

**Higher-dimensional evolving wormholes satisfying the null energy condition**Mahdi Kord Zangeneh,<sup>1,\*</sup> Francisco S. N. Lobo,<sup>2,†</sup> and Nematollah Riazi<sup>3,‡</sup><sup>1</sup>*Physics Department and Biruni Observatory, Shiraz University, Shiraz 71454, Iran*<sup>2</sup>*Centro de Astronomia e Astrofísica da Universidade de Lisboa,  
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In this work, we consider the possibility of expanding wormholes in higher dimensions, which is an important ingredient of modern theories of fundamental physics. An important motivation is that nontrivial topological objects such as microscopic wormholes may have been enlarged to macroscopic sizes in an expanding inflationary cosmological background. Since the Ricci scalar is only a function of time in standard cosmological models, we use this property as a simplifying assumption. More specifically, we consider a particular class of wormhole solutions corresponding to the choice of a spatially homogeneous Ricci scalar. The possibility of obtaining solutions with normal and exotic matter is explored and we find a variety of solutions including those in four dimensions that satisfy the null energy condition in specific time intervals. In particular, for five dimensions, we find solutions that satisfy the null energy condition throughout the respective evolution.

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**I. INTRODUCTION**

Wormhole physics dates back to the formulation of general relativity (GR). Indeed, in 1916, months after Einstein presented his gravitational field equations, Karl Schwarzschild found the first solution of Einstein's equations, which described the gravitational field of a vacuum nonrotating spherically symmetric solution. In the same year, Ludwig Flamm published a paper in which the geometry of the Schwarzschild solution was studied more closely. He pointed out a "tunnel-shaped" nature of space near the Schwarzschild radius, this being perhaps the first move towards the modern concept of the "throat" in wormholes [1].

Paging through the literature, one finds next that tunnel-like solutions were considered, in 1935, by Einstein and Rosen, where they constructed an elementary particle model represented by a "bridge" connecting two identical sheets [2]. They considered the possibility that fundamental particles such as the electron could be represented as microscopic spacetime tunnels that convey fluxes of the electric field. These tunnels were later denoted the Einstein-Rosen bridge. In fact, the Einstein-Rosen bridge is a coordinate artifact arising from choosing a coordinate patch, which is defined to double-cover the asymptotically flat region exterior to the black hole event horizon.

The field had lain dormant for about twenty years when, in 1955, John Wheeler, who was beginning to be interested in topological issues in GR, explored solutions of the

coupled Einstein-Maxwell equations, which he denoted gravitational-electromagnetic entities (geons) [3]. These were considered to be objects of the quantum foam connecting different regions of spacetime at the Planck scale. However, the term wormhole was only used for the first time in 1957 [4], when Misner and Wheeler presented a tour de force wherein Riemannian geometry of manifolds of nontrivial topology was investigated with an ambitious view to explaining all of physics. The aim was to use the source-free Maxwell equations, coupled to Einstein gravity, with nontrivial topology, to build models for classical electrical charges and all other particlelike entities in classical physics.

Subsequently to the geon concept, several wormhole solutions were obtained and discussed within different contexts [5]. However, it was only in 1988 that the full-fledged renaissance of wormhole physics took place through the seminal Morris-Thorne paper [6], and the theme is still in full flight. Morris and Thorne considered static and spherically symmetric traversable wormholes, and thoroughly analyzed their fundamental properties. It was found that these traversable wormholes possess a stress-energy tensor that violates the null energy condition (NEC), a property that was denoted exotic matter. Besides being hypothetical shortcuts in spacetime and consequently useful for inter- and inrauniverse travel, they were found to possess other intriguing applications, such as the usage for time travel [7] and investigating the interior of a black hole [8], amongst others.

Thus, a fundamental ingredient for the Morris-Thorne wormhole, i.e., for static and spherically symmetric wormhole solutions, is the violation of the NEC [6,9]. Exotic matter is particularly troublesome for measurements made

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by observers traversing through the throat with a radial velocity close to the speed of light, as for sufficiently high velocities,  $v \rightarrow c$ , the observer will measure a negative energy density [6]. Although classical forms of matter are believed to obey these energy conditions, it is a well-known fact that they are violated by certain quantum fields, such as the Casimir effect. In fact, the recent discovery that the Universe is undergoing an accelerated expansion [10] may be due to an exotic cosmic fluid that lies in the phantom regime. The realization of this fact has led to the study of wormhole solutions supported by different kinds of phantom fluids (see, for instance, [11]). Indeed, the violation of the energy conditions is a subtle issue, as almost all known and physically possible forms of matter satisfy these energy conditions, and we recall that their imposition is one of the necessary assumptions for proving the Hawking-Penrose singularity theorems [12,13].

