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Electromagnetic properties of neutrinos in the left-right model

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Within the left-right model contributions to the neutrino dipole magnetic moments coming from the charged gauge bosons $W^{\pm}_{1,2}$ and the singly charged Higgs bosons $\tilde{\delta}^{(\pm)}$ are considered. Calculations show that the Higgs sector contributions to the dipole magnetic moments could exceed the contributions caused by the charged gauge bosons. The resonance transitions in the light left-handed neutrino beam moving in a matter and a magnetic field are investigated in two flavor approximations. Analysis leads to the conclusion that the structure of the heavy neutrino sector admits only three possibilities: (i) the light-heavy neutrino mixing angles θ_{ii} $(i = 1, 2)$ are arbitrary but equal each other whereas the heavy neutrino masses are quasidegenerate; (ii) the heavy neutrino masses are hierarchical $(m_{N_1} < m_{N_2})$ while the angles θ_{ii} are equal to zero; (iii) the light-heavy mixing angles θ_{ii} are equal to each other and the heavy-heavy neutrino mixing is maximal.

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I. INTRODUCTION

At the end of 2002 as a consequence of a series of experiments with solar, atmospheric, and reactor neutrinos the existence of the neutrino oscillations has been established. This, in turn, meant that the neutrino has a mass and the partial lepton flavor violation takes place. At the same time monitoring of the Galaxy by the net of neutrino telescopes aimed to detect a neutrino signal from the expected galactic supernova explosion has been started. Neutrinos also find a use for a solution of applied problems as evidenced by the application of antineutrino detectors for nuclear reactor monitoring in the "on-line" regime and the appearance of a neutrino geotomography (for review see [1]). All this puts forward the neutrino physics in the forefront of natural sciences. However, in spite of achieved progress, there is a series of unsolved problems in the neutrino physics. Among these are the following: (i) the smallness of the neutrino mass $m_\nu \approx 10^{-6} m_e$ (m_e is an electron mass); (ii) electromagnetic neutrino properties; (iii) the neutrino nature (Dirac or Majorana).

Models with the seesaw mechanism give successful explanation of the first problem. In these models heavy right-handed neutrinos being seesaw partners of light left-handed neutrinos appear. The introduction of heavy neutrinos N_i $(i = 1, 2, 3)$ helps to solve some cosmological problems as well. For example, these neutrinos are used for explanation of the observed baryon asymmetry in the Universe thanks to the leptogenesis [2].

The existence of nonzero neutrino multipole moments is a theoretically interesting issue in neutrino physics. Whether they are also experimentally relevant quantities depends on their magnitudes. Our interest in electromagnetic neutrino properties is primarily caused by the fact that there exist plenty of astrophysical systems with intensive magnetic fields where neutrino physics plays an important part. Large magnetic fields are present in supernovas, neutron stars, and white dwarfs, and fields as large as $B_e = m_e^2/e \simeq 4.41 \times 10^{13}$ G can arise in supernova explosions or coalescing neutron stars. The remnants of such astrophysical cataclysms are magnetars, young neutron stars with magnetic fields 10^{14}–10^{16} G. It has been suggested that during the electroweak phase transition local magnetic fields much stronger than those of a magnetar could have existed, with field strength as high as 10^{22}–10^{24} G [3]. Unveiling the interconnection between the star magnetic field and its particle current flows could shed new light on the question of the star evolution. Thus, neutrinos drive supernova dynamics from beginning to end. Neutrino emissions and interactions play a crucial role in core collapse supernova. Their eventual emission from a protoneutron star contains nearly all the energy released in the star explosion. The neutrino behavior also explains the tremendous pulsar velocity [4]. Therefore, investigation of the neutrino electromagnetic properties will give very important information for better understanding of particle physics and cosmology.

In the $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model (SM) neutrinos are massless particles and, as a result, the mixing of neutrino states does not take place. Reconstruction of the neutrino sector of the SM is usually achieved by introducing a right-handed neutrino singlet (minimally extended SM) to make neutrinos massive Dirac particles. However, as this takes place, the explanation of the neutrino mass smallness is absent. Neutrino dipole magnetic moments (DMMs) predicted by the SM are so small that they are not of any physical interest. It also should be noted that in the SM the satisfactory mechanism to produce a baryon asymmetry in the universe is absent. All this taken together provides strong evidence of physics beyond the SM.

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The purpose of this work is to investigate neutrino electromagnetic properties in the context of the left-right model. In the next section a short description of the model is given. In Sec. II the motion of the high-energy beam of the left-handed neutrinos in a matter and a twisting magnetic field is examined. In Sec. III contributions to the neutrino DMMs coming both from charged gauge bosons and from singly charged Higgs bosons are considered. Finally in Sec. IV we summarize the results obtained.

II. THE LEFT-RIGHT MODEL

For the first time the model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group (LRM) was proposed at the beginning of the 1970s [5]. Then several versions of this model, which are distinguished by the choice of the transformation to the mass eigenstate basis in the space of neutral gauge bosons [6–10], appeared. This choice is determined both by the Higgs sector structure and by the gauge coupling constant values of the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ gauge groups. In Refs. [11,12] it was shown that all versions of the LRMs can be unified into the so-called continuous LRM, which is characterized by the orientation angle of the $SU(2)_R$ generator in the group space.

There are two possibilities of defining the left-right (LR) symmetry, namely, as a generalized parity P and as a generalized charge conjugation C. In Ref. [13] these two cases have been investigated in order to determine the precise lower limit on the LR symmetry scale conventionally identified with the mass of the additional charged gauge boson $W_2^{\pm}$. It was found that $m_{W_2} > 2.5$ TeV if $LR = C$ and $m_{W_2} > 4$ TeV if $LR = P$. Recall, for $g_R \approx g_L$ there is the theoretical relation connecting the masses of the additional charged and neutral gauge bosons

$$m_{Z_2} \approx 1.7 m_{W_2},$$

so that indirect limits via the bounds on the $W_2^{\pm}$ boson mass also yield more stringent constraints on the Z_2 boson mass.

In the LRM quarks and leptons enter into the left- and right-handed doublets

$$\left.\begin{array}{llll}
Q_L^i\left(\frac{1}{2},0,\frac{1}{3}\right): & \begin{pmatrix} u_L \\ d_L \end{pmatrix}^{\alpha}, & \begin{pmatrix} c_L \\ s_L \end{pmatrix}^{\alpha}, & \begin{pmatrix} t_L \\ b_L \end{pmatrix}^{\alpha}, \\
Q_R^i\left(0,\frac{1}{2},\frac{1}{3}\right): & \begin{pmatrix} u_R \\ d_R \end{pmatrix}^{\alpha}, & \begin{pmatrix} c_R \\ s_R \end{pmatrix}^{\alpha}, & \begin{pmatrix} t_R \\ b_R \end{pmatrix}^{\alpha}, \\
\Psi_L^a\left(\frac{1}{2},0,-1\right): & \begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix}, & \begin{pmatrix} \nu_{\mu L} \\ \mu_L^- \end{pmatrix}, & \begin{pmatrix} \nu_{\tau L} \\ \tau_L^- \end{pmatrix}, \\
\Psi_R^a\left(0,\frac{1}{2},-1\right): & \begin{pmatrix} N_{eR} \\ e_R^- \end{pmatrix}, & \begin{pmatrix} N_{\mu R} \\ \mu_R^- \end{pmatrix}, & \begin{pmatrix} N_{\tau R} \\ \tau_R^- \end{pmatrix},
\end{array}\right\} \tag{1}$$

where $i = 1, 2, 3$, $\alpha = R, G, B$, $a = e, \mu, \tau$, in brackets the values of S_L^W, S_R^W and $B - L$ are given, S_L^W (S_R^W) is the weak left (right) isospin while B and L are the baryon and lepton numbers, respectively. The LRM has three gauge coupling constants: g_L, g_R, and g' for the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ groups, respectively. The Higgs sector structure of the LRM determines the neutrino nature. The mandatory element of the Higgs sector is the bidoublet $\Phi(1/2, 1/2, 0)$

$$\Phi = \begin{pmatrix} \Phi_1^0 & \Phi_2^+ \\ \Phi_1^- & \Phi_2^0 \end{pmatrix}. \tag{2}$$

Its nonequal vacuum expectation values (VEVs) of the electrically neutral components bring into existence the masses of quarks and leptons. Then, to ensure that the neutrino is a Majorana particle, the Higgs sector has to contain two triplets $\Delta_L(1, 0, 2)$, $\Delta_R(0, 1, 2)$

$$(\sigma \cdot \Delta_L) = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix},$$
$$(\sigma \cdot \Delta_R) = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}. \tag{3}$$

For the neutrino to be a Dirac particle the Higgs sector instead of Δ_L and Δ_R must include two doublets $\chi_L(1/2, 0, 1)$, $\chi_R(0, 1/2, 1)$ and one bidoublet $\Phi(1/2, 1/2, 0)$. In what follows we shall consider the LRM version with Majorana neutrinos.

