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Lorentz symmetry violating low energy dispersion relations from a dimension-five photon scalar mixing operator

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Dimension-five photon (γ) scalar (ϕ) interaction terms usually appear in the bosonic sector of unified theories of electromagnetism and gravity. In these theories the three propagation eigenstates are different from the three field eigenstates. The dispersion relation in an external magnetic field shows that, for a nonzero energy (ω), out of the three propagating eigenstates one has superluminal phase velocity v_p. During propagation, another eigenstate undergoes amplification or attenuation, showing signs of an unstable system. The remaining one maintains causality. In this paper, using techniques from optics as well as gravity, we identify the energy (ω) interval outside which $v_p \leq c$ for the field eigenstates $|\gamma_{\parallel}\rangle$ and $|\phi\rangle$, and stability of the system is restored. The behavior of group velocity v_g is also explored in the same context. We conclude by pointing out its possible astrophysical implications.

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Scalar $\phi(x)$ photon γ interaction through the dimension-five operator originates in many theories beyond the standard model of particle physics, usually in the unified theories of electromagnetism and gravity [1]. The scalars involved can be moduli fields of string theory, Kaluza-Klein particles from extra dimension, scalar components of the gravitational multiplet in extended supergravity models etc., to name a few [2–9].

Usually these models predict optical activity where the vacuum is turned into a birefringent and dichroic one [10]. As a polarized light beam passes through such a medium, its plane of polarization gets rotated. This particular aspect has been explored and exploited extensively in the literature to explain and predict many interesting physical phenomena [11].

In this paper we point out other interesting aspects of such interactions encountered in the low energy sector of the theory, in a magnetized background of field strength $\mathcal{B}$. The theory under consideration has a tree-level interaction term $g_{\phi\gamma\gamma}\phi F_{\mu\nu}F^{\mu\nu}$, where $g_{\phi\gamma\gamma}$ is a dimensionful coupling constant between ϕ and the electromagnetic (EM) field. This term is Lorentz invariant (LI) and remains invariant under charge conjugation (C), parity transformation (P) and time reversal (T) symmetry transformations. Renormalizability of the theory is compromised because of the presence of dimensionful coupling constant $g_{\phi\gamma\gamma}$. However, in the presence of an external background magnetic field, all the good [both continuous and discrete (i.e., LI and CPT)] symmetries of the theory get compromised. Since theories violating CPT are known to violate Lorentz invariance [12] hence causality; therefore, the explicit violation of both in a nontrivial background introduces modifications to the dispersion relations affecting phase velocity (v_p) and group velocity (v_g). The same also introduces presence of unstable modes in a certain energy (ω) domain. Some of these issues are explored below.

The theory under consideration has three *propagation eigenstates*, a scalar $|\phi\rangle$ and two transversely polarized photons $|\gamma_{\|,\perp}\rangle$. One of the eigenstates, $|\gamma_{\parallel}\rangle$, has polarization vector parallel and the other one, $|\gamma_{\perp}\rangle$, has the same orthogonal to the background magnetic field $\mathcal{B}$.

We point out in this paper the two interesting possibilities that may emerge from the solutions of the two eigenstates, $|\phi\rangle$ and $|\gamma_{\parallel}\rangle$, (i) their phase velocity may become superluminal, (ii) their respective amplitudes may undergo attenuation or amplification, provided their energies lie in a certain interval. Within this energy interval the amplitudes of $|\gamma_{\parallel}\rangle$ and $|\phi\rangle$ may get amplified or damped, thus they are nonpropagating modes.

Naively, though this phenomena seems to get ameliorated, only at energy $\omega = \infty$, but through a careful analysis we show that there exists a finite energy interval outside which the individual field states $|\gamma_{\parallel}\rangle$ and $|\phi\rangle$ are cured of this malady. In other words outside that interval, the solutions of the same are well behaved as far as their stability and the magnitudes of the phase velocities (i.e., v_p for $|\phi\rangle$ and $|\gamma_{\parallel}\rangle$, both) are concerned.

