PHYSICAL REVIEW D **90,** 024065 (2014)

de Sitter vacua from nonperturbative flux compactifications

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(Received 22 January 2014; revised manuscript received 12 May 2014; published 23 July 2014)

We present stable de Sitter solutions of $\mathcal{N} = 1$ supergravity in a geometric type IIB duality frame with the addition of nonperturbative contributions. Contrary to the standard approach, we retain the moduli dependence of both the tree-level superpotential and its nonperturbative contribution. This provides the possibility for a single-step stabilization of all moduli simultaneously in a de Sitter vacuum. Using a genetic algorithm we find explicit solutions with different features.

DOI: 10.1103/PhysRevD.90.024065 PACS numbers: 04.50.-h, 04.65.+e, 11.25.Mj, 98.80.-k

I. INTRODUCTION

The importance of accelerating space-times in cosmology, both for inflation and dark energy, makes it critical to understand the role of de Sitter (dS) vacua in string theory. Many such constructions have been criticized as being rather *ad hoc*. In the Kachru-Kallosh-Linde-Trivedi (KKLT) setup [1], one adds a nonperturbative contribution as well as explicit, supersymmetry breaking (~~SUSY~~) uplift terms to achieve a dS vacuum. These are necessary additions to $\mathcal{N} = 1$ compactifications with only IIB gauge fluxes, which only lead to Minkowski vacua with flat directions [2]. On the IIA side, the situation regarding moduli stabilization is better, as the inclusion of gauge fluxes alone leads to anti–de Sitter (AdS) vacua [3]. However, it is not possible to obtain dS solutions in this vein [4]. Adding metric fluxes does lead to dS solutions [5,6], but all known examples are unstable. In this paper, we show that geometric and isotropic fluxes with nonperturbative contributions are enough to stabilize simultaneously all moduli in a dS vacuum, in the simplest scenario possible, widening the dS landscape.

We focus on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ compactification with fluxes in type IIB supergravity in ten dimensions. The number of untwisted moduli is $(h^{(1,1)}, h^{(2,1)}) = (3,3)$ plus the dilaton. We concentrate on the isotropic case with a single Kähler and complex structure moduli, that is, an STU-type of model. The Kähler potential takes the form

$$K = -\log[-i(S-\bar{S})] - 3\log[-i(T-\bar{T})] - 3\log[-i(U-\bar{U})]. \quad (1)$$

The scalar potential takes the usual form (we are setting $M_P^{-2} = 8\pi G = 1$):

$$V = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2), \quad (2)$$

$D_I W = \partial_I W + \partial_I K W$ with I, J labeling all moduli.

The tree-level superpotential depends on the dilaton S and complex structure U (jointly referred to as complex structure moduli). These are generated by the presence of Ramond–Ramond (RR) flux F_3 and Neveu-Schwarz–Neveu-Schwarz (NSNS) flux H_3 (with coefficients a_i, b_i, respectively):

$$W_{\text{tree}} = P(a_i, U) - SP(b_i, U), \quad (3)$$

where $P(f_i, U)$ are polynomials in U of the form

$$P(f_i, U) = f_0 - 3f_1 U + 3f_2 U^2 - f_3 U^3. \quad (4)$$

Thus, these fluxes generate a potential for the complex structure moduli stabilizing them [2]. However, the Kähler modulus remains as a flat direction.

To stabilize the T-modulus, the standard approach has been to first use the tree-level flux contributions to fix S and U in a supersymmetry (SUSY) vacuum, and, second, to introduce a nonperturbative term $W_{\text{NP}}(T)$ for the Kähler modulus, allowing its stabilization. It is assumed that the first step results in a constant contribution to the superpotential, $W_0 = \text{const}$, and a constant coefficient for the nonperturbative term, $A_0 = \text{const}$:

$$W = W_0 + A_0 e^{ixT}, \quad (5)$$

where $x = 2\pi/K$ for gaugino condensation with gauge group rank K. Using this superpotential, only AdS minima can be obtained. Therefore, a final third step has been taken by adding a suitable *uplifting* term [1], lifting the AdS minimum to a dS.