The spontaneous symmetry breaking (SSB) according to the chain

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$$

is realized for the following choice of the VEVs:

$$\langle \delta_{L,R}^0 \rangle = \frac{v_{L,R}}{\sqrt{2}}, \qquad \langle \Phi_1^0 \rangle = k_1, \qquad \langle \Phi_2^0 \rangle = k_2. \tag{4}$$

To achieve agreement with experimental data, it is necessary to ensure fulfillment of the conditions

$$v_L \ll \max(k_1, k_2) \ll v_R. \tag{5}$$

The structure of the Higgs potential V_H is the essential element of the theory because it defines the physical states basis of Higgs bosons, Higgs masses, and interactions between Higgses. We shall use the most general form of V_H proposed in Ref. [14]. After the SSB we are left with 14 physical Higgs bosons: four doubly charged scalars $\Delta_{1,2}^{(\pm\pm)}$, four singly charged scalars $\tilde{\delta}^{(\pm)}$ and $h^{(\pm)}$, four neutral scalars $S_{1,2,3,4}$ (S_1 is an analog of the SM Higgs boson), and two neutral pseudoscalars $P_{1,2}$. The detailed discussion of the Higgs sector structure has been done in Ref. [15]. In the third order of the perturbation theory contributions to the neutrino DMMs could give the singly-charged Higgs bosons only. Let us concentrate our attention on them. These bosons are defined as follows

$$h^{(\pm)} = b\Phi_+^{(\pm)} + \frac{ak_0}{v_R}\delta_R^\pm + \frac{d\beta k_0^2}{(\alpha+\rho_1-\rho_3/2)v_R^2}\delta_L^\pm, \quad (6)$$

$$\tilde{\delta}^{(\pm)} = \frac{a\beta k_0}{(\alpha+\rho_1-\rho_3/2)v_R}\delta_R^\pm - d\delta_L^\pm, \quad (7)$$

where

$$\Phi_+^{(\pm)} = \frac{k_1\Phi_1^{(\pm)} + k_2\Phi_2^{(\pm)}}{k_+}, \qquad k_\pm = \sqrt{k_1^2 \pm k_2^2}, \qquad k_0 = \frac{k_-^2}{\sqrt{2}k_+}, \qquad \alpha = \frac{\alpha_3 k_+^2}{2k_-^2}, \qquad \beta = \frac{k_+^2(\beta_1 k_1 + 2\beta_3 k_2)}{2k_-^2 k_0},$$

$$b = \left(1 + \frac{k_0^2}{v_R^2}\right)^{-1/2}, \quad a = \left\{1 + \left[1 + \frac{\beta^2}{(\alpha+\rho_1-\frac{\rho_3}{2})^2}\right]\frac{k_0^2}{v_R^2}\right\}^{-1/2}, \qquad d = \left[1 + \frac{\beta^2 k_0^2}{(\alpha+\rho_1-\frac{\rho_3}{2})^2 v_R^2}\right]^{-1/2},$$

$k_+ = 174$ GeV, and $\beta_1, \beta_3, \alpha_3, \rho_1, \rho_3$ are the constants entering into the Higgs potential. The Lagrangians we need are as follows (for details, see the book [1]):

$$\mathcal{L}_l^h + \mathcal{L}_l^{\tilde{\delta}} = [\alpha_{\nu_a h l_b}\bar{l}_b(x)(1-\gamma_5)\nu_a(x) - \alpha_{N_a h l_b}\bar{l}_b(x)(1+\gamma_5)N_a(x)]h(x)$$
$$- [\alpha_{\nu_a\tilde{\delta}l_b}\overline{l_b^c}(x)(1-\gamma_5)\nu_a(x) - \alpha_{N_a\tilde{\delta}l_b}\overline{l_b^c}(x)(1+\gamma_5)N_a(x)]\tilde{\delta}^*(x) + \text{H.c.}, \quad (8)$$

$$\mathcal{L}_\gamma^h + \mathcal{L}_\gamma^{\tilde{\delta}} = ie\{[\partial_\mu h^*(x)h(x) - h^*(x)\partial_\mu h(x)]A^\mu(x) + (h(x) \longrightarrow \tilde{\delta}(x))\} + \text{H.c.} \quad (9)$$

$$\mathcal{L}_{W\gamma}^h + \mathcal{L}_{W\gamma}^{\tilde{\delta}} = \{i\alpha_{Wh\gamma}[W_{1\mu}(x)\sin\xi + W_{2\mu}(x)\cos\xi]h^*(x) + (h(x) \longrightarrow \tilde{\delta}(x))\}A^\mu(x) + \text{H.c.}, \quad (10)$$

$$\mathcal{L}_l^{cc} = \frac{g_L}{2\sqrt{2}}\bar{l}_a(x)\gamma^\mu(1-\gamma_5)\nu_a(x)W_{L\mu}(x) + \frac{g_R}{2\sqrt{2}}\bar{l}_a(x)\gamma^\mu(1+\gamma_5)N_a(x)W_{R\mu}(x), \quad (11)$$

where

$$\alpha_{\nu_a h l_b} = \frac{h'_{ab}k_2 - h_{ab}k_1}{2k_+}, \qquad \alpha_{N_a h l_b} = \frac{h_{ab}k_2 - h'_{ab}k_1}{2k_+}, \qquad \alpha_{\nu_a\tilde{\delta}l_b} = \frac{f_{ab}}{\sqrt{2}}, \qquad a,b = e,\mu,\tau, \qquad \alpha_{N_a\tilde{\delta}l_b} = \frac{f_{ab}\beta_1 k_+}{2(\alpha+\rho_1-\rho_3/2)v_R},$$

$$\alpha_{W\tilde{\delta}\gamma} = e\beta_1 m_{W_1}, \qquad \alpha = \frac{\alpha_3 k_+^2}{2k_-^2}, \qquad \alpha_{Wh\gamma} = \frac{em_{W_1}(1-\tan^2\beta)(\alpha+\rho_1+1-\rho_3/2)}{1+\tan^2\beta},$$

$$W_1 = W_L\cos\xi + W_R\sin\xi, \qquad W_2 = -W_L\sin\xi + W_R\cos\xi,$$

f_{ab} are triplet Yukawa coupling constants, the superscript c means the charge conjugation operation, the symbols h.c. describe Hermitian conjugate terms, $\tan\beta = k_1/k_2$. In the expressions (8) and (11) the connection between the flavor and mass eigenstate bases, $\Psi^f(x)$ and $\Psi^m(x)$, will look like

$$\Psi^f(x) = \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \\ N_e(x) \\ N_\mu(x) \\ N_\tau(x) \end{pmatrix} = \mathcal{U}\Psi^m(x) = \mathcal{U}\begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \nu_3(x) \\ N_1(x) \\ N_2(x) \\ N_3(x) \end{pmatrix} \quad (12)$$

or, in components

$$\nu_a = \mathcal{U}_{\nu_a,i}\nu_i + \mathcal{U}_{\nu_a,i+3}N_i, \qquad N_a = \mathcal{U}_{N_a i}\nu_i + \mathcal{U}_{N_a i+3}N_i, \qquad (i = 1,2,3),$$

where

$$\mathcal{U} = \mathcal{M}^{\nu N}\begin{pmatrix} \mathcal{D}^{\nu\nu} & 0 \\ 0 & \mathcal{D}^{NN} \end{pmatrix}, \quad \mathcal{M}^{\nu N} = \begin{pmatrix} c_{11} & 0 & 0 & s_{11} & 0 & 0 \\ 0 & c_{22} & 0 & 0 & s_{22} & 0 \\ 0 & 0 & c_{33} & 0 & 0 & s_{33} \\ -s_{11} & 0 & 0 & c_{11} & 0 & 0 \\ 0 & -s_{22} & 0 & 0 & c_{22} & 0 \\ 0 & 0 & -s_{33} & 0 & 0 & c_{33} \end{pmatrix},$$

$$\mathcal{D}^{\eta\eta} = \begin{pmatrix} c^\eta_{12}c^\eta_{13}e^{i\alpha_\eta} & s^\eta_{12}c^\eta_{13}e^{i\beta_\eta} & s^\eta_{13}e^{-i\delta_\eta} \\ -(s^\eta_{12}c^\eta_{23} + c^\eta_{12}s^\eta_{23}s^\eta_{13}e^{i\delta_\eta})e^{i\alpha_\eta} & (c^\eta_{12}c^\eta_{23} - s^\eta_{12}s^\eta_{23}s^\eta_{13}e^{i\delta_\eta})e^{i\beta_\eta} & s^\eta_{23}c^\eta_{13} \\ (s^\eta_{12}s^\eta_{23} - c^\eta_{12}c^\eta_{23}s^\eta_{13}e^{i\delta_\eta})e^{i\alpha_\eta} & -(c^\eta_{12}s^\eta_{23} + s^\eta_{12}c^\eta_{23}s^\eta_{13}e^{i\delta_\eta})e^{i\beta_\eta} & c^\eta_{23}c^\eta_{13} \end{pmatrix},$$

$c_{ii} = \cos\theta_{ii}$, $s_{ii} = \sin\theta_{ii}$, θ_{ii} is the mixing angle between the light and heavy neutrinos in the i generation (light-heavy neutrino mixing), $c^\eta_{ik} = \cos\theta^\eta_{ik}$, $s^\eta_{ik} = \sin\theta^\eta_{ik}$, θ^η_{ik} is the mixing angle between the i and k generations in the sector of the light $\eta = \nu$ (heavy $\eta = N$) neutrinos, δ_η are the CP violating Dirac phases, while the phases α_η, and β_η are known as Majorana phases. For the light neutrinos δ_ν varies between 0 and 2π while α_ν and β_ν vary between 0 and π. The Dirac phases can lead to observable effects in oscillation experiments, whereas the Majorana phases have no effect in those experiments [16]. In its turn the Majorana phases, for example, influence: (i) neutrinoless double-beta decay; (ii) neutrino $\leftrightarrow$ antineutrino oscillation, (iii) rare leptonic decays of K and B mesons, such as $K^\pm \to \pi^\mp l^\pm l^\pm$ and similar modes for the B meson; (iv) leptogenesis in the early universe, which may be responsible for the present matter-antimatter asymmetry, and (v) values of neutrino multipole moments.