The same however cannot be endorsed for the group velocity v_g for these two states. The same (i.e., v_g) for $|\phi\rangle$ and $|\gamma_{\parallel}\rangle$ reach the luminal limit at $\omega = \infty$ only.

We note in passing that the phase velocities of $|\phi\rangle \pm |\gamma_{\parallel}\rangle$ do exhibit velocity selection rules as had been discussed in [13].

The other eigenstate, $|\gamma_{\perp}\rangle$, is free from any pathological problems. It poses a stable solution as well as causal group and phase velocities, i.e., $v_g = v_p = c$, $\forall x$, t, and ω.

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In this article we analyze the issues involved, from three different angles: (a) using differential geometric arguments involving the properties of a metric [$g^{\mu\nu}_{(\mathrm{eff})}(\omega)$ in our context] related to the stability of a manifold, as used in the context of relativity, (b) analyzing the dispersion relations (DR), and (c) by explicit evaluation of the phase velocities (v_p) from the solutions of the eigenstates, $|\gamma_{||,\perp}\rangle$ and $|\phi\rangle$, using principles of optics [14].

A critical analysis of the DR actually conforms with the findings, obtained from the stability analysis of the effective metric, $g^{\mu\nu}_{(\mathrm{eff})}(\omega)$. The interesting part however is v_p turns out to be complex exactly in the same energy domain, as is predicted from the stability analysis of $g^{\mu\nu}_{(\mathrm{eff})}(\omega)$ as well as the dispersion relations. This indicates the system to be in an unstable state in the relevant energy domain. A detailed further analysis reveals that, outside this energy range, some of the Lorentz invariance violating (LIV) pieces in the expression of v_p cancel out giving a LI and causal result. We conclude by pointing out the possible implications of this result in astrophysical or cosmological contexts.

Equations of motion.—The action for coupled scalar photon system, in four-dimensional flat space, is given by

$$S = \int d^4x\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}g_{\phi\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]. \quad (1)$$

The equations of motion can be obtained from Eq. (1) by employing the usual variational principles. However, in what follows, we would rewrite Eq. (1) by decomposing the EM field tensor $F_{\mu\nu}$ into two parts, a slowly varying background mean field $\bar{F}_{\mu\nu}$, and an infinitesimal fluctuation $f_{\mu\nu}$ (i.e., $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$), and then derive the equations of motion from the modified action. Without loss of generality, we would consider a local inertial frame, where the only nonzero component of $\bar{F}^{\mu\nu}$ is $\bar{F}^{12} = \mathcal{B}$. Assuming the magnitude of the scalar field to be of the order of the fluctuating EM field $f_{\mu\nu}$, one can linearize the resulting equations. The resulting equations of motion for the EM and the scalar fields turn out to be

$$\partial_\mu f^{\mu\nu} = -g_{\phi\gamma\gamma}\partial_\mu\phi\tilde{\bar{F}}^{\mu\nu}, \quad (2)$$

$$\partial_\mu\partial^\mu\phi = -\frac{1}{2}g_{\phi\gamma\gamma}\tilde{\bar{F}}^{\mu\nu}f_{\mu\nu}. \quad (3)$$

Equation (2) describes the evolution of the 2 degrees of freedom associated with the gauge fields and Eq. (3) describes the same for the scalar field. Since Eq. (2) provides four equations for 2 degrees of freedom of the gauge fields, one has to get rid of the extra relations by fixing a gauge and using the constraint equation.

However, there is another way, i.e., by working in terms of the field strength tensors and making use of the Bianchi identity. In this paper we will follow the second method. We will start with the Bianchi identity $\partial_\mu f_{\nu\lambda} + \partial_\nu f_{\lambda\mu} + \partial_\lambda f_{\mu\nu} = 0$ and multiply the same by $\bar{F}^{\nu\lambda}$; after this we operate ∂^μ on the resulting expression, to arrive at

$$\partial_\mu\partial^\mu(f_{\lambda\rho}\bar{F}^{\lambda\rho}) = -2\partial^\lambda\partial_\mu(f^{\mu\rho}\bar{F}_{\rho\lambda}). \quad (4)$$