Thus, one may adopt the approach of minimizing the usage of exotic matter [14,15]. In this context, a plethora of solutions have been investigated, using a wide variety of approaches [16–20]. More specifically, in rotating solutions it was found that the exotic matter lies in specific regions around the throat, so that it is possible for a certain class of infalling observers to move around the throat as to avoid the exotic matter supporting the wormhole [21]. Using the thin shell formalism, solutions where the exotic matter is concentrated at the throat have also been extensively investigated [22]. In the context of modified gravity it was shown that one may impose that the matter threading the wormhole satisfies the energy conditions, so that it is the higher order curvature terms that sustain these exotic geometries [23]. Astrophysical signatures have also been explored in the literature [24].

For dynamic wormholes, the NEC, or more precisely the averaged null energy condition, can be avoided in certain regions [25–30]. A particularly interesting case is that of a wormhole in a time-dependent inflationary background [31], in which the primary goal was to use inflation to enlarge an initially small and possibly submicroscopic wormhole. It is also possible that the wormhole will continue to be enlarged by the subsequent Friedmann-Robertson-Walker (FRW) phase of expansion. One could perform a similar analysis to [31] by replacing the de Sitter scale factor by a FRW scale factor [27–29]. In particular, in [27,28] specific examples for evolving wormholes that exist only for a finite time were considered, and a special class of scale factors that exhibit “flashes” of the weak energy condition (WEC) violation was also analyzed.

The present paper investigates the possibility and naturalness of expanding wormholes in higher dimensions, which is an important ingredient of the modern theories of fundamental physics, such as string theory, supergravity, Kaluza-Klein, and others. One of our motivations for considering wormhole solutions in an expanding cosmological background refers to the inflation theory [32] where

the quantum fluctuations in the inflaton field are considered as the seed of large scale structures in the Universe. As mentioned above, the nontrivial topological objects such as microscopic wormholes may have been formed during inflation and enlarged to macroscopic ones as the Universe expanded [31]. We also explore the possibility that these higher-dimensional wormholes satisfy the NEC, and we explicitly show that this is indeed the case.

This paper is organized in the following way: in Sec. II, we present the  $(n + 1)$ -dimensional field equations for the specific case of a spatially independent curvature scalar. In Sec. III, we analyze the two-way traversability conditions of the wormhole structure. In Sec. IV, we explore wormhole solutions in different expansionary regimes, and finally in Sec. V we conclude.

## II. ACTION, FIELD EQUATIONS, AND $(n + 1)$ -DIMENSIONAL SOLUTIONS

The action of GR in  $(n + 1)$  dimensions is written as

$$S = \int d^{n+1}x \sqrt{-g} \left( \frac{1}{2} \mathcal{R} + \mathcal{L}_m \right), \quad (1)$$

where  $\mathcal{R}$  is the scalar curvature and  $\mathcal{L}_m$  is the matter Lagrangian density; we have considered  $c = 8\pi G = 1$ . Varying this action with respect to the metric, we obtain the  $(n + 1)$ -dimensional Einstein equations  $G_{AB} = T_{AB}$ , where  $(A, B = 0 \dots n)$ , and  $T_{AB}$  is the matter stress-energy tensor.

Since we are looking for expanding wormhole solutions in a cosmological background, we use the metric

$$ds^2 = -dt^2 + R(t)^2 \left[ \frac{dr^2}{1 - a(r)} + r^2 d\Omega_{n-1}^2 \right], \quad (2)$$

in which  $R(t)$  is the scale factor and  $a(r)$  is an unknown dimensionless function, defined as  $a(r) = b(r)/r$ , where  $b(r)$  denotes the shape function [6]. Note that this metric is a generalization of the FRW metric, although being less symmetric than the latter. With this generalization, metric (2) is still isotropic about the center of the symmetry, though not necessarily homogeneous. When the dimensionless shape function vanishes,  $a(r) \rightarrow 0$ , the metric (2) reduces to the flat FRW metric; and as  $R(t) \rightarrow \text{const}$  it approaches the static wormhole metric.

To see that the wormhole form of the metric is preserved with time, consider an embedding of  $t = \text{const}$  and  $\theta_{(n-2)} = \pi/2$  slices of the spacetime given by Eq. (2), in a flat three-dimensional Euclidean space with metric

$$ds^2 = d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2 d\phi^2. \quad (3)$$

In this context, the metric of the wormhole slice is

$$ds^2 = \frac{R^2(t)dr^2}{1-a(r)} + R^2(t)r^2d\phi^2. \quad (4)$$

Now, comparing the coefficients of  $d\phi^2$ , one has

$$\bar{r} = R(t)r|_{t=\text{const}}, \quad (5)$$

$$d\bar{r}^2 = R^2(t)dr^2|_{t=\text{const}}. \quad (6)$$

It is important to keep in mind, in particular when considering derivatives, that Eqs. (5)–(6) do not represent a “coordinate transformation,” but rather a “rescaling” of the  $r$  coordinate on each  $t = \text{constant}$  slice [31].