The mass squared of the $h^{(\pm)}$- and $\tilde{\delta}^{(\pm)}$-bosons are defined by the relations

$$m_h^2 = \alpha(v_R^2 + k_0^2) + \frac{\beta^2 k_0^2}{\alpha + \rho_1 - \rho_3/2}, \tag{13}$$

$$m_{\tilde{\delta}}^2 = (\rho_3/2 - \rho_1)v_R^2 - \frac{\beta^2 k_0^2}{\alpha + \rho_1 - \rho_3/2}. \tag{14}$$

It is obvious that, depending on the values of the Higgs potential parameters, the masses of the singly charged Higgs bosons may lie on the electroweak scale (EWS) and beyond it. In order for the $h^{(\pm)}$-boson to lie on the EWS the parameter α must be much less than 1. However, among the physical Higgs bosons there is the neutral Higgs boson S_2

$$S_2 = -\Phi_-^{0r}\sin\theta_0 + \Phi_+^{0r}\cos\theta_0, \tag{15}$$

where

$$\Phi_-^0 = \frac{k_1\Phi_2^0 - k_2\Phi_1^0}{k_+}, \qquad \Phi_+^0 = \frac{k_1\Phi_1^0 + k_2\Phi_2^0}{k_+},$$

θ_0 is the mixing angle in the sector of the neutral Higgs bosons ($\theta_0 < k_+^2/v_R^2$) and the superscript r means the real part of the corresponding quantity. The parameter α also enters into the expression for m_{S_2}

$$m_{S_2}^2 = \alpha v_R^2 + \frac{4k_-^4[2(2\lambda_2 + \lambda_3)k_1k_2/k_+^2 + \lambda_4]^2}{\alpha v_R^2}. \tag{16}$$

Since the Lagrangian describing the interaction between quarks and neutral Higgs bosons takes the form

$$\mathcal{L}_q^n = -\frac{1}{\sqrt{2}k_+}\sum_{i,k=1,2,3}\bar{u}_i\left\{\left[m_{u_i}\left(c_{\theta_0} - \frac{2k_1k_2}{k_-^2}s_{\theta_0}\right)S_1 - m_{u_i}\left(s_{\theta_0} + \frac{2k_1k_2}{k_-^2}c_{\theta_0}\right)S_2 - im_{d_i}\gamma_5P_1\right]\delta_{ik} + \frac{k_+^2}{k_-^2}(\mathcal{K}\mathcal{M}_d\mathcal{K}^*)_{ik}(S_1s_{\theta_0} + S_2c_{\theta_0})\right\}u_k + (u_i \to d_i, m_{u_i} \leftrightarrow m_{d_i}, \gamma_5 \to -\gamma_5), \tag{17}$$

where $\mathcal{M}_u$ ($\mathcal{M}_d$) is the diagonal matrix for the up (down) quarks and $\mathcal{K}$ is the Cabibbo-Kobayashi-Maskawa matrix, then in order to suppress large flavor changing neutral currents caused by the S_2-boson, we must demand [13]

$$m_{S_2} \geq 10 \text{ TeV}. \tag{18}$$

Therefore, α must be a finite number and as a result the scenario when the $h^{(\pm)}$ boson lies on the EWS is excluded.

So we see that only the scenario when the $\tilde{\delta}^{(\pm)}$ boson lies on the EWS while the $h^{(\pm)}$ boson occurs on Tev scale is possible. It is realized under conditions

$$\alpha \approx 1, \qquad (\rho_3/2 - \rho_1) \approx \frac{k_+^2 + 3\beta^2 k_0^2}{3v_R^2}. \tag{19}$$

We draw attention to the fact that in this case two physical Higgs bosons S_4 and P_2

$$S_4 = \delta_L^{0r}, \qquad P_2 = \delta_L^{0i}$$

whose masses are as follows

$$m_{S_4}^2 = (\rho_3/2 - \rho_1) v_R^2, \qquad m_{P_2}^2 = (\rho_3/2 - \rho_1) v_R^2 \tag{20}$$

lie on the EWS too. In what follows we shall consider this very scenario.

In the LRM the Higgs sector proves to be connected with the neutrino sector. In the case of two generations a and b, in order to define $\alpha_{\nu_a \tilde{\delta} l_a}$, $\alpha_{N_a \tilde{\delta} l_a}$, $\alpha_{\nu_a h l_a}$, and $\alpha_{N_a h l_a}$ one should use the formulas

$$\left.\begin{aligned} f_{aa} v_R &= s_{11}^2 [(c_{12}^\nu)^2 m_{\nu_1} + (s_{12}^\nu)^2 m_{\nu_2}] + c_{11}^2 [(c_{12}^N)^2 m_{N_1} + (s_{12}^N)^2 m_{N_2}], \\ f_{bb} v_R &= f_{aa} v_R (\theta_{11} \longrightarrow \theta_{22}, \theta_{12}^{\nu,N} \longrightarrow \theta_{12}^{\nu,N} + \pi/2), \end{aligned}\right\} \tag{21}$$

$$\alpha_{\nu_a h l_a} = \frac{1 + \tan^2\beta}{2k_+(1 - \tan^2\beta)} \left(m_D^a - \frac{2 m_{l_a} \tan\beta}{1 + \tan^2\beta} \right), \qquad \alpha_{N_a h l_a} = \alpha_{\nu_a h l_a} (m_{l_a} \leftrightarrow -m_D^a), \tag{22}$$

where

$$\left.\begin{aligned} m_D^a &= c_{aa} s_{aa} [(c_{ab}^N)^2 m_{N_1} + (s_{ab}^N)^2 m_{N_2} - (c_{ab}^\nu)^2 m_{\nu_1} - (s_{ab}^\nu)^2 m_{\nu_2}], \\ m_D^b &= m_D^a (\theta_{aa} \to \theta_{bb}, \theta_{ab}^{\nu,N} \to \theta_{ab}^{\nu,N} + \pi/2). \end{aligned}\right\} \tag{23}$$

with v_R to be estimated by the relation

$$v_R = \sqrt{\frac{(m_{W_2}^2 - m_{W_1}^2) \cos 2\xi}{g_L^2}}. \tag{24}$$

III. NEUTRINO OSCILLATIONS IN MATTER AND MAGNETIC FIELD

It is clear that some information about the neutrino sector structure of the model under consideration could be obtained under oscillation experiments. Let us examine the motion of the high-energy beam of the left-handed electron neutrinos in a matter and a twisting magnetic field

$$B_x \pm i B_y = B_\perp \exp\{\pm i \Phi(z)\}. \tag{25}$$

In the two flavor approximation the object of our investigation represents the system with the wave function $\Psi^T = (\nu_{eL}, \nu_{\mu L}, N_{eR}, N_{\mu R})$ and with the mixing matrix of the form

$$\mathcal{U} = \begin{pmatrix} \cos\theta_{12}^\nu & \sin\theta_{12}^\nu & 0 & 0 \\ -\sin\theta_{12}^\nu & \cos\theta_{12}^\nu & 0 & 0 \\ 0 & 0 & \cos\theta_{12}^N & \sin\theta_{12}^N \\ 0 & 0 & -\sin\theta_{12}^N & \cos\theta_{12}^N \end{pmatrix} \begin{pmatrix} \cos\theta_{11} & 0 & \sin\theta_{11} & 0 \\ 0 & \cos\theta_{22} & 0 & \sin\theta_{22} \\ -\sin\theta_{11} & 0 & \cos\theta_{11} & 0 \\ 0 & -\sin\theta_{22} & 0 & \cos\theta_{22} \end{pmatrix}. \tag{26}$$

The corresponding Hamiltonian is determined by the expression

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_{\nu\nu} & \mathcal{H}_{\nu N} \\ \mathcal{H}_{\nu N}^\dagger & \mathcal{H}_{NN} \end{pmatrix}, \tag{27}$$

where

$$\mathcal{H}_{\nu\nu} = \begin{pmatrix} c^2_{\theta_{11}}\Delta^\nu_c + s^2_{\theta_{11}}\Delta^N_c + c_{2\theta_{11}}\Sigma + V_{eL} - \dot{\Phi}/2 & c_{\theta_{11}}c_{\theta_{22}}\Delta^\nu_s + s_{\theta_{11}}s_{\theta_{22}}\Delta^N_s + \mu_{\nu_e\nu_\mu}B_\perp \\ c_{\theta_{11}}c_{\theta_{22}}\Delta^\nu_s + s_{\theta_{11}}s_{\theta_{22}}\Delta^N_s + \mu_{\nu_e\nu_\mu}B_\perp & -c^2_{\theta_{22}}\Delta^\nu_c - s^2_{\theta_{22}}\Delta^N_c + c_{2\theta_{22}}\Sigma + V_{\mu L} - \dot{\Phi}/2 \end{pmatrix},$$

$$\mathcal{H}_{\nu N} = \begin{pmatrix} \frac{s_{2\theta_{11}}}{2}(\Delta^N_c - \Delta^\nu_c - 2\Sigma) + \mu_{\nu_e N_e}B_\perp & s_{\theta_{11}}c_{\theta_{22}}\Delta^N_s - c_{\theta_{11}}s_{\theta_{22}}\Delta^\nu_s + \mu_{\nu_e N_\mu}B_\perp \\ c_{\theta_{11}}s_{\theta_{22}}\Delta^N_s - s_{\theta_{11}}c_{\theta_{22}}\Delta^\nu_s + \mu_{\nu_e N_\mu}B_\perp & \frac{s_{2\theta_{22}}}{2}(\Delta^\nu_c - \Delta^N_c - 2\Sigma) + \mu_{\nu_\mu N_\mu}B_\perp \end{pmatrix},$$

$$\mathcal{H}_{NN} = \mathcal{H}_{\nu\nu}\left(\theta_{11} \to \theta_{11} + \frac{\pi}{2}, \theta_{22} \to \theta_{22} + \frac{\pi}{2}, V_{eL} \to V_{eR}, V_{\mu L} \to V_{\mu R}, \dot{\Phi} \to -\dot{\Phi}\right), \qquad \dot{\Phi} = \frac{d\Phi}{dt},$$

$$\Delta^\nu_{c(s)} = \frac{m^2_{\nu_2} - m^2_{\nu_1}}{4E}\cos 2\theta^\nu_{12}(\sin 2\theta^\nu_{12}), \qquad \Delta^N_{c(s)} = \frac{m^2_{N_2} - m^2_{N_1}}{4E}\cos 2\theta^N_{12}(\sin 2\theta^N_{12}),$$

$$\Sigma = \frac{m^2_{\nu_1} + m^2_{\nu_2} - m^2_{N_1} - m^2_{N_2}}{8E}, \qquad c_{\theta_{ii}} = \cos\theta_{ii}, \qquad s_{\theta_{ii}} = \sin\theta_{ii}, \qquad c_{2\theta_{ii}} = \cos 2\theta_{ii}, \qquad i = 1, 2,$$