Next we can multiply Eq. (2) by $\bar{F}_{\nu\lambda}$, and subsequently operate ∂^λ on the same to obtain

$$\partial^\lambda\partial_\mu(f^{\mu\nu}\bar{F}_{\nu\lambda}) = -g_{\phi\gamma\gamma}\partial^\lambda\partial_\mu\phi\tilde{\bar{F}}^{\mu\nu}\bar{F}_{\nu\lambda}.$$

Now using the relation given by Eq. (4), on the last equation, we find the equation for the eigenstate $|\gamma_{||}\rangle$, given by

$$\partial_\mu\partial^\mu(f_{\rho\sigma}\bar{F}^{\rho\sigma}/2) = g_{\phi\gamma\gamma}\partial^\lambda\partial_\alpha\phi(\tilde{\bar{F}}^{\alpha\nu}\bar{F}_{\nu\lambda}). \quad (5)$$

The equation for the eigenstate $|\gamma_\perp\rangle$ can be obtained by performing the same steps leading to Eq. (5), except the multiplication of Eq. (2) by the factor $\bar{F}_{\nu\lambda}$. In this step, instead of $\bar{F}_{\nu\lambda}$ we have to use the multiplicative factor $\tilde{\bar{F}}_{\nu\lambda}$. This would lead us to

$$\partial_\mu\partial^\mu(f_{\nu\lambda}\tilde{\bar{F}}^{\nu\lambda}/2) = 0. \quad (6)$$

It is easy to perform a *consistency check* on Eq. (6) using Eq. (5). If we replace $\bar{F}_{\nu\lambda}$ by $\tilde{\bar{F}}_{\nu\lambda}$ in Eq. (5) then we immediately recover Eq. (6), because the right-hand side of Eq. (5) vanishes; since $\tilde{\bar{F}}^{\alpha\nu}\tilde{\bar{F}}_{\nu\lambda} = 0$, because of our assumption that, for the background EM field, only $\bar{F}^{12} \neq 0$. Hence Eq. (6) is consistent.

Now we introduce the new set of variables, $\psi = \frac{f_{\nu\lambda}\bar{F}^{\nu\lambda}}{2}$ and $\tilde{\psi} = \frac{f_{\nu\lambda}\tilde{\bar{F}}^{\nu\lambda}}{2}$, and use them in Eqs. (5) and (6), and subsequently go to momentum space, to obtain the dispersion relations. Those are given by

$$k^2\psi - g_{\phi\gamma\gamma}(k_\alpha\tilde{\bar{F}}^{\alpha\nu}\bar{F}_{\nu\lambda}k^\lambda)\phi = 0, \qquad k^2\tilde{\psi} = 0,$$
$$\text{and} \quad k^2\phi - g_{\phi\gamma\gamma}\psi = 0. \quad (7)$$

Since we have assumed that only $\bar{F}^{12} = \mathcal{B} = \tilde{\bar{F}}^{03} \neq 0$, then it follows from there that $(\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}) = 2\mathcal{B}^2$ and $(k_\alpha\tilde{\bar{F}}^{\alpha\nu}\bar{F}_{\nu\lambda}k^\lambda) = k_\perp^2\mathcal{B}^2$. Furthermore, if the angle between $\mathcal{B}$ and $\vec{K}$ is Θ, then the component of $\vec{K}$ normal to $\mathcal{B}$ is $\vec{K}_\perp = \vec{K}\sin\Theta$. Hence, using the same one can denote

$$k_\perp^2\mathcal{B}^2 = K^2\sin^2\Theta\mathcal{B}^2 \simeq (\omega\mathcal{B}\sin\Theta)^2. \quad (8)$$

While rewriting Eq. (8), it was assumed that $\omega \simeq K$ to zeroth order in the coupling constant $g_{\phi\gamma\gamma}$. From now on, for the sake of brevity, we may denote $\mathcal{B}\sin\Theta = \mathcal{B}_T$, at times.

