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# Extended tachyon field using form invariance symmetry

Iván E. Sánchez G.[*]

*Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires and IMAS, CONICET, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*

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In this work we illustrate how form-invariance transformations (FITs) can be used to construct phantom and complementary tachyon cosmologies from standard tachyon field universes. First we show how these transformations act on the Hubble expansion rate, and the energy density and pressure of the tachyon field. Then we use the FIT to generate three different families of the tachyon field. In other words, the FIT generates new cosmologies from a known "seed" one; in particular, we apply the FIT to the ordinary tachyon field to obtain two types of tachyon species, denoted as the phantom and complementary tachyons. We see that the FIT allows us to pass from a nonstable cosmology to a stable one and vice versa. Finally, as an example, we apply the transformations to an inverse-square potential, $V \propto \phi^{-2}$, and generate the extended tachyon field.

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## I. INTRODUCTION

The tachyon field can play an important role in inflationary models [1–7] as well as in the present accelerated expansion, simulating the effect of dark energy [1–3,8–11] depending upon the form of the tachyon potential [1–3,12–15]. The tachyon is an unstable field which has becomes important in string theory through its role in the Dirac-Born-Infeld Lagrangian, because it is used to describe the D-brane action [16]. It was shown that the tachyon field could play a useful role in cosmology independent of the fact that it can be an unstable field [17]. Besides, it was pointed out in Ref. [1] that the tachyon Lagrangian can be accommodated into a quintessence form when the derivatives of the fields are small.

Several years ago, it was proposed that the tachyon Lagrangian could be extended in such a way as to allow the barotropic index to take any value [18], generating new species of tachyons called phantom and complementary tachyons in addition to the ordinary one [9,11,18]. The standard tachyon field can also describe a transition from an accelerated to a decelerated regime, behaving as an inflaton field at early times and as a matter field at late times. The complementary tachyon field always behaves as a matter field. The phantom tachyon field is characterized by a rapid expansion where its energy density increases with time [11,19–21].

On the other hand, form-invariance transformations involve internal or external variables in such a way that the transformations preserve the form of the dynamical equations, i.e., they have a form-invariance symmetry (FIS) [22]. Particularly useful are the $T$ duality [23] and "scale-factor duality" [24].

A new kind of internal symmetry that preserves the form of the spatially flat Friedmann cosmology was found in Refs. [25–27]. There it was shown that the equations governing the evolution of Friedmann-Robertson-Walker (FRW) cosmologies have a FIS group. The form-invariance transformation (FIT) that preserves the form of these equations relates the fluid, energy density, and pressure with geometrical quantities, such as the scale factor and Hubble expansion rate. The FIS introduces an alternative concept of equivalence between different physical problems, meaning that essentially a set of cosmological models are equivalent when their dynamical equations are form invariant under the action of some internal symmetry group [28].

The FIS makes it possible to find exact solutions in several contexts and generate new cosmologies from a known "seed" one [27–30].

In this paper we will show that the FIT applied to the standard tachyon field, used as a seed, can generate the complementary tachyon field and the phantom tachyon field. Our main goal is to show that the extended tachyon field is a consequence of the internal symmetry that preserves the form of the Einstein equations in a FRW spacetime. Additionally, we will show that the FIT allows us to pass from a nonstable cosmology to a stable one and vice versa. In particular, we will analyze the tachyon field when driven by a potential depending inversely on the square of the scalar field. We will start with some seed cosmology and use the FIT to obtain a different one, for example, passing from an accelerated to a superaccelerated scenario.

## II. FIS IN FLAT FRW COSMOLOGY AND LINEAR FIT

We will investigate an internal symmetry contained in the Einstein equations for a spatially flat FRW spacetime,

[*]isg.cos@gmail.com

 

$$3H^2 = \rho, \tag{1}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble expansion rate and $a(t)$ is the scale factor. We assume that the universe is filled with a perfect fluid having energy density $\rho$ and pressure $p$. The two independent Einstein equations have three unknown quantities $(H, p, \rho)$, and hence the system of equations (1)–(2) has one degree of freedom. This allows us to introduce a FIT which involves these quantities,

$$\bar{\rho} = \bar{\rho}(\rho), \tag{3}$$

$$\bar{H} = \left(\frac{\bar{\rho}}{\rho}\right)^{1/2} H, \tag{4}$$

$$\bar{p} + \bar{\rho} = \left(\frac{\rho}{\bar{\rho}}\right)^{1/2} \frac{d\bar{\rho}}{d\rho}(\rho + p). \tag{5}$$