This three-step process has been criticized in the literature since, in general, not only W_0 but also the coefficient A_0 depends on the complex structure moduli [7]. Therefore, the second step can lead to complications since heavy modes could mix with light modes [8,9] and create instabilities [10] (see however [11] for a discussion on the consistency conditions for this step). Moreover, adding an uplifting term by hand is under limited theoretical control (e.g., adding an antibrane is an explicit heavy breaking of supersymmetry and it is not clear that one can still use a supergravity

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description). An alternative was proposed to uplift using D-terms in [12,13]. Finally, the third step has been relaxed in [14], where it was shown how to obtain dS minima without the need of an artificial uplifting term.[1] However, the second step has been so far assumed in order to obtain stabilization of all moduli.[2] This paper addresses the natural question of whether a single-step process can give rise to (meta)stable dS vacua, stabilizing all moduli simultaneously, even in simple models.[3]

II. A NOVEL MECHANISM OF MODULI STABILIZATION

To study this possibility, we consider the usual tree-level superpotential augmented with a nonperturbative term. The coefficient of the latter will generically depend on the complex structure moduli S and U in a nontrivial but unknown way. Following the reasoning for nongeometric fluxes of [23], it seems natural to model this dependence in a way that respects the S and U duality covariance.[4] This leads to the following ansatz:

$$W = W_{\mathrm{tree}} + [P(\tilde{a}_i, U) - P(\tilde{b}_i, U)S]e^{ixT}, \qquad (6)$$

where all four polynomials have the structure (4). Although we do not provide a rigorous derivation of this moduli dependence in the nonperturbative term, it is constrained to be such a polynomial in S, U by duality arguments; indeed, a similar form has been studied in an explicit string theory setting [25]. An alternative interpretation of this ansatz is that the polynomials represent a Taylor expansion in terms of small S, U, as we will justify below.

The coefficients of the nonperturbative term appear in complete analogy with the gauge fluxes in the superpotential. Thus we refer to $\tilde{a}_i$ and $\tilde{b}_i$ as nonperturbative fluxes. This leads to a total set of 16 fluxes. However, as we explain later, it will be sufficient to have 12 fluxes. We therefore set the fourth polynomial equal to zero.

To find solutions, we use the property that any solution to the equations of motion can be represented by a solution in the origin of moduli space (in our conventions located at $S = T = U = i$ with $x = 1$). This technique was first proposed in the context of half-maximal supergravity [26] and subsequently used to explore the vacuum structure of maximal supergravity [27,28], but can be applied to any theory with a homogeneous scalar manifold. This avoids an overcounting of solutions and reduces dramatically the complexity of the equations of motion. While these in general can be high-degree polynomials for the fields, in the origin these reduce to quadratic equations in terms of fluxes. Solutions correspond to flux configurations for which these quadratic combinations vanish. The origin is however not a configuration that should be considered a valid supergravity limit, since the volume of the internal space and the string coupling are both equal to one. Below we explain how to deal with this issue.

To solve the resulting quadratic equations in the fluxes $\{a_i, b_i, \tilde{a}_i\}$, we use the fact that when supersymmetry is preserved, the equations of motion are implied [29]:

$$D_I W \equiv A_I + iB_I = 0 \Rightarrow \partial_I V = 0, \qquad (7)$$

where the six ~~SUSY~~ parameters A_I and B_I are linear combinations of the superpotential couplings $\{a_i, b_i, \tilde{a}_i\}$. It will be advantageous to split up the latter in (linear combinations of) two sets: there are six ~~SUSY~~ parameters while the orthogonal combinations preserve SUSY. Via this approach, in general all moduli take part in ~~SUSY~~ and contribute to the uplifting of the potential. This is to be contrasted to for example [1], where uplifting is only considered in the direction of $\mathrm{Im}(T)$ while S and U are in SUSY minima.

Next, one exploits the fact that the equations of motion are implied by SUSY. Therefore, the equations of motion become quadratic in the ~~SUSY~~ parameters or bilinear in the ~~SUSY~~ and SUSY parameters. For this to work, the total set of parameters must be at least equal to twice the number of (real) fields, in our case $6 + 6 = 12$. Type IIB tree-level flux contributions to the superpotential consist of eight parameters a_i, b_i (3), (4). In [29] the extra couplings were taken to be so-called nongeometric fluxes. Here, we add the nonperturbative fluxes $\tilde{a}_i$ (6).

Given these sets of solutions parametrized by the six ~~SUSY~~ parameters, we follow [30] in using a genetic algorithm to scan this parameter space to look for stable dS solutions (for similar applications of genetic algorithms, see [31–33]). We thus require both the cosmological constant as well as all the scalar masses, obtained by diagonalizing the mass matrix

$$(m^2)^I_J = \frac{K^{IK}\partial_K \partial_J V}{V}, \qquad (8)$$

to be positive.

