$ symmetry maps to a two-torus. An $SL(2,\mathbb{R})$ transformation acting on a type IIB supergravity solution compactified on the two-torus produces the gravity dual of the γ-deformation. The gravity dual of the β-deformation is an $SL(3,\mathbb{R})$ transformation, which consists of the $SL(2,\mathbb{R})$ transformation and an S-duality transformation $SL(2,\mathbb{R})_s$[2] [2].

The charges of the chiral superfields under $U(1) \times U(1)$ in the gauge theory correspond to the angular momenta along the two-torus. In the marginally deformed $\mathrm{AdS}_5 \times S^5$, the $SO(6)$ isometry is broken to $U(1) \times U(1) \times U(1)$. The angle coordinates (ϕ_1, ϕ_2, ϕ_3) of the S^5 are linear combinations of the coordinates of the $U(1) \times U(1) \times U(1)$. The Bogomol'nyi-Prasad-Sommerfield geodesics are chosen with angular momenta

$$(J_{\phi_1}, J_{\phi_2}, J_{\phi_3}) \sim (J,0,0), (0,J,0), (0,0,J), (J,J,J). \tag{1.1}$$

For undeformed $\mathrm{AdS}_5 \times S^5$, the geodesics can be transformed to one another by $SO(6)$, which is the isometry of the S^5. Therefore the Penrose limits for the geodesics produce one pp wave. For deformed $\mathrm{AdS}_5 \times S^5$, which has a $U(1) \times U(1) \times U(1)$ symmetry, the geodesics are not isometrically equivalent. The first three geodesics and the fourth geodesic are two distinct geodesics. The Penrose limit for the first three cases is studied in [2,3]. The Penrose limit for the fourth case is studied in [4] where it is also shown that the spectrum of the string in this pp-wave limit is independent of the parameter γ. The pp-wave limits of marginally deformed geometries which include the σ-deformation are discussed in [5]. Giant gravitons on the deformed pp waves are investigated in [5,6].

The $SL(3,\mathbb{R})$ transformation for the β-deformation can be generalized by incorporating the $SL(2,\mathbb{R})$ symmetry of type IIB theory, which is also the symmetry of the toroidal compactification [7]. In [8], torus deformation is considered for the generalization. In this work, we apply the generalized β-deformation to $\mathrm{AdS}_5 \times S^5$ and take the Penrose limit of the deformed $\mathrm{AdS}_5 \times S^5$ along the (J,J,J) geodesic. We study the spectrum of the string in the deformed pp-wave background.

In Sec. II, we review the generalization of the β-deformation and present the deformed $\mathrm{AdS}_5 \times S^5$ geometry. In Sec. III, we study the pp-wave limit of the β-deformed $\mathrm{AdS}_5 \times S^5$ with the axion-dilaton, which is identified with the torus modulus of the rectangular torus before the torus deformation. We present the bosonic part of the string sigma model and compute the spectrum. In Sec. IV, we summarize our results.

II. GENERALIZATION OF THE β-DEFORMED GEOMETRY

The β-deformation [2] acting on a type IIB supergravity solution which has a two-torus symmetry is derived from an $SL(3,\mathbb{R})$ transformation acting on an 11-dimensional supergravity solution which has a three-torus symmetry. The coordinates of the three-torus are $(\varphi_1, \varphi_2, \varphi_3)$. The type IIB supergravity solution is obtained by a dimensional reduction along φ_3 and a T-duality transformation along φ_1. The $SL(3,\mathbb{R})$ matrix for the β-deformation is

$$\Lambda^T_{\mathrm{LM}} = \begin{pmatrix} 1 & 0 & 0 \\ \gamma & 1 & \sigma \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.1}$$

The type IIB supergravity solution can be generalized by an $SL(3,\mathbb{R})$ transformation

$$L = \begin{pmatrix} L_{11} & 0 & L_{13} \\ 0 & 1 & 0 \\ L_{31} & 0 & L_{33} \end{pmatrix}, \qquad L_{11}L_{33} - L_{13}L_{31} = 1. \tag{2.2}$$

*ariunzul.d@gmail.com

†sihnsy@gmail.com

[1] We use γ for the real parameter and β for the complex parameter.