In order to make the mass dimension of $\psi, \tilde{\psi}$ and ϕ the same, we can multiply Eq. (7) by $\omega\mathcal{B}\sin\Theta$ and redefine $\Phi = \omega\mathcal{B}\sin\Theta\phi$. Upon doing the same, the coupled dispersion relations can be cast as a matrix equation:

$$\begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & -g_{\phi\gamma\gamma}(\omega\mathcal{B}_T) \\ 0 & -g_{\phi\gamma\gamma}(\omega\mathcal{B}_T) & k^2 \end{bmatrix}\begin{bmatrix} \tilde{\psi} \\ \psi \\ \Phi \end{bmatrix} = 0. \quad (9)$$

The real symmetric matrix, in Eq. (9), can be diagonalized by a orthogonal rotation through angle θ, in the $\psi - \Phi$ plane.

Propagation eigenstates.—We already have explained that $\tilde{\psi}$ and ψ have their respective polarization vectors $\perp$ and $\|$ to $\mathcal{B}$. The off-diagonal elements in Eq. (9) make ψ and Φ to mix during their space-time evolution; while $\tilde{\psi}$ remains unaffected. Next we diagonalize Eq. (9), by the orthogonal transformation discussed before, and express the same as

$$\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 - g_{\phi\gamma\gamma}\mathcal{B}_T\omega & 0 \\ 0 & 0 & k^2 + g_{\phi\gamma\gamma}\mathcal{B}_T\omega \end{pmatrix}\begin{pmatrix} \tilde{\psi} \\ \frac{\Phi+\psi}{\sqrt{2}} \\ \frac{\Phi-\psi}{\sqrt{2}} \end{pmatrix} = 0. \quad (10)$$

It is easy to see from Eq. (10) that the *propagating eigenstates* $\tilde{\psi}$, $\frac{\Phi+\psi}{\sqrt{2}}$ and $\frac{\Phi-\psi}{\sqrt{2}}$, satisfy the following dispersion relations:

$$\omega = K, \quad (11)$$

$$\omega_+ = \pm\sqrt{K^2 + g_{\phi\gamma\gamma}\mathcal{B}_T\omega}, \quad (12)$$

$$\omega_- = \pm\sqrt{K^2 - g_{\phi\gamma\gamma}\mathcal{B}_T\omega}. \quad (13)$$

We point out that the dispersion relations obtained from Eqs. (9) and (10) are identical to those obtained in [15–17], provided appropriate limits are taken.

Upon dividing Eqs. (12) and (13) by K, we arrive at the expressions for the phase velocities, $v_p^\pm = \sqrt{1 \pm g_{\phi\gamma\gamma}\mathcal{B}_T/\omega}$, corresponding to the propagation eigenstates, $[\frac{\Phi\pm\Psi}{\sqrt{2}}]$. It is easy to verify that, for $g_{\phi\gamma\gamma}\mathcal{B}_T > \omega$, the magnitude of $v_p^+ > 1$ that is, phase velocity of the eigenstate $[\frac{\Phi+\Psi}{\sqrt{2}}]$ propagates with superluminal speed, and v_p^- is complex so the amplitude of the corresponding eigenstate $[\frac{\Phi+\Psi}{\sqrt{2}}]$ would be attenuated or damped, *as was mentioned in the beginning.*

Effective metric.—To understand more about the Lorentz invariance violating (LIV) dispersion relation in the magnetized vacuum for the mixed propagation eigenstate $[\frac{\Psi+\Phi}{\sqrt{2}}]$, we note that the dispersion relation for the same can be written as $g^{\mu\nu}_{(\mathrm{eff})}(\omega)k_\mu k_\nu = 0$, where $g^{\mu\nu}_{(\mathrm{eff})}(\omega) = \mathrm{diag}([1 - \frac{g_{\phi\gamma\gamma}\mathcal{B}\sin\Theta}{\omega}], -1, -1, -1)$ and k_μ is the usual wave 4-vector. The form of the effective metric given above is similar to the ones discussed in the context of doubly special relativity (DSR) [18]. We clarify at the outset that the same has been obtained, here, by demanding that the dispersion relation can be written as a quadratic of k_μ's, like the same for massless particles. One may interpret this effective metric as the metric of the underlying spacetime over which $[\frac{\Psi+\Phi}{\sqrt{2}}]$ is propagating. The inverse of the same is $g_{\mu\nu(\mathrm{eff})}(\omega)$, given by $g_{\mu\nu(\mathrm{eff})}(\omega) = \mathrm{diag}(\frac{1}{[1-\frac{g_{\phi\gamma\gamma}\mathcal{B}\sin\Theta}{\omega}]}, -1, -1, -1)$. Next we would perform stability analysis of the system using this metric.