III. PERTURBATIVE RELIABILITY

In order to get to a regime for the values of the moduli that is a reliable supergravity approximation of string theory, we need to ensure that we work at large volume

[1] Further uplifting alternatives using perturbative corrections have been discussed in [15,16].

[2] A one-step stabilization in heterotic orbifold compactifications using the explicit modular covariance of the superpotential has been discussed in [17]. In the large volume scenario [18], an alternative single-step stabilization giving rise to dS vacua has been achieved term by term in an expansion in inverse powers of the volume [19–21]. These solutions include perturbative corrections to the Kähler potential, D-terms, and chiral matter, and thus go beyond our present discussion.

[3] A similar approach in the IIA duality frame is discussed in [22].

[4] Indeed, in a duality covariant formulation of $\mathcal{N} = 1$ supergravity with nongeometric fluxes, it is possible to find stable dS solutions [24].

and small string coupling, such that higher string mode contributions and loop corrections are suppressed: (a) Large volume: $\mathcal{V} \sim r^6 \gg 1$. (b) Small string coupling: $g_s^{-1} \gg 1$, where $\mathcal{V} = \mathrm{Vol.}/\ell_s$, with ℓ_s being the string scale, and we have introduced a characteristic (dimensionless) radius of the internal space, r. The volume is further given in terms of the overall Kähler modulus as $\mathrm{Im}T = \mathcal{V}^{2/3}$ and the string coupling in terms of the dilation as $g_s = (\mathrm{Im}S)^{-1}$.

Consider the following rescaling of the volume and the string coupling $r \to N^{\alpha} r$, $g_s \to N^{-\beta} g_s$, for α and β some positive numbers with $N \gg 1$. From the expression for the scalar potential, the fluxes and x have to be rescaled as

$$\begin{aligned} a_i, \tilde{a}_i &\to N^{6\alpha+\beta/2+\gamma} a_i, \tilde{a}_i, \qquad x \to N^{-4\alpha} x, \\ b_i &\to N^{6\alpha-\beta/2+\gamma} b_i \end{aligned} \tag{9}$$

for the solution to be preserved. We have also introduced a parameter γ that represents an overall scaling of the fluxes that is always possible to perform. The potential scales as $V \to N^{2\gamma} V$, and the normalized masses remain invariant. For the special case of gaugino condensation, where $x = \frac{2\pi}{K}$, the scaling (9) implies that we need to scale K as $K \to N^{4\alpha} K$.

Given a solution, we can achieve a large volume and small string coupling regime, via a suitable rescaling of the parameters. A drawback may be that this rescaling requires a small value of the parameter x, which, in the case of gaugino condensation, translates into a large rank of the gauge group K. In the context of noncompact Calabi-Yaus, it has been discussed that arbitrarily high gauge group ranks are possible [34]. In the compact case the situation turns out to be more restrictive, but relatively large values are possible [16].

Finally, we should also consider the tadpole cancellation condition, which is a quadratic combination of flux parameters, $H_3 \wedge F_3$:

$$N_{\mathrm{D3}} = a_3 b_0 - 3a_2 b_1 + 3a_1 b_2 - a_0 b_3, \tag{10}$$

scaling according to $N_{\mathrm{D3}} \to N^{12\alpha+2\gamma} N_{\mathrm{D3}}$. As the tadpole is bounded from below by the orientifold contribution, one should worry about this rescaling in the case of negative N_{D3}. Indeed, in all our examples below, the tadpole will be negative. In order to avoid that the large volume limit pushes the tadpole below its lower limit, one can choose the γ parameter suitably.

Notice that there is no particular requirement of the value for the complex structure modulus U at the minimum. Therefore, we keep this field to the origin. However, we could rescale it as well to small values in such a way that the power expansion in the nonperturbative function P_3 can be truncated at the third power consistently.

We next consider the relevance of possible perturbative corrections to the Kähler potential since these could dominate over the nonperturbative contributions to the superpotential, rendering the present setup inconsistent. Perturbative contributions scale with $K_{\mathrm{P}} \sim 1/(\mathcal{V} g_s^{3/2})$. Since our method to find solutions starts with all fields at the origin and fluxes of the same order, all contributions in the superpotential are of the same order. After making the above described rescalings, all terms in the potential scale in the same way and, once perturbative Kähler contributions are added, we can write

$$V_{\mathrm{full}} \to N^{2\gamma}(V + K_{\mathrm{P}} V) \sim N^{2\gamma}\left(V + N^{-6\alpha+\frac{3\beta}{2}} V\right); \tag{11}$$

hence, K_{P} contributions will be suppressed with a factor $N^{-6\alpha+\frac{3\beta}{2}}$ compared to the potential calculated here. We can therefore safely neglect these by a suitable choice of α, β.

