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Colliding p-branes in the dynamical intersecting brane system

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We discuss the dynamics of intersecting p-branes with cosmological constants in the higher-dimensional gravity theories. For the delocalized brane case, these solutions describe an asymptotically de Sitter or power-law expanding universe, while for the partially localized intersecting branes, they describe homogeneous and isotropic universes at each position of the overall transverse space. We then apply these time-dependent branes to the study on the collision of two 0-branes and show that the $0-8$-brane system or the smeared $0-p_I$-brane system can provide an example of colliding branes if they have the same brane charges and only one overall transverse space. Finally, we argue some applications of the solutions in supergravity models.

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I. INTRODUCTION

In many recent developments involving cosmological models and brane collisions in higher-dimensional gravity theories [1–33], the dynamical p-branes carrying charges have played important roles. In the classical solution of a single p-brane, the coupling of the dilaton to the field strength includes the parameter N. Since these brane solutions with $N = 4$ are related to well-known D-branes and M-branes in supergravity theories, they certainly exhibit many attractive properties in the higher-dimensional spacetime. Some static solutions with $N \neq 4$ also have supersymmetry after dimensional reductions to lower-dimensional theory [34,35]. The time-dependent generalizations of these solutions are thus important examples of higher-dimensional gravity theories. The dynamical brane solution with the cosmological constant can be obtained by choosing the coupling constant appropriately [20,21,24]. For a single 2-form field strength and a nontrivial dilaton, we have found that the dynamical single 0-brane solution describes the Milne universe [21,24]. The field equations give an asymptotically de Sitter solution if the scalar field is trivial [24], which is a generalization of the Kastor-Traschen solution in the four-dimensional Einstein-Maxwell theory [36,37]. The construction of intersecting branes with a cosmological constant is a natural generalization of the single cosmological brane solutions. The time-dependent intersecting branes we have mainly discussed are localized only along the relative or overall transverse directions in a higher-dimensional background, which are delocalized intersecting brane systems. However, in the higher-dimensional gravity theories, one of the branes is localized along the relative transverse directions but delocalized along the overall transverse directions, which are partially localized branes solutions. If the background has the cosmological constant, there is little known about the dynamics of the intersecting brane system for not only the delocalized case but also the partially localized one.

In the present paper, we will explore the possible generalization of these solutions to the case of the intersecting brane systems with cosmological constants, although similar single brane solutions have been analyzed in Ref. [24]. We recall these arguments for constructions of the solution and modify the ansatz of the fields. A brane configuration has to satisfy an intersection rule which is an algebraic equation that relates the coupling of the dilaton to the dimensionality of the branes. The intersection rule implies that only the 0-brane can depend on time and the dynamical 0-brane commutes with the static p-branes. We will study the dynamical intersecting brane solutions for not only the delocalized case but also the partially localized one.

The paper is constructed as follows: In Secs. II and III, we derive the dynamical intersecting brane solutions with cosmological constants in a D-dimensional theory following the approach developed in Ref. [24]. We then illustrate how the dynamical solution of two or n intersecting branes arise under the condition of $N \neq 4$ in the D-dimensional theory. The spacetime starts with the structure of the combined 0-branes. If they do not have the same charges, a singularity hypersurface appears before they meet as the time decreases for $D > 4$. We then discuss the dynamics of two 0-branes with static p-branes (or the dynamics of two black holes) in Sec. IV. If there exists one uncompactified extra dimension [$0-8$-brane system or $0-(D-1)$-brane systems ($p \leq 7$)] and two brane systems have the same brane charges, the solution describes a collision of two branes (or two black holes), which is similar to the result in Refs. [3,21,28]. In Sec. V, applications of these solutions to five- or six-dimensional supergravity models are discussed. We consider in detail the construction yielding the dynamical 0- or 1-brane in the Nishino-Salam-Sezgin model. We also provide brief discussions for a time-dependent brane system in Romans' supergravity model. We describe how our Universe could be represented in the present formulation via an appropriate compactification and give the application to cosmology. We show that there exists no accelerating expansion of our Universe, although the conventional power-law expansion of the Universe is possible. We then discuss the dynamics of two 0- or 1-branes with smeared branes. If two brane systems have the same brane charges with smearing some dimensions, the solution describes a collision of two brane backgrounds.

There is a curvature singularity in the dynamical brane background if we set a particular value for the constant parameters. Then the solution implies that the presence of the singularities is signaling possible instabilities, making the solutions sick or unphysical. We study the classical stability of the solutions in Sec. VI. Our preliminary analysis will present that the energy of Klein-Gordon scalar fields in the dynamical brane background grows with time for inertial observers approaching the singularity. In terms of using the preliminary analysis performed in Refs. [38–41], the Klein-Gordon modes will be studied, arriving at the preliminary conclusion of instability. Section VII will be devoted to the summary and conclusions.

II. DYNAMICAL PARTIALLY LOCALIZED INTERSECTING BRANE BACKGROUNDS WITH COSMOLOGICAL CONSTANTS

In this section, we will construct the partially localized time-dependent brane systems in D dimensions with cosmological constants.

We consider a D-dimensional theory composed of the metric g_{MN}, the scalar field ϕ, cosmological constants $\Lambda_I(I=r,s)$, and two antisymmetric tensor field strengths of rank (p_r+2) and (p_s+2). The action in D dimensions is given by

$$S=\frac{1}{2\kappa^2}\int\left[(R-2\mathrm{e}^{\alpha_r\phi}\Lambda_r-2\mathrm{e}^{\alpha_s\phi}\Lambda_s)\ast\mathbf{1}_D-\frac{1}{2}\ast d\phi\wedge d\phi\right.$$
$$-\frac{1}{2}\frac{1}{(p_r+2)!}\mathrm{e}^{\epsilon_r c_r\phi}\ast F_{(p_r+2)}\wedge F_{(p_r+2)}$$
$$\left.-\frac{1}{2}\frac{1}{(p_s+2)!}\mathrm{e}^{\epsilon_s c_s\phi}\ast F_{(p_s+2)}\wedge F_{(p_s+2)}\right], \tag{1}$$

where R denotes the Ricci scalar constructed from the D-dimensional metric g_{MN}, $\alpha_I(I=r,s)$ are constants, κ^2 denotes the D-dimensional gravitational constant, $\ast$ is the Hodge operator in the D-dimensional spacetime, and $F_{(p_r+2)}$ and $F_{(p_s+2)}$ are (p_r+2) and (p_s+2)-form field strengths, respectively. The constant parameters c_I and $\epsilon_I(I=r,s)$ are defined by

$$c_I^2=N_I-\frac{2(p_I+1)(D-p_I-3)}{D-2}, \tag{2a}$$

$$\epsilon_I=\begin{cases}+ & \text{if the}\quad p_I\text{-brane is electric},\\ - & \text{if the}\quad p_I\text{-brane is magnetic},\end{cases} \tag{2b}$$

respectively. Here N_I is constant. The (p_r+2)-form and (p_s+2)-form field strengths $F_{(p_r+2)}$ and $F_{(p_s+2)}$ are given by the (p_r+1)-form and (p_s+1)-form gauge potentials $A_{(p_r+1)}$ and $A_{(p_s+1)}$, respectively:

$$F_{(p_r+2)}=dA_{(p_r+1)},\qquad F_{(p_s+2)}=dA_{(p_s+1)}. \tag{3}$$

For the D-dimensional action (1), the field equations read

$$R_{MN}=\frac{2}{D-2}(\mathrm{e}^{\alpha_r\phi}\Lambda_r+\mathrm{e}^{\alpha_s\phi}\Lambda_s)g_{MN}+\frac{1}{2}\partial_M\phi\partial_N\phi+\frac{1}{2}\frac{\mathrm{e}^{\epsilon_r c_r\phi}}{(p_r+2)!}\left[(p_r+2)F_{MA_2\ldots A_{(p_r+2)}}F_N{}^{A_2\ldots A_{(p_r+2)}}-\frac{p_r+1}{D-2}g_{MN}F^2_{(p_r+2)}\right]$$
$$+\frac{1}{2}\frac{\mathrm{e}^{\epsilon_s c_s\phi}}{(p_s+2)!}\left[(p_s+2)F_{MA_2\ldots A_{(p_s+2)}}F_N{}^{A_2\ldots A_{(p_s+2)}}-\frac{p_s+1}{D-2}g_{MN}F^2_{(p_s+2)}\right], \tag{4a}$$

$$\triangle\phi-\frac{1}{2}\frac{\epsilon_r c_r}{(p_r+2)!}\mathrm{e}^{\epsilon_r c_r\phi}F^2_{(p_r+2)}-\frac{1}{2}\frac{\epsilon_s c_s}{(p_s+2)!}\mathrm{e}^{\epsilon_s c_s\phi}F^2_{(p_s+2)}-2\alpha_r\mathrm{e}^{\alpha_r\phi}\Lambda_r-2\alpha_s\mathrm{e}^{\alpha_s\phi}\Lambda_s=0, \tag{4b}$$

$$d[\mathrm{e}^{\epsilon_r c_r\phi}\ast F_{(p_r+2)}]=0, \tag{4c}$$

$$d[\mathrm{e}^{\epsilon_s c_s\phi}\ast F_{(p_s+2)}]=0, \tag{4d}$$

where $\triangle$ denotes the Laplace operator with respect to the D-dimensional metric g_{MN}.

The D-dimensional metric involving the intersecting branes with a cosmological constant can be put in the general form

$$ds^2=h_r^{a_r}(x,y,z)h_s^{a_s}(x,v,z)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu+h_r^{b_r}(x,y,z)h_s^{a_s}(x,v,z)\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j$$
$$+h_r^{a_r}(x,y,z)h_s^{b_s}(x,v,z)w_{mn}(\mathrm{Y}_2)dv^m dv^n+h_r^{b_r}(x,y,z)h_s^{b_s}(x,v,z)u_{ab}(\mathrm{Z})dz^a dz^b, \tag{5}$$

TABLE I. Intersections of $p_r - p_s$-branes in the metric (5), where $p' = p_s + p_r - p$.

Case		0	1	$\cdots$	p	$p+1$	$\cdots$	p_s	p_s+1	$\cdots$	p'	$p'+1$	$\cdots$	$D-1$
	p_r	∘	∘	∘	∘				∘	∘	∘			
$p_r - p_s$	p_s	∘	∘	∘	∘	∘	∘	∘						
	x^N	t	x^1	$\cdots$	x^p	y^1	$\cdots$	y^{p_s-p}	v^1	$\cdots$	v^{p_r-p}	z^1	$\cdots$	$z^{D-p'-1}$

where $q_{\mu\nu}$ is the $(p+1)$-dimensional metric depending only on the $(p+1)$-dimensional coordinates x^μ, γ_{ij} is the (p_s-p)-dimensional metric depending only on the (p_s-p)-dimensional coordinates y^i, w_{mn} is the (p_r-p)-dimensional metric depending only on the (p_r-p)-dimensional coordinates v^m, and u_{ab} is the $(D+p-p_r-p_s-1)$-dimensional metric depending only on the $(D+p-p_r-p_s-1)$-dimensional coordinates z^a. Here we assume that the parameters $a_I (I=r,s)$ and $b_I (I=r,s)$ in the metric (5) are given by

$$a_I = -\frac{4(D-p_I-3)}{N_I(D-2)}, \qquad b_I = \frac{4(p_I+1)}{N_I(D-2)}. \tag{6}$$

The brane configuration is illustrated in Table I.

The dynamical brane solutions are characterized by two warp factors h_r and h_s, depending on the $(D+p-p_r-p_s-1)$-dimensional coordinates transverse to the corresponding brane as well as the $(p+1)$-dimensional world-volume coordinate. In the case of intersection involving two branes, the powers of warp factors have to obey the intersection rule and then split the coordinates in four parts [42–44]. One is coordinates of the world-volume spacetime, x^μ, which are common to the p_r-, p_s-branes. The others are coordinates of the overall transverse space z^a and coordinates of the relative transverse y^i and v^m, which are transverse to only one of the p_r-, p_s-branes. In this section, we consider the intersections of a p_r- and a p_s-brane with the following conditions in D dimensions. We assume that the functions h_r and h_s depend not only on overall transverse coordinates but also on the corresponding relative coordinates and world-volume coordinates. We therefore may write $h_r = h_r(x,y,z), h_s = h_s(x,v,z)$.

We give the expression for the field strengths $F_{(p_r+2)}$ and $F_{(p_s+2)}$ and scalar field ϕ of a p_r-brane intersecting with a p_s-brane over a p-brane configuration:

$$\mathrm{e}^\phi = h_r^{2\epsilon_r c_r/N_r} h_s^{2\epsilon_s c_s/N_s}, \tag{7a}$$

$$F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d[h_r^{-1}(x,y,z)]\wedge\Omega(\mathrm{X})\wedge\Omega(\mathrm{Y}_2), \tag{7b}$$

$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d[h_s^{-1}(x,v,z)]\wedge\Omega(\mathrm{X})\wedge\Omega(\mathrm{Y}_1), \tag{7c}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y}_1)$, and $\Omega(\mathrm{Y}_2)$ are the volume $(p+1)$-form, (p_s-p)-form, and (p_r-p)-form, respectively:

$$\Omega(\mathrm{X}) = \sqrt{-q}dx^0\wedge dx^1\wedge\cdots\wedge dx^p, \tag{8a}$$

$$\Omega(\mathrm{Y}_1) = \sqrt{\gamma}dy^1\wedge dy^2\wedge\cdots\wedge dy^{p_s-p}, \tag{8b}$$

$$\Omega(\mathrm{Y}_2) = \sqrt{w}dv^1\wedge dv^2\wedge\cdots\wedge dv^{p_r-p}. \tag{8c}$$

Here, q, γ, and w denote the determinants of the metrics $q_{\mu\nu}$, γ_{ij}, and w_{mn}, respectively.

A. Power-law expanding universe

In this subsection, we consider the field equations (7) with $c_I(I=r,s)\neq 0$. The parameters $\alpha_I(I=r,s)$ are assumed to be

$$\alpha_r = -\epsilon_r c_r, \qquad \alpha_s = -\epsilon_s c_s. \tag{9}$$

Let us first consider the gauge field equations (4c) and (4d). Using the assumptions (5) and (7), we have

$$d[h_s^{4(\chi+1)/N_s}\partial_i h_r(*_{\mathrm{Y}_1}dy^i)\wedge\Omega(\mathrm{Z}) + h_s^{4\chi/N_s}\partial_a h_r(*_{\mathrm{Z}}dz^a)\wedge\Omega(\mathrm{Y}_1)] = 0, \tag{10a}$$

$$d[h_r^{4(\chi+1)/N_r}\partial_m h_s(*_{\mathrm{Y}_2}dv^m)\wedge\Omega(\mathrm{Z}) + h_r^{4\chi/N_r}\partial_a h_s(*_{\mathrm{Z}}dz^a)\wedge\Omega(\mathrm{Y}_2)] = 0, \tag{10b}$$

where $*_{\mathrm{Y}_1}$, $*_{\mathrm{Y}_2}$, and $*_{\mathrm{Z}}$ denote the Hodge operator on Y_1, Y_2, and Z, respectively, and χ is given by

$$\chi = p+1-\frac{(p_r+1)(p_s+1)}{D-2}+\frac{1}{2}\epsilon_r\epsilon_s c_r c_s. \tag{11}$$

In the following, we discuss the case of $\chi=0$, because the relation $\chi=0$ is consistent with the intersection rule which has been found in Refs. [11,15,22,24,45–60].

Setting $\chi=0$, Eq. (10a) gives

$$h_s\triangle_{\mathrm{Y}_1}h_r + \triangle_{\mathrm{Z}}h_r = 0, \qquad \partial_\mu\partial_i h_r + \frac{4}{N_s}\partial_\mu \ln h_s \partial_i h_r = 0, \qquad \partial_\mu\partial_a h_r = 0, \tag{12}$$

where $\triangle_{\mathrm{Y}_1}$ and $\triangle_{\mathrm{Z}}$ are the Laplace operators on the space of Y_1 and Z, respectively.

On the other hand, Eq. (10b) leads to

$$h_r \triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s = 0, \qquad \partial_\mu \partial_m h_s + \frac{4}{N_r}\partial_\mu \ln h_r \partial_m h_s = 0,$$
$$\partial_\mu \partial_a h_s = 0, \tag{13}$$

where we used Eq. (11) and $\triangle_{\mathrm{Y}_2}$ is the Laplace operators on the space of Y_2.

Now we consider the Einstein equation (4a). Using the ansatz (5) and (7) and the intersection rule $\chi = 0$, the Einstein equations become

$$\begin{aligned}
&R_{\mu\nu}(\mathrm{X}) - \frac{4}{N_r} h_r^{-1} D_\mu D_\nu h_r - \frac{4}{N_s} h_s^{-1} D_\mu D_\nu h_s + \frac{2}{N_r}\partial_\mu \ln h_r \left[\left(1 - \frac{4}{N_r}\right)\partial_\nu \ln h_r - \frac{4}{N_s}\partial_\nu \ln h_s\right] \\
&\quad + \frac{2}{N_s}\partial_\mu \ln h_s \left[\left(1 - \frac{4}{N_s}\right)\partial_\nu \ln h_s - \frac{4}{N_r}\partial_\nu \ln h_r\right] - \frac{2}{D-2}\left(\Lambda_r h_r^{-2+a_r p_r} h_s^{a_s - 2\epsilon_r\epsilon_s c_r c_s/N_s} + \Lambda_s h_r^{a_r - 2\epsilon_r\epsilon_s c_r c_s/N_r} h_s^{-2+a_s p_s}\right) q_{\mu\nu} \\
&\quad - \frac{1}{2} q_{\mu\nu} h_r^{-4/N_r} h_s^{-4/N_s}\left[a_r h_r^{-1}\left(h_s^{4/N_s}\triangle_{\mathrm{Y}_1} h_r + \triangle_{\mathrm{Z}} h_r\right) + a_s h_s^{-1}\left(h_r^{4/N_r}\triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s\right)\right] \\
&\quad - \frac{1}{2} q_{\mu\nu}\left[a_r h_r^{-1}\triangle_{\mathrm{X}} h_r - a_r q^{\rho\sigma}\partial_\rho \ln h_r \left\{\left(1 - \frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\}\right. \\
&\quad \left. + a_s h_s^{-1}\triangle_{\mathrm{X}} h_s - a_s q^{\rho\sigma}\partial_\rho \ln h_s \left\{\left(1 - \frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] = 0,
\end{aligned} \tag{14a}$$

$$\frac{2}{N_r} h_r^{-1}\left(\partial_\mu \partial_i h_r + \frac{4}{N_s}\partial_\mu \ln h_s \partial_i h_r\right) = 0, \qquad \frac{2}{N_s} h_s^{-1}\left(\partial_\mu \partial_m h_s + \frac{4}{N_r}\partial_\mu \ln h_r \partial_m h_s\right) = 0, \tag{14b}$$

$$\frac{2}{N_r} h_r^{-1}\partial_\mu \partial_a h_r + \frac{2}{N_s} h_s^{-1}\partial_\mu \partial_a h_s = 0, \tag{14c}$$

$$\begin{aligned}
&R_{ij}(\mathrm{Y}_1) - \frac{1}{2} h_r^{4/N_r}\gamma_{ij}\left[b_r h_r^{-1}\triangle_{\mathrm{X}} h_r - b_r q^{\rho\sigma}\partial_\rho \ln h_r \left\{\left(1 - \frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\}\right. \\
&\quad \left. + a_s h_s^{-1}\triangle_{\mathrm{X}} h_s - a_s q^{\rho\sigma}\partial_\rho \ln h_s \left\{\left(1 - \frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] \\
&\quad - \frac{1}{2}\gamma_{ij} h_s^{-4/N_s}\left\{b_r h_r^{-1}\left(h_s^{4/N_s}\triangle_{\mathrm{Y}_1} h_r + \triangle_{\mathrm{Z}} h_r\right) + a_s h_s^{-1}\left(h_r^{4/N_r}\triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s\right)\right\} \\
&\quad - \frac{2}{D-2}\left[\Lambda_r h_r^{-2+a_r p_r + 4/N_r} h_s^{a_s - 2\epsilon_r\epsilon_s c_r c_s/N_s} + \Lambda_s h_r^{a_r - 2(\epsilon_r\epsilon_s c_r c_s - 2)/N_r} h_s^{-2+a_s p_s}\right]\gamma_{ij} = 0,
\end{aligned} \tag{14d}$$

$$\frac{8}{N_r N_s (D-2)^2}\left[(p_r + 1)(p_s + 1) - (D-2)(p_r + p_s + 2)\right]\partial_i \ln h_r \partial_m \ln h_s = 0, \tag{14e}$$

$$\begin{aligned}
&R_{mn}(\mathrm{Y}_2) - \frac{1}{2} h_s^{4/N_s} w_{mn}\left[a_r h_r^{-1}\triangle_{\mathrm{X}} h_r - a_r q^{\rho\sigma}\partial_\rho \ln h_r \left\{\left(1 - \frac{4}{N_r}\right)\partial_\sigma \ln h_r - \frac{4}{N_s}\partial_\sigma \ln h_s\right\}\right. \\
&\quad \left. + b_s h_s^{-1}\triangle_{\mathrm{X}} h_s - b_s q^{\rho\sigma}\partial_\rho \ln h_s \left\{\left(1 - \frac{4}{N_s}\right)\partial_\sigma \ln h_s - \frac{4}{N_r}\partial_\sigma \ln h_r\right\}\right] \\
&\quad - \frac{1}{2} w_{mn} h_r^{-4/N_r}\left\{a_r h_r^{-1}\left(h_s^{4/N_s}\triangle_{\mathrm{Y}_1} h_r + \triangle_{\mathrm{Z}} h_r\right) + b_s h_s^{-1}\left(h_r^{4/N_r}\triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s\right)\right\} \\
&\quad - \frac{2}{D-2}\left[\Lambda_r h_r^{-2+a_r p_r} h_s^{a_s - 2(\epsilon_r\epsilon_s c_r c_s - 2)/N_s} + \Lambda_s h_r^{a_r - 2\epsilon_r\epsilon_s c_r c_s/N_r} h_s^{-2+a_s p_s + 4/N_s}\right] w_{mn} = 0,
\end{aligned} \tag{14f}$$

$$
\begin{aligned}
&R_{ab}(\mathrm{Z})-\frac{1}{2}h_r^{4/N_r}h_s^{4/N_s}u_{ab}\left[b_r h_r^{-1}\triangle_{\mathrm{X}}h_r-b_r q^{\rho\sigma}\partial_\rho\ \ln h_r\left\{\left(1-\frac{4}{N_r}\right)\partial_\sigma\ \ln h_r-\frac{4}{N_s}\partial_\sigma\ \ln h_s\right\}\right.\\
&\left.+b_s h_s^{-1}\triangle_{\mathrm{X}}h_s-b_s q^{\rho\sigma}\partial_\rho\ \ln h_s\left\{\left(1-\frac{4}{N_s}\right)\partial_\sigma\ \ln h_s-\frac{4}{N_r}\partial_\sigma\ \ln h_r\right\}\right]\\
&-\frac{1}{2}u_{ab}[b_r h_r^{-1}(h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_r+\triangle_{\mathrm{Z}}h_r)+b_s h_s^{-1}(h_r^{4/N_r}\triangle_{\mathrm{Y}_2}h_s+\triangle_{\mathrm{Z}}h_s)]\\
&-\frac{2}{D-2}[\Lambda_r h_r^{-2+a_r p_r+4/N_r}h_s^{a_s-2(\epsilon_r\epsilon_s c_r c_s-2)/N_s}+\Lambda_s h_r^{a_r-2(\epsilon_r\epsilon_s c_r c_s-2)/N_r}h_s^{-2+a_s p_s+4/N_s}]u_{ab}=0,
\end{aligned}\tag{14g}
$$

where D_μ is the covariant derivative constructed from the metric $q_{\mu\nu}$, $\triangle_{\mathrm{X}}$, $\triangle_{\mathrm{Y}_1}$, $\triangle_{\mathrm{Y}_2}$, and $\triangle_{\mathrm{Z}}$ are the Laplace operators on X, Y_1, Y_2, and Z, respectively, and $R_{\mu\nu}(\mathrm{X})$, $R_{ij}(\mathrm{Y}_1)$, $R_{mn}(\mathrm{Y}_2)$, and $R_{ab}(\mathrm{Z})$ are the Ricci tensors with respect to the metrics $q_{\mu\nu}(\mathrm{X})$, $\gamma_{ij}(\mathrm{Y}_1)$, $w_{mn}(\mathrm{Y}_2)$, and $u_{ab}(\mathrm{Z})$, respectively.

From Eqs. (14b) and (14c), the warp factors h_r and h_s can be expressed as

$$h_r=h_0(x)+h_1(y,z),\qquad h_s=h_s(v,z),\quad \text{for }\partial_\mu h_s=0,\tag{15a}$$

$$h_r=h_r(y,z),\qquad h_s=k_0(x)+k_1(v,z),\qquad \text{for }\partial_\mu h_r=0.\tag{15b}$$

If we require that the background satisfies

$$\partial_\mu h_s=0,\qquad p=p_r=0,\qquad \Lambda_s=0,\qquad \chi=0,\tag{16}$$

the Einstein equations (14) reduce to

$$
\begin{aligned}
&-\frac{2}{N_r}\left[2h_r^{-1}\frac{d^2h_0}{dt^2}-\left(1-\frac{4}{N_r}\right)(\partial_t\ \ln h_r)^2\right]+\frac{2}{D-2}\Lambda_r h_r^{-2}\\
&+\frac{1}{2}h_r^{-4/N_r}h_s^{-4/N_s}[a_r h_r^{-1}(h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1)+a_s h_s^{-1}\triangle_{\mathrm{Z}}h_s]-\frac{1}{2}a_r\left[h_r^{-1}\frac{d^2h_0}{dt^2}-\left(1-\frac{4}{N_r}\right)(\partial_t\ \ln h_r)^2\right]=0,
\end{aligned}\tag{17a}
$$

$$
\begin{aligned}
&R_{ij}(\mathrm{Y}_1)+\frac{1}{2}b_r h_r^{4/N_r}\gamma_{ij}\left[h_r^{-1}\frac{d^2h_0}{dt^2}-\left(1-\frac{4}{N_r}\right)(\partial_t \ln h_r)^2\right]\\
&-\frac{2}{D-2}\Lambda_r h_r^{-2+\frac{4}{N_r}}\gamma_{ij}-\frac{1}{2}\gamma_{ij}h_s^{-4/N_s}[b_r h_r^{-1}(h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1)+a_s h_s^{-1}\triangle_{\mathrm{Z}}h_s]=0,
\end{aligned}\tag{17b}
$$

$$
\begin{aligned}
&R_{ab}(\mathrm{Z})+\frac{1}{2}b_r h_r^{4/N_r}h_s^{4/N_s}u_{ab}\left[h_r^{-1}\frac{d^2h_0}{dt^2}-\left(1-\frac{4}{N_r}\right)(\partial_t \ln h_r)^2\right]\\
&-\frac{2\Lambda_r}{D-2}h_r^{-2+\frac{4}{N_r}}h_s^{\frac{4}{N_s}}u_{ab}-\frac{1}{2}u_{ab}[b_r h_r^{-1}(h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1)+b_s h_s^{-1}\triangle_{\mathrm{Z}}h_s]=0.
\end{aligned}\tag{17c}
$$

Note that Eq. (14f) becomes trivial for $p=p_r=0$. By combining the above equations and setting $p=p_r=0$, the Einstein equations for $N_r\neq 4$ lead to

$$R_{ij}(\mathrm{Y}_1)=0,\qquad R_{ab}(\mathrm{Z})=0,\tag{18a}$$

$$h_r=h_0(t)+h_1(y,z),\qquad h_s=h_s(z),\tag{18b}$$

$$\left(\frac{dh_0}{dt}\right)^2+N_r\left(1-\frac{4}{N_r}\right)^{-1}\Lambda_r=0,\qquad h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1=0,\tag{18c}$$

$$\triangle_{\mathrm{Z}}h_s=0.\tag{18d}$$

Finally, we check the scalar field equation for the case of $p = p_r = 0$. Substituting Eqs. (7), (15), and (16) and the intersection rule $\chi = 0$ into Eq. (4b), we have

$$\frac{\epsilon_r c_r}{N_r} h_r^{-b_r 4/N_r} h_s^{-b_s 4/N_s}\left[-h_r^{-1}\frac{d^2 h_0}{dt^2}+\left(1-\frac{4}{N_r}\right)(\partial_t \ln h_r)^2+N_r\Lambda_r h_r^{-2}\right]+\frac{\epsilon_r c_r}{N_r}h_r^{-1}(h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1)+\frac{\epsilon_s c_s}{N_s}h_s^{-1}\triangle_{\mathrm{Z}}h_s=0. \tag{19}$$

Hence the scalar field equation (19) reads

$$\frac{d^2 h_0}{dt^2}=0, \qquad \left(\frac{dh_0}{dt}\right)^2+N_r\left(1-\frac{4}{N_r}\right)^{-1}\Lambda_r=0,$$
$$h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1=0, \tag{20a}$$

$$\triangle_{\mathrm{Z}}h_s=0. \tag{20b}$$

These are consistent with the Einstein equations (18). The function h_r can depend on the coordinate t only if $N_r \neq 4$. For $N_r = 4$, the scalar field equation leads to $\Lambda_r = 0$.