With respect to the  $\bar{z}, \bar{r}, \phi$  coordinates, the wormhole form of the metric will be preserved if the metric on the embedded slice has the form

$$ds^2 = \frac{d\bar{r}^2}{1-\bar{a}(\bar{r})} + \bar{r}^2d\phi^2, \quad (7)$$

where  $\bar{a}(\bar{r}_0) = 1$ , i.e.,  $\bar{b}(\bar{r})$  has a minimum at some  $\bar{b}(\bar{r}_0) = \bar{r}_0$ . Equation (4) can be rewritten in the form of Eq. (7) by using Eqs. (5)–(6) and

$$\bar{a}(\bar{r}) = R(t)a(r). \quad (8)$$

The evolving wormhole will have the same overall size and shape relative to the  $\bar{z}, \bar{r}, \phi$  coordinate system, as the initial wormhole had relative to the initial  $z, r, \phi$  embedding space coordinate system. This is due to the fact that the embedding space corresponds to  $z, r$  coordinates that scale with time (each embedding space corresponds to a particular value of  $t = \text{constant}$ ). Following the embedding procedure [6], using Eqs. (3) and (7), one deduces that

$$\frac{d\bar{z}}{d\bar{r}} = \pm \left( \frac{1}{\bar{a}(\bar{r})} - 1 \right)^{-1/2} = \frac{dz}{dr}, \quad (9)$$

which implies

$$\bar{z}(\bar{r}) = \pm R(t)z(r). \quad (10)$$

Therefore, we see that the relation between the embedding space at any time  $t$  and the initial embedding space at  $t = 0$ , from Eqs. (6) and (10), is given by the following:

$$ds^2 = d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2d\phi^2 = R^2(t)[dz^2 + dr^2 + r^2d\phi^2]. \quad (11)$$

Relative to the  $\bar{z}, \bar{r}, \phi$  coordinate system the wormhole will always remain the same size, as the scaling of the embedding space compensates for the evolution of the wormhole. However, the wormhole will change size relative to the initial  $t = 0$  embedding space.

Writing the analog of the “flaring out condition” [6] for the evolving wormhole, we have  $d^2\bar{r}(\bar{z})/d\bar{z}^2 > 0$ , at or

near the throat. From Eqs. (5), (6), (8), and (9) it follows that

$$\frac{d^2\bar{r}(\bar{z})}{d\bar{z}^2} = \frac{1}{R(t)} \left( -\frac{a'}{2a^2} \right) = \frac{1}{R(t)} \frac{d^2r(z)}{dz^2} > 0, \quad (12)$$

at or near the throat, where the prime denotes the derivative with respect to  $r$ . Note that this also implies  $a' < 0$ , at or near the throat. Taking into account Eqs. (5), (8), and  $\bar{b}'(\bar{r}) = d\bar{b}/d\bar{r} = b'(r) = db/dr$ , one may rewrite the right-hand side of Eq. (12) relative to the barred coordinates as

$$\frac{d^2\bar{r}(\bar{z})}{d\bar{z}^2} = -\frac{\bar{a}'}{2\bar{a}^2} > 0, \quad (13)$$

or  $\bar{a} < 0$  at or near the throat. One verifies that, using the barred coordinates, the flaring out condition, Eq. (13), has the same form as for the static wormhole.

Thus, it can be shown that metric (2) represents a traversable wormhole provided

$$a(r_0) = 1, \quad a(r) < 1, \quad a'(r) < 0, \quad (14)$$

where  $r_0$  is the wormhole throat, which represents a minimum radius in the wormhole spacetime [6]. The second condition is imposed in order to avoid a change in the metric signature. The third condition is the flaring out condition and plays a fundamental role in the analysis of the violation of the energy conditions.

Note that the comoving radial distance defined by

$$l(r) = \pm \int_{r_0}^r \frac{dr}{\sqrt{1-a(r)}} \quad (15)$$

should be real and finite everywhere in spite of the fact that the  $rr$ -component of the covariant metric diverges at the throat; the  $\pm$  signs denote the upper and lower parts of the wormhole.