Stability analysis using $g_{\mu\nu(\mathrm{eff})}(\omega)$.—It has been pointed out in [19] that, for a space-time to be stable, the determinant of its metric must be negative, else the system is unstable and would decay to a stable ground state. The purpose of writing the effective metric was to find out if there exists a bound or interval over which determinant of the same is negative indicating possibility of attenuation or growth of the amplitudes of the eigenmodes.

If we take a critical look at $g_{\mu\nu(\mathrm{eff})}(\omega)$, it is clearly seen that unless $\omega > g_{\phi\gamma\gamma}\mathcal{B}\sin\Theta = \omega_c$ the value of $\mathrm{Det}|g_{\mu\nu(\mathrm{eff})}(\omega)| > 0$, hence there would be growth (instability) or damping (attenuation) in the system. Now if we go back to Eq. (12), one can verify that the same can be recast in the following form: $(\omega - \frac{g_{\phi\gamma\gamma}\mathcal{B}\sin\Theta}{2})^2 - (\frac{g_{\phi\gamma\gamma}\mathcal{B}\sin\Theta}{2})^2 = K^2$. Accordingly, for $\omega < \omega_c$, wave vector K becomes imaginary, signaling attenuation or growth of amplitude. Therefore, we are tempted to conclude that the deductions of [19] hold even for the effective metric $g_{\mu\nu(\mathrm{eff})}(\omega)$.

Causal stability.—It has been pointed out in [20–22] that the stability of causal manifolds are governed by two conditions: (a) the underlying metric has to be Lorenzian and (b) there should exists a scalar timelike function $T(x)$, i.e., continuous and infinitely differentiable everywhere on the manifold; and covariant derivative of $T(x)$ i.e., $D_\mu T(x) \neq 0$, and $g^{\mu\nu}_{(\mathrm{eff})}(\omega)D_\mu T(x)D_\nu T(x) > 0$ [23,24]. In our case both conditions are satisfied, provided we take $T(x) = t$ as the time coordinate (i.e., illustrating the absence of closed timelike or spacelike curves).

Inhomogeneous wave equations.—It is possible to get the solutions for the *propagating eigenstates* $\tilde{\psi}$, $\frac{\Phi+\psi}{\sqrt{2}}$ and $\frac{\Phi-\psi}{\sqrt{2}}$, from the dispersion relations given by Eqs. (11)–(13), which follows from Eq. (10).

Sometimes, presenting results in its full generality becomes a fruitful and instructive exercise in many areas of exact science. It helps in pointing out potential sources of new scientific features. Keeping this philosophy in mind, we express the solutions of the coupled set of equations, as an explicit function of the rotation angle θ, in the $\psi - \Phi$ plane. They have the following form:

$$\begin{pmatrix} \tilde{\psi} \\ \cos\theta\psi + \sin\theta\Phi \\ -\sin\theta\psi + \cos\theta\Phi \end{pmatrix} = \begin{pmatrix} A_0 e^{i(\omega t - kx)} \\ A_1 e^{i(\omega_+ t - kx)} \\ A_2 e^{i(\omega_- t - kx)} \end{pmatrix}. \quad (14)$$

It is not difficult to see that, for $\theta = \pi/4$, one recovers back the expressions for propagating eigenstates, $\frac{\Phi \pm \psi}{\sqrt{2}}$. We would like to mention here that we would not consider $\theta = \frac{\pi}{4}$ until we reach an appropriate point.