We can find the solution in which the p_s-brane part depends on x^μ. For $p = p_s = 0$, $\Lambda_r = 0$, and $\partial_t h_r = 0$, we have

$$R_{mn}(\mathrm{Y}_2)=0, \qquad R_{ab}(\mathrm{Z})=0, \tag{21a}$$

$$h_r=h_r(z), \qquad h_s=k_0(t)+k_1(v,z), \tag{21b}$$

$$\frac{d^2 k_0}{dt^2}=0, \qquad \left(\frac{dk_0}{dt}\right)^2+N_s\left(1-\frac{4}{N_s}\right)^{-1}\Lambda_s=0,$$
$$h_r^{4/N_r}\triangle_{\mathrm{Y}_2}k_1+\triangle_{\mathrm{Z}}k_1=0, \tag{21c}$$

$$\triangle_{\mathrm{Z}}h_r=0. \tag{21d}$$

It is clear that there is a solution for $k_0(t)$ such as $\partial_t h_s \neq 0$ unless $N_s = 4$. For $N_r = 4$, the field equations lead to $\Lambda_r = 0$.

If $F_{(p_r+2)} = 0$ and $F_{(p_s+2)} = 0$, the warp factors h_1 and k_1 are trivial functions. Then the D-dimensional spacetime is no longer warped [11]. Moreover, Eqs. (18) and (21) imply the two cases. First, p_r-, p_s-branes are delocalized. These are localized only along the overall transverse directions. Second, the 0-brane is completely localized on the p_s- (or p_r-) brane which is localized only along the overall transverse directions, which is a partially localized $p_r - 0$ (or $0 - p_s$) brane system.

As an example, we set

$$p=p_r=0, \qquad \gamma_{ij}=\delta_{ij}, \qquad u_{ab}=\delta_{ab},$$
$$h_s=h_s(z), \tag{22}$$

where δ_{ij} and δ_{ab} are the p_s- and $(D-p_s-1)$-dimensional Euclidean metrics, respectively. Equation (18c) gives

$$h_0(t)=c_0 t+c_1, \qquad c_0=\pm\sqrt{N_r\left(\frac{4}{N_r}-1\right)^{-1}\Lambda_r}, \tag{23}$$

where c_0 and c_1 are constants. Hence, solutions exist for $N_r < 4$ if $\Lambda_r > 0$ and vice versa.

If the functions h_1 and h_s satisfy the coupled partial differential equations

$$h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1=0, \qquad \triangle_{\mathrm{Z}}h_s=0, \tag{24}$$

the harmonic function h_s that satisfies the equation in (18d) takes the form

$$h_s(z)=1+\sum_\ell \frac{M_\ell}{|z^a-z^a_\ell|^{D-p_s-3}}, \tag{25}$$

where z^a_ℓ is the location of the ℓth p_s-brane and M_ℓ is constant. Since we consider the case in which the p_s-branes coincide at the same location in the overall transverse directions, the harmonic function h_s can be written by the following form [27,61,62]:

$$h_s(z)=\frac{M}{|z^a-z^a_0|^{D-p_s-3}}, \tag{26}$$

where M is constant and the stack of p_s-branes is located at the same points z^a_0 along the z directions. We can find solutions for the harmonic function h_1 in the case where each of the p_s-branes does not coincide at the same location in the overall transverse directions.

If we set $D - p_s \neq 3$ and $2 - 4N_s^{-1}(D - p_s - 3) \neq 0$ for the overall transverse space, Eq. (24) can be solved as

$$h_1(y,z) = 1 + \sum_\ell \frac{M_\ell}{[|y^i - y^i_\ell|^2 + \frac{4M^{4/N_s}}{\{2-4N_s^{-1}(D-p_s-3)\}^2}|z^a - z^a_0|^{2-4N_s^{-1}(D-p_s-3)}]^{\zeta_r}}, \tag{27}$$

where M_ℓ is constant and ζ_r is given by

$$\zeta_r = \frac{1}{2}\left[p_s - 1 + \frac{(2-4N_s^{-1})(D-p_s-3)+2}{2-4N_s^{-1}(D-p_s-3)}\right]. \tag{28}$$

Hence, the functions h_r and h_s can be expressed as

$$h_r(t,y,z) = c_0 t + c_1 + \sum_\ell \frac{M_\ell}{[|y^i - y^i_\ell|^2 + \frac{4M^{4/N_s}}{\{2-4N_s^{-1}(D-p_s-3)\}^2}|z^a - z^a_0|^{2-4N_s^{-1}(D-p_s-3)}]^{\zeta_r}}, \tag{29a}$$

$$h_s(z) = \frac{M}{|z^a - z^a_0|^{D-p_s-3}}, \tag{29b}$$

where c_0, c_1, M_ℓ, and M are constant parameters and y^i_ℓ and z^a_0 are constants representing the positions of the branes. The curvature singularities appear at $h_r = 0$ in the D-dimensional metric (5). Moreover, there is also a singularity at $z^a = z^a_0$ unless the scalar field is trivial.

Upon setting $D - p_s = 3$ and $N_s = 4$, the solutions of Eq. (24) are given by

$$h_r(t,y,z) = c_0 t + c_1 + \sum_\ell \frac{M_\ell}{[|y^i - y^i_\ell|^2 + M|z^a - z^a_0|^2]^{\frac{1}{2}(p_s+1)}}, \tag{30a}$$

$$h_s(z) = M \ln|z^a - z^a_0|. \tag{30b}$$

In the case of $D - p_s = 5$ and $N_s = 4$, the functions h_r and h_s can be written by

$$h_r(t,y,z) = c_0 t + c_1 + \sum_\ell M_\ell[|y^i - y^i_\ell|^2 - p_s M \ln|z^a - z^a_0|], \tag{31a}$$

$$h_s(z) = \frac{M}{|z^a - z^a_0|^2}. \tag{31b}$$

The solutions (30) and (31) have a singular hypersurface at infinity as well as at $h_r = 0$, because the D-dimensional metric depends on the logarithmic function of the transverse coordinates. These solutions also give a singularity at $z^a = z^a_0$ if the dilaton is nontrivial.

It is possible to find the solution for $\partial_t h_r = 0$ and $\partial_t h_s \neq 0$ if the roles of Y_1 and Y_2 are exchanged. The solution of the field equations for $D - p_r \neq 3$ and $D - p_r \neq 5$ can be written by

$$h_s(t,v,z) = c_0 t + c_1 + \sum_\ell \frac{M_\ell}{[|v^m - v^m_\ell|^2 + \frac{4M^{4/N_r}}{\{2-4N_r^{-1}(D-p_r-3)\}^2}|z^a - z^a_0|^{2-4N_r^{-1}(D-p_r-3)}]^{\zeta_s}}, \tag{32a}$$

$$h_r(z) = \frac{M}{|z^a - z^a_0|^{D-p_r-3}}, \tag{32b}$$

where ζ_s is given by

$$\zeta_s = \frac{1}{2}\left[p_r - 1 + \frac{(2-4N_r^{-1})(D-p_r-3)+2}{2-4N_r^{-1}(D-p_r-3)}\right]. \tag{33}$$

If we set $D - p_r = 3$, $D - p_r = 5$, and $N_r = 4$, the harmonic functions h_r and h_s have logarithmic spatial dependence like (30) and (31).

Assuming $\Lambda_r > 0$ and introducing a new time coordinate τ by

$$\frac{\tau}{\tau_0} = (c_0 t + c_1)^{\frac{(N_r-2)(D-2)+2}{N_r(D-2)}}, \quad \tau_0 = \frac{N_r(D-2)}{c_0[(N_r-2)(D-2)+2]}, \tag{34}$$

we find the D-dimensional metric (5) as

$$ds^2 = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{N_r(D-2)}{(N_r-2)(D-2)+2}} h_1\right]^{-\frac{4(D-3)}{N_r(D-2)}} h_s^{a_s}\left[-d\tau^2 + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{N_r(D-2)}{(N_r-2)(D-2)+2}} h_1\right\}^{\frac{4}{N_r}} \left(\frac{\tau}{\tau_0}\right)^{\frac{4}{(N_r-2)(D-2)+2}} \left(\gamma_{ij}dy^i dy^j + h_s^{\frac{4}{N_s}} u_{ab} dz^a dz^b\right)\right]. \tag{35}$$

Since h_s does not approach constant in any region, the whole spacetime cannot be homogeneous and isotropic. But on each $z^a = \text{const}$ slice the spacetime becomes a homogeneous and isotropic universe. In the limit $\tau \to \infty$, the function h_1 can be negligible in the warp factor. This is guaranteed by a scalar field with the exponential potential. The accelerating universe is obtained on each $z^a = \text{const}$ slice if $N_r < 2$, which corresponds to the case of a positive cosmological constant. For $\frac{2(D-3)}{D-2} < N_r < 2$, the solution provides a power-law inflationary universe, and for $N_r > \frac{2(D-3)}{D-2}$, the scale factor diverges at $\tau = \tau_\infty > 0$, taking the involution $\tau \to \tau_\infty - \tau$. Finally, for $N_r = \frac{2(D-3)}{D-2}$, we obtain a de Sitter universe which will be discussed in the next subsection.

B. de Sitter universe

Next, we consider the solution with a dilaton which is the case of $c_I = 0 (I = r \,\text{or}\, s)$. In terms of $c_I = 0$, Eq. (2a) gives

$$N_I = \frac{2(D - p_I - 3)(p_I + 1)}{(D-2)}. \tag{36}$$

If we assume

$$c_r = 0, \qquad c_s \neq 0, \qquad p = p_r = 0,$$
$$N_r = \frac{2(D-3)}{(D-2)}, \qquad \alpha_r = -\frac{N_s a_s}{2\epsilon_s c_s}, \qquad \Lambda_s = 0, \tag{37}$$

the field equations reduce to

$$R_{ij}(\mathrm{Y}_1) = 0, \qquad R_{ab}(\mathrm{Z}) = 0, \tag{38a}$$

$$h_r(t,y,z) = h_0(t) + h_1(y,z),$$
$$\left(\frac{dh_0}{dt}\right)^2 - \frac{2(D-3)^2}{(D-2)(D-1)}\Lambda_r = 0, \tag{38b}$$

$$h_s^{4/N_s} \triangle_{\mathrm{Y}_1} h_1 + \triangle_{\mathrm{Z}} h_1 = 0, \qquad \triangle_{\mathrm{Z}} h_s = 0. \tag{38c}$$

Then Eq. (38b) gives

$$h_0 = c_0 t + c_1, \tag{39}$$

where c_1 is an integration constant and c_0 is given by

$$c_0 = \pm(D-3)\sqrt{\frac{2}{(D-2)(D-1)}\Lambda_r}. \tag{40}$$

Thus there is no solution for $\Lambda_r < 0$. If the metric $u_{ab}(\mathrm{Z})$ is assumed to be Eq. (22), the function h_1 is given by Eq. (27). Now we introduce a new time coordinate τ by

$$c_0 \tau = \ln t, \tag{41}$$

where we have set $c_0 > 0$ for simplicity. Then the D-dimensional metric (5) can be expressed as

$$\begin{aligned} ds^2 = h_s^{a_s}[&-(1 + c_0^{-1}\mathrm{e}^{-c_0\tau}h_1)^{-2}d\tau^2 \\ &+ (1 + c_0^{-1}\mathrm{e}^{-c_0\tau}h_1)^{2/(D-3)}(c_0\mathrm{e}^{c_0\tau})^{2/(D-3)} \\ &\times \{\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j + h_s^{4/N_s}u_{ab}(\mathrm{Z})dz^a dz^b\}]. \end{aligned} \tag{42}$$

The function h_s does not become constant in any region. Then, the D-dimensional spacetime cannot be de Sitter spacetime. However, the spacetime gives a homogeneous and isotropic universe on each $y^i = \text{const}$, $z^a = \text{const}$ slice. If we set $h_s = \text{const}$ and $h_1 = h_1(z)$, Eq. (42) becomes the solution which has been discussed by Refs. [63,64] (see also [65]). Furthermore, for $D = 4$ and by setting $h_s = 1$, the solution is the Kastor-Traschen one [36].

III. THE INTERSECTION INVOLVING n BRANE BACKGROUNDS

The construction that we have analyzed in Sec. II is a special case of a more general construction of intersecting branes with a cosmological constant. In effect, we have been studying the special case of intersections involving a two-brane. The time-dependent brane with a cosmological constant property is a 0-brane, represented by a 2-form. To describe more general intersections on a time-dependent background, one simply incorporates additional branes in a dynamical background. Without loss of the time dependence, it is possible to also add n delocalized branes. This also has one important further refinement. Instead of power-law expansion, the support of a 0-brane might be accelerated expansion, where D-dimensional geometry is an asymptotically de Sitter spacetime. The n intersection allows the time dependence of only 0-branes but not of the p-branes ($p \neq 0$). The reason for this is that the time-dependent brane we have obtained can be performed in the case of $\chi = 0$, where χ is defined by (11). So the coefficient of the time dependence is simply proportional to the cosmological constant that we have explored in Sec. II: the Einstein equations give $p = 0$.

In this section, we discuss the intersection of the delocalized n branes in the higher-dimensional gravity theory with the cosmological constants. The general action describing the intersection involving the n brane system is given by

$$S = \frac{1}{2\kappa^2}\int \left[\left(R - 2\sum_I \mathrm{e}^{\alpha_I\phi}\Lambda_I\right) * \mathbf{1}_D - \frac{1}{2} * d\phi\wedge d\phi - \frac{1}{2}\sum_I \frac{\mathrm{e}^{\epsilon_I c_I \phi}}{(p_I+2)!} * F_{(p_I+2)}\wedge F_{(p_I+2)}\right], \qquad (43)$$

where κ^2 denotes the D-dimensional gravitational constant, R is the D-dimensional Ricci scalar constructed from the D-dimensional metric g_{MN}, ϕ is a scalar field, $F_{(p_I+2)}$ is the antisymmetric tensor fields of rank $(p_I + 2)$, $*$ is the Hodge dual operator in the D-dimensional spacetime, and c_I and ϵ_I are constants defined by

$$c_I^2 = N_I - \frac{2(p_I+1)(D-p_I-3)}{D-2}, \qquad (44a)$$

$$\epsilon_I = \begin{cases} + & \text{for the electric brane,} \\ - & \text{for the magnetic brane.} \end{cases} \qquad (44b)$$

Here I denotes the type of the corresponding branes.

The D-dimensional action (43) gives the field equations

$$R_{MN} = \frac{2}{D-2}\sum_I \mathrm{e}^{\alpha_I\phi}\Lambda_I g_{MN} + \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}\sum_I \frac{1}{(p_I+2)!}\mathrm{e}^{\epsilon_I c_I\phi}\left[(p_I+2)F_{MA_2\ldots A_{p_I+2}}F_N{}^{A_2\ldots A_{p_I+2}} - \frac{p_I+1}{D-2}g_{MN}F^2_{(p_I+2)}\right], \qquad (45a)$$

$$\triangle\phi - 2\sum_I \alpha_I \mathrm{e}^{\alpha_I\phi}\Lambda_I - \frac{1}{2}\sum_I \frac{\epsilon_I c_I}{(p_I+2)!}\mathrm{e}^{\epsilon_I c_I\phi}F^2_{(p_I+2)} = 0, \qquad (45b)$$

$$d[\mathrm{e}^{\epsilon_I c_I\phi} * F_{(p_I+2)}] = 0, \qquad (45c)$$

where $\triangle$ denotes the Laplace operator with respect to the D-dimensional metric g_{MN}.

We adopt the ansatz that the D-dimensional metric can be written by

$$ds^2 = -A(t,z)dt^2 + \sum_{\alpha=1}^{p} B^{(\alpha)}(t,z)(dx^\alpha)^2 + C(t,z)u_{ab}(\mathrm{Z})dz^a dz^b, \qquad (46)$$

where $u_{ab}(\mathrm{Z})$ denotes the metric of the $(D-p-1)$-dimensional Z space which depends only on the $(D-p-1)$-dimensional coordinates z^a. Concerning the functions A, $B^{(\alpha)}$, and C, we assume

$$A = \prod_I [h_I(t,z)]^{a_I}, \qquad B^{(\alpha)} = \prod_I [h_I(t,z)]^{\delta_I^{(\alpha)}}, \qquad C = \prod_I [h_I(t,z)]^{b_I}, \qquad (47)$$

where the constants a_I, b_I, and $\delta_I^{(\alpha)}$ are given, respectively, by

$$a_I = -\frac{4(D-p_I-3)}{N_I(D-2)}, \qquad b_I = \frac{4(p_I+1)}{N_I(D-2)}, \qquad \delta_I^{(\alpha)} = \begin{cases} a_I & \text{for } \alpha\in I, \\ b_I & \text{for } \alpha\notin I. \end{cases} \qquad (48)$$

The function $h_I(t,z)$ is a straightforward generalization of the static solution associated with the brane I in an intersecting brane system [54,55] to the dynamical one.

We further require that the dilaton ϕ and the form fields $F_{(p+2)}$ satisfy the following conditions:

$$\mathrm{e}^\phi = \prod_I h_I^{2\epsilon_I c_I/N_I}, \qquad F_{(p_I+2)} = \frac{2}{\sqrt{N_I}}d(h_I^{-1})\wedge\Omega(\mathrm{X}_I), \qquad (49)$$

where X_I is the space associated with the brane I, and the volume (p_I+1)-form $\Omega(\mathrm{X}_I)$ is written by

$$\Omega(\mathrm{X}_I) = dt\wedge dx^{p_1}\wedge\cdots\wedge dx^{p_I}. \qquad (50)$$

A. Power-law expanding universe

Firstly, we consider the Einstein equations (45a) with $c_I \neq 0 (I = 0, \ldots, n-1)$. We assume that the parameters $\alpha_I (I = 0, \ldots, n-1)$ are given by

$$\alpha_I = -\epsilon_I c_I. \qquad (51)$$

We impose the condition with respect to the components of D-dimensional metric [55]

$$A^{(D-p-3)}\prod_{\alpha=1}^{p} B^{(\alpha)}C = 1, \qquad A^{-1}\prod_{\alpha\in I}(B^{(\alpha)})^{-1}\mathrm{e}^{\epsilon_I c_I\phi} = h_I^2. \qquad (52)$$

The Einstein equations (45a) become

$$\sum_{I,I'}\left(\frac{2}{N_I}\delta_{II'}-M_{II'}\right)\partial_t \ln h_I\partial_t \ln h_{I'}+\frac{2}{D-2}\sum_I\Lambda_I h_I^{-2+a_Ip_I}\prod_{I'\neq I}h_{I'}^{a_{I'}-\frac{2\epsilon_I\epsilon_{I'}c_Ic_{I'}}{N_{I'}}}$$
$$+\frac{1}{2}\sum_I b_I\left[\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I-\sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right]\partial_t \ln h_I-\frac{1}{2}\sum_I\left(\frac{4}{N_I}+b_I\right)h_I^{-1}\partial_t^2h_I+\frac{1}{2}\prod_{I'}h_{I'}^{-4/N_{I'}}\sum_I a_Ih_I^{-1}\triangle_Z h_I=0, \tag{53a}$$

$$\sum_I\frac{2}{N_I}h_I^{-1}\partial_t\partial_a h_I+\sum_{I,I'}\left(M_{II'}-\frac{2}{N_{I'}}\delta_{II'}\right)\partial_t \ln h_I\partial_a \ln h_{I'}=0, \tag{53b}$$

$$\prod_{J'}h_{J'}^{-a_{J'}}\sum_\gamma\prod_J h_J^{\delta_J^{(\gamma)}}\sum_I\delta_I^{(\gamma)}\left[h_I^{-1}\partial_t^2h_I-\left\{\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I-\sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\}\partial_t \ln h_I\right]$$
$$-\prod_{J'}h_{J'}^{-b_{J'}}\sum_\gamma\prod_J h_J^{\delta_J^{(\gamma)}}\sum_I\delta_I^{(\gamma)}h_I^{-1}\triangle_Z h_I-\frac{4}{D-2}\sum_I\Lambda_I h_I^{-2+\delta_I^{(\gamma)}p_I}\prod_{I'\neq I}h_{I'}^{\delta_{I'}^{(\gamma)}-\frac{2\epsilon_I\epsilon_{I'}c_Ic_{I'}}{N_{I'}}}=0, \tag{53c}$$

$$R_{ab}(\mathrm{Z})+\frac{1}{2}u_{ab}\prod_J h_J^{4/N_J}\sum_I b_I\left[h_I^{-1}\partial_t^2h_I-\left\{\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I-\sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\}\partial_t \ln h_I\right]$$
$$-\frac{1}{2}u_{ab}\sum_I b_Ih_I^{-1}\triangle_Z h_I-\sum_{I,I'}\frac{2}{N_I}\left(M_{II'}-\frac{2}{N_{I'}}\delta_{II'}\right)\partial_a \ln h_I\partial_b \ln h_{I'}-\frac{2}{D-2}\sum_I\Lambda_I h_I^{-2+a_Ip_I+\frac{4}{N_I}}\prod_{I'\neq I}h_{I'}^{a_{I'}-\frac{2(\epsilon_I\epsilon_{I'}c_Ic_{I'}-2)}{N_{I'}}}u_{ab}=0, \tag{53d}$$

where $R_{ab}(\mathrm{Z})$ is the Ricci tensor with respect to the metric $u_{ab}(\mathrm{Z})$ and $M_{II'}$ is defined by

$$M_{II'}\equiv\frac{1}{4}\left[a_Ia_{I'}+\sum_\alpha\delta_I^{(\alpha)}\delta_{I'}^{(\alpha)}+(D-p-3)b_Ib_{I'}\right]+\frac{2}{N_IN_{I'}}\epsilon_I\epsilon_{I'}c_Ic_{I'}. \tag{54}$$

Equation (53b) can be rewritten as

$$\sum_{I,I'}\left[M_{II'}+\frac{2}{N_I}\delta_{II'}\frac{\partial_t\partial_a \ln h_I}{\partial_t \ln h_I\partial_a \ln h_I}\right]\partial_t \ln h_I\partial_a \ln h_{I'}=0. \tag{55}$$

One can find that Eq. (55) is equivalent to satisfying that

$$\frac{\partial_t\partial_a \ln h_I}{\partial_t \ln h_I\partial_a \ln h_I}=k_I. \tag{56}$$

Then we have

$$M_{II'}+\frac{2}{N_I}k_I\delta_{II'}=0. \tag{57}$$

Equations (44a), (48), and (55) give

$$M_{II}=\frac{1}{4}[(p_I+1)a_I^2+(p-p_I)b_I^2+(D-p-3)b_I^2]+\frac{2}{N_I^2}c_I^2=\frac{2}{N_I}. \tag{58}$$

By combining (58) with (57), the constant k_I in (57) is $k_I=-1$, which implies

$$M_{II'}=\frac{2}{N_{I'}}\delta_{II'}. \tag{59}$$

By taking account of these results, Eq. (53b) yields

$$\partial_t\partial_a[h_I(t,z)]=0. \tag{60}$$

Hence we find

$$h_I(t,z)=K_I(t)+H_I(z). \tag{61}$$

For $I\neq I'$, (59) provides the intersection rule on the dimension $\bar{p}$ of the intersection for each pair of branes I and I' $(\bar{p}\leq p_I,p_{I'})$ [57,58]:

$$\bar{p} = \frac{(p_I+1)(p_{I'}+1)}{D-2} - 1 - \frac{1}{2}\epsilon_I c_I \epsilon_{I'} c_{I'}. \tag{62}$$

Under the assumptions given above, we next reduce the gauge field equations. In terms of the ansatz (49), the Bianchi identity $dF_{(p_I+2)} = 0$ is automatically satisfied:

$$h_I^{-1}(2\partial_a \ln h_I \partial_b \ln h_I + h_I^{-1}\partial_a\partial_b h_I)dz^a \wedge dz^b \wedge \Omega(\mathrm{X}_I) = 0. \tag{63}$$

By utilizing (49), the gauge field equation becomes

$$d[\partial_a H_I(*_{\mathrm{Z}} dz^a) \wedge *_{\mathrm{X}}\, \Omega(\mathrm{X}_I)] = 0, \tag{64}$$

where we used Eqs. (52) and (61) and $*_{\mathrm{X}}$, $*_{\mathrm{Z}}$ are the Hodge dual operators on $\mathrm{X}(\equiv \cup_I X_I)$ and Z, respectively. Hence, (64) gives (61), and we find

$$\triangle_{\mathrm{Z}} H_I = 0. \tag{65}$$

The roles of the Bianchi identity and field equations are interchanged for the magnetic ansatz [55,57,58]. Then the net result is the same.

In order to complete the system of equations, we must also consider the scalar field equation. Substituting the ansatz for fields (49) and the metric (46) and (61), the equation of motion for the scalar field (45b) reduces to

$$-\prod_{I''} h_{I''}^{-a_{I''}} \sum_I \frac{1}{N_I}\epsilon_I c_I \left[h_I^{-1}\frac{d^2 K_I}{dt^2} - \left\{\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\}\partial_t \ln h_I - N_I \Lambda_I h_I^{-2}\right] + \prod_{I''} h_{I''}^{-b_{I''}} \sum_I \frac{1}{N_I} h_I^{-1}\epsilon_I c_I \triangle_{\mathrm{Z}} H_I = 0. \tag{66}$$

Furthermore, (66) reads

$$\frac{d^2 K_I}{dt^2} = 0, \tag{67a}$$

$$\triangle_{\mathrm{Z}} H_I = 0, \tag{67b}$$

$$\sum_I \frac{\epsilon_I c_I}{N_I}\left[\left\{\left(1-\frac{4}{N_I}\right)\partial_t \ln h_I - \sum_{I'\neq I}\frac{4}{N_{I'}}\partial_t \ln h_{I'}\right\} \times \partial_t \ln h_I + N_I \Lambda_I h_I^{-2}\right] = 0. \tag{67c}$$

From Eq. (67a), we obtain

$$K_I = A_I t + B_I, \tag{68}$$

where A_I and B_I are constants.

1. The intersection involving the same brane

Let us first consider the case that all cosmological constants become nonvanishing. If we set $\Lambda_I \neq 0$, the field equations imply that all functions are equal:

$$h_I(t,z) = K(t,z) = K_0(t) + K_1(z), \qquad N_I = N_{I'} \equiv N. \tag{69}$$

We can find the solutions if the function h and N satisfy

$$K_0(t) = At + B, \qquad A = \pm\sqrt{N_I \Lambda_I \Big/ \sum_I \left(\frac{4}{N_I} - 1\right)}, \tag{70}$$

where B denotes a constant. Then the remaining Einstein equations (53) are

$$R_{ab}(\mathrm{Z}) = 0. \tag{71}$$

Now we set

$$u_{ab} = \delta_{ab}, \tag{72}$$

where δ_{ab} is the $(D-p-1)$-dimensional Euclidean metric. In this case, the solution for h_I can be obtained explicitly as

$$K(t,z) = At + B + \sum_k \frac{M_k}{|z^a - z_k^a|^{D-p-3}}, \tag{73}$$

where M_k's are constant parameters and z_k^a represents the positions of the branes in Z space. If the functions h_I coincide, the locations of the p_I-brane will also coincide. In this case, all branes have the same total amount of charge at the same position.

Let us consider the intersection rule in the D-dimensional gravity theory. If we set $p_I = \tilde{p}$ for all p_I, the intersection rule (62) leads to

$$\bar{p} = \tilde{p} - \frac{N}{2}. \tag{74}$$

Then, we find the intersection involving two $\tilde{p}$-branes:

$$\tilde{p} \cap \tilde{p} = \tilde{p} - \frac{N}{2}. \tag{75}$$

Since the number of intersections for $\tilde{p} < \frac{N}{2}$ is negative, there is no solution in these brane backgrounds.

If we choose $K_0 = 0 (A = B = 0)$, the metric describes the known static and extremal multi-black-hole solution with black hole charges M_k [54,55,57,58].

2. A dynamical brane in the intersecting brane system

In the following, we consider the case that there is only one function h_I which depends on both z^a and t. We denote it with the subscript $\tilde{I}$, while other functions of $I' \neq \tilde{I}$ are either dependent on z^a or constant. If we assume $N_{\tilde{I}} \neq 4$, we have

$$\partial_t h_{I'} = 0, \qquad p_{\tilde{I}} = 0, \qquad \Lambda_{I'} = 0, \quad \text{for } I' \neq \tilde{I}. \tag{76}$$

We can find the solutions if the function $h_{\tilde{I}}$ and $N_{\tilde{I}}$ satisfy

$$h_{\tilde{I}}(t,z) = K_{\tilde{I}}(t) + H_{\tilde{I}}(z),$$
$$K_{\tilde{I}}(t) = \pm\left[\left(\frac{4}{N_{\tilde{I}}} - 1\right)^{-1} N_{\tilde{I}}\Lambda_{\tilde{I}}\right]^{\frac{1}{2}} t + c_{\tilde{I}}, \qquad N_{\tilde{I}} \neq 4, \tag{77}$$

where $c_{\tilde{I}}$ is constant. Then the remaining Einstein equations (53) are

$$R_{ab}(\mathrm{Z}) = 0. \tag{78}$$

Now we set

$$u_{ab} = \delta_{ab}, \tag{79}$$

where δ_{ab} is the $(D-p-1)$-dimensional Euclidean metric. In this case, the solution for h_I can be written explicitly as

$$h_{\tilde{I}}(t,z) = \pm\left[\left(\frac{4}{N_{\tilde{I}}} - 1\right)^{-1} N_{\tilde{I}}\Lambda_{\tilde{I}}\right]^{\frac{1}{2}} t + \tilde{c}_{\tilde{I}} + \sum_k \frac{M_{\tilde{I},k}}{|z^a - z^a_k|^{D-p-3}}, \tag{80a}$$

$$h_{I'}(z) = \tilde{c}_{I'} + \sum_l \frac{M_{I',l}}{|z^a - z^a_l|^{D-p-3}}, \tag{80b}$$

where $\tilde{c}_{\tilde{I}}$, $\tilde{c}_{\tilde{I}}$, $M_{\tilde{I},k}$, and $M_{I',l}$ are constant parameters and z^a_k and z^a_l denote the positions of the branes in Z space. $N_{\tilde{I}} < 4$ leads to $\Lambda_{\tilde{I}} > 0$ and vice versa. Since the functions h_I coincide, the locations of the p_I-brane also coincide. This physically means that all branes have the same total amount of charge at the same position. Here we have discussed the solution without compactification of Z space. If we consider the case that q dimensions of Z space are smeared, we can find the different power of harmonics, i.e., $|z^a - z^a_k|^{-(D-p-3-q)}$ $(q \leq D-p-2)$.