The energy-momentum tensor is  $T_B^A = \text{diag}(-\rho, P_r, P_t, P_t, \dots)$ , so that using the Einstein field equations and Eq. (2), the  $(n+1)$ -dimensional field equations are satisfied by the following stress-energy profile:

$$\rho(r, t) = \frac{(n-1)(n-2)a(r)}{2R(t)^2r^2} + \frac{(n-1)a'(r)}{2R(t)^2r} + \frac{n(n-1)\dot{R}(t)^2}{2R(t)^2}, \quad (16)$$

$$P_r(r, t) = -\frac{(n-1)\ddot{R}(t)}{R(t)} - \frac{(n-1)(n-2)\dot{R}(t)^2}{2R(t)^2} - \frac{(n-1)(n-2)a(r)}{2R(t)^2r^2}, \quad (17)$$

$$P_t(r, t) = -\frac{(n-2)a'(r)}{2R(t)^2 r} - \frac{(n-1)(n-2)\dot{R}(t)^2}{2R(t)^2} - \frac{(n-1)\ddot{R}(t)}{R(t)} - \frac{(n-2)(n-3)a(r)}{2R(t)^2 r^2}, \quad (18)$$

where the overdot denotes a derivative with respect to time.

The Ricci scalar will play a fundamental role in our analysis, so we write it down explicitly as

$$\mathcal{R} = \frac{2n\ddot{R}(t)}{R(t)} + \frac{(n-1)a'(r)}{R(t)^2 r} + \frac{n(n-1)\dot{R}(t)^2}{R(t)^2} + \frac{(n-1)(n-2)a(r)}{R(t)^2 r^2}. \quad (19)$$

Since the Ricci scalar is only a function of time in standard cosmological models, it provides a motivation to use this property as a simplifying assumption in our calculations, in the presence of a wormhole. In other words, we are looking for classes of solutions corresponding to the choice of a homogeneous Ricci scalar, i.e.,  $\partial\mathcal{R}/\partial r = 0$ , which implies

$$r^2 a''(r) + (n-3)ra'(r) - 2(n-2)a(r) = 0. \quad (20)$$

The above differential equation yields the following solution:

$$a(r) = \frac{r_0^{n-2} - kr_0^n}{r^{n-2}} + kr^2, \quad (21)$$

where the condition  $a(r_0) = 1$  was used to eliminate the integration constant. We point out that although  $k$  can, in principle, be a continuous variable, we have used the fact that the spacetime is asymptotically FRW and applied the normalization  $k = 0, \pm 1$  for the curvature constant. It is worthwhile to mention that it is common to consider static wormholes supported by radiation that have a traceless stress-energy tensor [33]. In such a case, the Ricci scalar vanishes if there is no cosmological constant within the framework of GR. Our assumption leads to the same situation if the scale factor is assumed to be independent of time, i.e., for a static case.

As mentioned before, the dimensionless shape function  $a(r)$  should satisfy the conditions (14). It is easy to show that for  $k = 0$  and  $-1$  these conditions are satisfied, whereas for  $k = +1$  they are not. Therefore, we continue our discussions using  $k = 0$  and  $-1$ , which present flat and open universes, respectively.

With  $a(r)$  in hand, given by Eq. (21), one can rewrite the field equations for the spatially flat background ( $k = 0$ ) as

$$\rho = \rho_{(\text{fb})}, \quad (22)$$

$$P_r = -\frac{(n-1)(n-2)r_0^{n-2}}{2r^n R^2} + P_{(\text{fb})}, \quad (23)$$

$$P_t = \frac{(n-2)r_0^{n-2}}{2r^n R^2} + P_{(\text{fb})}, \quad (24)$$

where  $\rho_{(\text{fb})}$  and  $P_{(\text{fb})}$  are the respective flat background components given by

$$\rho_{(\text{fb})} = \frac{n(n-1)\dot{R}^2}{2R^2}, \quad (25)$$

$$P_{r(\text{fb})} = P_{t(\text{fb})} = P_{(\text{fb})} \quad (26)$$

$$= -\frac{(n-1)\ddot{R}}{R} - \frac{(n-1)(n-2)\dot{R}^2}{2R^2}, \quad (27)$$

respectively.

For the specific case of the open background ( $k = -1$ ), we have

$$\rho = \rho_{(\text{ob})}, \quad (28)$$

$$P_r = -\frac{(n-1)(n-2)(r_0^n + r_0^{n-2})}{2r^n R^2} + P_{(\text{ob})}, \quad (29)$$

$$P_t = \frac{(n-2)(r_0^n + r_0^{n-2})}{2r^n R^2} + P_{(\text{ob})}, \quad (30)$$

where the  $\rho_{(\text{ob})}$  and  $P_{(\text{ob})}$  components correspond to the open background, and are given by

$$\rho_{(\text{ob})} = \rho_{(\text{fb})} - \frac{n(n-1)}{2R^2}, \quad (31)$$

$$P_{(\text{ob})} = P_{r(\text{ob})} = P_{t(\text{ob})} = P_{(\text{fb})} + \frac{(n-1)(n-2)}{2R^2}. \quad (32)$$

Since our solutions are in a spherically symmetric cosmological background, the components of the stress-energy tensor should be asymptotically independent of  $r$ . It is easy to see this expected behavior is obeyed by them.