[2] We use σ for the S-duality transformation.

This corresponds to the $SL(2,\mathbb{R})$ symmetry of the type IIB theory, which is also the symmetry of the toroidal compactification [7]. The axion-dilaton $\tau = \tau_1 + i\tau_2$ transforms as

$$\tau' = \frac{L_{11}\tau + L_{31}}{L_{13}\tau + L_{33}}. \tag{2.3}$$

The $SL(3,\mathbb{R})$ transformation $L\Lambda_{\rm LM}^T$, therefore, produces the β-deformed geometry incorporating the $SL(2,\mathbb{R})$ symmetry of the type IIB supergravity. By applying Lunin and Maldacena's solution generating technique with the $SL(3,\mathbb{R})$ matrix $L\Lambda_{\rm LM}^T$ to the type IIB supergravity solution in the form given by (A.7) in [2], the generalized β-deformation is obtained in [8] as[3]

$$\begin{aligned}
ds'^2 &= F'\left[\frac{1}{\sqrt{\Delta}}(D\varphi_1 - CD\varphi_2)^2 + \sqrt{\Delta}(D\varphi_2)^2\right] + \frac{e^{2\Phi'/3}}{F'^{1/3}} g_{\mu\nu}dx^\mu dx^\nu,\\
F' &= FG\sqrt{H}, \qquad e^{\Phi'} = \sqrt{G}H\tau_2^{-1}, \qquad \chi' = H^{-1}(h + \gamma\sigma\tau_2^2F^2),\\
B_2' &= GF^2(\gamma f - \sigma h)D\varphi_1 \wedge D\varphi_2 + \frac{\sigma}{2}\tilde{d}_{\mu\nu}dx^\mu \wedge dx^\nu,\\
C_2' &= GF^2(\gamma h - \sigma g)D\varphi_1 \wedge D\varphi_2 + \frac{\gamma}{2}\tilde{d}_{\mu\nu}dx^\mu \wedge dx^\nu,\\
F_5' &= \tilde{F}_5 + \star\tilde{F}_5,
\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}
G^{-1} &= 1 + (\gamma^2 f - 2\gamma\sigma h + \sigma^2 g)F^2,\\
H &= f + \tau_2^2\sigma^2F^2,
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
f &= (L_{33} + L_{13}\tau_1)^2 + L_{13}{}^2\tau_2^2,\\
g &= (L_{31} + L_{11}\tau_1)^2 + L_{11}{}^2\tau_2^2,\\
h &= (L_{33} + L_{13}\tau_1)(L_{31} + L_{11}\tau_1) + L_{11}L_{13}\tau_2^2;
\end{aligned} \tag{2.6}$$

$\tilde{F}_5$ in (2.4) has no indices along the torus (φ_1, φ_2). The star is taken with the new metric.

We consider $\mathrm{AdS}_5 \times S^5$ defined by

$$\begin{aligned}
ds^2 &= R^2\left[-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2 + \sum_{i=1}^3 d\mu_i^2 + \sum_{i=1}^3 \mu_i^2 d\phi_i^2\right],\\
e^{-\Phi_0} &= \tau_2, \qquad \chi_0 = \tau_1, \qquad B_2 = 0, \qquad C_2 = 0,\\
C_4 &= 4R^4\tau_2(\omega_4 + \omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3),\\
F_5 &= 4R^4\tau_2(\omega_{\mathrm{AdS}_5} + \omega_{S^5}),\\
\omega_{\mathrm{AdS}_5} &= d\omega_4, \qquad \omega_{S^5} = d\omega_1 \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3,\\
d\omega_1 &= \cos\alpha\sin^3\alpha\cos\theta\sin\theta d\alpha \wedge d\theta,
\end{aligned} \tag{2.7}$$

with

$$\mu_1 = \cos\alpha, \qquad \mu_2 = \sin\alpha\cos\theta, \qquad \mu_3 = \sin\alpha\sin\theta. \tag{2.8}$$