Hence, the FIT (3)–(5), generated by the invertible function $\bar{\rho}(\rho)$, preserves the form of the system of equations (1)–(2) and ensures that the FRW cosmology has a FIS. The FIT (3)–(5) maps solutions of a definite cosmology, through the variables $(H, p, \rho)$, into solutions of other system of equations that define a different cosmology that is identified with the barred variables $(\bar{H}, \bar{p}, \bar{\rho})$, forming a Lie group structure as was demonstrated in Ref. [28].

We present the FIT induced by the linear generating function $\bar{\rho} = n^2\rho$, with $n$ being a constant. After this choice, Eqs. (3)–(5) become

$$\bar{\rho} = n^2\rho, \tag{6}$$

$$\bar{H} = nH \Rightarrow \bar{a} = a^n, \tag{7}$$

$$(\bar{\rho} + \bar{p}) = n(\rho + p). \tag{8}$$

Hence, the linear transformation (6) leads to a linear combination of the variables $\rho, H, p$ and a power transformation of the scale factor, obtained after integrating $\bar{H} = nH$. Finally, Eq. (8) gives the transformation rule for the pressure of the fluid,

$$\bar{p} = -n^2\rho + n(\rho + p). \tag{9}$$

In the case of two universes, each is filled with a perfect fluid for which we assume the equations of state $\bar{p} = (\bar{\gamma} - 1)\bar{\rho}$ and $p = (\gamma - 1)\rho$, respectively. The barotropic index $\gamma$ transforms as

$$\bar{\gamma} = \frac{(\bar{\rho} + \bar{p})}{\bar{\rho}} = \frac{\rho + p}{n\rho} = \frac{\gamma}{n}, \tag{10}$$

after using Eq. (6) along with Eq. (8).

The existence of a Lie group structure opens the possibility of connecting the scale factor $a$ of a seed cosmology with the scale factor $\bar{a} = a^n$ of a different cosmology.

## III. THE EXTENDED TACHYON COSMOLOGY

We turn our attention to the tachyon field and show how it transforms under the FIT (6)–(8). We consider a scalar field $\phi$ of the tachyon type with the self-interaction potential $V(\phi)$. The background energy density and pressure of the tachyon condensate, for a flat FRW cosmology, are

$$\rho_\phi = \frac{V}{\sqrt{1 - \dot{\phi}^2}}, \qquad p_\phi = -V\sqrt{1 - \dot{\phi}^2}, \tag{11}$$

respectively. The corresponding Einstein-Klein-Gordon equations are

$$3H^2 = \frac{V}{\sqrt{1 - \dot{\phi}^2}}, \tag{12}$$

$$\ddot{\phi} + 3H\dot{\phi}(1 - \dot{\phi}^2) + \frac{1 - \dot{\phi}^2}{V}\frac{dV}{d\phi} = 0. \tag{13}$$

The equation of state for the tachyon is $p = (\gamma - 1))\rho$, so the barotropic index is

$$\gamma = \dot{\phi}^2, \tag{14}$$

with $0 < \gamma < 1$ for Eq. (11). The sound speed is $c_s^2 = 1 - \gamma > 0$, and using Eq. (14) we can write

$$c_s^2 = 1 - \dot{\phi}^2. \tag{15}$$

From Eqs. (6) and (9), the transformed energy density and pressure of the tachyon field are given by

$$\bar{\rho} = \frac{\bar{V}}{\sqrt{1 - \dot{\bar{\phi}}^2}} = \frac{n^2 V}{\sqrt{1 - \dot{\phi}^2}}, \tag{16}$$

$$\bar{p} = -\bar{V}\sqrt{1 - \dot{\bar{\phi}}^2} = -\left(1 - \frac{\dot{\phi}^2}{n}\right)\frac{n^2 V}{\sqrt{1 - \dot{\phi}^2}}. \tag{17}$$

So, we find that the tachyon field, the potential, the barotropic index, and the sound speed transform linearly under the FIT (6)–(8),

$$\dot{\bar{\phi}}^2 = \frac{\dot{\phi}^2}{n}, \qquad \bar{V} = n^2 V\sqrt{\frac{1 - \dot{\phi}^2/n}{1 - \dot{\phi}^2}}, \tag{18}$$

$$\bar{\gamma} = \frac{\gamma}{n}, \qquad \bar{c}_s^2 = \frac{n - \dot{\phi}^2}{n}, \tag{19}$$

and the scalar field transforms as $\bar{\phi} = \phi/\sqrt{n}$.