The constants A_0, A_1 and A_2 appearing in Eq. (14) are to be derived from the boundary conditions one imposes on the dynamical degrees of freedom. The solutions for the dynamical variables, from Eq. (14), turn out to be

$$\tilde{\psi}(t,x) = A_0 e^{i(\omega t - k.x)},$$

$$\psi(t,x) = [A_1 \cos\theta e^{i\omega_+ t} - A_2 \sin\theta e^{i\omega_- t}]e^{-ikx},$$

and $$\Phi(t,x) = [A_1 \sin\theta e^{i\omega_+ t} + A_2 \cos\theta e^{i\omega_- t}]e^{-ikx}. \quad (15)$$

In the following we consider the boundary conditions, $\Phi(0,0) = 0$ and $\psi(0,0) = 1$. With these boundary conditions, we have $\frac{A_2}{\sin\theta} = -1$ and the solution for ψ turns out to be

$$\psi(t,x) = [\cos^2\theta e^{i(\omega_+ t - kx)} + \sin^2\theta e^{i(\omega_- t - kx)}]. \quad (16)$$

Defining, $a_x^2(t) = (\mathcal{R}e[\psi(t,0)])^2 + (\mathcal{I}m[\psi(t,0)])^2$, we get the following form for $\psi(t,x)$:

$$\psi(t,x) = a_x(t) e^{i\left(\tan^{-1}\left[\frac{\cos^2\theta \sin\omega_+ t + \sin^2\theta \sin\omega_- t}{\cos^2\theta \cos\omega_+ t + \sin^2\theta \cos\omega_- t}\right] - kx\right)}. \quad (17)$$

A wave equation of this type is usually called an inhomogeneous wave equation [14]. The phase velocity for such a system, where the solution is represented by $a(t)e^{i(\varphi(t) - kx)}$ is defined by $v_p = \frac{1}{K}\frac{\partial\varphi(t)}{\partial t}$.

In more complicated physical situations, when medium effects, polarization effects due to strong external fields etc., are taken into account, the angle θ would depend on those parameters. Hence $\varphi(t)$ may become a complicated function of time. As a result, the phase velocity v_p may become a function of time with varied physical implications.

However, for the simple case in hand, substituting $\theta = \frac{\pi}{4}$ in Eq. (17), followed by some algebra, it is easy to demonstrate that $\varphi(t) = \frac{(\omega_+ + \omega_-)t}{2}$. Now using the same in the expression for phase velocity v_p yields

$$v_p = \left(\frac{\omega_+ + \omega_-}{2K}\right). \quad (18)$$

Using Eqs. (12) and (13) in Eq. (18) and considering the dispersion relation to zeroth order in $g_{\phi\gamma\gamma}$, i.e., $\omega \simeq K$, we obtain

$$v_p = \frac{1}{2}\left[\sqrt{\left(1 - \frac{g_{\phi\gamma\gamma}\mathcal{B}_T}{\omega}\right)} + \sqrt{\left(1 + \frac{g_{\phi\gamma\gamma}\mathcal{B}_T}{\omega}\right)}\right]. \quad (19)$$

The expression for phase velocity, as given by Eq. (19), provides an interesting limit for ω; in order to have a real phase velocity, one must have $\omega \geq g_{\phi\gamma\gamma}\mathcal{B}_T$. So in principle, one can define an expansion parameter $\delta = \frac{g_{\phi\gamma\gamma}\mathcal{B}_T}{\omega}$, and perform an all order expansion of $(1 \pm \frac{g_{\phi\gamma\gamma}\mathcal{B}_T}{\omega})^{1/2}$, in powers of δ, for $\delta < 1$, and be convinced that the magnitude of v_p stays less than c, i.e., phase velocity is causal.