$\mu_{\nu_e\nu_\mu}, \mu_{N_e N_\mu}, \mu_{\nu_e N_e}$, and $\mu_{\nu_e N_\mu}$ are transit dipole neutrino magnetic moments, V_{eL} (V_{eR}) and $V_{\mu L}$ ($V_{\mu R}$) are the matter potentials (MPs) describing the matter interaction with the left(right)-handed electron neutrino and muon neutrino, respectively. In what follows we shall assume that $E_\nu \ll m^2_W/2m_e$, that is, we are constrained by energies to be no higher than 6×10^4 GeV. Then, when calculating the MP by means of the Feynman diagrams, one may neglect the momentum terms in the denominators of the gauge boson propagators. The calculations result in

$$\left.\begin{aligned} &V_{eL} = \sqrt{2}G_F(N_e - N_n/2) + V^H_{eL}, \qquad V_{\mu L} = -\sqrt{2}G_F N_n/2 + V^H_{\mu L}, \\ &V_{eR} = \frac{g^2_R N_e}{4m^2_{W_2}} - \frac{g^2_R c^2_{\theta_W} N_n}{8(c^2_{\theta_W} - s^2_{\theta_W})m^2_{Z_2}}, \qquad V_{\mu R} = -\frac{g^2_R c^2_{\theta_W} N_n}{8(c^2_{\theta_W} - s^2_{\theta_W})m^2_{Z_2}}, \\ &V^H_{aL} = \left(\frac{\alpha^2_{\nu_a he}}{2m^2_h} - \frac{\alpha^2_{\nu_a \tilde{\delta} e}}{2m^2_{\tilde{\delta}}}\right)N_e, \end{aligned}\right\} \tag{28}$$

where N_e (N_n) is the density of electrons (neutrons), $c_{\theta_W} = \cos\theta_W$, $s_{\theta_W} = \sin\theta_W$, θ_W is the Weinberg angle and we have neglected the mixing in the gauge boson sector. As it follows from (27) the sectors of the light and heavy neutrinos prove to be connected. One may neglect this connection only in the situation when

$$|\mathcal{H}_{\nu N}| \ll |\mathcal{H}_{\nu\nu}| \quad \text{and} \quad |\mathcal{H}_{\nu N}| \ll |\mathcal{H}_{NN}|. \tag{29}$$

It is evident that in the simplest case the inequalities (28) take place provided

$$\theta_{11} \approx \theta_{22} \approx 0, \qquad |\mu_{\nu_l N_l}B_\perp| \approx 0. \tag{30}$$

Equalling the corresponding elements of the Hamiltonian (27), we can find all the totality of the resonance conversions in the case under consideration. Under fulfillment of the condition

$$V_{eL} - V_{\mu L} = -(c^2_{\theta_{22}} + c^2_{\theta_{11}})\Delta^\nu_c - (s^2_{\theta_{22}} + s^2_{\theta_{11}})\Delta^N_c + (c_{2\theta_{22}} - c_{2\theta_{11}})\Sigma \tag{31}$$

the $\nu_{eL} \to \nu_{\mu L}$-resonance [Mikheyev-Smirnov-Wolfenstein (MSW) resonance] occurs. Investigation of this resonance with the solar and reactor neutrinos gives the information concerning the mixing parameters of the electron and muon neutrinos. Since the description of the MSW-resonance within the SM is sufficiently successful, then corrections to the SM predictions must be small in any SM extensions. Then, from Eq. (31) it follows that only three versions of the heavy neutrino sector structure are possible: (i) the light-heavy neutrino mixing angles θ_{11} and θ_{22} are arbitrary but equal each other whereas the heavy neutrino masses are quasidegenerate (quasidegenerate masses—QDM), that is, the following must take place

$$\theta_{11} = \theta_{22} \quad \text{and} \quad V^H_{eL} - V^H_{\mu L} = \frac{m^2_{N_1} - m^2_{N_2}}{2E}\cos 2\theta^N_{12}\sin^2\theta_{11}; \tag{32}$$

(ii) the heavy neutrino masses are hierarchical ($m_{N_1} < m_{N_2}$) while the angles θ_{11} and θ_{22} are equal to zero (no masses degeneration—NMD); (iii) $\theta_{11} = \theta_{22}$ and the heavy-heavy neutrino mixing is maximal, $\theta^N_{12} = \pi/4$, and as a result the heavy neutrino masses are hierarchical (maximal heavy-heavy mixing—MHHM).

We are also interested whether heavy right-handed neutrinos can be produced at the expense of oscillations in the high-energy beam of the left-handed light neutrinos. To put this another way, whether the resonant conversions from the light neutrino sector to the heavy one are possible. With the help of the Hamiltonian (27) we conclude that the

$\nu_{eL} \to N_{eR}$ resonance transition would take place under fulfillment of the following condition

$$V_{eL} - V_{eR} - \dot{\Phi} = (s^2_{\theta_{11}} - c^2_{\theta_{11}})(\Delta^\nu_c - \Delta^N_c) - 2c_{2\theta_{11}}\Sigma. \quad (33)$$

It is clear that the QDM-, NMD-, and MHHM-schemes do not allow the existence of this resonance transition. Analogously, production of the heavy muon neutrino $N_{\mu R}$ due to the resonance transition $\nu_{eL} \to N_{\mu R}$ proves to be forbidden since the condition of its existence

$$V_{eL} - V_{eR} - \dot{\Phi} = -(c^2_{\theta_{11}} + s^2_{\theta_{22}})\Delta^\nu_c - (s^2_{\theta_{11}} + c^2_{\theta_{22}})\Delta^N_c - (c_{2\theta_{11}} + c_{2\theta_{22}})\Sigma \quad (34)$$

cannot be realized in all three schemes. So we can infer that, in spite of nonzero values of $\mu_{\nu_a N_b}$, in oscillation experiments with the light neutrinos beam we have no chance to observe the heavy neutrinos production even at energies $E_\nu > m_N$.

IV. NEUTRINO DIPOLE MAGNETIC MOMENTS

Since a neutrino is a neutral particle then its total Lagrangian does not incorporate any multipole moments. These moments arise due to vacuum effects. The vacuum structure, in turn, is governed by the choice of model describing the elementary particle interactions. Electromagnetic properties of a massive Dirac neutrino are determined by four form factors. In this case the most general form of the matrix element for the conserved neutrino electromagnetic current J^{em}_μ is given by the expression [17]:

$$\langle \nu^D_i(p')|J^{em}_\mu|\nu^D_j(p)\rangle = \langle \nu^D_i(p')|i\sigma_{\mu\lambda}q^\lambda[F_M(q^2) + F_E(q^2)\gamma_5] + (q^2\gamma_\mu - q_\mu\hat{q})[F_V(q^2) + F_A(q^2)\gamma_5]|\nu^D_j(p)\rangle, \quad (35)$$

where $q = p' - p$, $F_M(q^2)$, $F_E(q^2)$, $F_A(q^2)$, and $F_V(q^2)$ are the magnetic, electric, anapole and reduced Dirac form factors, respectively. In the static limit ($q^2 = 0$) $F_M(q^2)$ and $F_E(q^2)$ define (anomalous) dipole magnetic moment μ_{ij} and dipole electric moment d_{ij}, respectively. At $i = j$ and $q^2 = 0$, $F_A(q^2)$ represents the anapole neutrino moment.

As far as a Majorana neutrino $|\nu^M_i\rangle$ is concerned, the *CPT* invariance demands that all the form factors, except the axial one F_A, are identically equal to zero [18]. Regarding nondiagonal elements, the situation depends on the fact whether *CP*-parity is conserved or not. For the *CP* noninvariant case all the four form factors are nonzero. When *CP* invariance takes place and the $|\nu^M_i\rangle$- and $|\nu^M_j\rangle$-states have identical (opposite) *CP*-parities, then $(F_E)_{ij}$ and $(F_A)_{ij}$ [$(F_M)_{ij}$ and $(F_V)_{ij}$] are different from zero [19].

Let us briefly discuss the experimental bounds on the neutrino DMMs. The most sensitive and established method for the experimental investigation of the diagonal neutrino DMMs is provided by direct laboratory measurements of (anti)neutrino-electron elastic scattering. A detailed description of such experiments could be found in Ref. [20]. At the moment the world best limit on $\mu_{\bar{\nu}_e}$ is coming from the GEMMA experiment at the Kalinin nuclear power plant [21]

$$\mu_{\bar{\nu}_e} \leq 2.9 \times 10^{-11}\mu_B \quad (90\% \text{ C.L.}). \quad (36)$$

Several experiments at accelerators have searched for an effect due to DMMs of ν_μ in $\nu_\mu - e$ and $\bar{\nu}_\mu - e$ elastic scattering. The current best limit has been obtained in the LSND experiment [22]

$$\mu_{\nu_\mu} \leq 6.8 \times 10^{-10}\mu_B \quad (90\% \text{ C.L.}) \quad (37)$$

Investigating $\nu_\tau - e$ and $\bar{\nu}_\tau - e$ elastic scattering, the DONUT collaboration has found the following bound [23]

$$\mu_{\nu_\tau} \leq 3.9 \times 10^{-7}\mu_B \quad (90\% \text{ C.L.}). \quad (38)$$

As for a Majorana neutrino, the global fit of the reactor and solar neutrino data gives the following values for transit DMMs [24]

$$(\mu^{\nu\nu})_{12}, \quad (\mu^{\nu\nu})_{13}, \quad (\mu^{\nu\nu})_{23} \leq 1.8 \times 10^{-10}\mu_B. \quad (39)$$

Transit DMMs for the Dirac as well as Majorana neutrinos could be determined under observation of the processes

$$\nu_l + e^- \to \nu_{l'} + e^-, \qquad \bar{\nu}_l + e^- \to \bar{\nu}_{l'} + e^-, \qquad l \neq l' \quad (40)$$

which proceed with the partial lepton flavor violation.