For $K_{\tilde{I}} = 0 (A = B = 0)$, the solution describes the known static and extremal multi-black-hole solution with black hole charges $M_{\tilde{I},k}$ [55,57,58]. We can find the solution (80) for any $N_I \neq 4$. If we choose $N_I = 4$, the solutions have already discussed in Ref. [15].

Let us consider the intersection rule in the D-dimensional gravity theory. If we choose $p_{\tilde{I}} = \tilde{p} = 0$ for all $p_{\tilde{I}} \neq p_{I'}$, the intersection rule (62) leads to

$$\frac{p_{I'}+1}{D-2} - 1 - \frac{1}{2}\epsilon_{\tilde{I}} c_{\tilde{I}} \epsilon_{I'} c_{I'} = 0. \tag{81}$$

Now we discuss the application of the time-dependent solutions to study the cosmology. We assume an isotropic and homogeneous three-space in the Friedmann-Robertson-Walker (FRW) universe after compactification.

We set the $(D-p-1)$-dimensional Euclidean space with $u_{ab}(\mathrm{Z}) = \delta_{ab}(\mathrm{Z})$ and consider the case that there is only one function h_I depending on both z^a and t, which we denote it with the subscript $\tilde{I}$, and other functions are either dependent on z^a or constant. If we assume $N_{\tilde{I}} \neq 4$, the D-dimensional metric can be expressed as

$$\begin{aligned} ds^2 = &-\prod_{I\neq\tilde{I}} h_I^{a_I}\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{a_{\tilde{I}}+2}} H_{\tilde{I}}\right]^{a_{\tilde{I}}} d\tau^2 \\ &+ \sum_\alpha \prod_{I\neq\tilde{I}} h_I^{\delta_I^{(\alpha)}}\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{a_{\tilde{I}}+2}} H_{\tilde{I}}\right]^{\delta_{\tilde{I}}^{(\alpha)}} \left(\frac{\tau}{\tau_0}\right)^{\frac{2\delta_{\tilde{I}}^{(\alpha)}}{a_{\tilde{I}}+2}} (dx^\alpha)^2 \\ &+ \prod_{I\neq\tilde{I}} h_I^{b_I}\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{a_{\tilde{I}}+2}} H_{\tilde{I}}\right]^{b_{\tilde{I}}} \left(\frac{\tau}{\tau_0}\right)^{\frac{2b_{\tilde{I}}}{a_{\tilde{I}}+2}} \delta_{ab}(\mathrm{Z}) dz^a dz^b, \end{aligned} \tag{82}$$

where the function $H_{\tilde{I}}$ is given by

$$H_{\tilde{I}} = \sum_k \frac{M_{\tilde{I},k}}{|z^a - z^a_k|^{D-p-3}}, \tag{83}$$

and the cosmic time τ defined by

$$\frac{\tau}{\tau_0} = (At)^{(a_{\tilde{I}}+2)/2}, \qquad \tau_0 = \frac{2}{(a_{\tilde{I}}+2)A}. \tag{84}$$

If we can regard the three-dimensional part of the overall transverse space Z as our Universe, the power of the scale factor in the fastest expanding case is expressed as

$$\lambda = \frac{b_{\tilde{I}}}{a_{\tilde{I}}+2} = \left[-D + 3 + \frac{N_{\tilde{I}}}{2}(D-2)\right]^{-1}, \quad \text{for } D > 2, \tag{85}$$

where we used the D-dimensional metric (82). Hence, we cannot find the cosmological model which exhibits an accelerating expansion of our Universe. On the other hand, if our three-space is given by a three-dimensional subspace in relative transverse space, the power of the scale factor in the fastest expanding case is also given by (85).

By taking $\tau \to \tau_\infty - \tau$, where τ_∞ is constant, we have accelerated expansion for $\tau_\infty > \tau$ and $\lambda < 0$. This is equivalent to

$$N_{\tilde{I}} > 2, \qquad D > 2 - \frac{2}{N_{\tilde{I}} - 2}, \quad \text{or}$$
$$N_{\tilde{I}} < 2, 3 < D < 2 - \frac{2}{N_{\tilde{I}} - 2}, \tag{86}$$

for $D > 3$. However, the scale factor of our Universe diverges at $\tau = \tau_\infty$.

On the other hand, the power of the scale factor in the fastest expanding case is automatically positive for $D = 3$ and $N_{\tilde{I}} > 0$.

Next we discuss the cosmological solution in the lower-dimensional effective theories. We compactify $d(\equiv \sum_\alpha d_\alpha + d_z)$ dimensions to give our Universe, where d_α and d_z denote the compactified dimensions with respect to the relative and overall transverse space, respectively. The D-dimensional metric (46) is written by

$$ds^2 = ds^2(\mathrm{M}) + ds^2(\mathrm{N}), \tag{87}$$

where $ds^2(\mathrm{M})$ is a $(D-d)$-dimensional metric and $ds^2(\mathrm{N})$ is a metric of compactified dimensions.

In order to discuss the dynamics of the $(D-d)$-dimensional universe in the Einstein frame, we use the conformal transformation

$$ds^2(\mathrm{M}) = h_{\tilde{I}}^{B_{\tilde{I}}} \prod_{I \neq \tilde{I}} h_I^{C_I} ds^2(\bar{\mathrm{M}}), \tag{88}$$

where $B_{\tilde{I}}$ and C_I are expressed, respectively, as

$$B_{\tilde{I}} = -\frac{\sum_\alpha d_\alpha \delta_{\tilde{I}}^{(\alpha)} + d_z b_{\tilde{I}}}{D - d - 2},$$
$$C_I = -\frac{\sum_\alpha d_\alpha \delta_I^{(\alpha)} + d_z b_I}{D - d - 2}. \tag{89}$$

The $(D-d)$-dimensional metric in the Einstein frame is thus given by

$$ds^2(\bar{\mathrm{M}}) = h_{\tilde{I}}^{-B_{\tilde{I}}} \prod_{J \neq \tilde{I}} h_J{}^{-C_J} \left[-h^{a_{\tilde{I}}} \prod_{I \neq \tilde{I}} h_I^{a_I} dt^2 + \sum_{\alpha'} h_{\tilde{I}}^{\delta_{\tilde{I}}^{(\alpha')}} \prod_{I \neq \tilde{I}} h_I^{\delta_I^{(\alpha')}} (dx^{\alpha'})^2 + h_{\tilde{I}}^{b_{\tilde{I}}} \prod_{I \neq \tilde{I}} h_I^{b_I} \delta_{a'b'}(\mathrm{Z}') dz^{a'} dz^{b'} \right], \tag{90}$$

where $x^{\alpha'}$ denotes the coordinate of $(p - d_\alpha)$-dimensional relative transverse space and Z' is $(D - p - 1 - d_z)$-dimensional spaces.

If we set $K_{\tilde{I}} = At$, the $(D-d)$-dimensional metric (46) in the Einstein frame can be expressed as

$$ds^2(\bar{\mathrm{M}}) = \prod_{I \neq \tilde{I}} h_I^{-C_I} \left[-\prod_{I \neq \tilde{I}} h_I^{a_I} \left\{ 1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{B'_{\tilde{I}}+2}} H_{\tilde{I}} \right\}^{B'_{\tilde{I}}} d\tau^2 + \sum_{\alpha'} \prod_{I \neq \tilde{I}} h_I^{\delta_I^{(\alpha')}} \left\{ 1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{B'_{\tilde{I}}+2}} H_{\tilde{I}} \right\}^{-B_{\tilde{I}} + \delta_{\tilde{I}}^{(\alpha')}} \left(\frac{\tau}{\tau_0}\right)^{\frac{2(-B_{\tilde{I}} + \delta_{\tilde{I}}^{(\alpha')})}{B'_{\tilde{I}}+2}} (dx^{\alpha'})^2 \right.$$
$$\left. + \prod_{I \neq \tilde{I}} h_I^{b_I} \left\{ 1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{B'_{\tilde{I}}+2}} H_{\tilde{I}} \right\}^{B'_{\tilde{I}}+1} \left(\frac{\tau}{\tau_0}\right)^{\frac{2(B'_{\tilde{I}}+1)}{B'_{\tilde{I}}+2}} \delta_{a'b'}(\mathrm{Z}') dz^{a'} dz^{b'} \right], \tag{91}$$

where $B'_{\tilde{I}}$ is given by $B'_{\tilde{I}} = -B_{\tilde{I}} + a_{\tilde{I}}$ and we define the cosmic time τ:

$$\frac{\tau}{\tau_0} = (At)^{(B'_{\tilde{I}}+2)/2}, \qquad \tau_0 = \frac{2}{(B'_{\tilde{I}} + 2)A}. \tag{92}$$

Hence, in the Einstein frame, the power of the scale factor in the fastest expanding case is given by

$$0 < \frac{B'_{\tilde{I}} + 1}{B'_{\tilde{I}} + 2} < 1, \quad \text{for } D - d - 2 > 0. \tag{93}$$

If the physical parameters satisfy (93), the solutions do not give an accelerating expansion in our Universe. These are the similar results with the case of the other partially localized and delocalized intersecting brane backgrounds. Although we find the exact time-dependent brane solution, the power exponent of the scale factor is too small. Furthermore, in order to discuss a de Sitter solution in an intersecting brane background, one has to consider the trivial dilaton, which will be discussed in the next subsection.

B. de Sitter universe

In this subsection, we consider the Einstein equations (45a) with $c_{\tilde{I}} = 0$. Equation (44a) gives

$$N_{\tilde{I}} = \frac{2(D - p_{\tilde{I}} - 3)(p_{\tilde{I}} + 1)}{(D-2)}. \tag{94}$$

If we assume

$$p = p_{\tilde{I}} = 0, \qquad N_{\tilde{I}} = \frac{2(D-3)}{(D-2)},$$
$$\alpha_{I'} = -\frac{N_{I'} a_{I'}}{2\epsilon_{I'} c_{I'}}, \qquad \Lambda_{I'} = 0, \quad \text{for } I' \neq \tilde{I}, \tag{95}$$

the field equations reduce to

$$R_{ij}(\mathrm{Z}) = 0, \tag{96a}$$

$$h_{\tilde{I}}(t,z) = K_{\tilde{I}}(t) + H_{\tilde{I}}(z),$$
$$\left(\frac{dK_{\tilde{I}}}{dt}\right)^2 - \frac{2(D-3)^2}{(D-2)(D-1)}\Lambda_{\tilde{I}} = 0, \tag{96b}$$

$$\triangle_{\mathrm{Z}} H_{\tilde{I}} = 0, \qquad \triangle_{\mathrm{Z}} h_{I'} = 0. \tag{96c}$$

Then Eq. (96b) gives

$$K_{\tilde{I}}(t) = c_0 t + \tilde{c}, \tag{97}$$

where $\tilde{c}$ is an integration constant and c_0 is given by

$$c_0 = \pm(D-3)\sqrt{\frac{2}{(D-2)(D-1)}\Lambda_{\tilde{I}}}. \tag{98}$$

Thus, there is no solution for $\Lambda_{\tilde{I}} < 0$. If the metric $u_{ab}(\mathrm{Z})$ is assumed to be Eq. (79), the function $H_{\tilde{I}}$ is given by Eq. (83). Now we introduce a new time coordinate τ by

$$c_0 \tau = \ln t. \tag{99}$$

The D-dimensional metric (46) is then rewritten as

$$ds^2 = -\prod_{I' \neq \tilde{I}} h_{I'}^{a_{I'}}(z)(1 + c_0^{-1}\mathrm{e}^{-c_0\tau}H_{\tilde{I}})^{-2}d\tau^2 + (1 + c_0^{-1}\mathrm{e}^{-c_0\tau}H_{\tilde{I}})^{\frac{2}{D-3}}(c_0\mathrm{e}^{c_0\tau})^{\frac{2}{D-3}}$$
$$\times\left[\sum_{\alpha=1}^{p}\prod_{I'\neq\tilde{I}}\{h_{I'}(z)\}^{\delta_{I'}^{(\alpha)}}(dx^\alpha)^2 + \prod_{I'\neq\tilde{I}}\{h_{I'}(z)\}^{b_I}u_{ab}(\mathrm{Z})dz^a dz^b\right]. \tag{100}$$

The D-dimensional metric (100) implies that the spacetime describes an isotropic and homogeneous universe if $H_{\tilde{I}} = 0$. In the region where the terms with $H_{\tilde{I}}$ are negligible and $h_{I'}$ approaches a constant, which is realized in the limit $\tau \to \infty$ and for $c_0 > 0$, the D-dimensional spacetime becomes de Sitter universe. If we set $h_{I'}(z) =$ const and $u_{ab} = \delta_{ab}$, Eq. (100) becomes the solution which has been discussed by Ref. [63] (see also [65]). Furthermore, for $D = 4$ and by setting all $h_{I'} = 1$, the solution is the Kastor-Traschen one [36].

C. The behavior of the solutions

Now we will study the spacetime structure. The metric has singularities at $h_{\tilde{I}} = 0$ or $h_{I'} = 0$. The spacetime is thus not singular when it is restricted inside the domain specified by the conditions

$$h_{\tilde{I}}(t,z) = a_0 + a_1 t + K_{\tilde{I}}(z) > 0, \qquad h_{I'}(z) > 0, \tag{101}$$

where the function $K_{\tilde{I}}$ is defined in (83). The D-dimensional spacetime cannot be extended beyond this region, because a curvature singularity appears in the D-dimensional spacetime. The regular spacetime with branes ends up with the singularities.

Since the system with $a_1 > 0$ has the time reversal one of $a_1 < 0$, the dynamics of the spacetime depends on the signature of a_1.

Here we will consider the case with $a_1 > 0$. Then the function $h_{\tilde{I}}$ is positive everywhere for $t > 0$ and the spacetime is nonsingular. In the limit of $t \to \infty$ and apart from a position of the branes, near which the geometry takes a cylindrical form of an infinite throat, the solution is approximately described by a time-dependent uniform spacetime.

Now we discuss the time evolution for $t \leq 0$. The spacetime is regular everywhere and has a cylindrical topology near each brane at $t = 0$. As time slightly decreases, a curvature singularity appears as $|z^a - z_\alpha^a| \to \infty$. The singular hypersurface cuts off more and more of the space as time decreases further. When t continues to decrease, the singular hypersurface eventually splits and surrounds each of the p-brane throats individually. The spatial surface is finally composed of two isolated throats. For $t > 0$, the time evolution of the D-dimensional spacetime is the time reversal of $t < 0$.

For any values of fixed z^a in the regular domain in the D-dimensional spacetime (46), the overall transverse space tends to expand asymptotically like $t^{b_{\tilde{I}}}$. Thus, the solutions describe static intersecting brane systems composed of p-branes near the positions of the branes, while, in the far region as $|z^a - z_\alpha^a| \to \infty$, the solutions approach de Sitter or FRW universes with the power-law expansion $t^{b_{\tilde{I}}}$. The emergence of time-dependent universes is an important feature of the dynamical brane solutions.

1. Asymptotic structure

We study the asymptotic behavior of the solutions. The solution describes a charged black hole in the FRW or de Sitter universe in the limit of $|z^a| \to \infty$, and $H_{\tilde{I}}$ vanishes.

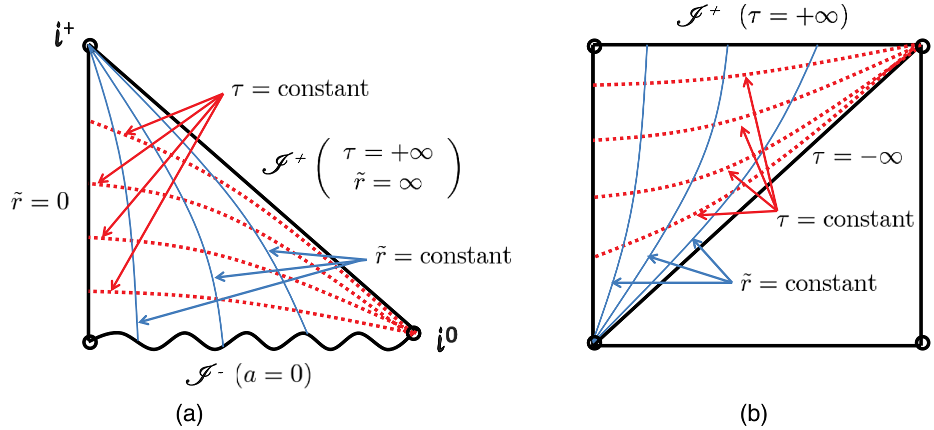

FIG. 1 (color online). Conformal diagrams of the D-dimensional spacetime for $p_{\tilde{I}} = 0$. The regions corresponding to $\tilde{r} \to \infty$ give the original spacetime, where $\tilde{r}^2 = \sum_\alpha (x^\alpha)^2 + \delta_{ab} z^a z^b$. (a) For the case of $a_{\tilde{I}} + 2 \neq 0$, the metric (46) approaches in the limit $r \to \infty$ to the D-dimensional flat FRW spacetime. (b) We also depict the conformal diagrams in the case of $a_{\tilde{I}} + 2 = 0$. One can recognize that the asymptotic region of the spacetime is the de Sitter universe.

First we consider the case of a power-law expanding universe. The function $h_{\tilde{I}}$ depends only on time t in the far region from branes, and the resulting metric (82) can be expressed as

$$ds^2 = -d\tau^2 + \sum_{\alpha=1}^{p} \left(\frac{\tau}{\tau_0}\right)^{\frac{2\delta_{\tilde{I}}^{(\alpha)}}{a_{\tilde{I}}+2}} (dx^\alpha)^2 + \left(\frac{\tau}{\tau_0}\right)^{\frac{2b_{\tilde{I}}}{a_{\tilde{I}}+2}} \delta_{ab}(\mathrm{Z}) dz^a dz^b. \tag{102}$$

The scale factor of the relative transverse space is given by $a_{\mathrm{r}}(\tau) = (\tau/\tau_0)^{b_{\tilde{I}}/a_{\tilde{I}}+2}$, while the expansion low for the overall transverse space is written by $a_{\mathrm{t}}(\tau) = (\tau/\tau_0)^{b_{\tilde{I}}/a_{\tilde{I}}+2}$. On the other hand, for $c_{\tilde{I}} = 0$ corresponding to de Sitter universe (100), the metric of D-dimensional spacetime in the far region from branes becomes

$$ds^2 = -d\tau^2 + (c_0 \mathrm{e}^{c_0 \tau})^{\frac{2}{D-3}} \left[\sum_{\alpha=1}^{p} (dx^\alpha)^2 + u_{ab}(\mathrm{Z}) dz^a dz^b \right]. \tag{103}$$

Figure 1 depicts the conformal diagrams of the FRW and de Sitter universes.

2. Near-horizon geometry

Next we discuss the near-horizon geometry of the solutions. We set the metric of $(D-p-1)$-dimensional overall transverse space:

$$\delta_{ab}(\mathrm{Z}) dz^a dz^b = dr^2 + r^2 d\Omega^2_{(D-p-2)}, \tag{104}$$

where δ_{ab} denotes the metric of $(D-p-1)$-dimensional flat space and the line elements of a $(D-p-2)$-sphere (S^{D-p-2}) are given by $d\Omega^2_{(D-p-2)}$. The harmonic function $K_{\tilde{I}}$ dominates in the limit of $r \to 0$, and the time dependence can be ignored. Thus the system becomes static near a position of branes. When all of the branes are located at the origin of the Z spaces, the solutions are rewritten as

$$h_{\tilde{I}}(t, r) = a_0 + a_1 t + \frac{M_{\tilde{I}}}{r^{D-p-3}}, \tag{105a}$$

$$h_{I'}(r) = 1 + \frac{L_{I'}}{r^{D-p-3}}. \tag{105b}$$

Here $M_{\tilde{I}}$ and $L_{I'}$ are the mass of $p_{\tilde{I}}$- and $p_{I'}$-branes, respectively. In the near-horizon region $r \to 0$, the dependence on t in (105) is negligible. Then the metric is reduced to the following form:

$$ds^2 = r^2 \left(\frac{M_{\tilde{I}}}{r^{D-p-3}}\right)^{b_{\tilde{I}}} \prod_{I'} \left(\frac{L_{I'}}{r^{D-p-3}}\right)^{b_{I'}} \left[-r^{-2} \left(\frac{M_{\tilde{I}}}{r^{D-p-3}}\right)^{-\frac{4}{N_{\tilde{I}}}} \prod_{I'} \left(\frac{L_{I'}}{r^{D-p-3}}\right)^{-\frac{4}{N_{I'}}} dt^2 \right.$$
$$\left. + r^{-2} \sum_{\alpha=1}^{p} \prod_{I'} \left(\frac{M_{\tilde{I}}}{r^{D-p-3}}\right)^{-b_{\tilde{I}} + \delta_{\tilde{I}}^{(\alpha)}} \left(\frac{L_{I'}}{r^{D-p-3}}\right)^{-b_{I'} + \delta_{I'}^{(\alpha)}} (dx^\alpha)^2 + \left(\frac{dr^2}{r^2} + d\Omega^2_{(D-p-2)}\right) \right]. \tag{106}$$

Thus the metric (106) describes a warped product of $(p+2)$-dimensional spacetime M_{p+2} and $(D-p-2)$-dimensional sphere S^{D-p-2}.

Hence, the near-brane geometry has the same metric form as the static one. If it has a horizon geometry, we can obtain a black hole solution in the time-dependent background. In fact, some solutions, for instance, the M2 – M2 – M2, M2 – M2 – M5 – M5 intersecting solution in 11 dimensions, give regular black hole spacetimes in the static limit [15].

Our solution approaches asymptotically the dynamical universe with the scale factor $a(\tau)$, while the static solution gives a black hole. Then we can regard the present solution as a black hole in the expanding universe.

IV. COLLISION OF 0-BRANES

In this section, we apply our dynamical intersecting brane solutions found in the previous section to brane collisions.

The functions $h_{\tilde{I}}$ and $h_{I'}$ are assumed to be

$$h_{\tilde{I}}(t,z) = c_0 t + \tilde{c}_{\tilde{I}} + H_{\tilde{I}}(z), \qquad h_{I'} = h_{I'}(z). \tag{107}$$

Here c_0 and $\tilde{c}_{\tilde{I}}$ are constants, and the function $H_{\tilde{I}}$ and $h_{I'}$ are expressed, respectively, as

$$H_{\tilde{I}}(z) = \sum_{k=1}^{m} \frac{M_{\tilde{I},k}}{|z^a - z^a_k|^{D-p-3-d}},$$
$$h_{I'}(z) = \tilde{c}_{I'} + \sum_{l=1}^{m} \frac{Q_{I',l}}{|z^a - z^a_l|^{D-p-3-d_{I'}}}, \tag{108}$$

where $\tilde{c}_{I'}$ is constant, d and $d_{I'}$ denote the number of smeared dimension for 0-brane and $p_{I'}$-brane, respectively, we assume $D \neq p+3+d$ and $D \neq p+3+d_{I'}$, and $M_{\tilde{I},k}(k = 1,\ldots,m)$ and $Q_{I',l}(l=1,\ldots,m)$ are mass constants of 0-brane and $p_{I'}$-branes located at z^a_k and z^a_l, respectively. Since $h_{I'}$ is the harmonic function on the $(D-p-1-d_{I'})$-dimensional Euclidean subspace in Z, we define

$$|z^a - z^a_k| = \sqrt{(z^1 - z^1_k)^2 + (z^2 - z^2_k)^2 + \cdots + (z^{D-p-1-d} - z^{D-p-1-d}_k)^2}, \tag{109a}$$

$$|z^a - z^a_l| = \sqrt{(z^1 - z^1_l)^2 + (z^2 - z^2_l)^2 + \cdots + (z^{D-p-1-d_{I'}} - z^{D-p-1-d_{I'}}_l)^2}. \tag{109b}$$

The metric, scalar, and gauge fields are given by Eqs. (46) and (49), respectively. For $D = p+3+d$ and $D = p+3+d_{I'}$, these become

$$H_{\tilde{I}}(z) = \sum_{k=1}^{m} M_{\tilde{I},k} \ln|z^a - z^a_k|,$$
$$h_{I'}(z) = \tilde{c}_{I'} + \sum_{l=1}^{m} Q_{I',l} \ln|z^a - z^a_l|. \tag{110}$$

Since the time dependence allows only for the 0-brane, we see that the $(D-3-d_{I'})$-brane background is critical case. If we consider the $(D-2-d_{I'})$-brane, the functions $h_{\tilde{I}}$ and $h_{I'}$ are written by the sum of linear functions of z. The possibility of brane collisions comes from the difference in the overall transverse dimension.

From the solution (108), there are curvature singularities at $h_{\tilde{I}} = 0$ or at $h_{I'} = 0$ in the D-dimensional background. Note that the regular D-dimensional spacetime is restricted to the region of $h_{\tilde{I}} > 0$ and $h_{I'} > 0$, which is bounded by curvature singularities. Hence, the D-dimensional metric (46) is regular if and only if $h_{\tilde{I}} > 0$ and $h_{I'} > 0$.

The solution with $0 - p_{I'}$ branes takes the form (82), where we set $K_{\tilde{I}} = c_0 t$ and the function $H_{\tilde{I}}$ is given by (83). We classify the behavior of the harmonic function $h_{I'}$ into two classes: $p_{I'} \leq (D-4-d_{I'})$ and $p_{I'} = (D-2-d_{I'})$. Since these depend on the dimensions of the $p_{I'}$-brane, we discuss them below separately. In the case of the $(D-3-d_{I'})$-brane, the harmonic function $h_{I'}$ diverges both at infinity and near $(D-3-d_{I'})$-branes. Since there is no regular spacetime region near branes due to $h_{I'} \to -\infty$, these solutions are not physically relevant. In the following, we discuss the collision involving the $0 - p_{I'}$ brane in D-dimensional spacetime.

A. Collision of the $0 - p_I$-brane in the asymptotically power-law expanding universe

The harmonic function $H_{\tilde{I}}$ becomes dominant in the limit of $z^a \to z^a_k$, while the function $h_{\tilde{I}}$ depends only on time τ in the limit of $|z^a| \to \infty$. Hence, we find a static structure of the $0 - p_{I'}$-brane system near branes. In the far region from branes, the function $H_{\tilde{I}}$ vanishes. Therefore, the metric can be written by

$$ds^2 = -\prod_{I' \neq \tilde{I}} h_{I'}^{a_{I'}} \bar{h}_{\tilde{I}}^{a_{\tilde{I}}} d\tau^2 + \sum_{\alpha=1}^{p} \prod_{I' \neq \tilde{I}} h_{I'}^{\delta^{(\alpha)}_{I'}} \left(\frac{\tau}{\tau_0}\right)^{\frac{2\delta^{(\alpha)}_{\tilde{I}}}{a_{\tilde{I}}+2}} \bar{h}_{\tilde{I}}^{\delta^{(\alpha)}_{\tilde{I}}} (dx^\alpha)^2$$
$$+ \prod_{I' \neq \tilde{I}} h_{I'}^{b_{I'}} \left(\frac{\tau}{\tau_0}\right)^{\frac{2b_{\tilde{I}}}{a_{\tilde{I}}+2}} \bar{h}_{\tilde{I}}^{b_{\tilde{I}}} \delta_{ab}(\mathrm{Z}) dz^a dz^b, \tag{111}$$

where $\bar{h}_{\tilde{I}}$ is defined by

$$\bar{h}_{\tilde{I}} = 1 + \left(\frac{\tau}{\tau_0}\right)^{-\frac{2}{a_{\tilde{I}}+2}} H_{\tilde{I}}. \tag{112}$$

In order to analyze the brane collision, we consider a concrete example, in which two $0 - p_{I'}$ branes are located at $z^a = (\pm L, 0, \ldots, 0)$. We will discuss the time evolution separately with respect to the signature of a constant τ_0, because the behavior of spacetime strongly depends on it.

Since the metric function is singular at $h_{\tilde{I}}(\tau, z) = 0$ and $h_{I'} = 0$, one can note that the regular spacetime exists inside the domain restricted by

$$h_{\tilde{I}}(\tau,z)=\left(\frac{\tau}{\tau_0}\right)^{\frac{2}{a_{\tilde{I}}+2}}+H_{\tilde{I}}(z)>0,\quad h_{I'}=h_{I'}(z)>0, \quad (113)$$

where the functions $H_{\tilde{I}}$ and $h_{I'}$ are defined in (107). The brane background evolves into a curvature singularity, because the dilaton ϕ diverges. Since the D-dimensional spacetime cannot be extended beyond this region, the regular spacetime with two 0-branes ($p + d \le 6$) ends on these singular hypersurfaces. The solution with $(\tau_0)^{-2/(a_{\tilde{I}}+2)} > 0$ is the time reversal one of $(\tau_0)^{-2/(a_{\tilde{I}}+2)} < 0$, because the time dependence appears only in the form of $(\tau/\tau_0)^{2/(a_{\tilde{I}}+2)}$. In the following, we consider the case with $(\tau_0)^{-2/(a_{\tilde{I}}+2)} < 0$.