R is the radius of AdS_5 and the radius of S^5. We apply the transformation (2.4) to (2.7). The angle coordinates (ϕ_1, ϕ_2, ϕ_3) are related to the coordinates (φ_1, φ_2) of the two-torus and the $U(1)$ R-symmetry direction ψ as

$$\phi_1 = \psi - \varphi_2, \qquad \phi_2 = \psi + \varphi_1 + \varphi_2, \qquad \phi_3 = \psi - \varphi_1. \tag{2.9}$$

The deformed $\mathrm{AdS}_5 \times S^5$ geometry with parameters $\hat{\gamma} = \gamma R^2$ and $\hat{\sigma} = \sigma R^2$ is

[3] We follow the formulas and the conventions of [2]. It is assumed that only the metric, the complex field $\chi_0 + ie^{-\Phi_0} = \tau = \tau_1 + i\tau_2$ and $\tilde{d}_{\mu\nu}$ are excited and the other fields are zero.

$$
\begin{aligned}
ds'^2 &= R^2H^{1/2}\Big[-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2 \\
&\quad + \sum_{i=1}^{3}(d\mu_i^2 + G\mu_i^2 d\phi_i^2) + G(\hat\gamma^2 f - 2\hat\gamma\,\hat\sigma\,h + \hat\sigma^2 g)\mu_1^2\mu_2^2\mu_3^2\Big(\sum_{i=1}^{3}d\phi_i\Big)^2\Big],\\
e^{\Phi'} &= \sqrt{G}H\tau_2^{-1},\\
\chi' &= H^{-1}(h+\tau_2^2\hat\gamma\,\hat\sigma\,g_0),\\
B_2' &= R^2G(\hat\gamma f - \hat\sigma h)\omega_2 - 4R^2\tau_2\hat\sigma\omega_1\wedge\sum_{i=1}^{3}d\phi_i,\\
C_2' &= R^2G(\hat\gamma h - \hat\sigma g)\omega_2 - 4R^2\tau_2\hat\gamma\omega_1\wedge\sum_{i=1}^{3}d\phi_i,\\
C_4' &= 4R^4\tau_2\omega_4 + 4R^4\tau_2 G[1-(\hat\gamma\,\hat\sigma\,h - \hat\sigma^2 g)g_0]\omega_1\wedge d\phi_1\wedge d\phi_2\wedge d\phi_3,\\
F_5' &= 4R^4\tau_2(\omega_{\mathrm{AdS}_5} + G\omega_{S^5}),
\end{aligned}
\tag{2.10}
$$

where

$$
\begin{aligned}
G^{-1} &= 1+(\hat\gamma^2 f - 2\hat\gamma\,\hat\sigma\,h + \hat\sigma^2 g)g_0,\\
H &= f+\tau_2^2\hat\sigma^2 g_0,\\
g_0 &= \mu_1^2\mu_2^2+\mu_2^2\mu_3^2+\mu_3^2\mu_1^2,\\
\omega_2 &= \mu_1^2\mu_2^2 d\phi_1\wedge d\phi_2 + \mu_2^2\mu_3^2 d\phi_2\wedge d\phi_3 + \mu_3^2\mu_1^2 d\phi_3\wedge d\phi_1.
\end{aligned}
\tag{2.11}
$$

f, g and h are the same as (2.6).

The transformation (2.2) can be related to torus deformation from an 11-dimensional viewpoint. The torus parameters considered in [8] are

$$
L_{11}=1,\qquad L_{13}=\frac{r_3}{R_1}\cos\xi,\qquad L_{31}=0,\qquad L_{33}=1,
\tag{2.12}
$$

where $R_i(i=1,3)$ are the radii of the torus before the deformation and $r_3=\frac{R_3}{\sin\xi}$ is the radius of the third direction after the deformation. ξ is the intersection angle between the direction along the first coordinate and the direction along the third coordinate of the slanted torus deformed by (2.2) with the components (2.12). We consider a simpler case in which the axion-dilaton is identified with the torus modulus of the rectangular torus before the deformation [8] as