The theoretical predictions of the minimally extended SM (in what follows we shall keep in mind just this version of the SM) are very far from upper experimental bounds. In the third order of the perturbation theory the contributions to the DMM of a Dirac neutrino are defined by the diagrams represented in Fig. 1. In the leading order on $\epsilon_a = m^2_a/m^2_W$ the diagonal and nondiagonal matrix elements of the neutrino DMMs are determined by the expressions [25]

$$\mu_{\nu_i} \equiv (\mu^{\nu\nu})_{ii} = \frac{3G_F m_e m_{\nu_i}}{4\sqrt{2}\pi^2}\left[1 - \frac{1}{2}\sum_a U^\dagger_{ia}U_{ai}\epsilon_a\right]\mu_B, \quad (41)$$

$$(\mu^{\nu\nu})_{if} = -\frac{3G_F m_e}{16\sqrt{2}\pi^2}(m_{\nu_i} + m_{\nu_f})\sum_a U^\dagger_{fa}U_{ai}\epsilon_a\mu_B, \quad (42)$$

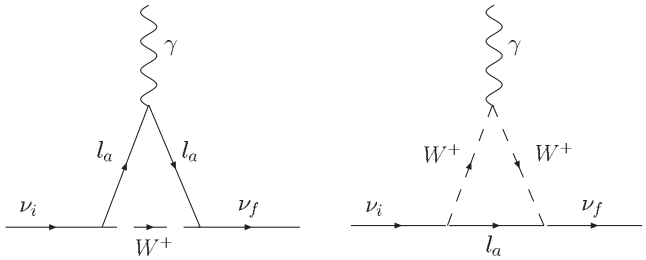

FIG. 1. The Feynman diagrams contributing to the neutrino DMM in the SM.

where U_{ai} is the neutrino mixing matrix in the SM and $i, f = 1, 2, 3$. From Eq. (41) it follows

$$\mu_{\nu_i} = 3.2 \times 10^{-19} \mu_B \left(\frac{m_{\nu_i}}{1 \text{ eV}} \right), \tag{43}$$

while Eq. (42) gives

$$(\mu^{\nu\nu})_{if} \approx 10^{-4} \mu_{\nu_i}. \tag{44}$$

So, in the case under consideration the neutrino DMMs are negligibly small.

For Majorana neutrinos the diagrams of Fig. 1 must be supplemented with those associated with the transitions $\nu_i^c \to \nu_f^c \gamma$. Making use of the Majorana condition

$$\nu_i = \lambda_{\odot} \nu_i^c, \tag{45}$$

where $\lambda_{\odot}$ is the phase factor of the Majorana neutrino production ($|\lambda_{\odot}|^2 = 1$), the properties of γ-matrices under the charge conjugation

$$C \gamma_\mu C^{-1} = -\gamma_\mu^T,$$

as well as the Hermiticity condition, we obtain the following relation between the magnetic form factor and the neutrino DMM

$$(\mu^{\nu\nu})_{if} = 2i \text{Im}[F_M(0)] \tag{46}$$

(the relation (46) could be derived using only the diagrams of Fig. 1 and taking into account the fact that the vector current is equal to zero for a Majorana neutrino [19]). Calculations fulfilled in the leading order on ϵ_a again lead to the negligibly small value for the neutrino DMMs [17]

$$(\mu^{\nu\nu})_{if} = -\frac{3iG_F m_e}{4\sqrt{2}\pi^2} (m_{\nu_i} + m_{\nu_f}) \sum_a \text{Im}[U_{fa}^\dagger U_{ai}] \epsilon_a \mu_B. \tag{47}$$

In the SM the smallness of μ_{ν_i} is caused by the fact that, at the Feynman diagram describing the neutrino DMM appearance, the W-boson interacts with left-handed currents only. Therefore, the chirality flip to be necessary for nonzero values of μ_{ν_i} has to do on the external neutrino line and, as a result, the DMM proves to be proportional to the neutrino mass. It is obvious that a sizeable increase of the DMM values would be expected in models with right-handed currents and heavy neutrinos. The LRM possesses the necessary properties.

Contributions to the neutrino DMMs coming from the diagrams with the virtual charged gauge bosons W_1 and W_2 (Fig. 1, where $W \to W_1, W_2$) were found in Refs. [17,26]. The obtained expressions are divided into two groups. The former describes the situation when in the initial and final states are either only light or only heavy neutrinos, that is, we are dealing with the vertices associated with the $\nu_i \to \nu_f \gamma$- and $N_i \to N_f \gamma$-transitions, respectively. The latter is connected with the $N_i \to \nu_f \gamma$-transitions. For the DMMs of the first group the neutrino DMMs have the form

$$(\mu^{\nu\nu})_{if} = -\frac{3ig_L^2 m_e (m_{\nu_i} + m_{\nu_f}) \mu_B}{64\pi^2} \sum_a \left[\frac{\cos^2\xi}{m_{W_1}^2} \epsilon_a^{(1)} + \frac{\sin^2\xi}{m_{W_2}^2} \epsilon_a^{(2)} \right] \times \text{Im}[(\mathcal{D}^{\nu\nu})_{fa}^\dagger \mathcal{D}_{ai}^{\nu\nu}], \tag{48}$$

$$(\mu^{NN})_{if} = \mu_{if}^{\nu\nu}(g_L \to g_R, \mathcal{D}_{ai}^{\nu\nu} \to \mathcal{D}_{ai}^{NN}, m_{\nu_i} \to m_{N_i}, \xi \to \xi + \pi/2), \tag{49}$$

while the neutrino DMMs belonging to the second group are defined by the expression

$$(\mu^{\nu N})_{if} = -\frac{ig_L g_R m_e \mu_B}{4\pi^2} \sin\xi \cos\xi \sum_a m_a \left\{ \sum_{k=1}^2 \frac{(-1)^k}{m_{W_k}^2} \left[1 + \epsilon_a^{(k)} \left(\ln \epsilon_a^{(k)} + \frac{9}{8} \right) \right] \right\} \times \text{Im}[e^{-i\phi} (\mathcal{D}^{\nu\nu})_{fa}^\dagger (\mathcal{D}^{NN})_{ai}], \tag{50}$$

where

$$\nu_a = \mathcal{D}_{ai}^{\nu\nu} \nu_i, \qquad N_a = \mathcal{D}_{ai}^{NN} N_i, \qquad \epsilon_a^{(k)} = \frac{m_a^2}{m_{W_k}^2},$$

and CP violating phase ϕ has been introduced into the charged gauge boson sector

$$W_L = W_1 \cos\xi + W_2 e^{i\phi} \sin\xi,$$
$$W_R = -W_1 e^{-i\phi} \sin\xi + W_2 \cos\xi.$$

In order to estimate the expressions (48)–(50) we need to know the value of the $W_L - W_R$ mixing angle ξ. The current experimental limits on it are as follows [27]

$$6\times 10^{-4} < |\xi| < 5.6\times 10^{-2}.$$

Now, supposing that

$$g_L = g_R = e s_{\theta_W}^{-1}, \qquad m_{W_2} = 2.5\ \text{TeV}, \qquad \xi = 2\times 10^{-2}, \tag{51}$$

and N_i, N_f are on the electroweak scale, we get

$$|(\mu^{\nu\nu})_{if}| \simeq 10^{-22}\mu_B, \qquad |(\mu^{NN})_{if}| \simeq 3\times 10^{-15}\mu_B,$$
$$|(\mu^{\nu N})_{if}| \simeq 2.7\times 10^{-11}\mu_B. \tag{52}$$

So, the found contributions of the charged gauge boson sector to $(\mu^{\nu\nu})_{if}$ and $(\mu^{NN})_{if}$ prove to be very small and are of no physical interest. However, one important point to remember is that in the above-mentioned works the mixing of the light and heavy neutrinos inside generation (light-heavy mixing) was not taken into account. Therefore, these results hold for the NMD case only.

Let us find the additions to the DMMs caused by the light-heavy neutrino mixing. The inclusion of this effect results in

$$\left.\begin{array}{ll}(\mu^{\nu\nu})_{if} \to (\mu^{\nu\nu})_{if} + (\mu^{\nu\nu})_{if}^{\text{add}}, & (\mu^{NN})_{if} \to (\mu^{NN})_{if} + (\mu^{NN})_{if}^{\text{add}}, \\ (\mu^{\nu N})_{if} \to (\mu^{\nu N})_{if} + (\mu^{\nu N})_{if}^{\text{add}}, & \end{array}\right\} \tag{53}$$

where

$$\begin{aligned}
(\mu^{\nu\nu})_{if}^{\text{add}} =& -\frac{3ig_R^2 m_e (m_{N_i}+m_{N_f})\mu_B}{64\pi^2}\sum_a\left[\frac{\sin^2\xi}{m_{W_1}^2}\epsilon_a^{(1)} + \frac{\cos^2\xi}{m_{W_2}^2}\epsilon_a^{(2)}\right]\times \text{Im}[\mathcal{U}^\dagger_{f,N_a}\mathcal{U}_{N_a,i}] \\
& - \frac{ig_L g_R m_e \mu_B}{4\pi^2}\sin\xi\cos\xi\sum_a m_a\left\{\sum_{k=1}^{2}\frac{(-1)^k}{m_{W_k}^2}\left[1+\epsilon_a^{(k)}\left(\ln\epsilon_a^{(k)}+\frac{9}{8}\right)\right]\right\}\times\text{Im}[\mathcal{U}^\dagger_{f,N_a}\mathcal{U}_{\nu_a,i}], \\
(\mu^{NN})_{if}^{\text{add}} =& -\frac{3ig_L^2 m_e (m_{\nu_i}+m_{\nu_f})\mu_B}{64\pi^2}\sum_a\left[\frac{\cos^2\xi}{m_{W_1}^2}\epsilon_a^{(1)} + \frac{\sin^2\xi}{m_{W_2}^2}\epsilon_a^{(2)}\right]\times \text{Im}[\mathcal{U}^\dagger_{f+3,\nu_a}\mathcal{U}_{\nu_a,i+3}] \\
& - \frac{ig_L g_R m_e \mu_B}{4\pi^2}\sin\xi\cos\xi\sum_a m_a\left\{\sum_{k=1}^{2}\frac{(-1)^k}{m_{W_k}^2}\left[1+\epsilon_a^{(k)}\left(\ln\epsilon_a^{(k)}+\frac{9}{8}\right)\right]\right\}\times\text{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{\nu_a,i+3}], \\
(\mu^{\nu N})_{if}^{\text{add}} =& -\frac{3ig_L^2 m_e (m_{\nu_i}+m_{N_f})\mu_B}{64\pi^2}\sum_a\left[\frac{\cos^2\xi}{m_{W_1}^2}\epsilon_a^{(1)} + \frac{\sin^2\xi}{m_{W_2}^2}\epsilon_a^{(2)}\right]\times \text{Im}[\mathcal{U}^\dagger_{f+3,\nu_a}\mathcal{U}_{\nu_a,i}] \\
& - \frac{3ig_R^2 m_e (m_{\nu_i}+m_{N_f})\mu_B}{64\pi^2}\sum_a\left[\frac{\sin^2\xi}{m_{W_1}^2}\epsilon_a^{(1)} + \frac{\cos^2\xi}{m_{W_2}^2}\epsilon_a^{(2)}\right]\times \text{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{N_a,i}].
\end{aligned}$$