For $(\tau)^{2/(a_{\tilde{I}}+2)} < 0$, the D-dimensional spacetime is nonsingular, because the function $h_{\tilde{I}}$ is positive everywhere. In the limit of $(\tau)^{2/(a_{\tilde{I}}+2)} \to -\infty$, the D-dimensional spacetime becomes asymptotically a time-dependent uniform background, while the cylindrical forms of infinite throats exist near branes.

For $\tau > 0$, the spatial metric is initially regular everywhere. The D-dimensional spacetime has a cylindrical topology near each brane. As τ increases slightly, a singular hypersurface appears from the spatial infinity ($|z^a - z_k^a| \to \infty$). As τ increases further, the singularity cuts the space off more and more. Since the singular hypersurface eventually splits and surrounds each of the brane throats, the spatial surface is finally composed of two isolated throats.

One notes that the transverse dimensions in the metric (111) expand asymptotically as $\tau^{b_{\tilde{I}}/(a_{\tilde{I}}+2)}$ for fixed spatial coordinates z^a. The D-dimensional spacetime becomes static near branes, while the background approaches a FRW universe in the far region ($|z^a - z_k^a| \to \infty$). Hence, the time evolution of the four-dimensional universe depends on the position of the observer. For $(\tau/\tau_0)^{2/(a_{\tilde{I}}+2)} < 0$, the behavior of D-dimensional spacetime is the time reversal of the period of $(\tau/\tau_0)^{2/(a_{\tilde{I}}+2)} > 0$.

Now we define

$$z_\perp = \sqrt{(z^2)^2 + (z^3)^2 + \cdots + (z^{D-1-p-d})^2}. \quad (114)$$

By using the above equation, the proper distance at $z_\perp = 0$ between two branes can be written by

$$d(\tau)=\int_{-L}^{L}dz^1\left[\left(\frac{\tau}{\tau_0}\right)^{2/(a_{\tilde{I}}+2)}+\frac{M_1}{|z^1+L|^{D-3-p_{I'}-d_{I'}}}+\frac{M_2}{|z^1-L|^{D-3-p_{I'}-d_{I'}}}\right]^{b_{\tilde{I}}/2}\left(1+\frac{Q_1}{|z^1+L|^{D-3-p_{I'}-d}}+\frac{Q_2}{|z^1-L|^{D-3-p_{I'}-d}}\right)^{b_{I'}/2}. \quad (115)$$

The proper distance is a monotonically increasing function of τ. We illustrate $d(\tau)$ for the case of the $0 - p_{I'}$ brane system in Fig. 2. We consider the case of $d = d_{I'} = 0$, $\tau_0 = -1$, $Q_1 = Q_2 = M_1 = M_2 = 1$, $L = 1$, and $D = 10$ or $D = 8$. It shows that two 0-branes are initially ($\tau < 0$) approaching, the distance $d(\tau)$ takes the minimum finite value at $\tau = 0$, and then two 0-branes segregate each other. Thus they will never collide. Hence, we cannot discuss a brane collision in this case.

B. Collision of the $0 - p_{I'}$-brane in the asymptotically de Sitter universe

Let us next discuss the collision in the $0 - p_{I'}$-brane with a trivial dilaton system. We consider the case that the harmonic function $H_{\tilde{I}}$ and $h_{I'}$ are linear in z and discuss in detail the $0 - (D - 2)$-brane in D dimensions as a example. In this case, we have one extra dimension z in Z space. The D-dimensional metric (100) can be rewritten by

$$ds^2 = -h_{I'}^{a_{I'}}(z)(1 + c_0^{-1}e^{-c_0\tau}H_{\tilde{I}})^{-2}d\tau^2 + h_{I'}^{a_{I'}}(z)(1 + c_0^{-1}e^{-c_0\tau}H_{\tilde{I}})^{\frac{2}{D-3}}(c_0e^{c_0\tau})^{\frac{2}{D-3}}\left[\sum_{\alpha=1}^{p}(dx^\alpha)^2 + h_{I'}^{4/N_{I'}}(z)dz^2\right], \quad (116)$$

where the function $H_{\tilde{I}}(z)$ is written by

$$H_{\tilde{I}}(z) = \sum_{k=1}^{m} M_{\tilde{I},k}|z - z_k|. \quad (117)$$

We consider the collision in the $0 - p_{I'}$-brane system with charges M_1 and Q_1 at $z^1 = -L$ and the other with charges M_2 and Q_2 at $z^1 = L$. The proper distance at $z_\perp = 0$ between the two 0-branes can be expressed as

$$d(\tau) = \int_{-L}^{L} dz(c_0e^{c_0\tau} + M_1|z^1 + L| + M_2|z^1 - L|)^{1/(D-3)} \times (1 + Q_1|z^1 + L| + Q_2|z^1 - L|)^{b_{I'}/2}. \quad (118)$$

In the period of $c_0 < 0$, the proper distance increases as τ increases. If $M_1 \neq M_2$, a singular hypersurface appears at $\tau = \tau_s \equiv \ln[-(M_1|z^1 + L| + M_2|z^1 - L|)c_0^{-1}]c_0^{-1} < 0$ when the distance is still finite.

However, in the case of the equal charges $Q_1 = Q_2 = M_1 = M_2 = M$, the situation is completely different,

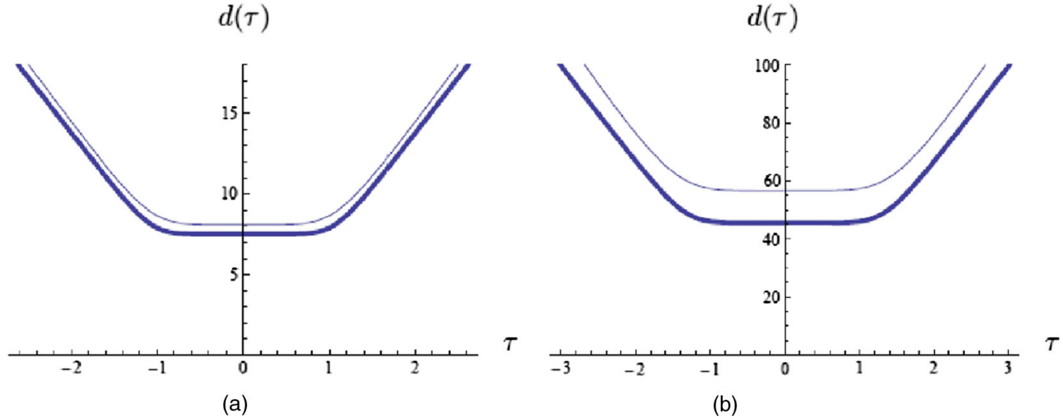

FIG. 2 (color online). (a) For the case of $M_1 = M_2$ in the asymptotically power-law expanding universe, the proper distance between two dynamical 0-branes given in (115) is depicted. We fix $d = d_{I'} = 0$, $D = 10$, $\tau_0 = -1$, $M_1 = 1$, $M_2 = 1$, $N = 2$, and $L = 1$ for the $0-8$- (bold curve) and $0-6$- (solid curve) branes. The distance decreases initially ($\tau < 0$) but turns to increase at $\tau = 0$, and then two 0-branes segregate each other. (b) We also show the proper distance between two dynamical 0-branes for $0-8$- (bold curve) and $0-6$- (solid curve) brane systems from the bottom in the case of $d = d_{I'} = 0$, $M_1 = 10$, $M_2 = 1$, $N = 2$, $L = 1$, and $D = 10$ in the asymptotically power-law expanding universe. Although the proper distance initially decreases as $\tau(< 0)$ increases, the distance increases as $\tau(> 0)$ increases.

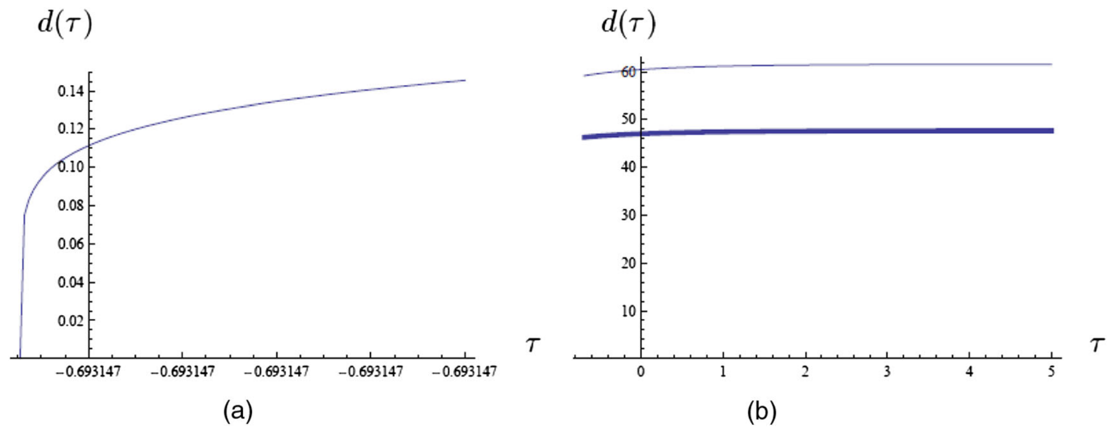

FIG. 3 (color online). (a) For the case of $M_1 = M_2$ in the asymptotically de Sitter universe, we show the proper distance between two dynamical 0-branes given in (118). We set $D = 10$, $c_0 = -1$, $M_1 = 1$, $M_2 = 1$, $N = 2$, and $L = 1$ for the $0-8$-brane. The proper distance rapidly vanishes near where two branes collide. (b) We also show the proper distance between two dynamical 0-branes for the $0-8$- (bold curve) and $0-6$- (solid curve) brane systems from the bottom in the case of $M_1 = 10$, $M_2 = 1$, $N = 2$, and $D = 10$ in the asymptotically de Sitter universe. The proper distance initially decreases as τ decreases and remains still finite when a singularity appears.

because the proper distance finally vanishes at $\tau_s = \ln(-2MLc_0^{-1})c_0^{-1} < 0$ as

$$d(\tau) = 2L(c_0 e^{c_0\tau} + 2LM)^{1/(D-3)}(1 + 2LM)^{b_{I'}/2}. \quad (119)$$

Then two branes can collide. A singularity is formed at the same point and time.

Let us consider the case $p_{I'} \neq D-2$. The D-dimensional metric (100) can be written as

$$ds^2 = -h_{I'}^{a_{I'}}(z)(1 + c_0^{-1}e^{-c_0\tau}H_{\tilde{I}})^{-2}d\tau^2 + h_{I'}^{a_{I'}}(z)(1 + c_0^{-1}e^{-c_0\tau}H_{\tilde{I}})^{\frac{2}{D-3}}(c_0 e^{c_0\tau})^{\frac{2}{D-3}} \times \left[\sum_{\alpha=1}^{p}(dx^\alpha)^2 + h_{I'}^{4/N_{I'}}(z)\delta_{ab}(\mathrm{Z})dz^a dz^b\right]. \quad (120)$$

Since the proper distance at $z_\perp = 0$ between two branes is given by

$$d(\tau) = \int_{-L}^{L} dz \left(c_0 e^{c_0 \tau} + \frac{M_1}{|z^1 + L|^{D-p_{I'}-3-d}} + \frac{M_2}{|z^1 - L|^{D-p_{I'}-3-d}}\right)^{1/(D-3)} \left(1 + \frac{Q_1}{|z^1 + L|^{D-3-p_{I'}-d}} + \frac{Q_2}{|z^1 - L|^{D-3-p_{I'}-d}}\right)^{b_{I'}/2}, \tag{121}$$

the distance increases monotonically with respect to τ.

In the case of $c_0 < 0$, initially ($\tau = 0$), D-dimensional space is regular except at $|z^a - z^a_k| \to 0$, while this is an asymptotically time-dependent spacetime and has the cylindrical form of an infinite throat near the 0-brane. At $\tau = \tau_s < 0$, a singularity appears from the spatial infinity ($|z^a - z^a_k| \to \infty$). As time decreases ($\tau < 0$), the singular hypersurface erodes the region with the large values of $|z^a - z^a_k|$. Since only the region near 0-branes remains regular, eventually it splits and each fragment surrounds each 0-brane individually. Figure 3 shows that this singularity appears before the proper distance $d(\tau)$ vanishes. Hence, the D-dimensional spacetime has the singularity before two branes collide. Although two 0-branes approach very slowly, a singularity suddenly appears at a finite distance. Then, the spacetime splits into two isolated 0-brane throats.

We show $d(\tau)$ integrated numerically in Fig. 3 for the case of $c_0 < 0$. In the future direction, the proper distance d increases. Then for $\tau > 0$, each brane gradually separates as τ increases.

V. APPLICATIONS TO SUPERGRAVITIES

In the case of ten or 11 dimensions with $N = 4$ and $\Lambda_I = 0$, Eq. (1) gives the action of supergravities. For instance, the bosonic part of the action of $D = 11$ supergravity includes only 4-form field strength, while, for $D = 10$, the constant c is precisely the dilaton coupling for the Ramond-Ramond $(p + 2)$-form in the type II supergravities. The dynamical solutions for the case of $N = 4$ have been already discussed in Ref. [21]. In this section, we will discuss the time-dependent solution in six-dimensional Nishino-Salam-Sezgin (NSS) gauged supergravity and Romans' gauged supergravity models. The bosonic part of the six-dimensional NSS model [66–69] is given by the expression (1) with $\Lambda_r > 0$, $\Lambda_s = 0$, while Romans' six-dimensional $\mathcal{N} = 4^g$ gauged supergravity [70] is expressed by the action (1) with $\Lambda_r < 0$, $\Lambda_s = 0$.

A. Nishino-Salam-Sezgin gauged supergravity

Now we consider the NSS model among the theories of $D = 6$. The couplings of the 2-form ($p_r = 0$) and the 3-form ($p_s = 1$) field strengths to the dilaton are given by $\epsilon_r c_r = -\frac{1}{\sqrt{2}}$ and $\epsilon_s c_s = -\sqrt{2}$, respectively:

$$S = \frac{1}{2\kappa^2} \int \left[(R - 2e^{\phi/\sqrt{2}}\Lambda) * \mathbf{1} - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2\cdot 2!} e^{-\phi/\sqrt{2}} * F_{(2)} \wedge F_{(2)} - \frac{1}{2\cdot 3!} e^{-\sqrt{2}\phi} * F_{(3)} \wedge F_{(3)} \right], \tag{122}$$

where R denotes the Ricci scalar constructed from the six-dimensional metric g_{MN}, κ^2 is the six-dimensional gravitational constant, $*$ is the Hodge operator in the six-dimensional spacetime, ϕ denotes the scalar field, $\Lambda > 0$ is the cosmological constant, and $F_{(2)}$ and $F_{(3)}$ are 2-form and 3-form field strengths, respectively. From Eq. (2a), the NSS model is realized by choosing $\Lambda_r = \Lambda > 0$, $\Lambda_s = 0$, $N_r = 2$, and $N_s = 4$.

The six-dimensional action (122) gives the field equations

$$R_{MN} = \frac{1}{2} e^{\phi/\sqrt{2}} \Lambda g_{MN} + \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{e^{-\phi/\sqrt{2}}}{2\cdot 2!} \left(2F_{MA} F_N{}^A - \frac{1}{4} g_{MN} F^2_{(2)} \right) + \frac{e^{-\sqrt{2}\phi}}{2\cdot 3!} \left(3F_{MAB} F_N{}^{AB} - \frac{1}{2} g_{MN} F^2_{(3)} \right), \tag{123a}$$

$$\triangle\phi + \frac{\sqrt{2}}{4\cdot 2!} e^{-\phi/\sqrt{2}} F^2_{(2)} + \frac{\sqrt{2}}{2\cdot 3!} e^{-\sqrt{2}\phi} F^2_{(3)} - \sqrt{2} e^{\phi/\sqrt{2}} \Lambda = 0, \tag{123b}$$

$$d[e^{-\phi/\sqrt{2}} * F_{(2)}] = 0, \tag{123c}$$

$$d[e^{-\sqrt{2}\phi} * F_{(3)}] = 0, \tag{123d}$$

where $\triangle$ denotes the Laplace operator with respect to the six-dimensional metric g_{MN}.

We construct solutions whose spacetime metric has the form

$$ds^2 = h_2^{1/2}(t,y,z) h_3^{1/2}(t,y,z) [-h_2^{-2}(t,y,z) h_3^{-1}(t,y,z) dt^2 + h_3^{-1}(t,y,z) dy^2 + u_{ab}(\mathrm{Z}) dz^a dz^b], \tag{124}$$

where $u_{ab}(\mathrm{Z})$ is the four-dimensional metric which depends only on the four-dimensional coordinates z^a. The scalar field ϕ and field strengths $F_{(2)}$ and $F_{(3)}$ are written, respectively, by

$$e^{\phi} = (h_2 h_3)^{-\sqrt{2}/2}, \tag{125a}$$

$$F_{(2)} = d[\sqrt{2} h_2^{-1}(t,y,z)] \wedge dt, \tag{125b}$$

$$F_{(3)} = d[h_3^{-1}(t, y, z)]\wedge dt\wedge dy. \tag{125c}$$

First we consider the Einstein equation (123a). By using the ansatz (124) and (125), the Einstein equations become

$$\frac{5}{4}h_2^{-1}\partial_t^2 h_2+\frac{3}{4}h_3^{-1}\partial_t^2 h_3+\frac{1}{4}h_2^{-2}(3h_2^{-1}\partial_y^2 h_2+h_3^{-1}\partial_y^2 h_3) +\frac{1}{4}h_2^{-2}h_3^{-1}(3h_2^{-1}\triangle_Z h_2+h_3^{-1}\triangle_Z h_3)-\frac{1}{2}h_2^{-2}h_3^{-1}\Lambda +\frac{1}{4}(\partial_t\ln h_2)^2+\frac{7}{4}\partial_t\ln h_2\partial_t\ln h_3-\frac{3}{4}h_2^{-2}(1-h_3)(\partial_y\ln h_2)^2 +\frac{3}{4}h_2^{-2}\partial_y\ln h_2\partial_y\ln h_3-\frac{3}{4}h_2^{-2}h_3^{-1}(1-h_3)u^{ab}\partial_a\ln h_2\partial_b\ln h_2 -\frac{1}{4}h_2^{-2}h_3^{-1}(1-h_2^2)u^{ab}\partial_a\ln h_3\partial_b\ln h_3=0, \tag{126a}$$

$$2h_2^{-1}\partial_t\partial_y h_2 + 2h_3^{-1}\partial_t\partial_y h_3 + \partial_t \ln h_2\partial_y \ln h_3 + 3\partial_t \ln h_3\partial_y \ln h_2 = 0, \tag{126b}$$

$$2h_2^{-1}\partial_t\partial_a h_2 + h_3^{-1}\partial_t\partial_a h_3 + \partial_t \ln h_2\partial_a \ln h_3 + \partial_t \ln h_3\partial_a \ln h_2 = 0, \tag{126c}$$

$$\frac{1}{4}h_2^2(h_2^{-1}\partial_t^2 h_2 - h_3^{-1}\partial_t^2 h_3) - \frac{1}{4}(h_2^{-1}\partial_y^2 h_2 + 3h_3^{-1}\partial_y^2 h_3) -\frac{1}{4}h_3^{-1}(h_2^{-1}\triangle_Z h_2 - h_3^{-1}\triangle_Z h_3) - \frac{1}{2}h_3^{-1}\Lambda + \frac{1}{4}(\partial_t h_2)^2 -\frac{1}{4}h_2^2\partial_t \ln h_2\partial_t \ln h_3 - \frac{3}{4}(1-h_3)(\partial_y \ln h_2)^2 -\frac{5}{4}\partial_y \ln h_2\partial_y \ln h_3 + \frac{1}{4}h_3^{-1}(1-h_3)u^{ab}\partial_a \ln h_2\partial_b \ln h_2 -\frac{1}{4}h_3^{-1}(1-h_2^2)u^{ab}\partial_a \ln h_3\partial_b \ln h_3 = 0, \tag{126d}$$

$$h_3^{-1}\partial_y\partial_a h_3 + 2\partial_y \ln h_2\partial_a \ln h_2 + \partial_y \ln h_2\partial_a \ln h_3 + \partial_y \ln h_3\partial_a \ln h_2 = 0, \tag{126e}$$

$$R_{ab}(Z) + \frac{1}{4}h_2^2 h_3 u_{ab}(h_2^{-1}\partial_t^2 h_2 + h_3^{-1}\partial_t^2 h_3) - \frac{1}{4}h_3 u_{ab}(h_2^{-1}\partial_y^2 h_2 + h_3^{-1}\partial_y^2 h_3) - \frac{1}{4}u_{ab}(h_2^{-1}\triangle_Z h_2 + h_3^{-1}\triangle_Z h_3) + \frac{1}{4}h_2^2 h_3 u_{ab}[(\partial_t \ln h_2)^2 + 3\partial_t \ln h_2\partial_t \ln h_3] + \frac{1}{4}h_3(1-h_3)u_{ab}(\partial_y \ln h_2)^2 - \frac{1}{4}h_3 u_{ab}\partial_y \ln h_2\partial_y \ln h_3 + \frac{1}{4}(1-h_3)u_{ab}u^{cd}\partial_c \ln h_2\partial_d \ln h_2 + \frac{1}{4}(1-h_2^2)u_{ab}u^{cd}\partial_c \ln h_3\partial_d \ln h_3 - (1-h_3)\partial_a \ln h_2\partial_b \ln h_2 -\frac{1}{2}(1-h_2^2)\partial_a \ln h_3\partial_b \ln h_3 - \frac{1}{2}(\partial_a \ln h_2\partial_b \ln h_3 + \partial_a \ln h_3\partial_b \ln h_2) - \frac{1}{2}u_{ab}\Lambda = 0, \tag{126f}$$

where $\triangle_Z$ denotes the Laplace operator on Z space and $R_{ab}(Z)$ is the Ricci tensor constructed from the metric $u_{ab}(Z)$.

We next consider the gauge field equations (123c) and (123d). Under the assumption (125), the gauge field equations are written by

$$d[h_3^2\partial_y h_2\Omega(Z) + h_3\partial_a h_2 dy\wedge(*_Z dz^a)] = 0, \tag{127a}$$

$$d[h_2\partial_a h_3(*_Z dz^a)] = 0, \tag{127b}$$

where $*_Z$ denotes the Hodge operator on Z and $\Omega(Z)$ is the volume 4-form on Z space:

$$\Omega(Z) = \sqrt{u}dz^1\wedge dz^2\wedge dz^3\wedge dz^4. \tag{128}$$

Here, u is the determinant of the metric u_{ab}.

Finally we consider the equation of motion for the scalar field. Substituting the ansatz (125) into Eq. (123b), we have

$$h_2^2 h_3(h_2^{-1}\partial_t^2 h_2 + h_3^{-1}\partial_t^2 h_3) + h_3(\partial_t h_2)^2 + 3h_2\partial_t h_2\partial_t h_3 - h_3(h_2^{-1}\partial_y^2 h_2 + h_3^{-1}\partial_y^2 h_3) + h_3(1-h_3)(\partial_y \ln h_2)^2 - h_2^{-1}\partial_y h_2\partial_y h_3 - h_2^{-1}\triangle_Z h_2 - h_3^{-1}\triangle_Z h_3 + (1-h_3)u^{ab}\partial_a \ln h_2\partial_b \ln h_2 + (1-h_2^2)u^{ab}\partial_a \ln h_3\partial_b \ln h_3 - 2\Lambda = 0. \tag{129}$$

Now we consider the two cases. One is $\partial_t h_2 \neq 0$ and $\partial_t h_3 = 0$. The other is $\partial_t h_2 = 0$ and $\partial_t h_3 \neq 0$. Upon setting $h_2 = 1$, the field equations reduce to

$$R_{ab}(Z) = 0, \tag{130a}$$

$$h_2 = 1, \qquad h_3 = k_0(t) + k_1(y) + k_2(z), \tag{130b}$$

$$\frac{d^2k_0}{dt^2}=\Lambda, \qquad \frac{d^2k_0}{dt^2}=-\frac{d^2k_1}{dy^2}, \qquad \triangle_{\rm Z}k_2=0. \tag{130c}$$

We can also choose the solution in which the 0-brane part depends on t. Then, we have

$$R_{ab}(\mathrm{Z})=0, \tag{131a}$$

$$h_2=h_2(t,v), \quad h_2=K_0(t)+K_1(v), \quad h_3=1, \tag{131b}$$

$$\left(\frac{dK_0}{dt}\right)^2=2\Lambda, \qquad \triangle_{\rm W}K_1=0, \tag{131c}$$

where $\triangle_{\rm W}$ denotes Laplace operator with respect to the metric w_{mn}:

$$\begin{aligned} &w_{mn}dv^mdv^n=dy^2+u_{ab}(\mathrm{Z})dz^adz^b,\\ &\triangle_{\rm W}K_1=\partial_y^2K_1+\triangle_{\rm Z}K_1. \end{aligned} \tag{132}$$

Here, w_{mn} is the five-dimensional metric, and v^m denotes the five-dimensional coordinate.

As a special example, we consider the case

$$u_{ab}=\delta_{ab}, \qquad h_2=1, \tag{133}$$

where δ_{ab} the four-dimensional Euclidean metric. Then, the solution for h_3 can be obtained explicitly as [20]

$$h_2=1, \tag{134a}$$

$$h_3(t,y,z)=\frac{\Lambda}{2}(t^2-y^2)+c_1t+c_2y+c_3+\sum_{l=1}^{N}\frac{M_l}{|z^a-z^a_l|^2}, \tag{134b}$$

where $c_i(i=1,2,3)$ and z^a_l are constants and the parameter M_l is the mass constant of 1-branes, which is located at $z^a=z^a_l$.

We can obtain the solution for $h_3=1$ and $\partial_th_2\neq 0$ if the roles of h_2 and h_3 are exchanged. The solution of the field equations is then written as

$$h_2(t,v)=\epsilon\sqrt{2\Lambda}t+c_4+\sum_{\alpha=1}^{N'}\frac{L_\alpha}{|v^m-v^m_\alpha|^3}, \tag{135a}$$

$$h_3=1, \tag{135b}$$

where c_4, v^m_α, and L_α are constants and $\epsilon=\pm1$. The delocalized brane solutions in the six-dimensional NSS supergravity [66–68,71,72] have been investigated in Refs. [20,73–80], including applications to cosmological models. According to the intersection rule, the number of the intersections dimensions involving the 0-brane and 1-brane is -1. Although meaningless in ordinary spacetime, these configurations are relevant in the Euclidean space, for instance, representing instantons.

In the following, we consider cosmological aspects of the solution describing time-dependent branes. We first study the time dependence of the scale factors in the 0-brane solutions after compactifying the extra directions, and our Universe is discussed. Next we discuss the dynamical 1-brane solution and apply it to the cosmology.

1. Cosmology in the 0-brane system

For the solution (135), we introduce a new time coordinate τ as

$$\left(\frac{\tau}{\tau_0}\right)\equiv(\epsilon\sqrt{2\Lambda}t+c_4)^{1/4}, \qquad \tau_0\equiv\frac{4}{\epsilon\sqrt{2\Lambda}}. \tag{136}$$

The six-dimensional metric is thus given by

$$ds^2=\left[1+\left(\frac{\tau}{\tau_0}\right)^{-4}\bar{h}_2(v)\right]^{-\frac{3}{2}}\left[-d\tau^2+\left\{1+\left(\frac{\tau}{\tau_0}\right)^{-4}\bar{h}_2(v)\right\}\times\left(\frac{\tau}{\tau_0}\right)^2\delta_{mn}(\mathrm{W})dv^mdv^n\right], \tag{137}$$

where δ_{mn} is the five-dimensional Euclidean metric and $\bar{h}_2(v)$ is defined by

$$\bar{h}_2(v)\equiv\sum_{\alpha=1}^{N'}\frac{L_\alpha}{|v^m-v^m_\alpha|^{3-d_s}}. \tag{138}$$

Here d_s denotes the number of smeared dimensions and should satisfy $0\leq d_s\leq 4$.

The six-dimensional spacetime implies $(\tau/\tau_0)^{-4}\bar{h}_2(v)=0$ in the limit $\tau\to\infty$. Then the scale factor of the six-dimensional space is proportional to τ. Although the dynamical 0-brane solutions cannot give a realistic universe such as an accelerating expansion, a matter-, or a radiation-dominated era, there is a possibility that appropriate compactification and smearing of the extra directions may lead to a realistic expansion. Now we will discuss this possibility.

We consider some compactification and smearing of the extra directions of the solutions. Our Universe has to be described by the 0-brane solution with six directions. Since the time direction is expressed as t, the remaining task is to identify the three spatial directions from the coordinates v^m.

In an approach such as the construction of the cosmological scenario on the basis of a dynamical brane background, three spatial directions are supposed to be on the overall transverse space to branes. If the spatial directions are specified with v^m, it also works in the present case. Then space is isotropic from the expression of the metric. Now we look for a way to realize an isotropic and homogeneous three-dimensional space in the 0-brane solutions.