### III. TWO-WAY TRAVERSABILITY OF WORMHOLE STRUCTURE

One of the most interesting properties of a wormhole as pointed out by Morris and Thorne [8] is its two-way traversability. In this section, some proofs will be presented to show that the wormholes discussed in this paper are indeed two-way traversable.

#### A. Redshift of a comoving source

Consider a radially moving light signal emitted from a comoving source. We assume that the signal is emitted at  $(t_1, l_1)$  ( $l_1$  is a comoving coordinate) and received by a

distant comoving observer at  $(t_0, l_0)$ . Using the metric (2) and Eq. (15) for a radial beam, we have

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_{l_1}^{l_0} dl, \quad (33)$$

where  $l(r)$  is the comoving radial distance defined in Eq. (15). Note that  $l_0$  and  $l_1$  can belong to either side of the throat. It is obvious that the rhs of Eq. (33) is independent of time. Therefore, the lhs should also be so and a signal that is emitted in an interval  $\tau_1$  should be received in an interval  $\tau_0$  such that  $(\tau_1, \tau_0 \ll t_1, t_0)$

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_{t_1+\tau_1}^{t_0+\tau_0} \frac{dt}{R(t)}. \quad (34)$$

Since  $\tau_0$  and  $\tau_1$  are very short time intervals, one deduces that

$$\frac{\tau_0}{\tau_1} = \frac{R(t_0)}{R(t_1)} = 1 + z, \quad (35)$$

where  $R(t_0)$  is the scale factor at the time of observation,  $R(t_1)$  is the scale factor at the time of emission, and  $z$  is the cosmological redshift. This shows that the redshift is the same as the cosmological redshift and no extra redshift is caused by the wormhole. It remains to examine whether the signal ever reaches the throat in a finite time or not. This will be addressed below.

### B. The behavior of radial geodesics

Since  $r$  is greater than  $r_0$  on both sides of the throat, one cannot trivially deduce whether the light signal passes through the throat or not using the  $r$  coordinate. Therefore, we need to transform from the radial coordinate  $r$  to the comoving radial coordinate  $l$  in order to analyze this behavior. Using Eq. (15), the analysis is more transparent. Consider radial motion so that the geodesic equation reads

$$\frac{d^2 l}{d\lambda^2} + \frac{2}{R} \frac{dR}{dt} \frac{dt}{d\lambda} \frac{dl}{d\lambda} = 0, \quad (36)$$

and

$$\frac{d^2 t}{d\lambda^2} + R \frac{dR}{dt} \left( \frac{dl}{d\lambda} \right)^2 = 0. \quad (37)$$

Equation (36) yields the first integral

$$\frac{dl}{d\lambda} = \frac{C}{R^2}. \quad (38)$$

Equation (38) shows that  $dl/d\lambda$  does not undergo a sign change along the path and neither does it vanish. This shows that the particle or signal continues its path, passes

the throat, and goes to the other side of the wormhole, and therefore the wormhole is two-way traversable.

### C. Reachability of the wormhole throat

In order to prove that there is a finite proper distance between a specific point and the throat, Eq. (15) should be solved explicitly. For arbitrary  $n$ , there is no analytical solution for the integral (15). Therefore, one could obtain  $l(r)$  in the vicinity of the throat, which is sufficient for our purpose. Using the approximate relation  $1 - (r_0/r)^{n-2} \approx (n-2)(r/r_0 - 1)$ , one obtains

$$l(r) \approx \begin{cases} 2\sqrt{\frac{r_0(r-r_0)}{n-2}} & \text{for } k = 0 \\ 2\sqrt{\frac{r_0(r-r_0)}{(n-2+nr_0^2)}} & \text{for } k = -1 \end{cases}, \quad (39)$$

which are clearly finite distances. Therefore, the throat is not located at spatial infinity and a finite time is required to reach it.

We should mention that horizons are theoretical constructs that qualitatively have two main specific properties: first, they are one-way membranes, and, second, the corresponding redshift (as observed by a distant observer) is infinite. More precise mathematical details can be found in [13,34]. Since it is common to consider the singularities of the metric as candidates of being horizons, one might ask whether the coordinate singularity at the throat forms a horizon. Based on the above-mentioned general qualitative features of horizons and according to the results obtained in Secs. (III A) and (III B), it is justified that there is no horizon at or around the throat. The possibility of the existence of a cosmological horizon, however, depends on the behavior of the scale factor  $a(t)$  and is not essentially affected by the presence or absence of the wormhole.