When the mixing angles between the light and heavy neutrinos are equal to zero then $\mathcal{M}^{\nu N}$ becomes identity matrix and, as a result, $(\mu^{\nu\nu})_{if}^{\text{add}}$, $(\mu^{NN})_{if}^{\text{add}}$, and $(\mu^{\nu N})_{if}^{\text{add}}$ vanish. To make an estimate of the derived additions it is necessary to have information concerning the value of the light-heavy neutrino mixing. Up to date there are a lot of papers devoted to the determination of experimental bounds on these quantities (see for review [28]). One way to find such bounds is connected with searches for the neutrinoless double beta decay ($0\nu\beta\beta$) and disentangle the heavy neutrino effect. Within the LRM the analysis of the $0\nu\beta\beta$ gave the upper bound on θ_{11} equal to 10^{-5} [29]. It should be stressed that this result was obtained in the assumption $m_{N_1} \gg m_{N_2}, m_{N_3}$, that is, for the maximal heavy-heavy mixing case only. However, there is the point of view that the $0\nu\beta\beta$ does not give the reliable answer on the value of the light-heavy mixing. Of course, the main uncertainties are connected with the determination of nuclear matrix elements. Furthermore, a number of approximations has to be made. For example, in Ref. [29] it was suggested that the Yukawa couplings of the triplets that define the Majorana mass terms for left-handed and right-handed neutrinos are equal. It is clear that in this case the Lagrangian, describing the interaction between the doubly charged Higgs bosons ($\Delta_{1,2}^{(\pm\pm)}$) and charged leptons, has very specific form and differs dramatically from the Lagrangian derived from the most general renormalizable Higgs potential proposed in Ref. [14].

The other way is to directly look for the presence of the light-heavy neutrino mixing, which can manifest in several ways, for example, (i) via departures from unitarity of the neutrino mixing matrix, which could be investigated in neutrino oscillation experiments as well as in lepton flavor violation searches, and (ii) via their signatures in collider experiments. The bounds obtained in this case prove to be less severe. As an illustration, in Ref. [30] the final states with same-sign dileptons plus two jets without missing energy ($l^\pm l^\pm jj$), arising from pp collisions were considered. This signal depends crucially on the light-heavy mixing. Analysis of the channel

$$p+p\to N_l^* l^\pm \to l^\pm + l^\pm + 2j \qquad (54)$$

led to the upper limit on θ_{11} equal to 2.23×10^{-2} (3.32×10^{-2}) for $m_{W_R}=3$ TeV ($m_{W_R}=4$ TeV) and $m_{N_l}=100$ GeV.

It is worth noting that, as was shown in Ref. [15], even at the fulfillment of the seesaw relation

$$m_{\nu_i} m_{N_i} \approx (m_D^a)^2,$$

the mixing angles between the light and heavy neutrinos belonging to the same generation θ_{ii} may reach the values 3×10^{-2} provided $v_L\neq0$ and the Higgs potential is chosen in the form suggested in Ref. [14].

Further, for the sake of simplicity, we shall assume that

$$\left.\begin{aligned}
&\mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{\nu_a,i}]\simeq \mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{N_a,i+3}]\simeq \mathrm{Im}[\mathcal{U}^\dagger_{f,\nu_a}\mathcal{U}_{\nu_a,i}]\simeq 1,\\
&\mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{\nu_a,i+3}]\simeq \mathrm{Im}[\mathcal{U}^\dagger_{f,N_a}\mathcal{U}_{N_a,i+3}]\simeq \mathrm{Im}[\mathcal{U}^\dagger_{f,\nu_a}\mathcal{U}_{N_a,i}]\simeq \sin\theta_{ff},\\
&\mathrm{Im}[\mathcal{U}^\dagger_{f,N_a}\mathcal{U}_{\nu_a,i+3}]\simeq \mathrm{Im}[\mathcal{U}^\dagger_{f+3,\nu_a}\mathcal{U}_{N_a,i}]\simeq \sin\theta_{ii}\sin\theta_{ff}.
\end{aligned}\right\} \qquad (55)$$

Supposing that $\sin\theta_{ii}\simeq\sin\theta_{ff}\simeq 2\times10^{-2}$ (it does not contradict the quasigenerate masses scheme), along with using Eqs. (51) and (55), we obtain

$$|(\mu^{\nu\nu})^{\mathrm{add}}_{if}|\approx|(\mu^{NN})^{\mathrm{add}}_{if}|\approx 5.7\times10^{-13}\mu_B, \qquad |(\mu^{\nu N})^{\mathrm{add}}_{if}|\approx 1.5\times10^{-13}\mu_B. \qquad (56)$$

Decreasing θ_{ii} up to 10^{-5} as is admitted by the maximal heavy-heavy mixing scheme, results in

$$|(\mu^{\nu\nu})^{\mathrm{add}}_{if}|\approx|(\mu^{NN})^{\mathrm{add}}_{if}|\approx 2.85\times10^{-16}\mu_B, \qquad |(\mu^{\nu N})^{\mathrm{add}}_{if}|\approx 0.75\times10^{-16}\mu_B. \qquad (57)$$

However in the LRM we also have contributions coming from the singly-charged Higgs bosons. Let us calculate them in the third order of the perturbation theory. In Fig. 2 the Feynman diagrams caused by the Lagrangian $\mathcal{L}_l^{\tilde\delta}$ are pictured. Calculations lead to the results

$$(\mu^{\nu\nu})_{if}=\frac{im_e\mu_B}{2\pi^2}\sum_a\{\alpha^2_{\nu_a\tilde\delta l_a}\Omega^{\nu_i\nu_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f,\nu_a}\mathcal{U}_{\nu_a,i}]+\alpha^2_{N_a\tilde\delta l_a}\Omega^{N_iN_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f,N_a}\mathcal{U}_{N_a,i}]+\alpha_{\nu_a\tilde\delta l_a}\alpha_{N_a\tilde\delta l_a}\Omega^{\nu_iN_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f,N_a}\mathcal{U}_{\nu_a,i}]\}, \qquad (58)$$

$$\begin{aligned}(\mu^{NN})_{if}=\frac{im_e\mu_B}{2\pi^2}\sum_a\{&\alpha^2_{N_a\tilde\delta l_a}\Omega^{N_iN_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{N_a,i+3}]+\alpha^2_{\nu_a\tilde\delta l_a}\Omega^{\nu_i\nu_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f+3,\nu_a}\mathcal{U}_{\nu_a,i+3}]\\
&+\alpha_{\nu_a\tilde\delta l_a}\alpha_{N_a\tilde\delta l_a}\Omega^{\nu_iN_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{\nu_a,i+3}]\},\end{aligned} \qquad (59)$$

$$\begin{aligned}(\mu^{\nu N})_{if}=\frac{im_e\mu_B}{2\pi^2}\sum_a\{&\alpha_{\nu_a\tilde\delta l_a}\alpha_{N_a\tilde\delta l_a}\Omega^{\nu_iN_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{\nu_a,i}]+\alpha^2_{\nu_a\tilde\delta l_a}\Omega^{\nu_i\nu_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f+3,\nu_a}\mathcal{U}_{\nu_a,i}]\\
&+\alpha^2_{N_a\tilde\delta l_a}\Omega^{\nu_iN_f}_{l_a\tilde\delta}\times\mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{N_a,i}]\},\end{aligned} \qquad (60)$$

where

$$\Omega^{\nu_i\nu_f}_{l_a\tilde{\delta}} = \int_0^1 \frac{xdx}{(m_{\nu_i}-m_{\nu_f})}\left[\ln\left|\frac{M_{\nu_i\tilde{\delta}}}{M_{\nu_f\tilde{\delta}}}\right| + \ln\left|\frac{M_{\nu_i l_a}}{M_{\nu_f l_a}}\right|\right], \qquad \Omega^{N_iN_f}_{l_a\tilde{\delta}} = \Omega^{\nu_i\nu_f}_{l_a\tilde{\delta}}(\nu_i \to N_i, \nu_f \to N_f),$$

$$\Omega^{\nu_iN_f}_{l_a\tilde{\delta}} = m_a\int_0^1 dx\left[\frac{x}{m^2_{N_f}(1-x)+m^2_{\nu_i}x}\ln\left|\frac{M_{\nu_i l_a}}{M_{N_f l_a}}\right| - \frac{1}{m^2_{N_f}}\ln\left|\frac{M_{\nu_i\tilde{\delta}}}{M_{N_f\tilde{\delta}}}\right|\right],$$

$$M_{\nu_i l_a} = (m_a^2 - m^2_{\nu_i})x + m^2_{\nu_i}x^2 + m^2_{\tilde{\delta}}(1-x), \qquad M_{N_i l_a} = M_{\nu_i l_a}(\nu_i \to N_i),$$

$$M_{\nu_i\tilde{\delta}} = (m^2_{\tilde{\delta}} - m^2_{\nu_i})x + m^2_{\nu_i}x^2 + m_a^2(1-x), \qquad M_{N_i\tilde{\delta}} = M_{\nu_i\tilde{\delta}}(\nu_i \to N_i).$$

In the NMD case the second and third terms in the expressions (58)–(60) turn into zero.