Since we set the coordinates (t, v^2, v^3, v^4) which describes our Universe, it is convenient to decompose the six-dimensional metric of the solutions into the following form:

$$ds^2 = ds_4^2 + ds_{\rm i}^2, \tag{139}$$

where each part of the six-dimensional metric is given by

$$ds_4^2 = -h_2^{-3/2}(t,v)dt^2 + h_2^{1/2}(t,v)\delta_{\alpha\beta}dv^\alpha dv^\beta, \tag{140a}$$

$$ds_{\rm i}^2 = h_2^{1/2}(t,v)\delta_{ij}dv^i dv^j. \tag{140b}$$

Here ds_4^2 is the metric of the four-dimensional spacetime with t, v^α $(\alpha = 3,4,5)$, while $ds_{\rm i}^2$ denotes the metric of the internal space. We can obtain the compactifications of the solutions depending on the internal space.

The internal space is described by the coordinates $v^i (i = 1,2)$, and the spatial part of our Universe $\delta_{\alpha\beta}$ is three-dimensional with $v^\alpha(\alpha = 3,4,5)$. Then $\delta_{\alpha\beta}$ and δ_{ij} are the three- and two-dimensional Euclidean metrics, respectively.

Now we derive the lower-dimensional effective theory by compactifying the extra directions. In order to find a realistic universe, we compactify the d-dimensional space to be a d-dimensional torus, where d is the compactified dimensions for the direction of internal space. The remaining noncompact space is the external space. The range of d is given by $0 \le d \le 1$, because the v^1 direction is preserved to measure the position of the universe in the overall transverse space. Hence the v^2 direction will be compactified, where the compactified direction has to be smeared out before the compactification.

Then the metric (124) with $h_3 = 1$ is recast into the following form:

$$ds^2 = ds_{\rm e}^2 + ds_{\rm i}^2, \tag{141}$$

where $ds_{\rm e}^2$ is the metric of $(6-d)$-dimensional external spacetime and $ds_{\rm i}^2$ is the metric of compactified dimensions. Upon setting $d = 1$, the compactified metric in the Einstein frame is

$$d\bar{s}_{\rm e}^2 = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-6}\bar{h}_2(v)\right]^{-\frac{5}{3}}\left[-d\tau^2 + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-6}\bar{h}_2(v)\right\}^2\left(\frac{\tau}{\tau_0}\right)^2 \times \{\delta_{\alpha\beta}dv^\alpha dv^\beta + (dv^1)^2\}\right], \tag{142}$$

where $d\bar{s}_{\rm e}^2$ is the five-dimensional metric in the Einstein frame and the constant parameters τ_0 and the cosmic time τ are defined, respectively, as

$$\frac{\tau}{\tau_0} \equiv (\epsilon\sqrt{2\Lambda}t)^{1/6}, \qquad \tau_0 \equiv \frac{6}{\epsilon\sqrt{2\Lambda}}. \tag{143}$$

Since the power exponent of the scale factor is given by 1, the metric of four-dimensional spacetime in the Einstein frame implies that the solutions gives rise to a Milne universe. To construct a realistic cosmological model such as in the inflationary scenario, it would be necessary to add some new ingredients in the background. Figure 4 depicts the conformal diagrams of the five-dimensional spacetime in the limit $\tau \to \infty$. Hence, the asymptotic regions of the present spacetime (142) resemble the five-dimensional Milne universe.

Finally, we discuss the near-horizon geometry of the 0-brane solution. When all of the 0-branes are located at the origin of the overall transverse space, the solution can be expressed as

$$h_2(t,r) = \epsilon\sqrt{2\Lambda}t + c_4 + \frac{L}{r^3}, \qquad r^2 \equiv \delta_{mn}v^m v^n, \tag{144}$$

where L is the total mass of 0-branes

$$L \equiv \sum_{\alpha=1}^{N'} L_\alpha. \tag{145}$$

In the near-horizon limit $r \to 0$, the dependence on t in (144) is negligible. The six-dimensional metric is thus reduced to the following form:

$$ds^2 = \left(\frac{L}{r^3}\right)^{-1/6}[ds_{{\rm AdS}_2}^2 + L^{2/3}d\Omega_{(4)}^2], \tag{146a}$$

$$ds_{{\rm AdS}_2}^2 \equiv -\left(\frac{L^{4/3}}{r^4}\right)^{-1}dt^2 + \frac{L^{2/3}}{r^2}dr^2, \tag{146b}$$

where $\delta_{mn}dv^m dv^n = dr^2 + r^2 d\Omega_{(4)}^2$ has been performed. The line elements of a two-dimensional AdS space (AdS_2) and a four-sphere with the unit radius (S^4) are given by $ds_{{\rm AdS}_2}^2$ and $d\Omega_{(4)}^2$, respectively. Then the six-dimensional metric (146) in the near-horizon limit of the 0-brane system describes a warped product of AdS_2 and S^4. Figure 4 shows the geometry of the AdS_2 and S^4.

2. *Cosmology in the 1-brane system*

Now we discuss the cosmological evolution for the time-dependent 1-brane solution (134). We define the cosmic time τ, which is given by

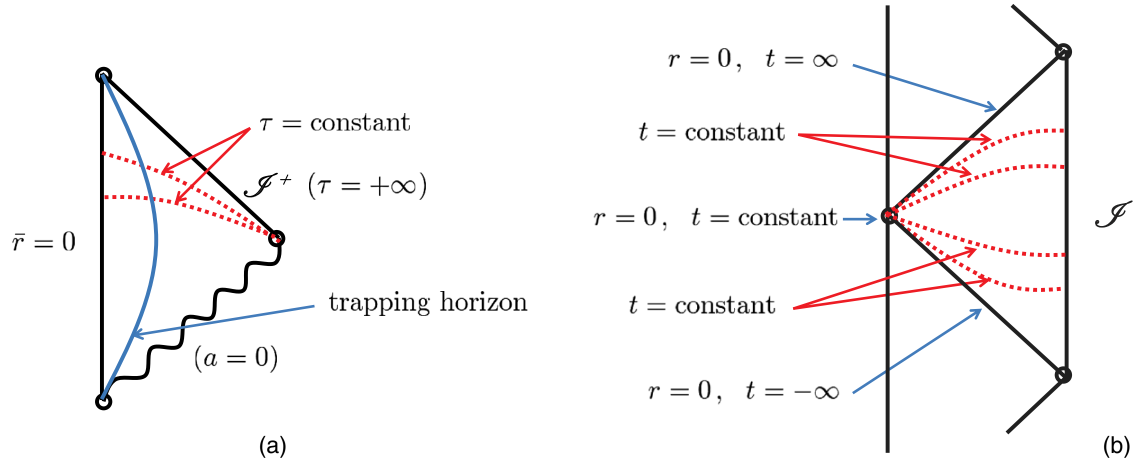

FIG. 4 (color online). (a) Conformal diagrams of the Milne universe are depicted. The solid line denotes the trapping horizon $\bar{r}_{\rm T} \equiv da(\tau)/d\tau$, where $a = (\tau/\tau_0)$ [19]. One can recognize that the asymptotic region of the spacetime in (142) corresponding to $\bar{r} \to \infty$ approximates the five-dimensional Milne universe, where $\bar{r}^2 = \delta_{\alpha\beta} v^\alpha v^\beta + (v^1)^2$. (b) The geometry of the 0-brane system (135) in the limit $r \to 0$ is depicted. The domain corresponding to $r \to 0$ with finite t describes warped $\mathrm{AdS}_2 \times \mathrm{S}^4$ spacetime [19].

$$\left(\frac{\tau}{\tau_0}\right) \equiv \left(\frac{\Lambda}{2} t^2\right)^{1/4}, \qquad \tau_0 \equiv \frac{2\sqrt{2}}{\sqrt{\Lambda}}, \qquad c_1 = 0. \quad (147)$$

The six-dimensional metric is expressed as

$$ds^2 = \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-4} \bar{h}_3(y,z)\right]^{-\frac{1}{2}} \left[-d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{-2} dy^2 + \left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-4} \bar{h}_3(y,z)\right\} \left(\frac{\tau}{\tau_0}\right)^2 \delta_{ab}(\mathrm{Z}) dz^a dz^b\right], \quad (148)$$

where $\bar{h}_3(y,z)$ is defined by

$$\bar{h}_3(y,z) \equiv -\frac{\Lambda}{2} y^2 + c_2 y + c_3 + \sum_{l=1}^{N} \frac{M_l}{|z^a - z^a_l|^{2-d_s}}. \quad (149)$$

Here d_s is the number of smeared dimensions and should satisfy $0 \le d_s \le 3$. In order to fix the location of our Universe in the transverse space, let us assume that at least one direction of $z^a (a = 1, \ldots, 4)$ is not smeared.

Now we apply the 1-brane solution to the lower-dimensional effective theory. Let us consider a compactification and smearing of the transverse space to the 0-brane of the 1-brane solution. First of all, our Universe is described by the solutions with the six-dimensional coordinates $t, y, z^a (a = 1, \ldots, 4)$. The time direction is identified with t. Our choice is to take the three-dimensional from the overall transverse space with z^a. The four-dimensional universe is spanned by t, z^2, z^3, and z^4, for instance. The z^1 direction is preserved to measure the position of our Universe in the overall transverse space of the 1-brane. Since the metric depends on z^a explicitly, we have to smear out z^2, z^3, and z^4 so as to define our Universe. Then the number of the smeared directions d_s should satisfy the condition $d_s = 3$.

It is necessary to take that $c_2 = 0$ and $\Lambda = 0$ in (124) to compactify the y direction. We compactify the y direction to fit our Universe, where y denotes the compactified dimensions with respect to the world volume of the 1-brane. The metric (124) with $h_2 = 1$ is then described by (141).

In terms of the conformal transformation

$$ds^2_{\rm e} = h_3^{1/6} d\bar{s}^2_{\rm e}, \quad (150)$$

we can rewrite the $(6-d)$-dimensional metric in the Einstein frame. If we set $d = 1$, the five-dimensional metric in the Einstein frame is

$$d\bar{s}^2_{\rm e} = -h_3^{-2/3}(t,z) dt^2 + h_3^{1/3}(t,z) \delta_{ab}(\mathrm{Z}) dz^a dz^b, \quad (151)$$

where $d\bar{s}^2_{\rm e}$ is the metric of five-dimensional external spacetime in the Einstein frame. For $h_3 = c_1 t + \bar{h}_3(z)$, the metric (151) is thus rewritten as

$$d\bar{s}^2_{\rm e} = -\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-3/2} \bar{h}_3(z)\right]^{-2/3} d\tau^2 + \left[1 + \left(\frac{\tau}{\tau_0}\right)^{-3/2} \bar{h}_3(z)\right]^{1/3} \left(\frac{\tau}{\tau_0}\right)^{1/2} \times [\delta_{\alpha\beta} dz^\alpha dz^\beta + (dz^1)^2], \quad (152)$$

where the spatial part of our Universe $\delta_{\alpha\beta}$ is three-dimensional with $z^\alpha (\alpha = 2, 3, 4)$, and the constant parameters τ_0 and the cosmic time τ are defined, respectively, as

$$\frac{\tau}{\tau_0} \equiv (c_1 t)^{2/3}, \qquad \tau_0 \equiv \frac{3}{2c_1}. \quad (153)$$

Unfortunately, the power exponent of the four-dimensional universe in the Einstein frame becomes $1/4$.

Hence, we have to conclude that, in order to obtain a realistic expansion of the universe in this type of models, one has to include additional fields on the background.

Let us finally consider the case of the near-horizon limit that the spacetime metric and the functions h_2 and h_3 satisfy (134). If we consider the case where N 1-branes are located at the origin of the Z space, we have

$$h_3(t, r) = \frac{\Lambda}{2}(t^2 - y^2) + c_1 t + c_2 y + c_3 + \frac{M}{r^2}, \quad r^2 \equiv \delta_{ab} z^a z^b, \tag{154}$$

where M is the total mass of 1-branes

$$M \equiv \sum_{l=1}^{N} M_l. \tag{155}$$

Since the dependence on t and y in (154) is negligible in the near-horizon limit $r \to 0$, the six-dimensional metric is reduced to the following form:

$$ds^2 = \left(\frac{M}{r^2}\right)^{-1/2}\left[-dt^2 + dy^2 + M\left\{\frac{dr^2}{r^2} + d\Omega_{(3)}^2\right\}\right], \tag{156}$$

where $\delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega_{(3)}^2$ has been used. The line elements of a three-dimensional space (M_3) and a three-sphere are given by $ds_{M_3}^2$ and $d\Omega_{(3)}^2$, respectively. Thus we see that the near-horizon limit of the 1-brane system is a warped product of M_3 with a certain internal 3-space with a circle.

B. Collision of the 0-brane in Nishino-Salam-Sezgin gauged supergravity

We next study the behavior of the time-dependent 0-brane solution (135). By substituting (135) into the metric (124), the six-dimensional metric is expressed as

$$ds^2 = -[\epsilon\sqrt{2\Lambda} t + c_4 + \bar{h}_2(v)]^{-3/2} dt^2 + [\epsilon\sqrt{2\Lambda} t + c_4 + \bar{h}_2(v)]^{1/2} w_{mn} dv^m dv^n, \tag{157}$$

where w_{mn} is given by (132) and the function $\bar{h}_2(v)$ is defined by (138). Since the time dependence appears through the function h_2, the next task is to study the time evolution of the solutions carefully. Hereafter we will consider it by focusing upon the collision of 0-branes. We also discuss smearing out some of the directions in the transverse space to decrease the number of transverse dimensions to the 0-brane effectively.

Now we consider the case that the number of the smeared directions is given by d_s. Then the function $\bar{h}_2(v)$ can be expressed as

$$\bar{h}_2(v) \equiv \sum_{\alpha} \frac{L_\alpha}{|v^m - v_\alpha^m|^{3-d_s}}, \tag{158}$$

where d_s is the number of smeared dimensions and should satisfy $0 \le d_s \le 4$, and we assume that at least one direction of $v^m (m = 1, \ldots, 5)$ is not smeared in order to fix the location of our Universe in the transverse space. In the following, we will use the function (158).

We will discuss the asymptotic behavior of the time-dependent solutions. In the limit of $v^m \to v_\alpha^m$, the time dependence in the function h_2 can be ignored, because the harmonic function $\bar{h}_2(v)$ dominates near a position of the 0-brane. On the other hand, the function $\bar{h}_2(v)$ vanishes in the limit of $v^m \to \infty$. Then the system becomes static near the 0-brane, while h_2 depends only on time t in the far region from 0-branes. Thus the six-dimensional metric in the limit of $v^m \to \infty$ is rewritten by

$$ds^2 = -(\epsilon\sqrt{2\Lambda} t + c_4)^{-3/2} dt^2 + (\epsilon\sqrt{2\Lambda} t + c_4)^{1/2} w_{mn} dv^m dv^n. \tag{159}$$

The metric has singularity at $h_2 = 0$. Then the spacetime is regular if it is restricted inside the domain specified by the conditions

$$h_2(t, v) = \epsilon\sqrt{2\Lambda} t + c_4 + \bar{h}_2(v) > 0, \tag{160}$$

where the function $\bar{h}_2(v)$ is defined in (158). Since the spacetime evolves into a curvature singularity, the six-dimensional spacetime cannot be extended beyond this region. The regular spacetime with 0-branes ends up with the singularities.

The evolution of the spacetime highly depends on the signature of $\tilde{\Lambda}(\equiv \epsilon\sqrt{2\Lambda})$. The system with $\tilde{\Lambda} > 0$ has the time reversal one of $\tilde{\Lambda} < 0$. Now we will discuss the case with $\tilde{\Lambda} < 0$. For $t < 0$, the spacetime is not singular, because the function h_2 is positive everywhere. In the limit of $t \to -\infty$, the solution is approximately given by a time-dependent uniform spacetime apart from a position of 0-branes. In the vicinity of branes, the geometry takes a cylindrical form of an infinite throat.

We study the time evolution for $t > 0$ and $c_4 = 0$. At $t = 0$, the spacetime is regular everywhere and has a cylindrical topology near each 0-brane. As time slightly evolves, a curvature singularity appears as $|v^m - v_\alpha^m| \to \infty$. The singular hypersurface cuts off more and more of the space as time increases further. When time continues to evolve, the singular hypersurface eventually splits and surrounds each of the 0-brane throats individually. Hence, the spatial surface is composed of each isolated throat. For $t < 0$, the time evolution of the six-dimensional spacetime is the time reversal of $t > 0$.

Since the metric (159) in the regular domain implies that the overall transverse space tends to expand asymptotically like $t^{1/4}$, for any values of fixed v^m, the solutions describe static 0-branes near the positions of the branes. In the far region as $|v^m - v^m_\alpha| \to \infty$, the solutions approach FRW universes with the power-law expansion $t^{1/4}$. The emergence of FRW universes is an important feature of the time-dependent 0-brane solutions.

We will discuss whether two 0-branes can collide or not. We put the two 0-branes at $v_1 = (0, 0, \ldots, 0)$ and $v_2 = (\xi, 0, \ldots, 0)$, where ξ is a constant. If we introduce the following quantity:

$$\tilde{v} = \sqrt{(v^2)^2 + (v^3)^2 + \cdots + (v^{5-d_s})^2}, \tag{161}$$

the proper distance at $\tilde{v} = 0$ between the two 0-branes is given by

$$d(t) = \int_0^\xi dv^1 \left(\epsilon\sqrt{2\Lambda}t + c_4 + \frac{L_1}{|v^1|^{3-d_s}} + \frac{L_2}{|v^1 - \xi|^{3-d_s}}\right)^{1/4}, \tag{162}$$

where L_1 and L_2 are the charges of the 0-brane. For $\epsilon = -1$, this is a monotonically decreasing function of t. The behavior of the proper length is different depending on the number of the smeared directions d_s. We will discuss it for each of the values of d_s below.

First we consider the case with $d_s \leq 3$. The proper length is plotted in Fig. 5 for the cases with $d_s = 0$ and $d_s = 2$. Since both cases show that a singularity appears before the proper distance becomes zero, the singularity between two 0-branes appears before collision. The two 0-branes approach very slowly, and then the singular hypersurface suddenly appears at a finite proper distance. The spacetime finally splits into two isolated 0-brane throats. Therefore one cannot see the collision of the 0-branes in these examples. For the other case with $d_s = 1$, the result is the same.

Next we consider the case with $d_s = 4$ and assume that the v^m directions apart from v^1 are smeared. Since the function $\bar{h}_2$ is linear in v, the behavior of the proper distance is different from the previous case. The six-dimensional metric is now given by (157). By choosing $v = v^1$, the harmonic function $\bar{h}_2$ is written by

$$\bar{h}_2(v) = \sum_{\alpha=1}^{N'} L_\alpha |v - v_\alpha|. \tag{163}$$

We discuss the time-dependent solutions in the case that one 0-brane charge L_1 is located at $v = 0$ and the other L_2 at $v = \xi$. The proper length between the two 0-branes is given by

$$d(t) = \int_0^\xi dv[\epsilon\sqrt{2\Lambda}t + c_4 + (L_1|v| + L_2|v - \xi|)]^{1/4}. \tag{164}$$

For $\epsilon = -1$, the proper distance decreases with time. If we set $L_1 \neq L_2$, a singularity appears again at a certain finite time $t = t_S$, while the proper distance is still finite, where t_S is defined as

$$t_S \equiv \frac{c_4 + L_1|v| + L_2|v - \xi|}{\sqrt{2\Lambda}}. \tag{165}$$

This is the same result as the case with $d_s \leq 3$.

On the other hand, two 0-branes have the same brane charges $L_1 = L_2 = L$, and the proper distance vanishes at a certain finite time $t = t_c$, where t_c is defined by

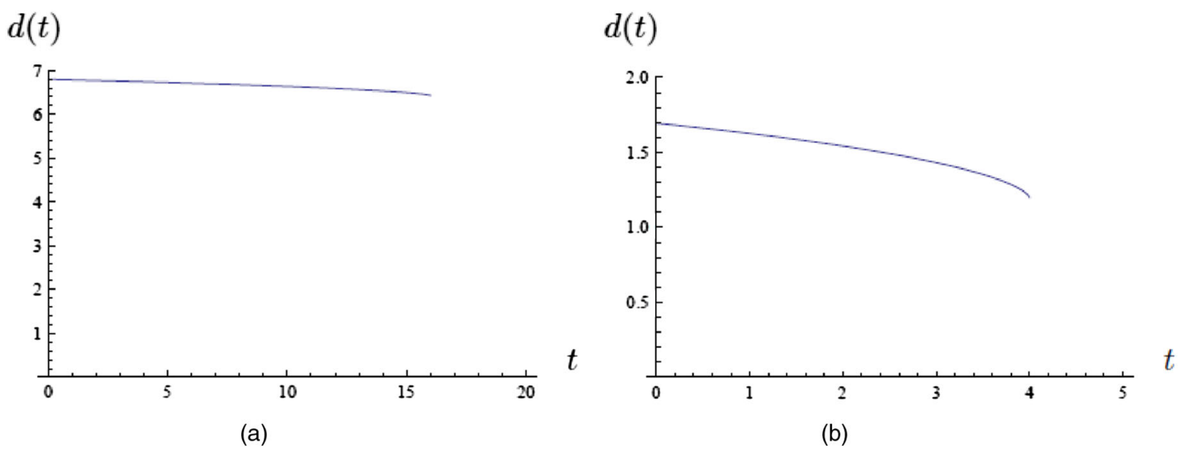

FIG. 5 (color online). The time evolution of the proper distance between two dynamical 0-branes for $d_s = 0$ (a) and $d_s = 2$ (b) in the six-dimensional Nishino-Salam-Sezgin gauged supergravity. For both cases, the two 0-brane charges are identical, $L_1 = L_2 = 1$, and the parameters are taken as $c_4 = 0$, $\Lambda = 0.5$, $\epsilon = -1$, and $\xi = 1$. The result is also the same, and a singularity develops before the collision of 0-branes.

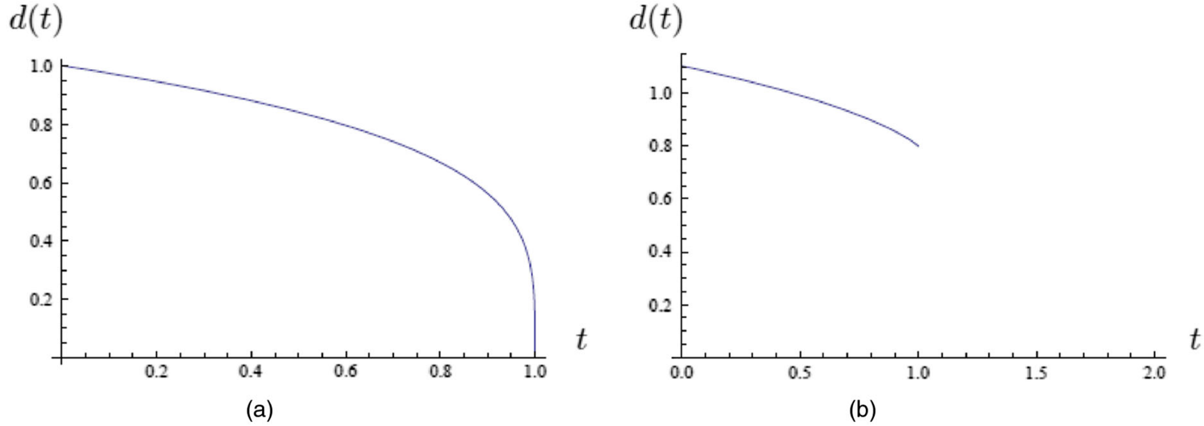

FIG. 6 (color online). The time evolution of the proper distance between two dynamical 0-branes for $L_1 = L_2 = 1$ (a) and $L_1 = 2$, $L_2 = 1$ (b) in the six-dimensional Nishino-Salam-Sezgin gauged supergravity. We fix $d_s = 4$, $c_4 = 0$, $\Lambda = 0.5$, $\epsilon = -1$, and $\xi = 1$. The proper distance rapidly vanishes near where two 0-branes collide for the case of $L_1 = L_2 = 1$, while for the case of $L_1 = 2$, $L_2 = 1$, it is still finite when a curvature singularity appears.

$$t_c \equiv \frac{c_4 + L\xi}{\sqrt{2\Lambda}}. \tag{166}$$

Hence two 0-branes can collide completely.

In terms of t_c, the proper length is expressed as

$$d(t) = L[-\sqrt{2\Lambda}(t - t_c)]^{1/4}. \tag{167}$$

If we choose the values as $c_4 = 0$, $\Lambda = 0.5$, $\xi = 1$, and $\epsilon = -1$, the proper distance $d(t)$ is plotted in Fig. 6 for the two cases (a) the same 0-brane charges $L_1 = L_2 = 1$ and (b) different charges $L_1 = 2$, $L_2 = 1$. In case (a), the two 0-branes can collide completely. However, in case (b), a singularity appears before collision, as we have already discussed analytically.

C. Collision of the 1-brane in Nishino-Salam-Sezgin gauged supergravity

Now we apply our time-dependent solutions to a collision of 1-brane systems. In the case of $h_2 = 1$, the function h_3 is assumed to be

$$h_3(t, y, z) = \frac{\Lambda}{2}(t^2 - y^2) + c_3 + \tilde{h}(z), \tag{168}$$

where c_3 is a constant parameter, we choose $c_1 = c_2 = 0$, and the harmonic function $\tilde{h}$ is expressed as

$$\tilde{h}(z) = \sum_{l=1}^{N} \frac{M_l}{|z^a - z_l^a|^{2-d_s}}, \quad \text{for } d_s \neq 2, \tag{169a}$$

$$\tilde{h}(z) = \sum_{l=1}^{N} M_l \ln|z^a - z_l^a|, \quad \text{for } d_s = 2. \tag{169b}$$

Here M_l are charges of 1-branes located at $z^a = z_l^a$ and

$$|z^a - z_l^a| = \sqrt{(z^1 - z_l^1)^2 + (z^2 - z_l^2)^2 + \cdots + (z^{4-d_s} - z_l^{4-d_s})^2}, \tag{170}$$

because the harmonic function $\tilde{h}$ is defined on the $(4 - d_s)$-dimensional Euclidean subspace in Z. The six-dimensional metric, scalar field, and gauge field of the solution are given by Eqs. (124) and (125), respectively. We see that $d_s = 2$ case is critical. For $d_s = 3$, the function $\tilde{h}$ is written by the sum of linear functions of z. The possibility of 1-brane collisions depends on the difference in the transverse dimensions, because the behavior of the gravitational field in the transverse space depends on the number of the transverse dimensions.

Although the six-dimensional metric (124) is regular if and only if $h_3 > 0$, the spacetime shows curvature singularities at $h_3 = 0$. Hence, the regular six-dimensional spacetime is restricted to the region of $h_3 > 0$, which is bounded by curvature singularities.

Let us study the time evolution for time-dependent 1-brane solution (134). We perform the following coordinate transformation:

$$t = \sqrt{\frac{2}{\Lambda}}\tilde{t}\cosh\tilde{y}, \qquad y = \sqrt{\frac{2}{\Lambda}}\tilde{t}\sinh\tilde{y}. \tag{171}$$

If we choose $c_1 = c_2 = 0$, we find

$$\begin{aligned} ds^2 &= \frac{2}{\Lambda}[1 + \tilde{t}^{-2}\tilde{h}(\tilde{z})]^{-1/2}\tilde{t}^{-1} \\ &\quad \times [-d\tilde{t}^2 + \tilde{t}^2\{d\tilde{y}^2 + (1 + \tilde{t}^{-2}\tilde{h}(\tilde{z}))\delta_{ab}d\tilde{z}^a d\tilde{z}^b\}] \\ &= \frac{2}{\Lambda}[1 + 16\bar{t}^{-4}\tilde{h}(\tilde{z})]^{-1/2}[-d\bar{t}^2 + \frac{1}{4}\bar{t}^2\{d\tilde{y}^2 \\ &\quad + (1 + 16\bar{t}^{-4}\tilde{h}(\tilde{z}))\delta_{ab}d\tilde{z}^a d\tilde{z}^b\}], \end{aligned} \tag{172}$$

where $\tilde{h}(\tilde{z})$, $\bar{t}$, and $\tilde{z}^a$ are defined, respectively, by

$$\tilde{h}(\tilde{z}) = c_3 + \frac{\Lambda}{2}\sum_{l=1}^{N}\frac{M_l}{|\tilde{z}^a - \tilde{z}_l^a|^2}, \qquad \bar{t} = 2\tilde{t}^{1/2},$$
$$\tilde{z}^a = \sqrt{\frac{\Lambda}{2}}z^a. \tag{173}$$

Here, $\tilde{t}$ obeys $\Lambda(t^2 - y^2)/2 = \tilde{t}^2$. The six-dimensional metric (172) represents a homogeneous and isotropic spacetime whose scale factor evolves as the cosmic time $\bar{t}$, which is described as the Milne universe. Hence, we can consider that the present solution with $\Lambda > 0$ gives a system of 1-branes in the Milne universe. The existence of the expanding Milne universe is guaranteed by the scalar field with the exponential potential in the six-dimensional action (122).

Now let us consider the collision of 1-branes. The solution (124) without 0-branes can be written in the form

$$ds^2 = \left[\frac{\Lambda}{2}(t^2 - y^2) + c_3 + \tilde{h}(z)\right]^{-\frac{1}{2}}(-dt^2 + dx^2)$$
$$+ \left[\frac{\Lambda}{2}(t^2 - y^2) + c_3 + \tilde{h}(z)\right]^{\frac{1}{2}}u_{ab}dz^a dz^b, \tag{174}$$

where we choose $c_1 = c_2 = 0$, u_{ab} denotes the four-dimensional metric, and the function $\tilde{h}(z)$ is given by (169). The behavior of the harmonic function $\tilde{h}(z)$ is divided into two classes depending on the dimensions of the 1-brane, that is, $d_s \neq 2$ and $d_s = 2$, which we will study below separately. For $d_s = 2$, the harmonic function $\tilde{h}(z)$ diverges both at infinity and near 1-branes. In particular, there is no regular spacetime region near 1-branes, because $\tilde{h}(z) \to -\infty$. Hence, such a 1-brane solution is not physically relevant.