$$
\tau=\tau_1+i\tau_2=i\frac{R_1}{R_3}=il.
\tag{2.13}
$$

The axion-dilaton (2.13) transforms under (2.3) with the components (2.12) as

$$
\tau'=\frac{R_1}{r_3}e^{i\xi}=l(\sin\xi\cos\xi+i\sin^2\xi).
\tag{2.14}
$$

This is the torus moduli of the deformed torus. By substituting (2.13) into (2.10) we find the β-deformed $\mathrm{AdS}_5\times S^5$ on the slanted torus

$$
\begin{aligned}
ds^2 &= R^2\tilde H^{1/2}\Big[-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2 \\
&\quad + \sum_{i=1}^{3}(d\mu_i^2+\tilde G\mu_i^2 d\phi_i^2) + 9\tilde G\mathcal{P}\mu_1^2\mu_2^2\mu_3^2 d\psi^2\Big],\\
e^{\Phi'} &= \sqrt{\tilde G}\tilde H l^{-1},\\
\chi' &= \tilde H^{-1}(l\cot\xi + \hat\gamma\,\hat\sigma\,l^2 g_0),\\
B_2' &= R^2\tilde G\mathcal{Q}\omega_2 - 12R^2\hat\sigma l\omega_1\wedge d\psi,\\
C_2' &= R^2\tilde G\mathcal{T}\omega_2 - 12R^2\hat\gamma l\omega_1\wedge d\psi,\\
C_4' &= 4R^4 l\omega_4 + 4R^4 l\tilde G(1-\mathcal{U}g_0)\omega_1\wedge d\phi_1\wedge d\phi_2\wedge d\phi_3,\\
F_5' &= 4R^4 l(\omega_{\mathrm{AdS}_5}+\tilde G\omega_{S^5}),
\end{aligned}
\tag{2.15}
$$

where

$$
\begin{aligned}
\tilde G^{-1} &= 1+\mathcal{P}g_0,\\
\tilde H &= \csc^2\xi + \hat\sigma^2 l^2 g_0,\\
\mathcal{P} &= \hat\gamma^2\csc^2\xi - 2\hat\gamma\,\hat\sigma\,l\cot\xi + \hat\sigma^2 l^2,\\
\mathcal{Q} &= \hat\gamma\csc^2\xi - \hat\sigma l\cot\xi,\\
\mathcal{T} &= \hat\gamma l\cot\xi - \hat\sigma l^2,\\
\mathcal{U} &= \hat\gamma\,\hat\sigma\,l\cot\xi - \hat\sigma^2 l^2.
\end{aligned}
\tag{2.16}
$$

This geometry contains four parameters. The parameters $\hat\gamma$ and $\hat\sigma$ arise from the marginal deformation. The parameters l and ξ arise from the axion-dilaton.

III. (J,J,J) GEODESIC

We investigate the Penrose limit of the geometry (2.15) along the geodesic with equal component angular momenta. This corresponds to $(\mu_1^2,\mu_2^2,\mu_3^2)=(1/3,1/3,1/3)$ in (2.8). In the vicinity of the geodesic with $\alpha_0=\arccos(1/\sqrt{3})$ and $\theta_0=\pi/4$, we set

$$\begin{aligned}
\alpha &= \alpha_0-\frac{1}{\Xi^{1/4}R}x^2,\\
\theta &= \theta_0+\sqrt{\frac{3}{2}}\frac{1}{\Xi^{1/4}R}x^1,\\
\rho &= \frac{1}{\Xi^{1/4}R}r,\\
\varphi_1 &= \sqrt{\frac{3+\mathcal{P}}{2}}\frac{1}{\Xi^{1/4}R}\left(x^3-\frac{1}{\sqrt{3}}x^4\right),\\
\varphi_2 &= \sqrt{\frac{2(3+\mathcal{P})}{3}}\frac{1}{\Xi^{1/4}R}x^4,\\
t &= x^+ +\frac{1}{(\Xi^{1/4}R)^2}x^-,\\
\psi &= -x^+ +\frac{1}{(\Xi^{1/4}R)^2}x^-,\\
\Xi &= \csc^2\xi+\frac{1}{3}\hat{\sigma}^2l^2,
\end{aligned}\tag{3.1}$$

and take the $R\to\infty$ limit of the geometry keeping $\hat{\gamma}$ and $\hat{\sigma}$ fixed. We also shift the coordinate x^- as $x^-\to x^-+\frac{\sqrt{3}}{2\sqrt{3+\mathcal{P}}}(x^1x^3+x^2x^4)$ to transform the resulting metric to a homogeneous pp wave [9–11].