We consider Eq. (11) with a barotropic index $0 < \gamma < 1$ as a seed tachyon, called the ordinary tachyon. Using the first equation of Eq. (19) we can get a barotropic index $\bar{\gamma} = \gamma/n < 0$. Then, the energy density and the pressure of the barred cosmology are given by

$$\bar{\rho} = \frac{\bar{V}}{\sqrt{1+\dot{\bar{\phi}}^2}}, \qquad \bar{p} = -\bar{V}\sqrt{1+\dot{\bar{\phi}}^2}. \tag{20}$$

These fluids—represented by Eq. (20) with negative pressure and negative barotropic index—describe phantom cosmologies. Moreover, we can get $1 < \bar{\gamma} = \gamma/n$ under the condition $n < \gamma$ by applying the transformed rule (19) to the barotropic index of the seed tachyon $0 < \gamma < 1$. So, the energy density and the pressure of the barred fluid are

$$\bar{\rho} = \frac{i|\bar{V}|}{i\sqrt{\dot{\bar{\phi}}^2 - 1}} = \frac{|\bar{V}|}{\sqrt{\dot{\bar{\phi}}^2 - 1}}, \tag{21}$$

$$\bar{p} = -i|\bar{V}|i\sqrt{\dot{\bar{\phi}}^2 - 1} = |\bar{V}|\sqrt{\dot{\bar{\phi}}^2 - 1}, \tag{22}$$

while those fluids described by Eqs. (21) and (22) give rise to nonaccelerated expanding evolutions.

We use the ordinary tachyon field (11) with $0 < \gamma < 1$ as a seed. With the application of the FIS (16)–(19) we find two species of tachyon fields, as in Ref. [18]: the phantom tachyon (20) with a $\gamma < 0$ and the complementary tachyon (21)–(22) with $1 < \gamma$. Therefore, the form-invariance transformations allow us to extend the family of the tachyon fields.

Following Gibbons [31] and Barrow *et al.* [32], an Einstein static universe containing a perfect fluid is always neutrally stable under the condition $c_s^2 > 1/5$. Therefore, the FIT (19) allows us to pass from a nonstable cosmology to a stable one and vice versa. For example, if we use a barotropic index $\gamma_0 = 6/7$ as a seed solution with $c_s^2 = 1/7$, by using the transformation rule (19) we can get a stable cosmology with $c_s^2 = 5/7 > 1/5$ if $n = 3$.

### A. Power-law expansion for the tachyon field

Let us assume that the potential is an inverse square in terms of the tachyon field,

$$V(\phi) = \frac{V_0}{\phi^2}, \tag{23}$$

with $V_0$ a constant. This potential, which diverges at $\phi = 0$, mimics the behavior of a typical potential in the condensate of bosonic string theory. Equation (23) leads to the power-law expansion $a(t) = kt^\delta$, with $k$ a constant, if $\phi$ is the only source [1,2]. The tachyon field and the barotropic index are

$$\phi = \left(\frac{2}{3\delta}\right)^{1/2} t, \qquad 0 < \gamma_0 < 1, \tag{24}$$

with

$$\delta = \frac{1}{3}[1 + \sqrt{1+4\beta}], \qquad \beta = \left(\frac{3V_0}{4}\right)^2. \tag{25}$$

For this reason the power-law expansion appears to be a good example for illustrating how—from a seed solution characterized by particular values of the parameters $V_0$, $\gamma_0$, and $k$—the FIS helps us find the scalar field and the scale factor driven by the inverse-square potential (23) for any other values of these parameters. Applying the FIT (6)–(8) to the seed solution (24)–(25) and using Eqs. (14), (18), and (19), we obtain the transformation rules for $V_0$ and $\gamma_0$,

$$\bar{\gamma}_0 = \frac{\gamma_0}{n}, \tag{26}$$

$$\bar{V}_0 = nV_0\sqrt{\frac{1-\frac{\gamma_0}{n}}{1-\gamma_0}}. \tag{27}$$

Therefore, the transformed tachyon field, for a barotropic index $\bar{\gamma} < 0$, is given by

$$\bar{\phi} = \left(\frac{2}{-3|\bar{\delta}|}\right)^{1/2} t, \qquad \bar{\delta} = \frac{1}{3}\left[1 - \sqrt{1+4\bar{\beta}}\right]. \tag{28}$$

These tachyon field solutions [Eq. (28)] describe phantom cosmologies. Note that if $n = -1$ in Eqs. (26) and (27) we can get the results of Ref. [9] for the phantom tachyon.