Since the harmonic function $\tilde{h}(z)$ becomes dominant in the limit of $z^a \to z_l^a$ (near 1-branes), we find a static structure of the 1-brane system. In the far region from 1-branes, that is, in the limit of $|z^a - z_l^a| \to \infty$, the function h_3 depends only on time t, because $\tilde{h}(z)$ vanishes. The metric is thus written by

$$ds^2 = \left[\frac{\Lambda}{2}(t^2 - y^2) + c_3\right]^{-\frac{1}{2}}(-dt^2 + dx^2)$$
$$+ \left[\frac{\Lambda}{2}(t^2 - y^2) + c_3\right]^{\frac{1}{2}}u_{ab}dz^a dz^b. \tag{175}$$

In the following, we will analyze one concrete example, in which two 1-branes are located at $z_1 = (0, 0, \ldots, 0)$ and $z_2 = (z_0, 0, \ldots, 0)$ in order to study in more detail. Since the metric function is singular at $h_3 = 0$, the regular spacetime exists inside the domain restricted by

$$h_3(t, z) = \frac{\Lambda}{2}(t^2 - y^2) + c_3 + \tilde{h}(z) > 0, \tag{176}$$

where the function $\tilde{h}(z)$ is given by (169). The six-dimensional spacetime cannot be extended beyond this region, because not only does the dilaton ϕ diverge but also the spacetime evolves into a curvature singularity.

The regular spacetime with two 1-branes ends on these singularities. The time dependence appears in the form of $\frac{\Lambda}{2}t^2$. For $t^2 > y^2$ and $c_3 = 0$, the function h is positive everywhere and the six-dimensional spacetime is not singular. It is asymptotically a time-dependent uniform spacetime except for near branes in the limit of $z^a \to z_l^a$, where the background geometry becomes the cylindrical forms of infinite throats.

When $t \leq 0$, the spatial metric is initially ($t \to -\infty$) regular apart from $y \to \pm\infty$, and the spacetime has a cylindrical topology near each 1-brane. As t evolves slightly, a curvature singularity appears at $y \to \pm\infty$ and from a far region ($|z^1| \to \infty$). As t evolves further, the singularity cuts off the space. As the time continues to increase, the singular hypersurface eventually splits and surrounds each of the 1-brane throats individually. Then the spatial surface is composed of two isolated throats.

The six-dimensional metric (175) implies that the transverse dimensions expand asymptotically as $t^{1/2}$ for fixed spatial coordinates y and z^a. However, this is observer dependent, because it becomes static near branes, and the spacetime approaches a Friedmann-Robertson-Walker universe in the far region ($|z^1| \to \infty$), which expands in the background isotropically.

If we define

$$\bar{z} = \sqrt{(z^2)^2 + (z^3)^2 + \cdots + (z^{4-d_s})^2}, \tag{177}$$

the proper length at $\bar{z} = 0$ between two 1-branes is written by

$$d(t, y) = \int_0^{z_0} dz^1 \left[\frac{\Lambda}{2}(t^2 - y^2) + c_3 + \frac{M_1}{|z^1|^{2-d_s}} + \frac{M_2}{|z^1 - z_0|^{2-d_s}}\right]^{\frac{1}{4}}. \tag{178}$$

This is a monotonically decreasing function of time for $t \leq 0$. In Fig. 7, we show $d(t, y)$ for the case of the 1-brane system. We set $\Lambda = 2$, $z_0 = 1$, $c_3 = 0$, and $M_1 = M_2 = 1$. All of the six-dimensional space is initially ($t = -\infty$) regular except at $|y| \to \infty$ and $|z^1 - z_0| \to \infty$. Although the spacetime becomes asymptotically time dependent and has the cylindrical form of an infinite throat near the 1-brane, the singularity appears from a far region ($|z^1 - z_0| \to \infty$) and $|y| \to \infty$. As time increases ($t < 0$), the singularity erodes the region with the large $|y|$ region. The region of transverse space is also invaded in time. As a result, only the region of small $|y|$ and near 1-branes remains regular. When we study

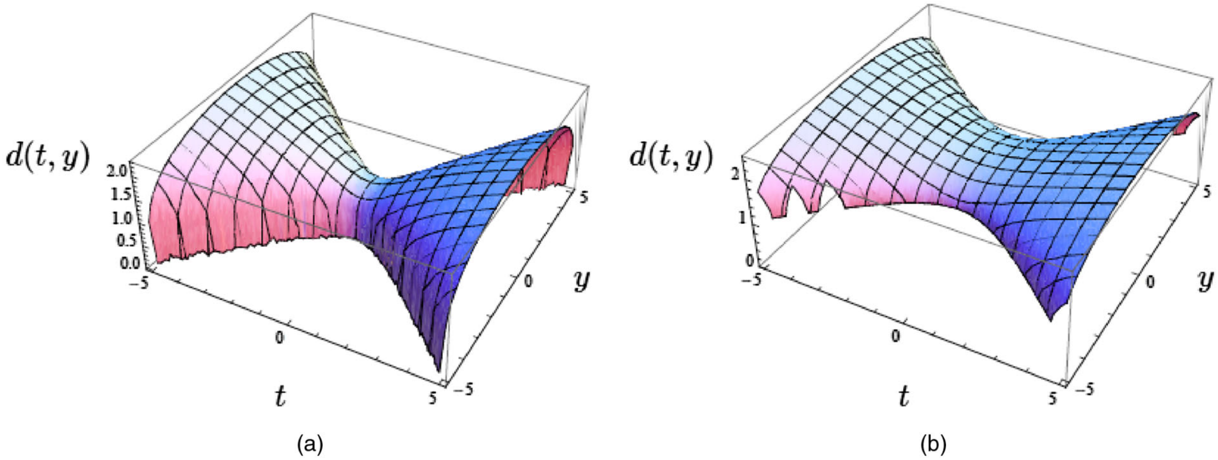

FIG. 7 (color online). The time evolution of the proper distance between two dynamical 1-branes for (a) $d_s = 3$ and (b) $d_s = 1$ in the six-dimensional Nishino-Salam-Sezgin gauged supergravity. We fix $c_3 = 0$, $M_1 = M_2 = 1$, $z_0 = 1$, and $\Lambda = 2$. The proper distance rapidly vanishes near where two 1-branes collide for the case of $d_s = 3$, while for the case of $d_s = 1$, it is still finite when a curvature singularity appears.

the evolution on the y and z^a plane, the singularity appears at infinity $|z^1| \to \infty$, $|y| \to \infty$, and comes to the region of two 1-branes. A singular hypersurface eventually surrounds each 1-brane individually, and then the regular regions near 1-branes split into two isolated throats. For the period of $t > 0$, we find the time-reversed behavior of the case of $t < 0$. Figures 7 and 8 show that this singularity appears before the distance d vanishes.

Then a singularity between two branes forms before their collision except for $d_s = 3$. Two 1-branes approach very slowly, a singularity suddenly appears at a finite distance, and the six-dimensional spacetime splits into two isolated 1-brane throats.

On the other hand, we can discuss a brane collision for $d_s = 3$ and $t < 0$. If $M_1 \neq M_2$, a singularity appears at $t = t_{\mathrm{S}} < 0$ when the distance is still finite (see Fig. 9). This is just the same as the case in Sec. V B. However, if $M_1 = M_2 = M$, the result completely changes (Fig. 7). Since the distance eventually vanishes at $t = t_{\mathrm{c}}$, two 1-branes collide with each other. The proper length for fixed y decreases as time increases from $t = -\infty$, and it eventually vanishes at $t = t_{\mathrm{c}}$. Hence, one 1-brane approaches the other as time evolves, causing the complete collision at $t = t_{\mathrm{c}}$. If we fix the 1-brane charges such that $M_1 = M_2 = M$, the branes first collide at larger $|y|$, and as time progresses, the subsequent collisions occur at the smaller $|y|$. We show $d(t, y)$ integrated numerically in Figs. 7, 8, and 9.

We also calculate the distance $d(t, y)$ at $y = 0$ and $\bar{z} = 0$ between two branes before the singularity appears except for the case of $d_s = 3$ if $M_1 = M_2$. The proper length is also given by Eq. (178). In the present case, d is a monotonically decreasing function of t^2 when $t < 0$.

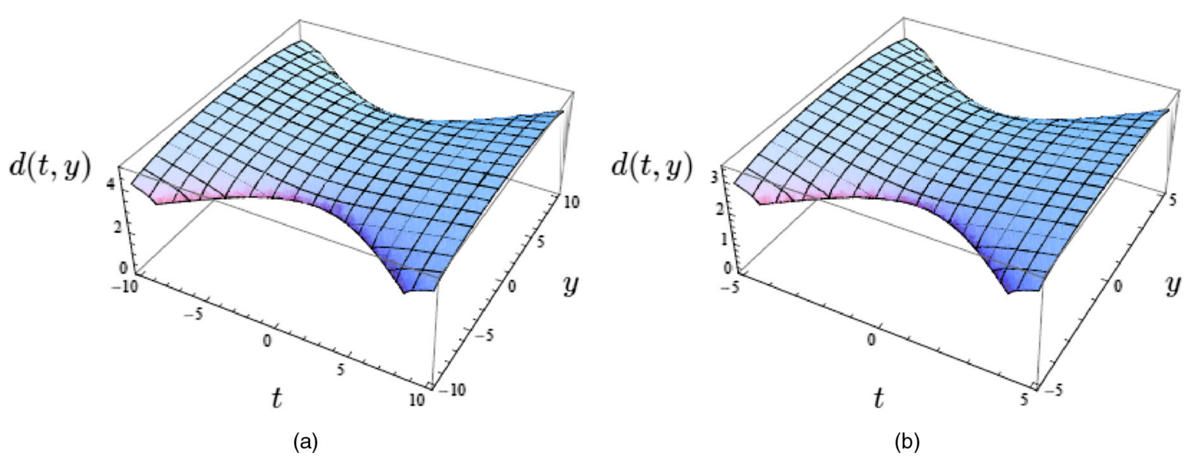

FIG. 8 (color online). The proper distance between two dynamical 1-branes given in (178) is depicted for (a) $M_1 = 10$, $M_2 = 1$ and (b) $M_1 = 2$, $M_2 = 1$ in the six-dimensional Nishino-Salam-Sezgin gauged supergravity. We fix $c_3 = 0$, $d_s = 0$, $z_0 = 1$, and $\Lambda = 2$. In both cases, a singularity appears at $t = t_{\mathrm{S}} < 0$ when the distance is still finite.

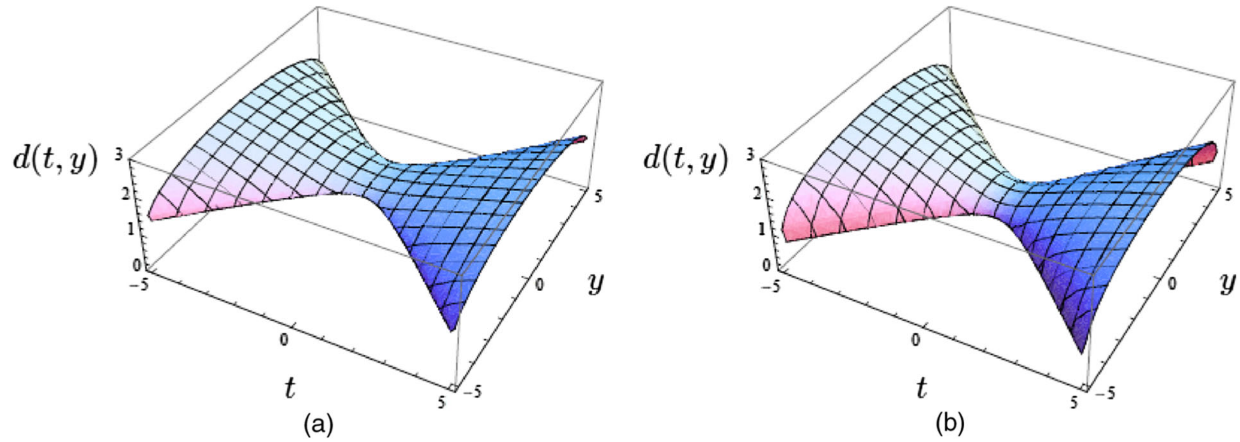

FIG. 9 (color online). The time evolution of the proper distance between two dynamical 1-branes for (a) $M_1 = 10$, $M_2 = 1$ and (b) $M_1 = 2$, $M_2 = 1$ in the six-dimensional Nishino-Salam-Sezgin gauged supergravity. We fix $c_3 = 0$, $d_s = 3$, $z_0 = 1$, and $\Lambda = 2$. For $M_1 \neq M_2$, a singularity appears at $t = t_S < 0$ when the distance is still finite. Then, the solution does not describe the collision of two 0-branes.

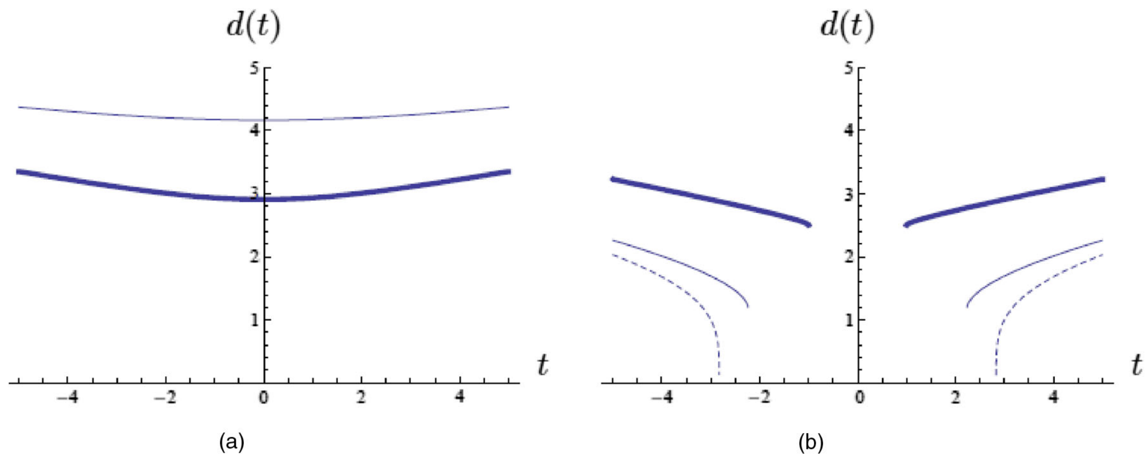

FIG. 10 (color online). (a) The proper distance between two dynamical 1-branes at $y = 0$ and $\bar{z} = 0$ for the case of $d_s = 0$ in the six-dimensional Nishino-Salam-Sezgin gauged supergravity is depicted. We fix $c_4 = 0$, $z_0 = 1$, and $\Lambda = 2$. For $t < 0$, the proper length decreases as time increases. The bold line denotes the case of $M_1 = M_2 = 1$, while the solid one corresponds to the $M_1 = 10$, $M_2 = 1$ case. (b) For the case of $M_1 = M_2$ in the six-dimensional Nishino-Salam-Sezgin gauged supergravity, the time evolution of the proper distance between two dynamical 1-branes at $y = 3$ and $\tilde{z} = 0$ given in (178) is depicted. We fix $c_4 = 0$, $z_0 = 1$, and $\Lambda = 2$. We show the lengths for $d_s = 0$ (bold line), $d_s = 1$ (solid line), and $d_s = 3$ (dashed line). The proper distance rapidly vanishes near where two 1-branes collide in the case of $d_s = 3$, while in the case of $d_s \neq 3$, it is still finite when a curvature singularity appears.

We show the time evolution of the distance in Fig. 10 for the case of $M_1 = M_2$.

On the other hand, for the case of $M_1 \neq M_2$, a singularity appears, when the proper distance is still finite. For the period of $t > 0$, the behavior of six-dimensional spacetime is the time reversal of the period of $t < 0$. We show the proper distance $d(t)$ integrated numerically in Fig. 10 for the cases of $d_s = 3$ and $d_s \neq 3$.

D. Romans' six-dimensional gauged supergravity

Similarly, for the six-dimensional Romans' theory [70], following the discussion in Ref. [81], the coupling of the 3-form and of the 2-form field strengths to the dilaton are given by $\epsilon_r c_r = -1/\sqrt{2}$ and $\epsilon_s c_s = \sqrt{2}$, respectively:

$$S = \frac{1}{2\kappa^2}\int \bigg[(R + 2e^{\phi/\sqrt{2}}\lambda) * \mathbf{1} - \frac{1}{2} * d\phi\wedge d\phi - \frac{1}{2\cdot 2!}e^{-\phi/\sqrt{2}} * F_{(2)}\wedge F_{(2)} - \frac{1}{2\cdot 3!}e^{\sqrt{2}\phi} * F_{(3)}\wedge F_{(3)}\bigg], \tag{179}$$

where R denotes the Ricci scalar constructed from the six-dimensional metric g_{MN}, κ^2 is the six-dimensional gravitational constant, $*$ is the Hodge operator in the six-dimensional spacetime, ϕ denotes the scalar field, $\lambda > 0$ is cosmological constants, and $F_{(3)}$ and $F_{(2)}$ are 3-form and 2-form field strengths, respectively. This has a negative scalar potential. In terms of Eq. (2a), Romans' model is given by choosing $\Lambda_r = -\lambda$, $\Lambda_s = 0$, $N_r = 2$, and $N_s = 4$.

From the six-dimensional action (179), we find the field equations:

$$R_{MN} = -\frac{1}{2}e^{\phi/\sqrt{2}}\lambda g_{MN} + \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{e^{-\phi/\sqrt{2}}}{2\cdot 2!}\left(2F_{MA}F_N{}^A - \frac{1}{4}g_{MN}F_{(2)}^2\right) + \frac{e^{\sqrt{2}\phi}}{2\cdot 3!}\left(3F_{MAB}F_N{}^{AB} - \frac{1}{2}g_{MN}F_{(3)}^2\right), \quad (180a)$$

$$\triangle\phi + \frac{\sqrt{2}}{4\cdot 2!}e^{-\phi/\sqrt{2}}F_{(2)}^2 - \frac{\sqrt{2}}{2\cdot 3!}e^{\sqrt{2}\phi}F_{(3)}^2 + \sqrt{2}e^{\phi/\sqrt{2}}\lambda = 0, \quad (180b)$$

$$d[e^{-\phi/\sqrt{2}} * F_{(2)}] = 0, \quad (180c)$$

$$d[e^{\sqrt{2}\phi} * F_{(3)}] = 0, \quad (180d)$$

where $\triangle$ denotes the Laplace operator with respect to the six-dimensional metric g_{MN}.

We assume the six-dimensional metric of the form (124). The scalar field and the gauge field strengths are assumed to be

$$e^{\phi} = h_2^{-\sqrt{2}/2}h_3^{\sqrt{2}/2}, \quad (181a)$$

$$F_{(2)} = d[\sqrt{2}h_2^{-1}(t,y,z)]\wedge dt, \quad (181b)$$

$$F_{(3)} = d[h_3^{-1}(t,y,z)]\wedge dt\wedge dy. \quad (181c)$$

The Einstein equations (180a) then reduce to

$$\frac{5}{4}h_2^{-1}\partial_t^2 h_2 + \frac{3}{4}h_3^{-1}\partial_t^2 h_3 + \frac{1}{4}h_2^{-2}(3h_2^{-1}\partial_y^2 h_2 + h_3^{-1}\partial_y^2 h_3) + \frac{1}{4}h_2^{-2}h_3^{-1}(3h_2^{-1}\triangle_Z h_2 + h_3^{-1}\triangle_Z h_3) + \frac{1}{2}h_2^{-2}\lambda + \frac{1}{4}(\partial_t \ln h_2)^2 + \frac{7}{4}\partial_t \ln h_2 \partial_t \ln h_3 + \frac{3}{4}h_2^{-2}\partial_y \ln h_2 \partial_y \ln h_3 = 0, \quad (182a)$$

$$h_2^{-1}\partial_t\partial_y h_2 + h_3^{-1}\partial_t\partial_y h_3 + \partial_t \ln h_3 \partial_y \ln h_2 = 0, \quad (182b)$$

$$2h_2^{-1}\partial_t\partial_a h_2 + h_3^{-1}\partial_t\partial_a h_3 = 0, \quad (182c)$$

$$\frac{1}{4}h_2^2(h_2^{-1}\partial_t^2 h_2 - h_3^{-1}\partial_t^2 h_3) - \frac{1}{4}(h_2^{-1}\partial_y^2 h_2 + 3h_3^{-1}\partial_y^2 h_3) - \frac{1}{4}h_3^{-1}(h_2^{-1}\triangle_Z h_2 - h_3^{-1}\triangle_Z h_3) + \frac{1}{2}\lambda + \frac{1}{4}(\partial_t h_2)^2 - \frac{1}{4}h_2^2\partial_t \ln h_2 \partial_t \ln h_3 - \frac{5}{4}\partial_y \ln h_2 \partial_y \ln h_3 = 0, \quad (182d)$$

$$h_3^{-1}\partial_y\partial_a h_3 + 2\partial_y \ln h_2 \partial_a \ln h_2 = 0, \quad (182e)$$

$$R_{ab}(Z) + \frac{1}{4}h_2^2 h_3 u_{ab}(h_2^{-1}\partial_t^2 h_2 + h_3^{-1}\partial_t^2 h_3) - \frac{1}{4}h_3 u_{ab}(h_2^{-1}\partial_y^2 h_2 + h_3^{-1}\partial_y^2 h_3) - \frac{1}{4}u_{ab}(h_2^{-1}\triangle_Z h_2 + h_3^{-1}\triangle_Z h_3) + \frac{1}{4}h_2^2 h_3 u_{ab}[(\partial_t \ln h_2)^2 + 3\partial_t \ln h_2 \partial_t \ln h_3] - \frac{1}{4}h_3 u_{ab}\partial_y \ln h_2 \partial_y \ln h_3 + \frac{1}{2}u_{ab}h_3\lambda = 0, \quad (182f)$$

where $\triangle_Z$ denotes the Laplace operator on Z space and $R_{ab}(Z)$ is the Ricci tensor constructed from the metric $u_{ab}(Z)$.

We next consider the gauge field. Under the ansatz (181), the Bianchi identity is automatically satisfied. Also the equation of motion for the gauge field becomes

$$d[h_3^{-1}\partial_y h_2 \Omega(Z) + \partial_a h_2 dy\wedge(*_Z dz^a)] = 0, \quad (183a)$$

$$d[\partial_a h_3(*_Z dz^a)] = 0, \quad (183b)$$

where $*_Z$ denotes the Hodge operator on Z.

Although the roles of the Bianchi identity and field equations are interchanged, the net result is the same. Finally, we consider the equation of motion for the scalar field. Substituting the scalar field and the gauge field in (181) into the equation of motion for the scalar field (180b), we have

$$h_2^2 h_3(h_2^{-1}\partial_t^2 h_2 - h_3^{-1}\partial_t^2 h_3) + h_3(\partial_t h_2)^2 - h_2\partial_t h_2\partial_t h_3 - h_3(h_2^{-1}\partial_y^2 h_2 - h_3^{-1}\partial_y^2 h_3) - h_2^{-1}\partial_y h_2\partial_y h_3 - h_2^{-1}\triangle_Z h_2 + h_3^{-1}\triangle_Z h_3 + 2h_3\lambda = 0. \quad (184)$$

Then, the functions h_2 and h_3 satisfy the equations

$$(\partial_t h_2)^2 + 2\lambda + h_2\partial_t^2 h_2 - h_2^{-1}\triangle_W h_2 = 0, \quad \text{for } h_3 = 1, \quad (185a)$$

$$-\partial_t^2 h_3 + \partial_y^2 h_3 + 2\lambda h_3 + h_3^{-1}\triangle_Z h_3 = 0, \quad \text{for } h_2 = 1, \quad (185b)$$

where Laplace operator $\triangle_W$ is defined in Eq. (132). If we set $h_2 = 1$, the field equations give

$$R_{ab}(\mathrm{Z}) = 0, \tag{186a}$$

$$h_2 = 1, \qquad \lambda = 0, \qquad \partial_t^2 h_3 = \partial_y^2 h_3 = 0, \qquad \triangle_{\mathrm{Z}} h_3 = 0. \tag{186b}$$

Now we will focus upon a case by imposing the conditions

$$u_{ab} = \delta_{ab}, \qquad h_2 = 1, \qquad \lambda = 0, \tag{187}$$

where δ_{ab} is the four-dimensional Euclidean metric. Then, the solution for h_3 can be obtained explicitly as

$$h_3(t, y, z) = c_1 t + c_2 y + c_3 + \sum_{l=1}^{N} \frac{M_l}{|z^a - z_l^a|^2}, \tag{188}$$

where $c_i (i = 1, 2, 3)$ are constants.

One can easily get the solution for $h_3 = 1$, $\lambda \neq 0$, and $\partial_t h_2 \neq 0$ if the roles of h_2 and h_3 are exchanged. The solution of field equations is thus expressed as

$$h_2(t, v) = \pm\sqrt{2i\lambda}t + c_5 + \sum_{\alpha=1}^{N'} \frac{L_\alpha}{|v^m - v_\alpha^m|^3}, \tag{189a}$$

$$h_3 = 1, \tag{189b}$$

where c_5, v_α^m, and L_α are constants and the five-dimensional coordinate v^m is defined by (132). Hence, there is no cosmological 0-brane solution in terms of the ansatz of fields (124) and (181) if $\lambda \neq 0$.

E. Romans' five-dimensional gauged supergravity

Finally, we consider the five-dimensional Romans' theory [82]. The five-dimensional action is given by

$$S = \frac{1}{2\kappa^2} \int \Big[(R + 2\mathrm{e}^{2\phi/\sqrt{6}}\bar{\lambda}) * \mathbf{1} - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2 \cdot 2!} \mathrm{e}^{-2\phi/\sqrt{6}} * F_{(2)} \wedge F_{(2)} - \frac{1}{2 \cdot 2!} \mathrm{e}^{4\phi/\sqrt{6}} * H_{(2)} \wedge H_{(2)} \Big], \tag{190}$$

where the expectation value of the Yang-Mills potential is assumed to vanish, R denotes the Ricci scalar constructed from the five-dimensional metric g_{MN}, κ^2 is the five-dimensional gravitational constant, $*$ is the Hodge operator in the five-dimensional spacetime, ϕ denotes the scalar field, $\bar{\lambda} > 0$ is the cosmological constant, $F_{(2)}$ and $H_{(2)}$ are 2-form field strengths, and the couplings of the 2-form field strengths and cosmological constant to the dilaton are given by $\epsilon_r c_r = -2/\sqrt{6}$, $\epsilon_s c_s = 4/\sqrt{6}$, and $\alpha_r = 2/\sqrt{6}$, in the action (1), respectively. This has also a negative scalar potential. In terms of Eq. (2a), Romans' five-dimensional model is given by setting $\Lambda_r = -\bar{\lambda}$, $\Lambda_s = 0$, $N_r = 2$, and $N_s = 4$.

The five-dimensional action (190) gives the field equations:

$$R_{MN} = -\frac{1}{2}\mathrm{e}^{2\phi/\sqrt{6}}\bar{\lambda} g_{MN} + \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{\mathrm{e}^{-2\phi/\sqrt{6}}}{2 \cdot 2!}\left(2F_{MA}F_N{}^A - \frac{1}{3}g_{MN}F_{(2)}^2\right) + \frac{\mathrm{e}^{4\phi/\sqrt{6}}}{2 \cdot 2!}\left(2H_{MA}H_N{}^A - \frac{1}{3}g_{MN}H_{(2)}^2\right), \tag{191a}$$

$$\triangle\phi + \frac{\sqrt{6}}{6 \cdot 2!}\mathrm{e}^{-2\phi/\sqrt{6}}F_{(2)}^2 - \frac{\sqrt{6}}{3 \cdot 2!}\mathrm{e}^{4\phi/\sqrt{6}}H_{(2)}^2 + \frac{2\sqrt{6}}{3}\mathrm{e}^{2\phi/\sqrt{6}}\bar{\lambda} = 0, \tag{191b}$$

$$d[\mathrm{e}^{-2\phi/\sqrt{6}} * F_{(2)}] = 0, \tag{191c}$$

$$d[\mathrm{e}^{4\phi/\sqrt{6}} * H_{(2)}] = 0, \tag{191d}$$

where $\triangle$ denotes the Laplace operator with respect to the five-dimensional metric g_{MN}.

We assume the five-dimensional metric of the form

$$ds^2 = h_2^{2/3}(t, z)k_2^{1/3}(t, z)[-h_2^{-2}(t, z)k_2^{-1}(t, z)dt^2 + u_{ab}(\mathrm{Z})dz^a dz^b], \tag{192}$$

where $u_{ab}(\mathrm{Z})$ is the four-dimensional metric which depends only on the four-dimensional coordinates z^a.