## IV. WORMHOLE SOLUTIONS IN DIFFERENT EXPANSION REGIMES

One of the properties of normal matter is that it satisfies the energy conditions, in particular, the NEC and the WEC. It was mentioned in the introduction that the matter that supports the static wormhole geometry violates the NEC and is therefore denoted exotic matter. The NEC requires that  $T_{\mu\nu} k^\mu k^\nu \geq 0$ , where  $k^\mu$  is any null vector. In terms of the energy density, radial pressure, and tangential pressure the NEC becomes [35]

$$\rho + P_r \geq 0, \quad \rho + P_t \geq 0. \quad (40)$$

Note that the WEC, in addition to the conditions considered above, also imposes a positive energy density,  $\rho \geq 0$ . In what follows, we investigate the NEC for the wormhole solutions in different expansion regimes.

### A. Flat background

In the case of a flat background, one can obtain two different solutions for the scale factor  $R(t)$  by applying the equation of state  $P_{(\text{fb})} = \omega\rho_{(\text{fb})}$ , given by

$$R(t) = \begin{cases} A_1 e^{\alpha t}, & \alpha > 0 \text{ for } \omega = -1 \\ A_2 t^{\frac{2}{n(1+\omega)}}, & \text{for } \omega \neq -1 \end{cases}. \quad (41)$$

The specific case of  $\omega = -1$  represents the inflationary regime. The case of  $\omega \neq -1$  represents the radiation dominated and matter dominated expansion regimes by considering  $\omega = 1/3$  and  $\omega = 0$ , respectively.

Consider first the inflationary expansion regime, where, by using Eqs. (22)–(27), one obtains

$$\rho = \frac{n(n-1)\alpha^2}{2}, \quad (42)$$

$$\rho + P_r = -\frac{(n-1)(n-2)r_0^{n-2}}{2r^n A_1^2} e^{-2\alpha t}, \quad (43)$$

$$\rho + P_t = \frac{(n-2)r_0^{n-2}}{2r^n A_1^2} e^{-2\alpha t}. \quad (44)$$

It is clear that  $\rho + P_r$  is always negative while  $\rho$  and  $\rho + P_t$  are always positive. Therefore, the NEC is always violated. But  $\rho + P_r$  tends to zero as  $t$  increases and therefore the wormhole matter ranges from an exotic matter regime to normal matter over time.

We continue our discussions with the second solution for the scale factor, for  $\omega \neq -1$ . In this case, we have

$$\rho = \frac{2(n-1)}{n(1+\omega)^2 t^2}, \quad (45)$$

$$\rho + P_r = -\frac{(n-1)(n-2)r_0^{n-2}}{2A_2^2 r^n} \frac{1}{t^{\frac{4}{n(1+\omega)}}} + \frac{2(n-1)}{(1+\omega)n} \frac{1}{t^2}, \quad (46)$$

$$\rho + P_t = \frac{(n-2)r_0^{n-2}}{2A_2^2 r^n} \frac{1}{t^{\frac{4}{n(1+\omega)}}} + \frac{2(n-1)}{(1+\omega)n} \frac{1}{t^2}. \quad (47)$$

For  $\omega < -1$ , it is clear that the NEC is violated due to  $\rho + P_r < 0$ . For  $\omega > -1$ , we have that  $\rho$  and  $\rho + P_t$  are always positive, while the quantity  $\rho + P_r$  should be analyzed in more detail. Figure 1 shows the behavior of  $\rho + P_r$  with respect to time at the throat  $r_0 = 1$  for  $A_2 = 1$ ,  $n = 3$ , and  $\omega = 0$  and  $1/3$ . It can be seen that, although the wormhole matter at the throat initially satisfies the NEC, the latter is violated as time passes. In Fig. 2, the quantity  $\rho + P_r$  is plotted against  $r$  and  $t$  with  $r_0 = 1$ ,  $A_2 = 1$ ,  $n = 3$  for  $\omega = 0$  and  $1/3$ . The figure shows that the region of exotic matter in the vicinity of the wormhole throat increases as time increases.