Expanding the expression for $\Omega^{\nu_i\nu_f}_{l_a\tilde{\delta}}$ as a power series in $m^2_\nu/m^2_{\tilde{\delta}}$, we get

$$\Omega^{\nu_i\nu_f}_{l_a\tilde{\delta}} \simeq -\frac{m_{\nu_i}+m_{\nu_f}}{2m^2_{\tilde{\delta}}}. \tag{61}$$

Analogously, the expansion of $\Omega^{N_iN_f}_{l_a\tilde{\delta}}$ as a power series in $m_{l_a}/m_{N_{i,f}}$ results in

$$\Omega^{N_iN_f}_{l_a\tilde{\delta}} \simeq \frac{1}{m_{N_i}-m_{N_f}}\left\{2\ln\left|\frac{m^2_{N_i}}{m^2_{N_f}}\right| + \frac{m^2_{\tilde{\delta}}}{m^2_{N_i}}\ln\left|\frac{m^2_{\tilde{\delta}}}{m^2_{N_i}}\right| + \frac{m^2_{N_i}-m^2_{\tilde{\delta}}}{m^2_{N_i}}\ln\left|\frac{m^2_{N_i}-m^2_{\tilde{\delta}}}{m^2_{N_i}}\right| - \frac{m^2_{\tilde{\delta}}}{m^2_{N_f}}\ln\left|\frac{m^2_{\tilde{\delta}}}{m^2_{N_f}}\right| - \frac{m^2_{N_f}-m^2_{\tilde{\delta}}}{m^2_{N_f}}\ln\left|\frac{m^2_{N_f}-m^2_{\tilde{\delta}}}{m^2_{N_f}}\right|\right\}. \tag{62}$$

In the QDM case the expression (62) takes the simple form

$$\Omega^{N_iN_f}_{l_a\tilde{\delta}} \simeq \frac{4m^2_{\tilde{\delta}}(m^2_{N_i}-m^2_{N_f})}{m^3_{N_i}(m^2_{\tilde{\delta}}-m^2_{N_i})}. \tag{63}$$

Taking into account the smallness of m_{l_a} compared with m_N and $m_{\tilde{\delta}}$, we could rewrite the expression for $\Omega^{\nu_iN_f}_{l_a\tilde{\delta}}$ as follows

$$\Omega^{\nu_iN_f}_{l_a\tilde{\delta}} \simeq -\frac{m_a}{m^2_{N_i}}\left[\ln\left|\frac{m^2_{\nu_f}}{m^2_{N_i}}\right|\ln\left|\frac{m^4_{\tilde{\delta}}}{m^2_{l_a}m_{\nu_f}m_{N_i}}\right| + \frac{1}{2}\left(\ln\left|\frac{m^2_{l_a}}{m^2_{\tilde{\delta}}}\right|\right)^2 + \ln\left|\frac{m^2_{N_i}-m^2_{\tilde{\delta}}}{m^2_{N_i}}\right|\ln\left|\frac{m^2_{\nu_f}}{m^2_{N_i}-m^2_{\tilde{\delta}}}\right|\right]. \tag{64}$$

It is clear that in the QDM case the main contributions in (58)–(60) are caused by the terms containing $\Omega^{\nu_iN_f}_{l_a\tilde{\delta}}$.

Let us give the numerical estimation of the expressions (58)–(60) in the QDM and NMD cases. In doing so for the singly charged Higgs boson mass we shall use the low bound 78.6 GeV obtained in the LEP-experiments [31]

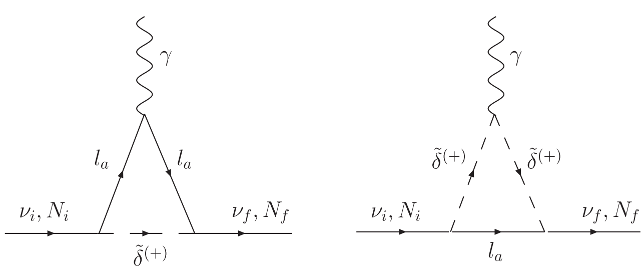

FIG. 2. The Feynman diagrams induced by the Lagrangian $\mathcal{L}^{\tilde{\delta}}_l$.

(note, that is the best model-independent limit). Regarding the coupling constants, they should be estimated for the QDM case and the NMD one separately. From Eq. (21) it follows that in the former case we can obtain the reasonably definite information about the f_{aa} value. Setting

$$m_{N_e} \simeq m_{N_\tau} \simeq 100 \text{ GeV}, \qquad m_{W_2} = 4 \text{ TeV}, \tag{65}$$

we get

$$f_{ee} \simeq f_{\tau\tau} \simeq 1.6\times 10^{-2}$$

to form

$$\alpha_{\nu_e\tilde{\delta}e} \simeq \alpha_{\nu_\tau\tilde{\delta}\tau} \simeq 1.2\times 10^{-2}, \qquad \alpha_{N_e\tilde{\delta}e} \simeq \alpha_{N_\tau\tilde{\delta}\tau} \simeq 4.6\times 10^{-4}. \tag{66}$$

Then, using (65), (66) we find

$$|(\mu^{\nu\nu})_{if}| \simeq |(\mu^{NN})_{if}| \simeq \begin{cases} 10^{-12}\mu_B, & \text{for } \theta_{ii} = 2\times 10^{-2}, \\ 10^{-15}\mu_B, & \text{for } \theta_{ii} = 10^{-5}, \end{cases} \quad (67)$$

and

$$|(\mu^{\nu N})_{if}| \simeq 4\times 10^{-11}\mu_B. \quad (68)$$

It should be stressed that the value of $(\mu^{\nu N})_{if}$ is independent of the light-heavy neutrino mixing (recall that $\mathcal{U}^{\dagger}_{f+3,N_a}\mathcal{U}_{\nu_a,i}$ does not hold $\sin\theta_{ii}$).

In the NMD case the relations (21) lose their predictive force. We must set the mixing angles in the heavy neutrino sector and make additional assumptions about the heavy neutrino mass differences. However, in any event the triplet Yukawa coupling constants f_{aa} may not exceed their upper bound obtained under investigating the direct and inverse τ-lepton decays [32]

$$\frac{f_{ee}f_{\tau\tau}}{m^2_{\tilde{\delta}}} \simeq 3.3\times 10^{-5}\ \text{GeV}^{-2}.$$

Using this bound we obtain

$$\alpha_{\nu_e\tilde{\delta}e} \simeq 0.33, \qquad \alpha_{N_e\tilde{\delta}e} \simeq 1.3\times 10^{-2},$$

to give

$$|(\mu^{\nu\nu})_{if}| \simeq 10^{-18}\mu_B, \qquad |(\mu^{\nu N})_{if}| \simeq 3\times 10^{-8}\mu_B,$$
$$|(\mu^{NN})_{if}| \simeq 10^{-10}\mu_B. \quad (69)$$

Attention is drawn to the fact that both in the QDM case and in the NMD one the expression for $(\mu^{\nu N})_{if}$ displays the weak logarithmic growth on the $\tilde{\delta}^{(\pm)}$-boson mass. For example, the enhancement of $m_{\tilde{\delta}}$ from 78 GeV to 200 GeV leads to the increase of $(\mu^{\nu N})_{if}$ of seven percent.

In the LRM there are contributions to the neutrino DMMs caused by the Lagrangian $\mathcal{L}^{\tilde{\delta}}_{W\gamma}$ as well. The corresponding Feynman diagrams are presented in Fig. 3. Let us work in the unitary gauge. When in the initial and final states the light neutrinos ν_i and ν_f present, the matrix element corresponding to the first diagram of Fig. 3 with the virtual W^+_1-boson has the form

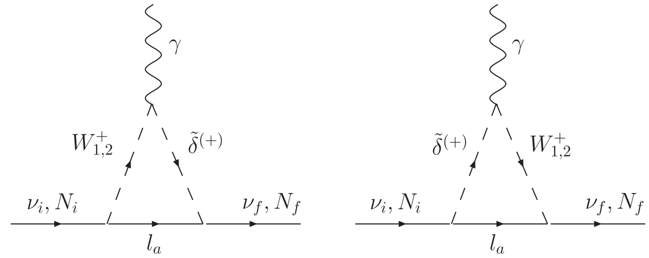

FIG. 3. The Feynman diagrams induced by $\mathcal{L}^{\tilde{\delta}}_{W\gamma}$.

$$\mathcal{M}_{\lambda}(p_1,p_2) = \frac{-ieg_L\alpha_{W\tilde{\delta}\gamma}\cos\xi\sin\xi}{\sqrt{2}(2\pi)^4}\sum_a \mathcal{U}^{\dagger}_{f,\nu_a}\mathcal{U}_{\nu_a,i}\alpha_{\nu_a\tilde{\delta}l_a}(1-\gamma_5) \times \int \frac{\hat{k}\gamma^{\sigma}[g_{\lambda\sigma}-(p_1-k)_{\lambda}(p_1-k)_{\sigma}/m^2_{W_1}]d^4k}{(k^2-m_a^2)[(p_2-k)^2-m^2_{\tilde{\delta}}][(p_1-k)^2-m^2_{W_1}]}. \quad (70)$$

Like any theory with the SSB, the LRM represents a renormalizable theory. Therefore, the matrix element (70) must be finite. However, a naive counting of moment degrees into the integrand indicates that $\mathcal{M}_{\lambda}(p_1,p_2)$ has a linear divergency (in fact the divergency is logarithmic). Now, unlike the diagrams of Fig. 2, contributions to the neutrino DMMs also give divergent parts of the matrix element $\mathcal{M}_{\lambda}(p_1,p_2)$. Obviously, infinities appearing in $\mathcal{M}_{\lambda}(p_1,p_2)$ cancel out when we take into consideration all divergent diagrams. In so doing one should keep in mind that the finite part of a divergent diagram is also transformed in the renormalization process, that is, one cannot simply throw away the divergent part in the expression in question. So, for the finite part of $M_{\lambda}(p_1,p_2)$ to be found, we should choose a specific procedure for removing infinities. In what follows we take advantage of Dyson's procedure [33], in which the expansion of the integrand in a power series in external momenta is followed by the substraction of divergent terms. Having done all the necessary calculations, we get the following additions to the neutrino DMMs

$$(\mu'^{\nu\nu})_{if} = \frac{im_e\mu_B}{8\sqrt{2}\pi^2}\sum_a \alpha_{W\tilde{\delta}\gamma}\{g_L\alpha_{\nu_a\tilde{\delta}l_a}\Lambda^{\nu_i\nu_f}_{W\tilde{\delta}} \times \text{Im}[\mathcal{U}^{\dagger}_{f,\nu_a}\mathcal{U}_{\nu_a,i}] + g_R\alpha_{N_a\tilde{\delta}l_a}\Lambda^{N_iN_f}_{W\tilde{\delta}} \times \text{Im}[\mathcal{U}^{\dagger}_{f,N_a}\mathcal{U}_{N_a,i}] + (g_L\alpha_{N_a\tilde{\delta}l_a}\Lambda^{\nu_iN_f}_{W\tilde{\delta}} + g_R\alpha_{\nu_a\tilde{\delta}l_a}\Lambda^{N_f\nu_i}_{W\tilde{\delta}}) \times \text{Im}[\mathcal{U}^{\dagger}_{f,N_a}\mathcal{U}_{\nu_a,i}]\}, \quad (71)$$