The bosonic part of the string sigma model is

$$\begin{aligned}
S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma[&\sqrt{\eta}\eta^{ab}g_{\mu\nu}\partial_aX^\mu\partial_bX^\nu\\
&+\epsilon^{ab}B_{\mu\nu}\partial_aX^\mu\partial_bX^\nu],
\end{aligned}\tag{3.2}$$

where $\alpha'=1/2\pi$, $0\le\sigma\le\pi$ and the world sheet metric η^{ab} is fixed as $\sqrt{\eta}\eta^{ab}=\text{diagonal}(-1,1)$ with $\eta=|\det\eta_{ab}|$. We impose the light cone gauge condition $x^+=\tau$. The Lagrangian density of the action (3.2) becomes

$$\begin{aligned}
\mathcal{L} = &-2x_\tau^- -\frac{1}{2}\Bigg\{\sum_{i=1}^{8}[-(x_\tau^i)^2+(x_\sigma^i)^2]+\sum_{i=5}^{8}(x^i)^2\\
&+\frac{4\mathcal{P}}{3+\mathcal{P}}[(x^1)^2+(x^2)^2]\Bigg\}\\
&+\frac{\sqrt{3}}{\sqrt{3+\mathcal{P}}}[-x^3x_\tau^1-x^4x_\tau^2+x^1x_\tau^3+x^2x_\tau^4]\\
&+\frac{2\mathcal{Q}}{\sqrt{3+\mathcal{P}}}\Xi^{-1/2}[x^2x_\sigma^3-x^1x_\sigma^4]\\
&-12\hat{\sigma}l\Xi^{-1/2}\left[\zeta x^2x_\sigma^1-\left(\frac{\sqrt{3}}{9}-\zeta\right)x^1x_\sigma^2\right],
\end{aligned}\tag{3.3}$$

where $x_\tau^i=\partial x^i/\partial\tau$, $x_\sigma^i=\partial x^i/\partial\sigma$. ζ is a gauge parameter, which arises from ω_1 in (2.8). The ζ terms cancel out in the equations of motion. We need to solve the equations of motion for x^i ($i=1,\ldots,4$) which belong to the deformed S^5. The equations of motion are

$$\frac{\partial^2x^i}{\partial\tau^2}-\frac{\partial^2x^i}{\partial\sigma^2}+f^{ij}\frac{\partial x^j}{\partial\tau}+h^{ij}\frac{\partial x^j}{\partial\sigma}+k^ix^i=0,\tag{3.4}$$

with nonzero coefficients

$$\begin{aligned}
f^{13} &= -f^{31}=f^{24}=-f^{42}=-\frac{2\sqrt{3}}{\sqrt{3+\mathcal{P}}},\\
h^{12} &= -h^{21}=-\frac{4}{\sqrt{3}}\hat{\sigma}l\Xi^{-1/2},\\
h^{14} &= -h^{41}=-h^{23}=h^{32}=\frac{2\mathcal{Q}\Xi^{-1/2}}{\sqrt{3+\mathcal{P}}},\\
k^1 &= k^2=\frac{4\mathcal{P}}{3+\mathcal{P}}.
\end{aligned}\tag{3.5}$$

We solve the differential equations by the mode expansion $x^i(\tau,\sigma)=\sum_{n=-\infty}^{\infty}x_n^i(\tau)e^{2in\sigma}$, $x_n^i=(x_{-n}^i)^*$ with a harmonic oscillator frequency ansatz $x_n^i(\tau)\sim u^i(\omega_n)e^{i\omega_n\tau}$ [4]. From the condition for the existence of nontrivial solutions, we obtain the equation