The scalar field and the gauge field strengths are assumed to be

$$\mathrm{e}^\phi = h_2^{-2/\sqrt{6}} k_2^{2/\sqrt{6}}, \tag{193a}$$

$$F_{(2)} = d[\sqrt{2}h_2^{-1}(t, z)] \wedge dt, \tag{193b}$$

$$H_{(2)} = d[k_2^{-1}(t, z)] \wedge dt. \tag{193c}$$

Then, the field equations are reduced to

$$R_{ab}(\mathrm{Z}) = 0, \tag{194a}$$

$$h_2(t, z) = h_0(t) + \bar{h}(z), \qquad k_2(t, z) = k_0(t) + \bar{k}(z), \tag{194b}$$

$$\left(\frac{dh_0}{dt}\right)^2 + 2\bar{\lambda} = 0, \qquad \frac{dh_0}{dt}\frac{dk_0}{dt} = 0, \qquad \frac{d^2h_0}{dt^2} = 0, \qquad \frac{d^2k_0}{dt^2} = 0, \tag{194c}$$

$$\triangle_Z \bar{h} = 0, \qquad \triangle_Z \bar{k} = 0, \tag{194d}$$

where $\triangle_Z$ is the Laplace operator on Z space and $R_{ab}(\mathrm{Z})$ is the Ricci tensor with respect to the metric $u_{ab}(\mathrm{Z})$. By setting $\bar{\lambda} \neq 0$, there is no cosmological solution because of Eq. (194d).

Let us consider the case

$$u_{ab} = \delta_{ab}, \qquad \bar{\lambda} = 0, \qquad \frac{dh_0}{dt} = 0, \tag{195}$$

where δ_{ab} is the four-dimensional Euclidean metric. Then we can construct the solution

$$h_2(z) = c_1 + \sum_{\alpha=1}^{N'} \frac{L_\alpha}{|z^a - z^a_\alpha|^2}, \tag{196a}$$

$$k_2(t, z) = c_2 t + c_3 + \sum_{l=1}^{N} \frac{M_l}{|z^a - z^a_l|^2}, \tag{196b}$$

where $c_i (i = 1, 2, 3)$, L_α, and M_l are the constants.

Now we discuss the cosmological evolution for time-dependent solution (196). We define the cosmic time τ, which is given by

$$\left(\frac{\tau}{\tau_0}\right) \equiv (c_2 t)^{2/3}, \qquad \tau_0 \equiv \frac{3}{2c_2}, \qquad c_3 = 0. \tag{197}$$

The five-dimensional metric can be expressed as

$$ds^2 = h_2^{-4/3}(z)\left[1 + \left(\frac{\tau}{\tau_0}\right)^{-3/2} \bar{k}(z)\right]^{-\frac{2}{3}} \left[-d\tau^2 + h_2^2(z)\left\{1 + \left(\frac{\tau}{\tau_0}\right)^{-3/2} \bar{k}(z)\right\}\left(\frac{\tau}{\tau_0}\right)^{1/2} \delta_{ab}(\mathrm{Z}) dz^a dz^b\right], \tag{198}$$

where the functions $h_2(z)$ and $\bar{k}(z)$ are given, respectively, by

$$h_2(z) = c_1 + \sum_{\alpha=1}^{N'} \frac{L_\alpha}{|z^a - z^a_\alpha|^{2-d_s}},$$
$$\bar{k}(z) \equiv c_3 + \sum_{l=1}^{N} \frac{M_l}{|z^a - z^a_l|^{2-d_s}}. \tag{199}$$

Here d_s is the number of smeared dimensions and should satisfy $0 \leq d_s \leq 3$. Here we assume that one direction of $z^a (a = 1, \ldots, 4)$ is not smeared in order to fix the location of our Universe in the transverse space. Our Universe is given by the solutions with the five-dimensional coordinates $t, z^a (a = 1, \ldots, 4)$. The time direction is written by t. Our choice is to take the three-dimensional from the overall transverse space with z^a. The four-dimensional universe is spanned by t, z^2, z^3, and z^4, for instance. The z^1 direction is preserved to measure the position of our Universe in the overall transverse space of 0-branes. Since the metric depends on z^a explicitly, we have to smear out z^2, z^3, and z^4 so as to define our Universe. Then the number of the smeared directions d_s should satisfy the condition $d_s = 3$.

Unfortunately, the power exponent of a four-dimensional universe becomes $1/4$. Hence, we have to conclude that, in order to obtain a realistic expansion of the universe in this type of models, one has to include additional fields on the background.

We study the asymptotic behavior of the dynamical 0-brane background. The time dependence in the function h_2 can be ignored in the limit of $z^a \to z^a_l$, because the harmonic function $\bar{k}(z)$ dominates near a position of the 0-brane. In the limit of $z^a \to \infty$, as function $\bar{k}(z)$ vanishes, the system becomes static near the 0-brane. Then, the function k_2 depends only on time in the far region from 0-branes. The five-dimensional metric in the limit of $z^a \to \infty$ is thus given by

$$ds^2 = -(c_2 t + c_3)^{-2/3} dt^2 + (c_2 t + c_3)^{1/3} u_{ab} dz^a dz^b. \tag{200}$$

The metric has a singularity at $t = -c_3/c_2$. Then the five-dimensional spacetime does not have any singularity if it is restricted inside the domain satisfied by the conditions

$$h_2(z) = c_1 + \sum_{\alpha=1}^{N'} \frac{L_\alpha}{|z^a - z^a_\alpha|^{2-d_s}} > 0,$$
$$k_2(t, z) = c_2 t + \bar{k}(z) > 0, \tag{201}$$

where the function $\bar{k}(z)$ is defined in (199). The five-dimensional spacetime cannot be extended beyond this region. Since the spacetime evolves into a curvature singularity, the regular spacetime with dynamical 0-branes ends up with the singularities.

Although the evolution of the dynamical 0-brane with $c_2 > 0$ has the time reversal one of $c_2 < 0$, the behavior of the background spacetime strongly depends on the signature of c_2. In the following, we will focus on the case with $c_2 < 0$. For $t < 0$, the function h_2 is positive everywhere. Then the spacetime is not singular. In the limit of $t \to -\infty$, the solution becomes a time-dependent uniform spacetime apart from a position of 0-branes. The five-dimensional background geometry can be described as a cylindrical form of an infinite throat near the dynamical 0-branes.

Let us consider the time evolution of the five-dimensional spacetime. At $t = 0$, the five-dimensional spacetime does not have any curvature singularity in the background. The background geometry has a cylindrical topology near each 0-brane. As time slightly increases, a curvature singularity appears far from 0-branes $|z^a - z^a_\alpha| \to \infty$. After that, the singular hypersurface cuts off more and more of the space as time increases further. The singular hypersurface splits and surrounds each of the 0-brane throats individually after time continues to evolve. The spatial surface is finally composed of two isolated throats. The time evolution of the five-dimensional spacetime for $t < 0$ is the time reversal of $t > 0$.

We find that the overall transverse space tends to expand asymptotically like $t^{1/6}$, for any values of fixed z^a, in the regular domain of the five-dimensional metric (200), while the solutions describe static 0-branes near the positions of the branes. In the far region from 0-branes where $|z^a - z^a_\alpha| \to \infty$, the background geometry becomes FRW universes with the power-law expansion $t^{1/6}$.

Next we consider the case of the near-horizon limit that the spacetime metric and the functions h_2 and k_2 are given by (196). If we consider the case where all 0-branes are located at the origin of the Z space, we have

$$h_2(r) = c_1 + \frac{L}{r^2}, \tag{202a}$$

$$k_2(t,r) = c_2 t + c_3 + \frac{M}{r^2}, \qquad r^2 \equiv \delta_{ab} z^a z^b, \tag{202b}$$

where L and M are the total masses of 0-branes

$$L \equiv \sum_{\alpha=1}^{N'} L_\alpha, \qquad M \equiv \sum_{l=1}^{N} M_l. \tag{203}$$

Since the dependence on t in (202) is negligible in the near-horizon limit $r \to 0$, the five-dimensional metric is reduced to the following form:

$$ds^2 = ds^2_{\mathrm{AdS}_2} + L^{2/3} M^{1/3} d\Omega^2_{(3)}, \tag{204a}$$

$$ds^2_{\mathrm{AdS}_2} \equiv L^{-4/3} M^{-2/3} \left(-r^4 dt^2 + \frac{L^2 M}{r^2} dr^2 \right), \tag{204b}$$

where $\delta_{ab} dz^a dz^b = dr^2 + r^2 d\Omega^2_{(3)}$ has been used. The line elements of a two-dimensional AdS space (AdS_2) and a three-sphere with the unit radius (S^3) are given by $ds^2_{\mathrm{AdS}_2}$ and $d\Omega^2_{(3)}$, respectively. Thus we see that the near-horizon limit of the 0-brane system is an AdS_2 with a certain internal 3-space.

Before closing this subsection, we discuss the collision of 0-branes. There are two kinds of 0-brane in the five-dimensional spacetime. One is a static 0-brane coming from the function $h_2(z)$. The other is a dynamical 0-brane given by $k_2(t,z)$. We set the two dynamical 0-branes at $z_1 = (0, 0, \ldots, 0)$ and $z_2 = (P, 0, \ldots, 0)$, where P is a constant. On the other hand, we suppose that N' static 0-branes are sitting at a point

$$z_1 = \cdots = z_{N'} \equiv z_0 = (z^1_0, 0, \ldots, 0). \tag{205}$$

Now we consider the following quantity:

$$\tilde{z} = \sqrt{(z^2)^2 + (z^3)^2 + \cdots + (z^{4-d_s})^2}. \tag{206}$$

Then the proper length at $\tilde{z} = 0$ between the two dynamical 0-branes is given by

$$d(t) = \int_0^P dz^1 \left(c_1 + \frac{L}{|z^1 - z^1_0|^{2-d_s}} \right)^{1/3} \times \left(c_2 t + c_3 + \frac{M_1}{|z^1|^{2-d_s}} + \frac{M_2}{|z^1 - P|^{2-d_s}} \right)^{1/6}, \tag{207}$$

where M_1 and M_2 are the charges of the dynamical 0-brane and L is defined by

$$L = \sum_{\alpha=1}^{N'} L_\alpha. \tag{208}$$

For $c_2 = -1$, the length $d(t)$ is a monotonically decreasing function of time. Since the time evolution of the proper length depends on the number of the smeared directions d_s, we shall analyze it for each of the values of d_s below.

First we consider the case with $d_s \le 2$. For $d_2 = 2$, the harmonic functions h_2 and k_2 diverge both at infinity and near 0-branes. In particular, there is no regular spacetime region near 0-branes because of $h_2 \to \infty$ and $\bar{k} \to \infty$. Then, these are not physically relevant. Hence, we show the proper length in Fig. 11 for the cases with $d_s = 0$ and $d_s = 1$. For both cases, the singularity between two dynamical 0-branes appears before collision, because a singularity appears before the proper distance becomes zero. Although two dynamical 0-branes initially approach very slowly, the singular hypersurface suddenly appears at a finite distance, and the spacetime finally splits into two isolated 0-brane throats. Therefore, we cannot analyze the collision of the dynamical 0-branes in these examples.

However, for the case with $d_s = 3$, the function h_2 and $\bar{k}$ are written by the linear function of z^a. If we assume that the z^a directions apart from z^1 are smeared, the time evolution of the proper length is different from the previous case. Hence, the harmonic functions h_2 and $\bar{k}$ are expressed, respectively, as

$$h_2(z^1) = c_1 + \sum_{\alpha=1}^{N'} L_\alpha |z^1 - z^1_\alpha|,$$
$$\bar{k}(z^1) = c_3 + \sum_{l=1}^{N} M_l |z^1 - z^1_l|. \tag{209}$$

We study the dynamics of 0-branes, where one 0-brane charge M_1 is located at $z^1 = 0$ and the other M_2 at $z^1 = P$. The proper distance between the two dynamical 0-branes is given by

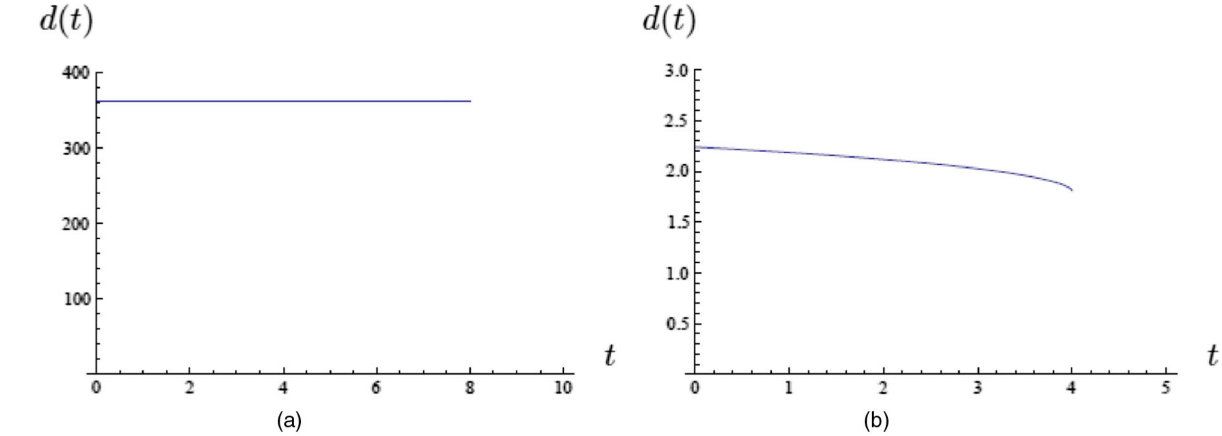

FIG. 11 (color online). The behavior of the proper distance between two dynamical 0-branes for $d_s = 0$ (a) and $d_s = 1$ (b) in the five-dimensional Romans' theory. For both cases, the two dynamical 0-brane charges are identical, $M_1 = M_2 = 1$, and the parameters are taken as $c_1 = 0$, $c_2 = -1$, $c_3 = 0$, $L = 1$, $z^1_0 = 0$, and $P = 1$. The result is also the same, and a singularity appears before the collision of dynamical 0-branes.

$$d(t) = \int_0^P dz^1 (c_1 + L|z^1 - z^1_0|)^{1/3} \times [c_2 t + c_3 + (M_1|z^1| + M_2|z^1 - P|)]^{1/6}, \qquad (210)$$

where we assume again that N' static 0-branes are sitting at a point $z^1 = z^1_0$ and L is defined by (208). For $c_2 < 0$, the proper distance decreases with time. By setting $M_1 \neq M_2$, a curvature singularity appears again at a certain finite time $t = t_S$ before the dynamical 0-branes collide. Then, t_S is written by

$$t_S \equiv -\frac{c_3 + M_1|z^1| + M_2|z^1 - P|}{c_2}. \qquad (211)$$

This is the same result as the case with $d_s \leq 2$.

On the other hand, two 0-branes have the same brane charges $M_1 = M_2 = M$, and the proper distance vanishes at a certain finite time $t = t_c$, where t_c is defined by

$$t_c \equiv -\frac{c_3 + MP}{c_2}. \qquad (212)$$

Then two dynamical 0-branes collide completely.

If we set $z^1_0 = 0$, for simplicity, the proper length between two dynamical 0-branes can be written by

$$d(t) = \frac{3}{4L}[-c_1^{4/3} + (c_1 + LP)^{4/3}][c_2(t - t_c)]^{1/6}. \qquad (213)$$

If we choose the physical parameters as $c_1 = 0$, $c_2 = -1$, $c_3 = 0$, $P = 1$, $z^1_0 = 0$, and $L = 1$, the proper distance $d(t)$

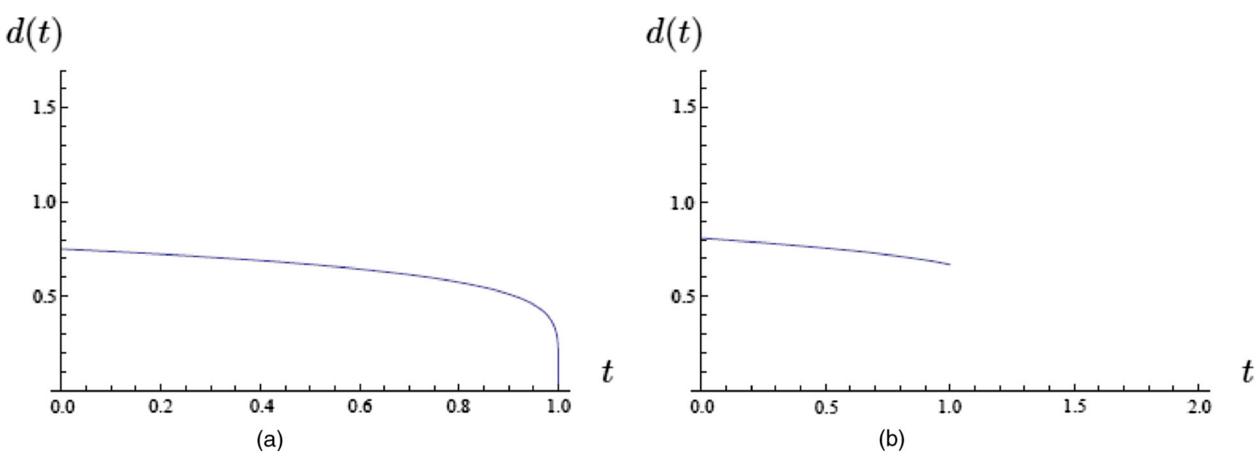

FIG. 12 (color online). The behavior of the proper distance between two dynamical 0-branes for $M_1 = M_2 = 1$ (a) and $M_1 = 2$, $M_2 = 1$ (b) in the five-dimensional Romans' theory. We fix $d_s = 3$, $c_1 = 0$, $c_2 = -1$, $c_3 = 0$, $z^1_0 = 0$, $L = 1$, and $P = 1$. The proper length rapidly vanishes near where two 0-branes collide for the case of $M_1 = M_2 = 1$. For the case of $M_1 = 2$, $M_2 = 1$, it is still finite when a curvature singularity appears.

is depicted in Fig. 12 for the two cases (a) the same 0-brane charges $M_1 = M_2 = 1$ and (b) different charges $M_1 = 2$, $M_2 = 1$. For case (a), the two dynamical 0-branes can collide completely. On the other hand, in case (b), a singularity appears before the collision of dynamical 0-branes, as we have already discussed in Sec. V B.

VI. THE INSTABILITY OF THE DYNAMICAL BRANE BACKGROUND

In this section, we briefly discuss the nature of the singularities appearing in the time-dependent solutions and present the stability analysis for the dynamical brane background. We follow the method used by Refs. [38–41] (see also [83–85]) and present the preliminary analysis performed, where the Klein-Gordon modes are analyzed. An analysis of such a possibility will definitely make the property of singularity more clear, even if it is just a simple preliminary study to assess this issue.

A. The dynamical 0-brane background in Nishino-Salam-Sezgin gauged supergravity

Let us first consider the stability for the 0-brane solution in Nishino-Salam-Sezgin gauged supergravity. The six-dimensional metric becomes static space near the 0-brane, while the background depends only on the times far from the 0-brane. We will study the stability in the 0-brane solution far from the branes.

For the limit $r \to \infty$ in the solution (144), the six-dimensional metric is expressed as

$$ds^2 = -(\epsilon\sqrt{2\Lambda}t + c_4)^{-3/2}dt^2 + (\epsilon\sqrt{2\Lambda}t + c_4)^{1/2}\delta_{mn}(\mathrm{W})dv^m dv^n, \tag{214a}$$

$$\delta_{mn}(\mathrm{W})dv^m dv^n \equiv dr^2 + r^2\omega_{ij}(\mathrm{S}^4)d\xi^i d\xi^j, \tag{214b}$$

where $\omega_{ij}(\mathrm{S}^4)$ denotes the metric of four-dimensional sphere. The six-dimensional metric has a curvature singularity at $t = -c_4/\epsilon\sqrt{2\Lambda}$. In order to study the stability, we consider the Klein-Gordon equation for a massive scalar field propagating in the background (214):

$$-\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\varphi) + m^2\varphi = 0, \tag{215}$$

where g denotes the determinant of the six-dimensional metric (214). In terms of the metric (214), the Klein-Gordon equation can be written by

$$\partial_t[(\epsilon\sqrt{2\Lambda}t + c_4)^2\partial_t\varphi] - r^{-4}\partial_r(r^4\partial_r\varphi) - \frac{1}{r^2}\triangle_{\mathrm{S}^4}\varphi + (\epsilon\sqrt{2\Lambda}t + c_4)^{1/2}m^2\varphi = 0, \tag{216}$$

where $\triangle_{\mathrm{S}^4}$ denotes the Laplace operator on the S^4. The six-dimensional metric involved permits separation of variables, so we take

$$\varphi = \varphi_0(t)\varphi_1(r)\varphi_2(\xi), \tag{217}$$

where the functions $\varphi_1(r)$ and $\varphi_2(\xi)$ obey the eigenvalue equations

$$\triangle_{\mathrm{W}}\varphi_1(r)\varphi_2(\xi) = -\lambda_{\mathrm{W}}^2\varphi_1(r)\varphi_2(\xi). \tag{218}$$

Here $\triangle_{\mathrm{W}}$ is the Laplace operator on the W space, λ_{W} is the eigenvalue, and $\varphi_1(r)$ and $\varphi_2(\xi)$ satisfy [86]

$$\varphi_1(r) = \frac{1}{r}[b_1 J_\nu(\lambda_{\mathrm{W}}r) + b_2 Y_\nu(\lambda_{\mathrm{W}}r)],$$
$$\triangle_{\mathrm{S}^4}\varphi_2(\xi) = -\lambda_{\mathrm{S}^4}^2\varphi_2(\xi), \tag{219}$$

where b_1 and b_2 are constants, J_ν and Y_ν denote the Bessel functions, and ν is related to the eigenvalue $\lambda_{\mathrm{S}^4}^2$ as

$$\nu^2 = \lambda_{\mathrm{S}^4}^2 + \frac{9}{4}. \tag{220}$$

The Klein-Gordon equation thus is rewritten by

$$\frac{d^2\varphi_0}{dt^2} + \frac{2\epsilon\sqrt{2\Lambda}}{(\epsilon\sqrt{2\Lambda}t + c_4)}\frac{d\varphi_0}{dt} + \frac{1}{(\epsilon\sqrt{2\Lambda}t + c_4)^2}[\lambda_{\mathrm{W}}^2 + (\epsilon\sqrt{2\Lambda}t + c_4)^{1/2}m^2]\varphi_0 = 0. \tag{221}$$

Then the solution for φ_0 is oscillatory, having the form

$$\varphi_0(t) = \frac{\Lambda}{2m^2(\epsilon\sqrt{2\Lambda}t + c_4)^{1/2}} \times \left[\eta_1\Gamma(1-\gamma)J_{-\gamma}\left\{\frac{4m(\epsilon\sqrt{2\Lambda}t + c_4)^{1/4}}{\epsilon\sqrt{2\Lambda}}\right\} + \eta_2\Gamma(1+\gamma)J_{\gamma}\left\{\frac{4m(\epsilon\sqrt{2\Lambda}t + c_4)^{1/4}}{\epsilon\sqrt{2\Lambda}}\right\}\right], \tag{222}$$

where η_1 and η_2 are constants, $J_{-\gamma}$ and J_γ denote the Bessel functions, and γ is given by

$$\gamma = 2\sqrt{1 - \frac{2\lambda_{\mathrm{W}}^2}{\Lambda}}. \tag{223}$$

Let us consider the energy of the Klein-Gordon modes to study whether the instability occurs or not. Using the asymptotic solution (222), we will see that $E \to \infty$ as the singularity is approached, where there is a curvature singularity at $t = -c_4/\epsilon\sqrt{2\Lambda}$ in the six-dimensional background (214). Since the velocity is well behaved besides the singularity, the energy of the Klein-Gordon modes can be estimated as

$$E = -u^M \partial_M \varphi, \qquad u = \alpha \partial_t + \beta \partial_r, \tag{224}$$

where u is velocity. In terms of the normalization condition $u^2 = -1$, the behavior of the α and β are determined in order to remain nonsingular. Then, we find

$$-\alpha^2 (\epsilon \sqrt{2\Lambda} t + c_4)^{-3/2} + \beta^2 (\epsilon \sqrt{2\Lambda} t + c_4)^{1/2} = -1. \tag{225}$$

As $t \to 0$, α and β have to behave as

$$\alpha \sim (\epsilon \sqrt{2\Lambda} t + c_4)^{3/4}, \qquad \beta \sim (\epsilon \sqrt{2\Lambda} t + c_4)^{-1/4}, \tag{226}$$

in the limit $r \to \infty$ for the dynamical 0-brane background. Upon setting (226), one then finds

$$-E = \alpha \partial_t \varphi + \beta \partial_r \varphi. \tag{227}$$

In terms of the asymptotic solution (222) with γ (223), we find that $E \to \infty$ as the singularity is approached if we set $\epsilon = 1$ and $\Lambda > 0$. Hence, the 0-brane solution implies that the energy-momentum tensor of the scalar field mode diverges far from the 0-brane. However, it is necessary to study a full analysis of the metric perturbations for whether the mode of Klein Gordon field is not likely to destabilize the metric modes near the singularity or not.

B. The dynamical 1-brane background in Nishino-Salam-Sezgin gauged supergravity

Next we consider the stability for the 1-brane solution in Nishino-Salam-Sezgin gauged supergravity. For the metric (168), the harmonic function $\tilde{h}(z)$ dominates in the limit of $z^a \to z_l^a$ (near a position of 1-branes) and the time dependence can be ignored. Thus the background becomes static. On the other hand, in the limit of $|z^a| \to \infty$, $\tilde{h}(z)$ vanishes. Then h_3 depends on t and y in the far region from 1-branes, and the resulting metric is given by

$$ds^2 = \left[\frac{\Lambda}{2}(t^2 - y^2) + c_1 t + c_2 y + c_3\right]^{-1/2} (-dt^2 + dy^2) + \left[\frac{\Lambda}{2}(t^2 - y^2) + c_1 t + c_2 y + c_3\right]^{1/2} \delta_{ab}(\mathrm{Z}) dz^a dz^b, \tag{228a}$$

$$\delta_{ab}(\mathrm{Z}) dz^a dz^b \equiv dr^2 + r^2 \tilde{\omega}_{ij}(\mathrm{S}^3) d\xi^i d\xi^j, \tag{228b}$$

where $\tilde{\omega}_{ij}(\mathrm{S}^3)$ is the metric of three-dimensional sphere. For the six-dimensional metric, a curvature singularity may appear at

$$\frac{\Lambda}{2}(t^2 - y^2) + c_1 t + c_2 y + c_3 = 0. \tag{229}$$

In the following, we will again discuss the stability in the 1-brane solution far from the branes. Let us consider the Klein-Gordon equation for a massive scalar field in the six-dimensional background (228):

$$-\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \psi) + m^2 \psi = 0, \tag{230}$$

where g denotes the determinant of the six-dimensional metric (228). Substituting the six-dimensional metric (228) into the Klein-Gordon equation for a massive scalar field (230), we find

$$\partial_t \left[\left\{\frac{\Lambda}{2}(t^2 - y^2) + c_1 t\right\} \partial_t \psi\right] - \partial_y \left[\left\{\frac{\Lambda}{2}(t^2 - y^2) + c_1 t\right\} \partial_y \psi\right] - r^{-3} \partial_r (r^3 \partial_r \psi) - \frac{1}{r^2} \triangle_{\mathrm{S}^3} \psi + \left[\left\{\frac{\Lambda}{2}(t^2 - y^2) + c_1 t\right\}\right]^{1/2} m^2 \psi = 0, \tag{231}$$

where we set $c_2 = c_3 = 0$ and $\triangle_{\mathrm{S}^3}$ denotes the Laplace operator on the S^3. The six-dimensional metric involved permits separation of variables, so we take

$$\psi = \psi_0(t) \psi_1(y) \psi_2(r) \psi_3(\xi), \tag{232}$$

where the functions $\psi_2(r)$ and $\psi_3(\xi)$ obey the eigenvalue equations

$$\triangle_{\mathrm{Z}} \psi_2(r) \psi_3(\xi) = -\lambda_{\mathrm{Z}}^2 \psi_2(r) \psi_3(\xi). \tag{233}$$

Here $\triangle_{\mathrm{Z}}$ is the Laplace operator on the Z space, λ_{Z} is the eigenvalue, for Eq. (233), and $\psi_2(r)$ and $\psi_3(\xi)$ are satisfy

$$\psi_2(r) = \frac{1}{r}[b_1 J_\nu(\lambda_{\mathrm{Z}} r) + b_2 Y_\nu(\lambda_{\mathrm{Z}} r)], \qquad \triangle_{\mathrm{S}^3} \psi_3(\xi) = -\lambda_{\mathrm{S}^3}^2 \psi_3(\xi), \tag{234}$$

where b_1 and b_2 are constants, J_ν and Y_ν denote the Bessel functions, and ν is related to $\lambda_{\mathrm{S}^3}^2$:

$$\nu^2 = \lambda_{\mathrm{S}^3}^2 + 1. \tag{235}$$

Hence, the Klein-Gordon equation is reduced to

$$\psi_1 \partial_t \left[\left\{\frac{\Lambda}{2}(t^2 - y^2) + c_1 t\right\} \frac{d\psi_0}{dt}\right] - \psi_0 \partial_y \left[\left\{\frac{\Lambda}{2}(t^2 - y^2) + c_1 t\right\} \frac{d\psi_1}{dy}\right] + \left[\lambda_{\mathrm{Z}}^2 + \left\{\frac{\Lambda}{2}(t^2 - y^2) + c_1 t\right\}^{1/2} m^2\right] \psi_0 \psi_1 = 0. \tag{236}$$

We shall discuss the massless cases in the following. In terms of $c_1 = 0$ and $m = 0$, the particular solutions of ψ_0 and ψ_1 are given, respectively, by

$$\psi_0(t) = \zeta_1 t^{-\frac{1}{2}+\rho} + \zeta_2 t^{-\frac{1}{2}-\rho},$$
$$\psi_1(y) = \sigma_1 y^{-\frac{1}{2}+\rho} + \sigma_2 y^{-\frac{1}{2}-\rho}, \tag{237}$$

where $\zeta_i (i = 1, 2)$ and $\sigma_i (i = 1, 2)$ are constants and ρ is defined by

$$\rho = \frac{1}{2}\sqrt{1 - \frac{4\lambda_{\rm Z}^2}{\Lambda}}. \tag{238}$$

We study the stability in terms of the energy of the Klein-Gordon modes. Using the asymptotic solution given by Eq. (237), we can estimate the energy near the singularity, where there is a curvature singularity at $t = \pm y$ in the six-dimensional background (228). Since the velocity is well defined besides the singularity, the energy of the Klein-Gordon modes can be written as

$$E = -v^M \partial_M \psi, \qquad v = \alpha_t \partial_t + \alpha_y \partial_y + \alpha_r \partial_r, \tag{239}$$

where v is velocity. By using the normalization condition $v^2 = -1$, the behavior of the α_t, α_y, and α_r are determined in order to remain nonsingular. Then, we find

$$(-\alpha_t^2 + \alpha_y^2)\left[\frac{\Lambda}{2}(t^2 - y^2)\right]^{-1/2} + \alpha_r^2\left[\frac{\Lambda}{2}(t^2 - y^2)\right]^{1/2} = -1. \tag{240}$$

In the limit $r \to \infty$ and $t \to 0$, for the dynamical 1-brane background (228), α_t, α_y, and α_r provided

$$(-\alpha_t^2 + \alpha_y^2)^{1/2} \sim \left[\frac{\Lambda}{2}(t^2 - y^2)\right]^{1/4},$$
$$\alpha_r \sim \left[\frac{\Lambda}{2}(t^2 - y^2)\right]^{-1/4}. \tag{241}$$

If we set the parameters (241), one then finds

$$-E = \alpha_t \partial_t \psi + \alpha_y \partial_y \psi + \alpha_r \partial_r \psi. \tag{242}$$

In terms of the asymptotic solution (237), we find that $E \to \infty$ as the singularity is approached at $t = \pm y$.