IV. EXPLICIT DE SITTER SOLUTIONS

We performed five individual searches where additional criteria were required. These additional criteria were chosen to be

1, 2: maximize and minimize $\tilde{\gamma} = |DW|^2/(3|W|^2)$ (while keeping $\tilde{\gamma} > 1$),

3, 4: maximize and minimize the scale between the fluxes $|b_i|/|a_i|$,

5: minimize the scale between the fluxes $|\tilde{a}_i|/|a_i|$.

The main properties of our solutions are presented in Table I, while the $\cancel{\mathrm{SUSY}}$ parameters for these solutions can be found in Table II. A number of general features can be extracted from these examples. First, it follows that the $\cancel{\mathrm{SUSY}}$ and AdS scales are always of the same order and cannot be separated. The maximum ratio between the scales is ≈1.0256, as follows from solution 1. Similarly, one can approach a ratio equal to one with very good accuracy, as illustrated by solution 2. Another observation from this solution is that the lowest mass can be made very large compared to the potential energy V_0. Finally, as the flux parameters are of decreasing order,

$$\{\tilde{a}_0, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3\} \approx \{0.0835, 0.0702, 0.0372, 0.00921\}; \tag{12}$$

the small-U expansion of (6) is justified in this case.

A second point is that we tried to achieve a hierarchy between the RR and NSNS fluxes. The reason for doing so is the small coupling limit; as the rescaling (9) acts differently on these two set of fluxes, we would like to start off with a hierarchy of values for these. After the rescaling, we end up at small coupling with fluxes of the same order. As can be seen from solutions 3 and 4, it is possible to achieve a small degree of separation between the two sets of fluxes. However, this separation is only due to large contributions from the nonperturbative fluxes.

Finally, in solution 5 we were able to create a hierarchy between the perturbative and nonperturbative fluxes. This hierarchy is only possible to achieve with the loss of a hierarchy of the NSNS and RR sector. The reason for this

TABLE I. Properties of the solutions (Sol.). The masses are normalized with the potential and all scales are given in Planck units.

	Sol. 1	Sol. 2	Sol. 3	Sol. 4	Sol. 5
V_0	0.00113	2.23×10^{-12}	0.0000251	0.0000234	8.61×10^{-12}
$\tilde{\gamma} - 1$	0.0256	5.11×10^{-11}	0.00248	0.0160	6.05×10^{-10}
$\frac{\lvert b_i \rvert}{\lvert a_i \rvert}$	0.298	0.599	1.32	0.208	0.997
$\frac{\lvert \tilde{a}_i \rvert}{\lvert a_i \rvert}$	0.611	0.274	0.528	0.621	0.000227
Masses	39.0	2.11×10^{10}	1140.	76.2	2.20×10^{9}
	19.7	8.71×10^{9}	387.	36.0	9.80×10^{8}
	12.4	7.00×10^{9}	106.	19.6	2.45×10^{8}
	9.74	3.41×10^{9}	18.4	11.4	490000.
	0.00236	1.26×10^{9}	6.16	0.774	101000.
	0.0000747	6.01×10^{8}	0.0000612	0.000252	100000.

TABLE II. The values of the ~~SUSY~~ parameters defined by (7) that give the solutions displayed in Table I (rounded to six digits).

	Sol. 1	Sol. 2	Sol. 3	Sol. 4	Sol. 5
A_1	−0.147286	−0.0859982	0.0590861	−0.0115235	0.000516097
A_2	0.449418	−1.58993	−0.483429	0.165447	1.15243
A_3	−0.907814	0.4631	−0.131249	−0.144582	−0.000587804
B_1	0.377918	−0.236806	0.0870739	0.0793589	0.00319387
B_2	1.6678	−1.12127	0.826607	0.259372	−0.196848
B_3	0.173821	−0.047207	−0.0614712	0.0902761	−0.00969035

lies in the structure of the equation of motion for S. On the level of the perturbative superpotential, this equation forces the so-called imaginary self-dual (ISD) condition for the flux $G_3 = F_3 + SH_3$ [2]. Via the addition of small nonperturbative contributions, it is only possible to perturb this condition. This is why we see in solutions 3 and 4 that the nonperturbative terms contribute much more than in solution 5. For the same reason, i.e., small nonperturbative contributions cannot significantly change the ISD condition, we are not able to find solutions without net O-planes, as is argued to be possible in type IIA [22].