Group velocity.—Group velocity for the situation under consideration is given by $v_g = |\frac{\partial\dot{\varphi}}{\partial K}|$. Using the expression for $\dot{\varphi}$, in the last relation, we obtain the expression for group velocity in terms of δ,

$$v_g = \frac{1}{2}\left[\frac{1 - \frac{\delta}{2}}{\sqrt{1-\delta}} + \frac{1 + \frac{\delta}{2}}{\sqrt{1+\delta}}\right]. \quad (20)$$

Expanding the right-hand side of Eq. (20) in powers of δ (assuming $\delta < 1$), one finds that $v_g > 1$, even when $0 < \delta < 1$. Of course, the problem of having complex v_g is avoided by considering $\delta < 1$, however the issue of superluminality remains. We believe that this is an artifact of the special background that violates Lorentz and CPT invariance. The presence of this special background may be responsible for making v_g of the $|\gamma_\parallel\rangle$ state superluminal.

In Fig. 1, we have plotted v_g and v_p, for various values of ω. As can be seen from the plots, that as energy, $\omega \to \infty$ the group (phase) velocity, $v_{g(p)} \to 1$.

The solution for $|\phi\rangle$ is similar to $|\gamma_\parallel\rangle$ modulo a constant phase factor. It seems that, for $\omega < \omega_c$, there is energy exchange between these two modes. A detailed understanding of the physics of energy transfer as well as how the system behaves once the backreaction of the propagating modes on the background is taken into account, following [25] and [26], seems to be an important issue. However addressing the same are beyond the aim and the scope of the current article and would not be dealt with any further here.

Signature.—In astrophysical situations synchrotron or curvature radiation is the most common process of nonthermal emission. As is well known, from [27], such

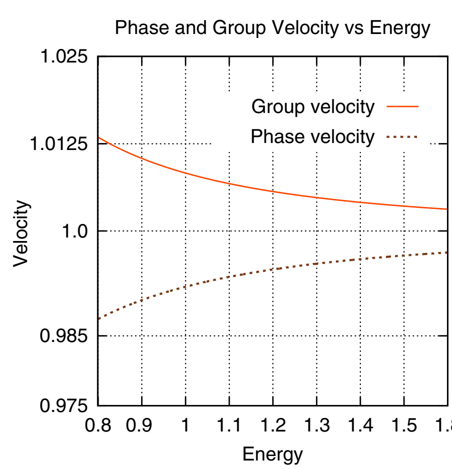

FIG. 1 (color online). Plot of velocity versus energy. Energy is plotted in units of 10^{-15} GeV. The other parameters are $g_{\phi\gamma\gamma} = (10^{-11}\ \text{GeV})^{-1}$ and $\mathcal{B} = 10^9$ Gauss.

radiations are always *polarized* along and orthogonal to the $\mathcal{B} - V$ plane, where V is the instantaneous velocity vector of the radiating charged particle. The synchrotron amplitudes of the electromagnetic radiation for these two polarized states are given by

$$A_\perp = \frac{\sqrt{3}\gamma_f^2\theta_e}{\omega_r}\sqrt{1+\gamma_f^2\theta_e^2}K_{1/3}\left(\frac{\omega}{2\omega_r}\right),$$
$$A_\parallel = i\frac{\sqrt{3}\gamma_f}{\omega_r}(1+\gamma_f^2\theta_e^2)K_{2/3}\left(\frac{\omega}{2\omega_r}\right). \quad (21)$$

In Eq. (21), γ_f is the Lorentz factor, $\omega_r = \frac{3\gamma_f^3}{\rho}$ is the cutoff frequency and ρ is the radius of curvature of the trajectory of the radiating particle. Lastly $\theta_c = \frac{1}{\gamma_f}$ is the opening angle of the radiating cone.

Since for $\omega < \omega_c$, the only evolving polarized state is $|\gamma_\perp\rangle$ when dimension-5, $\phi F_{\mu\nu}F^{\mu\nu}$ interaction is present, therefore, to a far-away observer, the synchrotron radiation would appear to be *linearly* polarized.