Let us next consider the case $m = 0$ and $\Lambda = 0$. In the limit $r \to \infty$ for the solution (154), the six-dimensional metric becomes

$$ds^2 = (c_1 t + c_4)^{-1/2}(-dt^2 + dy^2) + (c_1 t + c_4)^{1/2}\delta_{ab}({\rm Z})dz^a dz^b, \tag{243a}$$

$$\delta_{ab}({\rm Z})dz^a dz^b \equiv dr^2 + r^2\omega_{ij}({\rm S}^3)d\xi^i d\xi^j, \tag{243b}$$

where we set $c_2 = c_3 = 0$ and $\omega_{ij}({\rm S}^3)$ denotes the metric of the three-dimensional sphere. There is a curvature singularity at $t = -c_4/c_1$.

Now we study the behavior of the Klein-Gordon field. The scalar field equation (230) in the six-dimensional background (243) reads

$$\partial_t(c_1 t \partial_t \psi) - \partial_y(c_1 t \partial_y \psi) - r^{-3}\partial_r(r^3 \partial_r \psi) - \frac{1}{r^2}\triangle_{{\rm S}^3}\psi + (c_1 t)^{1/2} m^2 \psi = 0. \tag{244}$$

If we assume that the scalar field ψ is given by (232), where $\psi_2(r)$ and $\psi_3(\xi)$ can be written by (234), the function $\psi_1(y)$ is determined by the eigenvalue equation:

$$\frac{d^2\psi_1}{dy^2} = -\lambda_y^2 \psi_1. \tag{245}$$

Here, λ_y is constant. Then, the equation for ψ_0 becomes

$$\frac{d}{dt}\left(c_1 t \frac{d\psi_0}{dt}\right) + [\lambda_y^2 c_1 t + \lambda_{\rm Z}^2 + (c_1 t)^{1/2} m^2]\psi_0 = 0. \tag{246}$$

For the massless case, the solution of ψ_0 is given by the oscillatory form

$$\psi_0(t) = {\rm e}^{-i\lambda_y t}[f_1 U(\vartheta, 1, 2i\lambda_y t) + f_2 L_{-\vartheta}(2i\lambda_y t)], \tag{247}$$

where U denotes the hypergeometric function, $L_{-\vartheta}$ is the Laguerre polynomial, f_1 and f_2 are constants, and ϑ is defined by

$$\vartheta = \frac{1}{2} + \frac{i\lambda_{\rm Z}^2}{2c_1\lambda_y}. \tag{248}$$

We estimate the energy of the Klein-Gordon modes whether the instability exists or not. In terms of the asymptotic solution (247), we can present the behavior of the energy as the singularity is approached. Since the velocity is well behaved except for the singularity, the energy of the Klein-Gordon modes is given by (239). By using the normalization condition $v^2 = -1$, α_t, α_y, and α_r are determined by

$$(-\alpha_t^2 + \alpha_y^2)(c_1 t)^{-1/2} + \alpha_r^2 (c_1 t)^{1/2} = -1. \tag{249}$$

As $t \to 0$ and $r \to \infty$, the functions α_t, α_y, and α_r are set to be

$$(-\alpha_t^2 + \alpha_y^2)^{1/2} \sim (c_1 t)^{1/4}, \qquad \alpha_r \sim (c_1 t)^{-1/4}. \tag{250}$$

If we use Eq. (250), the energy can be expressed as

$$-E = \alpha_t \partial_t \psi + \alpha_y \partial_y \psi + \alpha_r \partial_r \psi. \tag{251}$$

Then, for the asymptotic solution (247), the energy becomes $E \to \infty$ as the singularity is approached, that is, $t \to 0$. Since the 1-brane solution gives that the energy-momentum tensor of the Klein-Gordon field mode diverges in this limit, the mode of the scalar field cannot stabilize the metric modes near the singularity.

C. The dynamical 0-brane background in five-dimensional Romans' gauged supergravity

In this subsection, we analyze the stability of the dynamical 0-brane background in the five-dimensional Romans' gauged supergravity. We will study the stability of the scalar field far from 0-branes. For $r \to \infty$ in the dynamical 0-brane background (202), the five-dimensional metric becomes

$$ds^2 = -(c_2 t + c_3)^{-2/3} dt^2 + (c_2 t + c_3)^{1/3} \delta_{ab}(\mathrm{Z}) dz^a dz^b, \tag{252a}$$

$$\delta_{ab}(\mathrm{Z}) dz^a dz^b \equiv dr^2 + r^2 \omega_{ij}(\mathrm{S}^3) d\xi^i d\xi^j, \tag{252b}$$

where we set $c_1 = 1$ and $\omega_{ij}(\mathrm{S}^3)$ denotes the metric of the three-dimensional sphere. There is a curvature singularity at $t = -c_3/c_2$.

Let us consider the behavior of the Klein-Gordon field:

$$-\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \varphi) + m^2 \varphi = 0, \tag{253}$$

where g denotes the determinant of the five-dimensional metric (252). The scalar field equation (253) in the five-dimensional background (252) reads

$$\partial_t[(c_2 t + c_3)\partial_t \varphi] - r^{-3}\partial_r(r^3 \partial_r \varphi) - \frac{1}{r^2} \triangle_{\mathrm{S}^3} \varphi + (c_2 t + c_3)^{1/3} m^2 \varphi = 0. \tag{254}$$

Here, $\triangle_{\mathrm{S}^3}$ denotes the Laplace operator on the S^3, and the scalar field φ is assumed to be

$$\varphi = \varphi_0(t)\varphi_1(r)\varphi_2(\xi), \tag{255}$$

where $\varphi_1(r)$ and $\varphi_2(\xi)$ are determined by the eigenvalue equation:

$$\triangle_{\mathrm{Z}} \varphi_1(r)\varphi_2(\xi) = -\lambda_{\mathrm{Z}}^2 \varphi_1(r)\varphi_2(\xi). \tag{256}$$

Here $\triangle_{\mathrm{Z}}$ is the Laplace operator on the Z space, λ_{Z} is the eigenvalue, and functions $\varphi_1(r)$ and $\varphi_2(\xi)$ obey [86]

$$\varphi_1(r) = \frac{1}{r}[\bar{b}_1 J_\nu(\lambda_{\mathrm{Z}} r) + \bar{b}_2 Y_\nu(\lambda_{\mathrm{Z}} r)], \qquad \triangle_{\mathrm{S}^3} \varphi_2(\xi) = -\lambda_{\mathrm{S}^3}^2 \varphi_2(\xi), \tag{257}$$

where $\bar{b}_1$ and $\bar{b}_2$ are constants, J_ν and Y_ν denote the Bessel functions, and ν is related to the eigenvalue $\lambda_{\mathrm{S}^3}^2$ as

$$\nu^2 = \lambda_{\mathrm{S}^3}^2 + 1. \tag{258}$$

By using Eq. (256), the equation for φ_0 becomes

$$\frac{d}{dt}\left[(c_2 t + c_3)\frac{d\varphi_0}{dt}\right] + [\lambda_{\mathrm{Z}}^2 + (c_2 t + c_3)^{1/3} m^2]\varphi_0 = 0. \tag{259}$$

For $m = 0$, the solution of φ_0 is given by the oscillatory form

$$\varphi_0(t) = \bar{f}_1 J_0\left(\frac{2\lambda_{\mathrm{Z}}}{c_2}\sqrt{c_2 t + c_3}\right) + \bar{f}_2 Y_0\left(\frac{2\lambda_{\mathrm{Z}}}{c_2}\sqrt{c_2 t + c_3}\right), \tag{260}$$

where $\bar{f}_1$ and $\bar{f}_2$ are constants. We calculate the energy of the Klein-Gordon modes to study the stability of the dynamical 0-brane background. By using the asymptotic solution (260), we can present the behavior of the energy as the singularity is approached. Since it is possible to calculate the velocity except for the singularity, the energy of the Klein-Gordon modes is given by

$$E = -u^M \partial_M \varphi, \qquad u = \alpha_t \partial_t + \alpha_r \partial_r, \tag{261}$$

where u denotes the velocity in the five-dimensional spacetime. In terms of the normalization condition $u^2 = -1$, α_t and α_r are given by

$$-\alpha_t^2 (c_2 t + c_3)^{-2/3} + \alpha_r^2 (c_2 t + c_3)^{1/3} = -1. \tag{262}$$

As $t \to -c_3/c_2$ and $r \to \infty$, the functions α_t and α_r are described as

$$\alpha_t \sim (c_2 t + c_3)^{1/3}, \qquad \alpha_r \sim (c_2 t + c_3)^{-1/6}. \tag{263}$$

By using Eq. (263), the energy of the scalar field can be expressed as

$$-E = \alpha_t \partial_t \varphi + \alpha_r \partial_r \varphi. \tag{264}$$

For the asymptotic solution (260), one can note that the energy becomes $E \to \infty$ as the singularity is approached. The dynamical 0-brane solution gives that the energy-momentum tensor of the scalar field mode diverges in the limit $t \to -c_3/c_2$. Hence, the mode of the scalar field cannot stabilize the metric modes near the singularity.

D. Intersection involving the $0 - p_I$-brane background in the D-dimensional asymptotically power-law expanding universe

Now we investigate the stability analysis for the dynamical $0 - p_I$-brane background. The geometry of the $0 - p_{I'}$-brane system becomes a static structure near branes, while the background geometry depends only on the time in the far region from branes. By setting $B = 0$ in the D-dimensional background (73), the metric in the limit $z^a \to \infty$ is thus given by

$$ds^2 = -(At)^{a_0} dt^2 + (At)^{b_0} \delta_{ab}(\mathrm{Z}) dz^a dz^b, \tag{265a}$$

$$\delta_{ab}(\mathrm{Z}) dz^a dz^b = dr^2 + r^2 \bar{\omega}_{ij}(\mathrm{S}^{D-2}) d\xi^i d\xi^j, \tag{265b}$$

where $\bar{\omega}_{ij}(\mathrm{S}^{D-2})$ denotes the metric of the $(D-2)$-dimensional sphere and a_0 and b_0 are defined, respectively, by

$$a_0 = -\frac{D-3}{D-2}, \qquad b_0 = \frac{1}{D-2}. \tag{266}$$

The D-dimensional spacetime has singularities at $t = 0$.

Let us consider the Klein-Gordon equation to discuss the stability analysis

$$-\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \varphi) + m^2 \varphi = 0, \tag{267}$$

where g denotes the determinant of the D-dimensional metric (265). Equation (267) on the D-dimensional background (265) becomes

$$\partial_t (At \partial_t \varphi) - r^{-(D-2)} \partial_r (r^{D-2} \partial_r \varphi) - \frac{1}{r^2} \triangle_{\mathrm{S}^{D-2}} \varphi + (At)^{b_0} m^2 \varphi = 0, \tag{268}$$

where $\triangle_{\mathrm{S}^{D-2}}$ denotes the Laplace operator on the S^{D-2}. We assume that the scalar field φ is given by

$$\varphi = \varphi_0(t) \varphi_1(r) \varphi_2(\xi), \tag{269}$$

where the functions $\varphi_1(r)$ and $\varphi_2(\xi)$ obey the eigenvalue equations

$$\triangle_{\mathrm{Z}} \varphi_1(r) \varphi_2(\xi) = -\lambda_{\mathrm{Z}}^2 \varphi_1(r) \varphi_2(\xi). \tag{270}$$

Here $\triangle_{\mathrm{Z}}$ denotes the Laplace operator on the Z space, and λ_{Y} is the eigenvalue for the equation.

The functions $\varphi_1(r)$ and $\varphi_2(\xi)$ also satisfy the equations [86]

$$\varphi_1(r) = \frac{1}{r}[b_3 J_{\bar{\nu}}(\lambda_{\mathrm{Z}} r) + b_4 Y_{\bar{\nu}}(\lambda_{\mathrm{Z}} r)],$$
$$\triangle_{\mathrm{S}^{D-2}} \varphi_2(\xi) = -\lambda_{\mathrm{S}^{D-2}}^2 \varphi_2(\xi), \tag{271}$$

where b_3 and b_4 are constants, $J_{\bar{\nu}}$ and $Y_{\bar{\nu}}$ denote the Bessel functions, and $\bar{\nu}$ is related to the eigenvalue $\lambda_{\mathrm{S}^{D-2}}^2$ as

$$\bar{\nu}^2 = \lambda_{\mathrm{S}^{D-2}}^2 + \frac{(D-3)^2}{4}. \tag{272}$$

By using Eqs. (265), (269), and (270), the field equation for φ_0 becomes

$$\frac{d}{dt}\left(At \frac{d\psi_0}{dt}\right) + [\lambda_{\mathrm{Z}}^2 + (At)^{b_0} m^2] \varphi_0 = 0. \tag{273}$$

Let us consider the case of $m = 0$. The solution of φ_0 is given by the oscillating form

$$\varphi_0(t) = f_3 J_0(2\lambda_{\mathrm{Z}} \sqrt{A^{-1} t}) + f_4 Y_0(2\lambda_{\mathrm{Z}} \sqrt{A^{-1} t}), \tag{274}$$

where f_3 and f_4 are constants and J_0 and Y_0 are the Bessel functions. The energy of the Klein-Gordon modes can be calculated by

$$E = -u^M \partial_M \varphi, \qquad u = \alpha \partial_t + \beta \partial_r, \tag{275}$$

where u is velocity. Then, α and β are determined by

$$-\alpha^2 (At)^{a_0} + \beta^2 (At)^{b_0} = -1, \tag{276}$$

where we used the normalization condition $u^2 = -1$. In the case of $t \to 0$ and $r \to \infty$, α and β must behave as

$$\alpha \sim (At)^{-a_0/2}, \qquad \beta \sim (At)^{-b_0/2}, \tag{277}$$

in order to remain nonsingular.

If we use the expression (277), the energy of the scalar field is given by

$$-E = \alpha \partial_t \varphi + \beta \partial_r \varphi. \tag{278}$$

For the asymptotic solution (274), one can note that the energy becomes $E \to \infty$ as the singularity is approached. Hence, the energy-momentum tensor of the Klein-Gordon field mode diverges. The classical solution gives the mode of the scalar field which cannot stabilize the metric modes near the singularity.

E. Intersection involving the $0 - p_{I'}$-brane background in the D-dimensional asymptotically de Sitter universe

Finally, we discuss the stability analysis for the $0 - p_{I'}$-brane in the asymptotically de Sitter universe. If we set $\tilde{c} = 0$ and take $z^a \to \infty$ in the background (97), the D-dimensional metric becomes

$$ds^2 = -d\tau^2 + (c_0 \mathrm{e}^{c_0 \tau})^{2/(D-3)} \delta_{ab}(\mathrm{Z}) dz^a dz^b, \tag{279a}$$

$$\delta_{ab}(\mathrm{Z}) dz^a dz^b = dr^2 + r^2 \bar{\omega}_{ij}(\mathrm{S}^{D-2}) d\xi^i d\xi^j, \tag{279b}$$

where $\bar{\omega}_{ij}(\mathrm{S}^{D-2})$ denotes the metric of the $(D-2)$-dimensional sphere, c_0 is given by (98), and the cosmic time τ is defined by (99). There is a curvature singularity at $\tau \to -\infty$ in the D-dimensional spacetime. In the following, we set $c_0 > 0$. Otherwise, the scale factor of D-dimensional spacetime becomes complex or negative.

We consider the Klein-Gordon field to analyze the stability

$$-\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\varphi) + m^2\varphi = 0, \tag{280}$$

where g is the determinant of the six-dimensional metric (279). Substituting the D-dimensional metric (279) into Eq. (280), we obtain

$$(c_0 e^{c_0\tau})^{-1}\partial_\tau[(c_0 e^{c_0\tau})^{\frac{D-1}{D-3}}\partial_\tau\varphi] - r^{-(D-2)}\partial_r(r^{D-2}\partial_r\varphi) - \frac{1}{r^2}\triangle_{S^{D-2}}\varphi + (c_0 e^{c_0\tau})^{\frac{2}{D-3}}m^2\varphi = 0, \tag{281}$$

where $\triangle_{S^{D-2}}$ denotes the Laplace operator on the S^{D-2}.

We assume an ansatz for the scalar field φ:

$$\varphi = \varphi_0(\tau)\varphi_1(r)\varphi_2(\xi), \tag{282}$$

where the functions $\varphi_1(r)$ and $\varphi_2(\xi)$ satisfy the eigenvalue equation

$$\triangle_Z\varphi_1(r)\varphi_2(\xi) = -\lambda_Z^2\varphi_1(r)\varphi_2(\xi), \tag{283}$$

and obey the equations

$$\varphi_1(r) = \frac{1}{r}[b_5 J_{\tilde\nu}(\lambda_Z r) + b_6 Y_{\tilde\nu}(\lambda_Z r)],$$
$$\triangle_{S^{D-2}}\varphi_2(\xi) = -\lambda^2_{S^{D-2}}\varphi_2(\xi). \tag{284}$$

Here $\triangle_Z$ is the Laplace operator on the Z space, b_5 and b_6 denote constants, $J_{\tilde\nu}$ and $Y_{\tilde\nu}$ are the Bessel functions, and $\tilde\nu$ is written by the eigenvalue $\lambda^2_{S^{D-2}}$ as

$$\tilde\nu^2 = \lambda^2_{S^{D-2}} + \frac{(D-3)^2}{4}. \tag{285}$$

In terms of Eqs. (270), (279), and (282), the field equation for φ_0 becomes

$$(c_0 e^{c_0\tau})^{-1}\frac{d}{d\tau}\left[(c_0 e^{c_0\tau})^{(D-1)/(D-3)}\frac{d\varphi_0}{d\tau}\right] + [\lambda_Z^2 + (c_0 e^{c_0\tau})^{2/(D-3)}m^2]\varphi_0 = 0. \tag{286}$$

Let us first consider the solution of φ_0 for $D = 5$ and $D = 6$. In the case of $D = 5$, the solution of φ_0 can be expressed as

$$\varphi_0(\tau) = \lambda_Z^2 c_0^{-3} e^{-c_0\tau}[f_5\Gamma(1-\ell_1)J_{-\ell_1}(2\lambda_Z c_0^{-3/2}e^{-c_0\tau/2}) + f_6\Gamma(1+\ell_1)J_{\ell_1}(2\lambda_Z c_0^{-3/2}e^{-c_0\tau/2})], \tag{287}$$

where f_5 and f_6 are constants, J_{ℓ_1} and $J_{-\ell_1}$ are the Bessel functions, and ℓ_1 is defined by

$$\ell_1 = 2\sqrt{1-\left(\frac{m}{c_0}\right)^2}. \tag{288}$$

On the other hand, setting $D = 6$, we can also find the solution of φ_0:

$$\varphi_0(\tau) = \frac{9}{4}\sqrt{\frac{3}{2}}\lambda_Z^{5/2}c_0^{-10/3}e^{-5c_0\tau/6} \times [f_7\Gamma(1-\ell_2)J_{-\ell_2}(3\lambda_Z c_0^{-4/3}e^{-c_0\tau/3}) + f_8\Gamma(1+\ell_2)J_{\ell_2}(3\lambda_Z c_0^{-4/3}e^{-c_0\tau/3})]. \tag{289}$$

Here f_7 and f_8 denote constants, J_{ℓ_2} and $J_{-\ell_2}$ are the Bessel functions, and the constant ℓ_2 is given by

$$\ell_2 = \frac{1}{2}\sqrt{25-36\left(\frac{m}{c_0}\right)^2}. \tag{290}$$

For $D \geq 4$, the solution of φ_0 can be written in the following form:

$$\varphi_0(\tau) = \left(\frac{D-3}{2}\right)^{\frac{D-1}{2}}\lambda_Z^{\frac{D-1}{2}}c_0^{-\frac{(D-1)(D-2)}{2(D-3)}}e^{-\frac{D-1}{2(D-3)}c_0\tau} \times [\bar f\Gamma(1-\ell)J_{-\ell}((D-3)\lambda_Z c_0^{-\frac{D-2}{D-3}}e^{-\frac{1}{D-3}c_0\tau}) + \tilde f\Gamma(1+\ell)J_\ell((D-3)\lambda_Z c_0^{-\frac{D-2}{D-3}}e^{-\frac{1}{D-3}c_0\tau})], \tag{291}$$

where $\bar f$ and $\tilde f$ are constants, J_ℓ and $J_{-\ell}$ denote the Bessel functions, and ℓ is defined by

$$\ell = \frac{1}{2}\sqrt{(D-1)^2 - 4(D-3)^2\left(\frac{m}{c_0}\right)^2}. \tag{292}$$

Since the energy of the scalar field can be written as (275), the energy of the Klein-Gordon modes can be given by the expression (278). Then, we can find α and β in this way:

$$-\alpha^2 + \beta^2(c_0 e^{c_0\tau})^{2/(D-3)} = -1, \tag{293}$$

where we used the normalization condition $u^2 = -1$. In the case of $\tau \to -\infty$ and $r \to \infty$, α and β have to behave as

$$\alpha \sim \text{const}, \qquad \beta \sim (c_0 e^{c_0\tau})^{-1/(D-3)}. \tag{294}$$

For the solution (291), the energy becomes $E \to \infty$ in the limit $\tau \to -\infty$. Since the energy is not convergent with the asymptotic solution, the mode of the scalar field does not stabilize the metric modes near the singularity.

VII. CONCLUSION AND DISCUSSION

In this paper, we have discussed the time-dependent intersecting branes with cosmological constants for not only the delocalized case but also the partially localized one in D-dimensional gravitational theory. We are everywhere brief, and on some points, we simply call attention to questions that might be investigated in the future. The function h_I depends on time as well as the coordinate of the relative and overall transverse spaces. The coupling constants between the field strengths and the dilaton are given by the assumptions (7) or (49) and depend on the parameter N. In the case of the 11- or ten-dimensional supergravity theory, the dilaton coupling requires $N = 4$. The power of the time dependence depends on the number of the brane and total dimensions with the parameter N of the dilaton coupling constant.

An exceptional case arises if the parameter N in the dilaton coupling takes another value than 4. There are static solutions with $N \neq 4$ in the lower-dimensional supergravity theories as well as Einstein-Maxwell theory [34,35]. In this case one gets asymptotically power-law expanding solutions if the dilaton is nontrivial. For the trivial dilaton, the Einstein equations give an asymptotically de Sitter solution for a single 2-form field strength. Since the cosmological constant is related to the field strength, the time derivative of the warp factor arises only from the Ricci tensor and can be compensated by the cosmological constant in the Einstein equations. This is the same structure as in Refs. [21,22,36] and the generalization of the solutions [36,63,64]. In $N = 4$ case, the equation of motion in the presence of the cosmological constant gives the static delocalized or partially localized intersecting brane solution because of the ansatz of the fields. Thus, one expects that the recipe for picking an accelerating expansion from the dynamical intersecting brane solutions depends on the dilaton coupling constant, and this is the case for the proposal in Ref. [22]. Once the de Sitter solution is obtained in the single p-brane solutions, it is possible to apply it to the intersecting brane systems.

An immediate point is that the time-dependent solutions make dynamical compactification more or less obvious, since cosmological evolution is a general property of the solution (with constant parameters) once the function h_I is properly endowed with the time dependence. The power of the scale factor in some solutions gives an accelerating expansion law even in the case that functions h_I depend on both the time and coordinates of overall transverse space, while the extra dimension will shrink as cosmic time increases. However, something is still missing, because the scale factor of our Universe diverges at $\tau = \tau_\infty$. At the moment, it is not clear how to do this.

We have discussed the dynamics of the brane collisions. As the spacetime is contracting in the D-dimensional spacetime, each 0-brane approaches others as the time evolves for $\tau < 0$ but separates for $\tau > 0$ in the asymptotically power-law expanding solutions. Thus 0-branes never collide. In the case of asymptotically de Sitter solutions, all domains between branes are connected at $\tau = 0$ ($c_0 < 0$). The domain shrinks as the time decreases, while the proper distance becomes constant as τ increases. For the $0 - p_{I'}$-brane system ($p \leq 7$), a singularity appears before 0-branes collide, and eventually the topology of the spacetime changes so that branes are separated by singular hypersurfaces surrounding each brane if branes are not smeared. Thus, we cannot describe the collision of two 0-branes in terms of these solutions. On the other hand, the $0 - 8$-brane system in ten dimensions or the smeared $0 - p_I$-brane system in D-dimensional theory can provide examples of colliding branes if they have the same brane charges and only one overall transverse space. We have analyzed the collision of the brane where the $p_0 - p_I$-branes are localized at the same position along the overall transverse directions, in the case of equal charges. The brane collision would not occur if the brane charges are different. Moreover, if these branes are localized at different positions, it raises the possibility that the curvature singularities appear.

We have also studied the dynamics of the five- or six-dimensional supergravity models with applications to cosmology and collision of branes. First we have discussed the brane solutions to study the time evolution in the NSS model. In the case of vanishing 3-form field strength in the five-dimensional effective theory, the scale factor of our four-dimensional spacetime is a linear function of the cosmic time which is the same evolution as the Milne universe. On the other hand, for the dynamical 1-brane without 2-form field strength, the solution tells us that the function h depends on all the world-volume coordinates of the 1-brane. Hence, the contribution of the field strength except for the 2-form leads to an inhomogeneous universe. We have investigated the dynamics of 0-branes and found that, when the spacetime is contracting in six dimensions, each 0-brane approaches the others as the time evolves. All domains between branes connected initially ($t = 0$), but it shrinks as t increases. However, for the 0-brane system without smearing branes, a singularity appears before 0-branes collide, and eventually the topology of the spacetime changes such that parts of the branes are separated by a singular region surrounding each brane. Thus, the solution cannot describe the collision of two 0-branes. In contrast, the smeared 0-brane system with $d_s = 4$ can provide an example of colliding 0-branes and collision of the universes, if they have the same brane charges.

We have next constructed the time-dependent 1-brane solution in the NSS supergravity model. In the asymptotic far 1-brane region, the 1-brane spacetime in the NSS model approaches the six-dimensional Milne universe. In regions close to the 1-branes, for concreteness, we have studied the case of two 1-branes in detail. The 1-brane is approaching the other as the time progress for $t < 0$. We have found that, in the case of $t < 0$, all of the domains between the 1-branes are initially connected, but some region (near small y) shrinks as the time increases, and eventually the topology of the spacetime changes such that parts of the branes are separated by a singular region surrounding each 1-brane. Thus, in the case of $d_s \neq 3$, 1-branes never collide. On the other hand, the case of $d_s = 3$, for $t < 0$, could provide an example of colliding 1-branes. We found that the collision time depends on both brane charges and the place in the world volume of the 1-brane. Since this case has the time-reversal symmetry, the evolution for $t > 0$ is obtained by the time-reversal transformation.

We also investigated the time-dependent solution in the five-dimensional supergravity model. The power of the scale factor is so small that the solutions cannot give a realistic expansion law. Then, it is necessary to include additional matter on the background in order to obtain a realistic expanding universe.

We finally analyzed the classical instability of the dynamical brane background towards singularity. In order to present the instability of the dynamical brane background, we have estimated whether an instability does exist by computing the energy of the Klein-Gordon modes. One can find that the energy seen by an observer diverges as the curvature singularity is approached. This implies that the mode of the scalar field is likely to destabilize the background metric modes near the singularity. Although this result has been given by preliminary analysis, it has made the property of singularity in the dynamical brane background more clear. It is also necessary for us to perform a more rigorous analysis by considering in detail the metric perturbation whether the stability analysis arrives to the same conclusion or not.

A recent study of intersecting systems depending on the time coordinate and overall transverse space shows that all warp factors in the solutions can depend on time [28]. It is interesting to study if similar more general solutions can be obtained by relaxing some of our assumptions. We hope to report on this subject in the near future elsewhere.

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