$$(\mu'^{NN})_{if} = \frac{im_e\mu_B}{8\sqrt{2}\pi^2}\sum_a \alpha_{W\tilde{\delta}\gamma}\{g_R\alpha_{N_a\tilde{\delta}l_a}\Lambda^{N_iN_f}_{W\tilde{\delta}} \times \text{Im}[\mathcal{U}^{\dagger}_{f+3,N_a}\mathcal{U}_{N_a,i+3}] + g_L\alpha_{\nu_a\tilde{\delta}l_a}\Lambda^{\nu_i\nu_f}_{W\tilde{\delta}} \times \text{Im}[\mathcal{U}^{\dagger}_{f+3,\nu_a}\mathcal{U}_{\nu_a,i+3}] + (g_L\alpha_{N_a\tilde{\delta}l_a}\Lambda^{\nu_iN_f}_{W\tilde{\delta}} + g_R\alpha_{\nu_a\tilde{\delta}l_a}\Lambda^{N_f\nu_i}_{W\tilde{\delta}}) \times \text{Im}[\mathcal{U}^{\dagger}_{f+3,N_a}\mathcal{U}_{\nu_a,i+3}]\}, \quad (72)$$

$$(\mu'^{\nu N})_{if} = -\frac{i m_e \mu_B}{8\sqrt{2}\pi^2}\sum_a \alpha_{W\tilde{\delta}\gamma} m_a \{ (g_L \alpha_{N_a \tilde{\delta} l_a} \Lambda^{\nu_i N_f}_{W\tilde{\delta}} + g_R \alpha_{\nu_a \tilde{\delta} l_a} \Lambda^{N_f \nu_i}_{W\tilde{\delta}}) \times \mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{\nu_a,i}] + g_R \alpha_{N_a\tilde{\delta} l_a} \Lambda^{N_i N_f}_{W\tilde{\delta}} \times \mathrm{Im}[\mathcal{U}^\dagger_{f+3,N_a}\mathcal{U}_{N_a,i}] + g_L \alpha_{\nu_a \tilde{\delta} l_a}\Lambda^{\nu_i\nu_f}_{W\tilde{\delta}} \times \mathrm{Im}[\mathcal{U}^\dagger_{f+3,\nu_a}\mathcal{U}_{\nu_a,i}]\}, \qquad (73)$$

where

$$\Lambda^{\nu_i\nu_f}_{W\tilde{\delta}} = \frac{1}{2}\sin 2\xi\left[\frac{1}{m^2_{W_1}}\Lambda^{\nu_i\nu_f}_{W_1\tilde{\delta}} + \frac{1}{m^2_{W_2}}\Lambda^{\nu_i\nu_f}_{W_2\tilde{\delta}}\right],$$

$$\Lambda^{\nu_i\nu_f}_{W_1\tilde{\delta}} = \int_0^1 dx\Bigg\{\left[\frac{1}{M_{\nu_f\tilde{\delta}} - M_{\nu_i W_1}}\ln\left|\frac{M_{\nu_f\tilde{\delta}}}{M_{\nu_i W_1}}\right|(x^2 m^2_{W_1} + 2(2x-3x^2)M_{\nu_i W_1} + (x^3 - x^4) \times (m^2_{\nu_i} + m^2_{\nu_f}) + x^3(m^2_{\nu_i} + m_{\nu_i}m_{\nu_f})) + 2(3x^2 - 2x)\left(\ln\left|\frac{l^{\nu_f}_{\tilde{\delta}}}{M_{\nu_f\tilde{\delta}}}\right| + \frac{l^{\nu_i}_{W_1}}{l^{\nu_f}_{\tilde{\delta}} - l^{\nu_i}_{W_1}}\ln\left|\frac{l^{\nu_f}_{\tilde{\delta}}}{l^{\nu_i}_{W_1}}\right|\right)\right] + (\nu_i \leftrightarrow \nu_f)\Bigg\},$$

$$l^{\nu_i}_{W_k} = (m^2_{W_k} - m^2_{\nu_i})x + m^2_{l_a}(1-x), \qquad l^{N_i}_{W_k} = l^{\nu_i}_{W_k}(\nu_i \to N_i),$$

$$l^{\nu_i}_{\tilde{\delta}} = (m^2_{\tilde{\delta}} - m^2_{\nu_i})x + m^2_{l_a}(1-x), \qquad l^{N_i}_{\tilde{\delta}} = l^{\nu_i}_{\tilde{\delta}}(\nu_i \to N_i), \qquad M_{\nu_i W_k} = l^{\nu_i}_{W_k} + m^2_{\nu_i}x^2,$$

$$M_{\nu_i\tilde{\delta}} = l^{\nu_i}_{\tilde{\delta}} + m^2_{\nu_i}x^2, \qquad M_{N_i W_k} = M_{\nu_i W_k}(\nu_i \to N_i), \qquad M_{N_i\tilde{\delta}} = l^{N_i}_{\tilde{\delta}} + m^2_{N_i}x^2$$

$$\Lambda^{N_iN_f}_{W\tilde{\delta}} = \left[\frac{\cos^2\xi}{m^2_{W_2}}\Lambda^{N_iN_f}_{W_2\tilde{\delta}} - \frac{\sin^2\xi}{m^2_{W_1}}\Lambda^{N_iN_f}_{W_1\tilde{\delta}}\right], \qquad \Lambda^{N_iN_f}_{W_k\tilde{\delta}} = \Lambda^{\nu_i\nu_f}_{W_k\tilde{\delta}}(\nu_i \to N_i, \nu_f \to N_f),$$

$$\Lambda^{\nu_iN_f}_{W\tilde{\delta}} = \frac{1}{2}\sin 2\xi\left[\frac{1}{m^2_{W_1}}\Lambda^{\nu_iN_f}_{W_1\tilde{\delta}} + \frac{1}{m^2_{W_2}}\Lambda^{\nu_iN_f}_{W_2\tilde{\delta}}\right],$$

$$\Lambda^{\nu_iN_f}_{W_1\tilde{\delta}} = \int_0^1 \frac{dx}{M_{N_f\tilde{\delta}} - M_{\nu_iW_1}}\ln\left|\frac{M_{N_f\tilde{\delta}}}{M_{\nu_iW_1}}\right|[x^3(m_{N_f} + m_{\nu_i}) - x^2(3m_{\nu_i} + m_{N_f}) + 2xm_{\nu_i}].$$

The approximate expressions for $\Lambda^{\nu_i\nu_f}_{W_k\tilde{\delta}}$ and $\Lambda^{N_iN_f}_{W_k\tilde{\delta}}$ have the form

$$\Lambda^{\nu_i\nu_f}_{W_k\tilde{\delta}} \simeq \frac{m^2_{W_k}}{m^2_{\tilde{\delta}} - m^2_{W_k}}\ln\left|\frac{m^2_{\tilde{\delta}}}{m^2_{W_k}}\right|, \qquad (74)$$

$$\Lambda^{N_iN_f}_{W\tilde{\delta}} \simeq \Lambda^{N_iN_f}_{W_2\tilde{\delta}} \simeq \frac{m^2_{W_2}}{m^2_{\tilde{\delta}} - m^2_{W_2}} \times \left[\ln\left|\frac{m^2_{\tilde{\delta}}}{m^2_{W_2}}\right| + 4\frac{m^6_{W_2}}{m^6_N}\ln\left|\frac{m^2_{W_2}}{m^2_{W_2} - m^2_N}\right| - 4\frac{m^4_{W_2}}{m^4_N}\right]. \qquad (75)$$

The expansion (74) is valid both for all three schemes while the expansion (75) holds for the QDM scheme only. As far as the expression for $\Lambda^{\nu_i\nu_f}_{W_k\tilde{\delta}}$ is concerned, there is no way to produce it in the form similar to (74) or (75) because the integral entering into it admits exclusively the numerical integration.

The numerical estimations of the obtained expressions demonstrate, that at the chosen values of the LRM parameters the contributions to the neutrino DMMs coming from the diagrams pictured in Fig. 3 are less than those shown in Fig. 2.

V. CONCLUSION

In the context of the LRM electromagnetic properties of Majorana neutrinos are studied. Investigation of the light left-handed neutrino beam moving in a condensed matter and a magnetic field has led to the conclusion that the structure of the heavy neutrino sector admits only three possibilities: (i) quasidegenerate masses case—the light-heavy neutrino mixing angles θ_{11} and θ_{22} are arbitrary but equal each other whereas the heavy neutrino masses are quasidegenerate; (ii) no mass degeneration case—the heavy neutrino masses are hierarchical ($m_{N_1} < m_{N_2}$) while the angles θ_{11} and θ_{22} are equal to zero; (iii) maximal heavy-heavy mixing case—mixing angles θ_{11} and θ_{22} are equal and the heavy-heavy neutrino mixing is maximal to give the hierarchy of the heavy neutrino masses. Investigation has also revealed that the resonance transitions between the sectors of the light and heavy neutrinos are forbidden.

Contributions to the neutrino dipole magnetic moments (DMMs) coming from the charged gauge bosons $W^{\pm}_{1,2}$ and the singly charged Higgs bosons $\tilde{\delta}^{(\pm)}$ have been considered. In so doing we have assumed that one of three heavy right-handed neutrinos and the $\tilde{\delta}^{(\pm)}$-boson are on the electroweak scale. The expressions for the DMMs are divided into two groups. The former is connected with the $\nu_i \to \nu_f \gamma$- and $N_i \to N_f \gamma$-transitions, $(\mu^{\nu\nu})_{if}$ and $(\mu^{NN})_{if}$, while the latter is associated with the $N_i \to \nu_f \gamma$-transitions, $(\mu^{\nu N})_{if}$. It was shown that contributions to $(\mu^{\nu\nu})_{if}$ and $(\mu^{NN})_{if}$ caused by the charged gauge bosons are maximal in the quasidegenerate masses case. As for the Higgs boson contributions to the DMMs, then, as the estimations have demonstrated, they are maximal in the no mass degeneration case and could exceed those caused by the charged gauge bosons. For example, at the definite values of the LRM parameters which are not contrary to experiments the upper limits on the magnetic moments $(\mu^{\nu N})_{if}$ and $(\mu^{NN})_{if}$ may reach the values of few $\times$ $10^{-8}\mu_B$ and $10^{-10}\mu_B$, respectively.

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