For the most interesting solution, 5, we observe also that because the nonperturbative contributions are suppressed, one may expect a separation of masses among S, U and T. Indeed, the last two smallest masses in Table I correspond to the eigenvectors that are dominated by the real and imaginary parts of T. The other small mass corresponds mostly to a combination of the S, U axions. Moreover, the lowest mass is still significantly larger than the potential.

Finally, it is interesting to consider the interplay between stability and dS solutions. For nongeometric stable dS solutions, the intersection regions of stability and dS are thin sheets because of small differences in the shape of these landscapes [29], thus requiring fine-tuning. This is not the case for the present nonperturbative solutions. The stability and potential landscapes, plotted in Fig. 1, have noticeably different shapes. This implies that there are sizeable intersection regions.[5]

This nontrivial overlap will be important when taking flux quantization into account. By scaling N large, the parameters A_I and B_I become approximately integers. One can then make a_i, b_i integers by an appropriate truncation. This will slightly modify the solution, (inversely) related to the order to which we rescale N. However, because of the large intersection areas of stability and dS, only a very coarse truncation would significantly modify the solution and possibly spoil stability and/or dS. In our case, where the orientifold tadpole gives a bound on how much rescaling can take place, the truncation would have to indeed be quite coarse. On account of the large stable dS regions, one can achieve quantization without losing stability or positive potential energy. We have explicitly checked this for solution 5, which can be rescaled and truncated to the flux parameters

$$\{a_0, a_1, a_2, a_3, b_0, b_1, b_2, b_3\} = \{-1, 4, 1, -12, 4, 0, -1, 0\}, \tag{13}$$

[5]This fact does not seem to hinge on the duality invariant ansatz (6); we expect it to hold for more general moduli dependence.

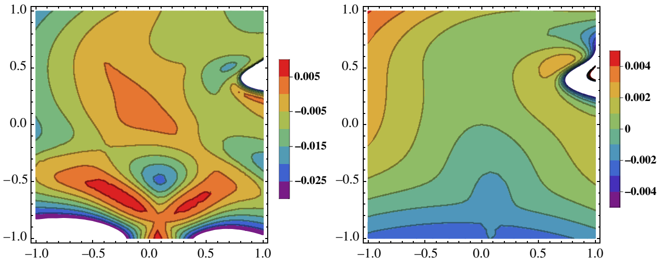

FIG. 1 (color online). The stability (non-normalized mass) (left) and dS (right) landscape of Sol. 2. The solution is located at the origin. The pictures are a two-dimensional slice of the parameters $x = -e(9B_1 - 2B_2 - B_2)/2$ and $y = -e(7A_1 - 2A_2 - A_3)/6$ that are part of a linear combination of A_I, B_I in Eq. (7).

which gives a tadpole $N_{O3} = 60$ and rank $K = 67$. This has a stable dS solution at

$$\{S, T, U\} \approx \{.00616 + ie^{1.32}, -.000456 + ie^{2.63}, -.117 + ie^{.0728}\}, \tag{14}$$

which is a perturbation of solution 5.

V. DISCUSSION

In summary, we considered a novel one-step mechanism to stabilize all geometric moduli of type IIB toroidal compactifications in a dS vacuum, using the nontrivial moduli dependence of the tree-level superpotential and the nonperturbative contributions. The latter is motivated by duality invariance of string theory, and can also be seen as a small-field expansion. Our approach improves the three-step KKLT mechanism by including the complex structure in the nonperturbative piece allowing us to stabilize all moduli at once in a dS vacua, avoiding also the introduction of explicitly ~~SUSY~~ terms, such as anti–D-branes. We have presented a number of explicit stable dS solutions, amongst one with quantized fluxes.

We view our results as very compelling arguments to extend the dS landscape in type IIB flux compactifications. They represent a first step towards a new direction allowing for a more complete landscape of stable dS vacua.

ACKNOWLEDGMENTS

The authors thank Ulf Danielsson, Giuseppe Dibitetto, Marc Andre Heller, Thomas Van Riet, and Bert Vercnocke for useful discussions. J. B. acknowledges support by the Swedish Research Council (VR) and the Göran Gustafsson Foundation.

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