On the other hand, if the transformed barotropic index is $1 < \bar{\gamma}$, we get

$$\bar{\phi} = \left(\frac{2}{3\bar{\delta}}\right)^{1/2} t, \qquad \bar{\delta} = \frac{1}{3}\left[1 \pm \sqrt{1-4|\bar{\beta}|}\right]. \tag{29}$$

This type of tachyon field solution with $1 < \gamma_0$ is called the complementary tachyon solution, which represents stiff matter with a deceleration cosmology.

Hence, the scale factor $a(t) = kt^\delta$ transforms as $\bar{a} = a^n$, so the transformed scalar field is $\bar{a} = \bar{k}t^{\bar{\delta}}$ with $\bar{k} = k^n$ and $\bar{\delta} = n\delta$. The condition to have an inflationary solution is that $1 < \delta$ and it is represented by the solutions (24). Notice that the exponent of the power-law solution can take positive or negative values provided that $n \in \Re$. We can see that this exponent is directly related with the barotropic index of the tachyon fluid, $\delta = 2/3\gamma_0$; therefore, changing $n$ is equivalent to allowing $\bar{\gamma}_0$ to vary over $\Re$. This simple fact leads us to a remarkable conclusion: there are new species of tachyons and the FIS has revealed their existence to us.

## IV. CONCLUSION

As part of a long-term investigation [25–28] we have shown here that form-invariance transformations can be used as tools for generating new solutions to the Einstein field equations. In this case the existence of two new kinds of extended tachyon fields were derived from the standard tachyon field ($0 < \gamma < 1$)—the complementary ($1 < \gamma$) and the phantom tachyon ($\gamma < 0$) fields—confirming the work in Ref. [18]. In addition, we see that the form-invariance transformations allow us to pass from a neutrally unstable universe to a stable one [31,32].

In particular, we have applied the method to obtaining phantom and complementary versions of FRW tachyon cosmologies, with an emphasis on power-law spacetimes generated by an inverse-square potential. We have found that the FIT transforms the seed scale factor $a = kt^{\delta}$ into the power-law solution $a = k^n t^{n\delta}$. For illustration purposes, if we start from a decelerated model with $2/3 < \delta < 1$, we can get a power-law inflation model with $\delta > 1$ or a superaccelerated model (phantom model) with $\delta < 0$. So, we have shown how the FIT generates new cosmologies from a seed one.