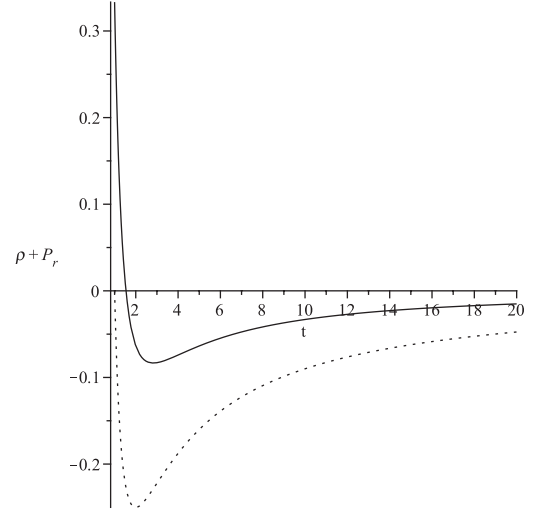


FIG. 1. For the flat background solution, the plot depicts the behavior of  $\rho + P_r$  at the throat, assuming  $r_0 = 1$ , with respect to time, with  $n = 3$  and  $A_2 = 1$  for  $\omega = 0$  (solid curve) and  $\omega = 1/3$  (dashed curve). The plot shows that, although initially there is normal matter at the throat, the throat matter tends to an exotic matter regime over time.

As we are considering higher-dimensional wormholes in an expanding spacetime, an interesting scenario to examine is whether these dynamic wormholes could be constructed from normal matter for  $n > 3$ . This is indeed the case for the solutions discussed here, by choosing suitable values for the constants. Figure 3 plots  $\rho + P_r$  against  $r$  and  $t$  for  $r_0 = 1$ ,  $A_2 = 2$ ,  $n = 4$ , and  $\omega = -1/2$ . As depicted in the figure,  $\rho + P_r$  is always positive for this choice of constants and therefore the NEC (and also WEC) is satisfied for the whole wormhole structure.

### B. Open background

In the case of the open background, by applying  $P_{(\text{ob})} = \omega\rho_{(\text{ob})}$ , we consider the following analytical solutions:

$$R(t) = \begin{cases} A_3 \sinh(\frac{t}{A_3}), & \text{for } \omega = -1 \\ A_4 t, & \text{for } \omega = \frac{2-n}{n} \end{cases}, \quad (48)$$

where the case  $\omega = -1$  corresponds to the inflationary regime.

Let us first investigate the solution corresponding to the inflationary expansion regime,  $R(t) = A_3 \sinh(t/A_3)$ . Using Eqs. (28)–(32), we have

$$\rho = \frac{n(n-1)}{2A_3^2}, \quad (49)$$

$$\rho + P_r = -\frac{(n-1)(n-2)(r_0^n + r_0^{n-2})}{2A_3^2 r^n \sinh(\frac{t}{A_3})^2}, \quad (50)$$

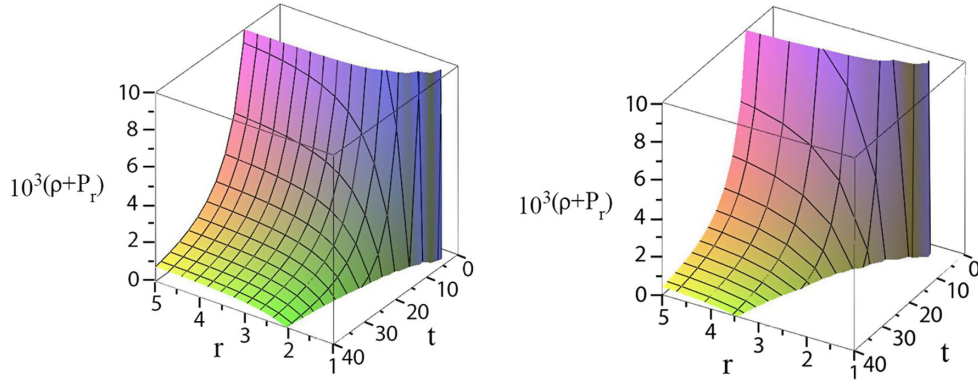


FIG. 2 (color online). For the flat background solution, the plots depict the behavior of  $10^3(\rho + P_r)$  with respect to  $r$  and  $t$ , with  $n = 3$  and  $A_2 = 1$ , for  $\omega = 0$  (left plot) and  $\omega = 1/3$  (right plot), respectively. As it is clear from the plots, the region of the exotic matter in the vicinity of the throat increases as time passes.

$$\rho + P_t = \frac{(n-2)(r_0^n + r_0^{n-2})}{2A_3^2 r^n \sinh(\frac{t}{A_3})^2}. \quad (51)$$

$$\rho + P_r = \left[ -\frac{(n-1)(n-2)(r_0^n + r_0^{n-2})}{2A_4^2 r^n} + \frac{(n-1)(A_4^2 - 1)}{A_4^2} \right] \frac{1}{t^2}, \quad (53)$$

As in the case of the flat background,  $\rho + P_r$  is always negative, implying the violation of the NEC throughout the spacetime, while  $\rho$  and  $\rho + P_t$  are always positive. However,  $\rho + P_r$  tends to zero as  $t$  increases and therefore during the inflationary era the wormhole matter tends from an exotic matter regime to a normal matter one, at temporal infinity.