So, the differential intensity spectrum per unit energy, per unit solid angle at the source for the $|\gamma_\perp\rangle$ state, following Eq. (21), is given by $\frac{d^2I}{d\omega d\Omega} = \frac{(e\omega)^2}{4\pi^2}(|A_\perp|^2)$. Furthermore, if all the astrophysical absorption mechanisms are negligible, then the magnitude of $\frac{d^2I}{d\omega d\Omega}$ at the source as well as at the observation point would remain the same. Therefore, the differential intensity spectrum for two different energies $(\omega_1, \omega_2 < \omega_c)$, would be related to the respective energies ω_1 and ω_2 by

$$\frac{\frac{d^2I(\omega_1)}{d\omega_1 d\Omega}}{\frac{d^2I(\omega_2)}{d\omega_2 d\Omega}} = \left[\frac{\omega_1 K_{\frac{1}{3}}(\frac{\omega_1}{2\omega_c})}{\omega_2 K_{\frac{1}{3}}(\frac{\omega_2}{2\omega_c})}\right]^2, \quad \text{implying} \quad \frac{\omega_2}{\omega_1} = \left[\frac{\frac{d^2I(\omega_1)}{d\omega_1 d\Omega}}{\frac{d^2I(\omega_2)}{d\omega_2 d\Omega}}\right]^{\frac{3}{4}}. \quad (22)$$

This is the *intensity-energy* relation. While deriving the same [i.e., Eq. (22)], we have used Eq. (21) and expanded $K_{1/3}(x)$ in descending powers of x.

Next we would like to relate this *intensity-energy* relation [Eq. (22)] with *the rotation measure*.

Since the intervening media between the source and the far-away observer is magnetized and composed of nonrelativistic, degenerate electrons; the plane of polarization (POP) of a polarized light (of energy ω) passing through the same would undergo Faraday rotation (FR), given by [28]

$$\varphi = \frac{\alpha\pi(\mathcal{B}\cos\Theta)}{\omega^2 m_e}nl + \zeta. \quad (23)$$

Here, ζ is the angle between POP and $\hat{\mathcal{B}}$ at source and l is the length of the path traveled. The rest of the symbols in Eq. (23) have their usual meaning.

Since the net rotation measure $(\varphi - \zeta)$ due to FR goes as $\frac{1}{\omega^2}$; therefore, for a multifrequency plane polarized light beam, the ratio of the two *rotation measures* at two distinct energies (ω_1 and ω_2), will be given by the following relation:

$$\omega_2/\omega_1 = \sqrt{(\varphi_1 - \zeta)/(\varphi_2 - \zeta)}. \quad (24)$$

Equation (24) may henceforth be termed as energy-dependent-rotation measure.

Now we can use Eqs. (22) and (24) to arrive at a relation between the rotation measure and the differential intensity spectrum, for $\tilde{\psi}$ (i.e., the solution for the $|\gamma_\perp\rangle$ state), and the same is

$$\frac{(\varphi_1 - \zeta)}{(\varphi_2 - \zeta)} = \left[\frac{d^2I(\omega_1)}{d\omega_1 d\Omega} \div \frac{d^2I(\omega_2)}{d\omega_2 d\Omega}\right]^{\frac{3}{2}}. \quad (25)$$

For magnetic field strength at the source, $\mathcal{B} \sim 10^9$ Gauss, and $g_{\phi\gamma\gamma} \sim (10^{11}\ \text{GeV})^{-1}$, we have $\omega_c \sim 10^{-5}$ eV which lies in the radio range.

So, the polarization versus (differential) intensity distribution pattern, for plane polarized light, in the energy range $0 < \omega < 10^{-5}$ eV, from distant astrophysical objects (with dominant synchrotron source), should behave according to Eqs. (22) and (25).

Conversely, for ω above ω_c, both $\tilde{\psi}$ and ψ would propagate in space-time and ψ would undergo amplitude modulation because of mixing with Φ. Hence, the emerging light beam may bear some appropriate polarimetric [29] and dispersive signatures of $g_{\phi\gamma\gamma}\phi\bar{F}_{\mu\nu}F^{\mu\nu}$ interaction, when the Faraday and the mixing effects are considered together, provided the same is realized in nature.

Similar signatures from different astrophysical radio sources were reported in [30] and [31] sometime back. They may have some implications for the situation we have discussed in this paper. However, one should work with the new data sets before coming to a definite conclusion.

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