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[1] T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002).
[2] A. Feinstein, Phys. Rev. D **66**, 063511 (2002).
[3] L. R. W. Abramo and F. Finelli, Phys. Lett. B **575**, 165 (2003).
[4] C. Campuzano, S. del Campo, and R. Herrera, Phys. Lett. B **633**, 149 (2006).
[5] S. Chattopadhyay, U. Debnath, and G. Chattopadhyay, Astrophys. Space Sci. **314**, 41 (2008).
[6] S. del Campo, R. Herrera, and A. Toloza, Phys. Rev. D **79**, 083507 (2009).
[7] R. K. Jain, P. Chingangbam, and L. Sriramkumar, Nucl. Phys. **B852**, 366 (2011).
[8] J. M. Aguirregabiria and R. Lazkoz, Phys. Rev. D **69**, 123502 (2004).
[9] J. M. Aguirregabiria, L. P. Chimento, and R. Lazkoz, Phys. Rev. D **70**, 023509 (2004).
[10] G. Calcagni, and A. R. Liddle, Phys. Rev. D **74**, 043528 (2006).
[11] L. P. Chimento, M. Forte, G. M. Kremer, and M. O. Ribas, Gen. Relativ. Gravit. **42**, 1523 (2010).
[12] J. S. Bagla, H. K. Jassal, and T. Padmanabhan, Phys. Rev. D **67**, 063504 (2003).
[13] Z. K. Guo and Y. Z. Zhang, J. Cosmol. Astropart. Phys. 08 (2004) 010.
[14] E. J. Copeland, M. R. Garousi, M. Sami, and S. Tsujikawa, Phys. Rev. D **71**, 043003 (2005).
[15] R. C. de Souza and G. M. Kremer, Phys. Rev. D **89**, 027302 (2014).
[16] A. Sen, J. High Energy Phys. 04 (2002) 048; 07 (2002) 065; Mod. Phys. Lett. A **17**, 1797 (2002); A. Sen, J. High Energy Phys. 10 (1999) 008; M.R. Garousi, Nucl. Phys. **B584**, 284 (2000); J. High Energy Phys. 05 (2003) 058; E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras, and S. Panda, J. High Energy Phys. 05 (2000) 009; D. Kutasov and V. Niarchos, Nucl. Phys. **B666**, 56 (2003).
[17] G. W. Gibbons, Phys. Lett. B **537**, 1 (2002); S. Mukohyama, Phys. Rev. D **66**, 024009 (2002); L. Kofman and A. Linde, J. High Energy Phys. 07 (2002) 004; M. Sami, Mod. Phys. Lett. A **18**, 691 (2003); A. Mazumdar, S. Panda, and A. Perez-Lorenzana, Nucl. Phys. **B614**, 101 (2001); Y. S. Piao, R. G. Cai, X. m. Zhang, and Y. Z. Zhang, Phys. Rev. D **66**, 121301 (2002); J. M. Cline, H. Firouzjahi, and P. Martineau, J. High Energy Phys. 11 (2002) 041; Z. K. Guo, Y. S. Piao, R. G. Cai, and Y. Z. Zhang, Phys. Rev. D **68**, 043508 (2003); S. Nojiri and S. D. Odintsov, Phys. Lett. B **571**, 1 (2003); E. Elizalde, J. E. Lidsey, S. Nojiri, and S. D. Odintsov, Phys. Lett. B **574**, 1 (2003); D. A. Steer and F. Vernizzi, Phys. Rev. D **70**, 043527 (2004); V. Gorini, A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Rev. D **69**, 123512 (2004); B. C. Paul and M. Sami, Phys. Rev. D **70**, 027301 (2004); L. R. Abramo, F. Finelli, and T. S. Pereira, Phys. Rev. D **70**, 063517 (2004); G. Calcagni and S. Tsujikawa, Phys. Rev. D **70**, 103514 (2004).
[18] L. P. Chimento, Phys. Rev. D **69**, 123517 (2004).
[19] S.-G. Shi, Y.-S. Piao, and C.-F. Qiao, J. Cosmol. Astropart. Phys. 04 (2009) 027.
[20] R. Rangdee and B. Gumjudpai, Astrophys. Space Sci. **349**, 975 (2014).
[21] B. Novosyadlyj, arXiv:1311.0227.
[22] Symmetries in Physics, edited by F. Gieres, M. Kibler, C. Lucchesi, and O. Piguet (Editions Frontieres, Gif sur Yvette, France, 1998).
[23] M. B. Green, J. H. Schwartz, and E. Witten, *Superstring theory* (Cambridge University Press, Cambridge, England, 1987); J. Polchinski, *String Theory, vol. I-II* (Cambridge University Press, Cambridge, England, 1998).
[24] G. Veneziano, Phys. Lett. B **265**, 287 (1991); A. A. Tseytlin, Mod. Phys. Lett. A **06**, 1721 (1991).
[25] L. P. Chimento, Phys. Rev. D **65**, 063517 (2002).
[26] L. P. Chimento, Phys. Lett. B **633**, 9 (2006).

[27] L. P. Chimento and R. Lazkoz, Int. J. Mod. Phys. D **14**, 587 (2005).
[28] L. P. Chimento, M. G. Richarte, and I. E. Sánchez, Mod. Phys. Lett. A **28**, 1250236 (2013).
[29] L. P. Chimento, M. Forte, R. Lazkoz, and M. G. Richarte, Phys. Rev. D **79**, 043502 (2009).
[30] T. Charters and J. P. Mimoso, J. Cosmol. Astropart. Phys. 08 (2010) 022.
[31] G. W. Gibbons, Nucl. Phys. **B292**, 784 (1987).
[32] J. D. Barrow, G. F. R. Ellis, R. Maartens, and C. G. Tsagas, Classical Quantum Gravity **20**, L155 (2003).