

# Global visibility of a singularity in spherically symmetric gravitational collapse

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We revisit the gravitational collapse of spherically symmetric Lemaître-Tolman-Bondi dust models. A sufficient condition for global visibility of singularity is given. This condition also allows us to extend the condition of local visibility to mass functions which are not Taylor expandable near the center.

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## I. INTRODUCTION

The gravitational collapse of a spherically symmetric ball of dust given by Oppenheimer and Snyder (OS) gives the most transparent demonstration of the formation of black holes in general relativity [1]. The key idea of an event horizon hiding the singularity at the center of a collapsing star is elucidated by infinite redshift of the signals from its boundary as seen by an asymptotic observer, far away from the boundary. Thus, an event horizon provides a protection from a singularity. However, a singularity does raise an important question about viability of the general theory of relativity, and covering it by a horizon only reduces it to a minor embarrassment for the theory. Earlier, singularities were seen as an artefact of underlying symmetry of the collapsing scenario, as the collapsing dust particles were aimed at the center. It was believed that any departure from spherical symmetry would make singularities disappear. However, two major developments in the 1960's changed our perception on singularities. First, the observation of cosmic microwave background which strongly supported the idea that the Universe had a singular beginning, and second a series of theorems by Hawking and Penrose indicating that singularities can occur under very generic conditions both in cosmology and gravitational collapse of stars [2]. This essentially meant that singularities could not be swept under a carpet (event horizon) but, in a way, should be stitched to it.

In the late 1960's, the OS model led to black hole formation as the “establishment viewpoint” in classical general relativity, if the remnant mass of the collapsing star was more than a few solar masses [3]. However, with the proof of singularity theorems there was an increasing need to protect general relativity, an otherwise very successful classical theory, from such a catastrophic consequence. Most of these attempts led to various conjectures shielding the singularity. Two of the most widely discussed proposals

were the cosmic censorship conjecture by Penrose [4], and the Hoop conjecture by Thorne [5]. To date both remain unproven, and the cosmic censorship conjecture is considered to be one of the most important unsolved problems in classical general relativity (see [6] for reviews).

With progress in our understanding of solutions to Einstein equations the evidence against cosmic censorship has only mounted. Nevertheless most of these counterexamples have been limited to spacetimes with assumptions on underlying symmetry (spherical/cylindrical) and choice matter models (dust/directed radiation/perfect fluids, etc.) [7–14]. With the advent of string/M theory as the best candidate theory for quantum gravity it is considered likely that in late stages of collapse we have higher order curvature corrections to Einstein equations. However, several studies initiated in this area have been unable to restore censorship [15–17].

The analysis of singularities in almost all the cases mentioned above has been limited to radial null geodesics in the neighborhood of singularity. This violates only the so-called weak censorship conjecture. The case of astrophysical interest would be a violation of strong censorship which requires the singular geodesic to reach the boundary of the collapsing cloud without getting trapped. This can allow for a possibility to model extreme high energy phenomenon (e.g., gamma ray bursts) on naked singularities. In this paper we derive a sufficient condition for the existence of a globally visible singularity forming in dust collapse.

In the next section we give a brief overview of marginally bound dust models. This is followed by a section on causal structure of singularity arising in gravitational collapse. The main results are summarized in the section on local and global visibility of singularity. We end with a section on discussion and concluding remarks.

## II. THE MODEL

The well-known Lemaître-Tolman-Bondi (LTB) space-time [18] is a spherically symmetric solution of Einstein field equations with metric of the form

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$$ds^2 = -dt^2 + \frac{(R')^2}{1+f(r)} dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

It is sourced by an energy-momentum tensor in the form of a pressure-free perfect fluid (equation of state  $p = 0$ ), given by

$$T^{\mu\nu} = \epsilon \delta_t^\mu \delta_r^\nu. \quad (2)$$

Here, energy density  $\epsilon$ , and area coordinate  $R$  are functions of  $r$  and  $t$ . As a reasonability condition on the energy-momentum tensor in GR we impose some energy condition on initial data. We impose a weak energy condition which, in case of dust, restricts energy density to positive values. The underlying symmetry of spacetime allows us to define an invariant mass function  $2m = R(1 - g^{\mu\nu} \partial_\mu R \partial_\nu R)$ , giving the mass inside a sphere of radius  $R$ . The Einstein field equations take a particularly simple form in terms of the mass function:

$$\epsilon(r, t) = \frac{m'}{4\pi R^2 R'}, \quad (3)$$

$$\dot{R}^2 = \frac{2m}{R} + f(r), \quad (4)$$

where superscripts ‘‘prime’’ and an ‘‘overdot’’ signify partial derivatives with respect to  $r$  and  $t$ , respectively. The first equation is the  $t, t$  component of the Einstein field equations, and second is the definition of mass function giving dynamics of the collapsing shells. Since dust particles follow geodesics, the mass inside a shell is conserved, hence mass function is a function of  $r$  only.

Note that the form of Eq. (2) allows us to interpret  $f(r)$  as the total energy function. From the Eq. (4) describing growth of energy density we see that there are both *shell-crossing* ( $R' = 0$ ), and *shell-focusing* ( $R = 0$ ) singularities in these models. The shell crossing singularities can be easily avoided in these models by choosing appropriate initial conditions, i.e., considering models with decreasing density away from the center, and by not giving any additional inward velocity to the outer shells. The first condition is physically realistic as we expect stars to have maximum density at the center. The marginally bound scenario ensures the second condition as all shells start at rest from infinity. Thus, we will be concerned with shell focusing singularities here.