Consider now the second case where  $\omega = (2-n)/n$  and  $R(t) = A_4 t$ . In this case, one obtains the following relationships:

$$\rho = \frac{n(n-1)(A_4^2 - 1)}{2A_4^2} \frac{1}{t^2}, \quad (52)$$

For  $A_4 < 1$ , it is clear that the NEC is violated due to  $\rho + P_r < 0$ ; note also that  $\rho < 0$ . However, this case  $A_4 < 1$  should be excluded, as  $\rho$  coincides with the background energy density. This would imply that the energy density of the Universe is negative, so it is not physically acceptable. For  $A_4 > 1$ , it is obvious that  $\rho$  and  $\rho + P_t$  are always positive while  $\rho + P_r$  should be investigated. Figure 4 depicts  $\rho + P_r$  in terms of  $r$  and  $t$  for  $r_0 = 0.5$ ,  $A_4 = 3$ , and  $n = 4$ . This figure shows that by choosing suitable

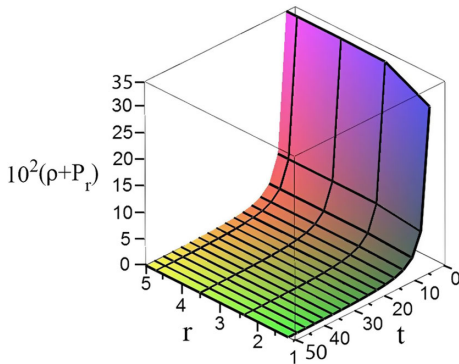


FIG. 3 (color online). For the flat background solution, the plot depicts the behavior of  $10^2(\rho + P_r)$  with respect to  $r$  and  $t$  for  $n = 4$ ,  $r_0 = 1$ ,  $A_2 = 2$ , and  $\omega = -1/2$ . This plot shows that, for suitable choices of constants, there are wormhole structures constructed from matter that satisfy the null energy condition.

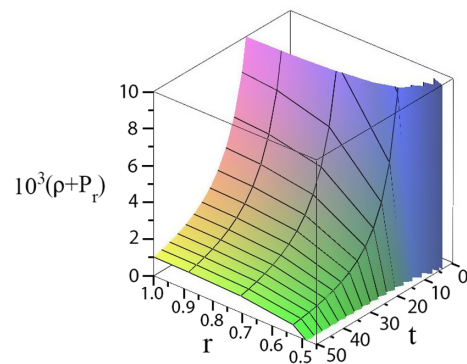


FIG. 4 (color online). For the open background, the plot depicts the behavior of  $10^3(\rho + P_r)$  with respect to  $r$  and  $t$  for  $n = 4$ ,  $r_0 = 0.5$ , and  $A_4 = 3$ . It shows that in the case of an open universe, it is possible to have wormholes constructed from normal matter.

constants, there is a wormhole structure constructed from normal matter.

## V. SUMMARY AND CONCLUSION

The present paper investigates the possibility and naturalness of expanding wormholes in higher dimensions, which is an important ingredient of modern theories of fundamental physics, for instance, string theory, supergravity, and Kaluza-Klein, amongst others. One of our motivations for considering wormhole solutions in an expanding cosmological background refers to the inflationary theory where the quantum fluctuations in the inflaton field may have served as the seed for the large scale structures in the Universe. Nontrivial topological objects such as microscopic wormholes may have been formed through the quantum foam and enlarged to macroscopic size during inflation and in the subsequent expansion of the Universe. Indeed, if most of the wormholes in the quantum foam survived enlargement through inflation, then the Universe might be far more inhomogeneous and topologically complicated than we observe.

Indeed, postulating higher-dimensional spacetimes is an important ingredient of modern theories of fundamental physics. In this context, the existence of higher dimensions

may help construct wormhole solutions that respect energy conditions. In particular, in a cosmological setup, microscopic, dynamical wormholes produced in the early Universe may be inflated to macroscopic scales and thus be—at least in principle—astrophysically observable. In this work, by assuming a homogeneous matter field (i.e., energy density depending only on the time coordinate), which holds in the standard cosmology, we arrived at interestingly simple and exact solutions. More specifically, we considered a particular class of wormhole solutions corresponding to a spatially homogeneous Ricci scalar. The possibility of obtaining solutions with normal and exotic matter was explored and we found new solutions including those that satisfy the NEC in specific time intervals. In particular, in five dimensions, we found solutions that satisfy the NEC everywhere.

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