$$\omega^8+c_6\omega^6+c_4\omega^4+c_2\omega^2+c_0=0,\tag{3.6}$$

with coefficients

$$\begin{aligned}
c_6 &= -8-16n^2,\\
c_4 &= 16+\frac{32}{3}\left(6-\frac{\hat{\sigma}^2l^2}{\Xi}\right)n^2+96n^4,\\
c_2 &= -\frac{128}{3}\frac{\hat{\sigma}^2l^2}{\Xi}n^2-128\left(1-\frac{2}{3}\frac{\hat{\sigma}^2l^2}{\Xi}\right)n^4-256n^6,\\
c_0 &= \frac{256}{9}\frac{\hat{\sigma}^4l^4}{\Xi^2}n^4-\frac{512}{3}\frac{\hat{\sigma}^2l^2}{\Xi}n^6+256n^8.
\end{aligned}\tag{3.7}$$

The solutions are

$$\omega=1\pm\sqrt{1+4n^2\pm\frac{4n\hat{\sigma}l}{\sqrt{3\csc^2\xi+\hat{\sigma}^2l^2}}}.\tag{3.8}$$

The spectrum does not depend on the deformation parameter $\hat{\gamma}$ while the spectrum depends on the deformation parameter $\hat{\sigma}$. When $\hat{\sigma}\neq0$, the axion-dilaton parameters l and ξ contribute to the spectrum. When $\hat{\sigma}=0$, we recover the result of [4].

IV. DISCUSSION

We have applied the β-deformation, which incorporates the $SL(2,\mathbb{R})$ symmetry of the type IIB theory, to

$\mathrm{AdS}_5 \times S^5$. The $SL(2,\mathbb{R})$ parameters can be related to torus parameters from an 11-dimensional viewpoint. The β-deformation becomes simpler when the axion-dilaton is identified with the torus modulus of the rectangular torus before the torus deformation. We have chosen the geodesic with equal component angular momenta to take the Penrose limit of the β-deformed $\mathrm{AdS}_5 \times S^5$, which contains four parameters arising from the marginal deformation and the axion-dilaton. We have presented the string sigma model and obtained the spectrum of the string in the deformed pp-wave limit. The spectrum does not depend on $\hat{\gamma}$ while the spectrum depends on $\hat{\sigma}$. The axion-dilaton parameters contribute to the spectrum when $\hat{\sigma} \neq 0$.

[1] R. G. Leigh and M. J. Strassler, Nucl. Phys. **B447**, 95 (1995).
[2] O. Lunin and J. M. Maldacena, J. High Energy Phys. 05 (2005) 033.
[3] V. Niarchos and N. Prezas, J. High Energy Phys. 06 (2003) 015.
[4] R. de Mello Koch, J. Murugan, J. Smolic, and M. Smolic, J. High Energy Phys. 08 (2005) 072; T. Mateos, J. High Energy Phys. 08 (2005) 026.
[5] S. D. Avramis, K. Sfetsos, and D. Zoakos, Nucl. Phys. **B787**, 55 (2007).
[6] A. Hamilton and J. Murugan, J. High Energy Phys. 06 (2007) 036.
[7] P. S. Aspinwall, Nucl. Phys. B, Proc. Suppl. **46**, 30 (1996); J. H. Schwarz, Phys. Lett. B **360**, 13 (1995); **364**, 252(E) (1995); Nucl. Phys. B, Proc. Suppl. **49**, 183 (1996).
[8] S. Shin, Phys. Rev. D **89**, 065026 (2014).
[9] M. Blau, J. M. Figueroa-O'Farrill, C. Hull, and G. Papadopoulos, J. High Energy Phys. 01 (2002) 047; Classical Quantum Gravity **19**, L87 (2002).
[10] G. Papadopoulos, J. G. Russo, and A. A. Tseytlin, Classical Quantum Gravity **20**, 969 (2003); M. Blau and M. O'Loughlin, Nucl. Phys. **B654**, 135 (2003).
[11] R. R. Metsaev, Nucl. Phys. **B625**, 70 (2002); R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D **65**, 126004 (2002); J. G. Russo and A. A. Tseytlin, J. High Energy Phys. 04 (2002) 021; M. Blau, M. O'Loughlin, G. Papadopoulos, and A. A. Tseytlin, Nucl. Phys. **B673**, 57 (2003).