The LTB solution for the marginally bound case can be written as

$$R^{\frac{3}{2}} = r^{\frac{3}{2}} - \frac{3}{2} \sqrt{2mt}. \quad (5)$$

This is the integrated form of Eq. (4), and where we have fixed the integration constant using scaling freedom  $R(0, r) = r$ , i.e., on initial hypersurface we equate the

proper and the coordinate distance. In what follows we shall analyze the marginally bound models [ $f(r) = 0$ ] only.

### III. SINGULARITY AND HORIZONS

As discussed in the previous section, in dust spacetime, the shell crossing singularities can be easily avoided by a suitable choice of initial data. From Eq. (5), the singularity curve describing a shell focusing singularity  $R = 0$ , in spacetime is

$$t_0(r) = \frac{2r^{3/2}}{3\sqrt{2m}}. \quad (6)$$

Since a physically reasonable density profile is a monotonic decreasing function away from the center, the singularity forms at the center first. Also, this restricts the spacetime coordinates to values

$$0 \leq r \leq r_c, \quad -\infty < t < t_0(r),$$

where  $r_c$  is the boundary of the collapsing cloud.

The characteristic feature of a black hole is that it represents a region of spacetime from where no information (matter or energy) can escape. This phenomenon is a consequence of the fact that propagation of light is influenced by gravitational field. If the gravitational field of an object is strong enough the light cannot escape eventually falling back on it. This allows us to define a wavefront which is moving outward yet decreasing in its area, a criterion for light being trapped. More precisely can we define a trapped surface as ‘‘a closed (compact without boundary) spacelike 2-surface which has the property that the null geodesics which meet this surface orthogonally are all converging in its neighborhood’’ [19].

The equation for outgoing radial null geodesics in the marginally bound dust spacetime is

$$\frac{dt}{dr} = \frac{\partial R}{\partial r}. \quad (7)$$

Using the outgoing null geodesic equation we can derive the following expression for a change in the proper distance  $R$  as

$$\frac{dR}{dr} = \left(1 - \sqrt{\frac{2m}{R}}\right) R'. \quad (8)$$

Since we are assuming no shell-crossings ( $R' > 0$ ) everywhere in the spacetime,  $dR/dr$  is either positive, negative, or 0, when  $2m < R$ ,  $2m > R$ , or  $2m = R$ , respectively. Therefore, the surface defined by

$$R = 2m \quad (9)$$

describes a locus of the turning points of outgoing radial null geodesics. This is the equation for an apparent horizon.

Thus, a second condition for a physically reasonable initial data is the absence of trapped surfaces ( $2m < r$ ) everywhere in the collapsing cloud.

It has been shown by several authors [7] that dust collapse models lead to violation of cosmic censorship conjecture. This revived a wide interest in the problem and in past two decades we have seen censorship violation in various collapse scenarios. Essentially, most of the analytical approaches in this area analyze null geodesics in the neighborhood of the singularity. What is shown is that depending on initial data we can have radial null geodesics in the spacetime with their past end points at singularity. Due to the vacuum exterior, the event and the apparent horizons meet at the boundary. When traced backwards the apparent horizon meets the singularity at the center. However, tracing the event horizon requires integrating the radial null geodesic equation. It is only in the homogeneous dust collapse model, studied by Oppenheimer and Snyder [1], that one can draw the event horizon curve.

In what follows we give a brief review of results on local visibility of singularities in dust collapse [10,20,21]. Consider outgoing radial null geodesics in the spacetime (7). The geodesics with past end points at singularity  $t_0(0)$ , take the following approximate form:

$$t = t_0(0) + Xr^\alpha, \quad (10)$$

near center. Here  $\alpha$  and  $X$  are both positive for geodesics to exist in the spacetime. Using solution (5) we expand  $R'$  near the center, and evaluate  $dt/dr$  using Eq. (10) above. Thus we can write null geodesic equation (7) near the center where the left-hand side has unknown parameters  $X$  and  $\alpha$  and its right-hand side has initial data in terms of mass (or energy density) expanded near  $r \sim 0$ . Thus, existence or otherwise of a naked singularity, which requires positive definite values for  $X$  and  $\alpha$  gets determined in terms of initial data. In marginally bound dust collapse if the leading inhomogeneity term in the density near the center is either linear or quadratic, the singularity is always naked (locally) [21]. If it is cubic we can have both black hole and naked singularity [21,22].

#### IV. LOCAL AND GLOBAL VISIBILITY

The role of apparent horizon in describing the final fate of gravitational collapse and its relation to initial data in dust spacetimes was shown in [22]. The analytical work in this area has been limited to local analyses of geodesics surrounding central singularity. An astrophysically interesting scenario would be to have a globally visible singularity [23]. Combining Eqs. (5), (6), and (9) we can write

$$t_0(r) - t_{ah}(r) = \frac{4}{3}m(r). \quad (11)$$

Since regular initial data ensures that mass should vanish at the center, singularity curve and apparent horizon curve meet there. The positivity of mass for any noncentral shell ( $r > 0$ ) ensures that all the noncentral points on the singularity curve are safely trapped. Therefore, it is only the central shell focusing singularity from where light may escape getting trapped. In what follows we shall focus on the central singularity ( $r = 0$ ).

In marginally bound dust collapse, initial data can be specified in terms of local expansion of density near central singularity (3),

$$\rho(r) = \epsilon(r, 0) = \rho_0 + \frac{1}{n!}\rho_n r^n + \dots = \frac{1}{4\pi r^2} \frac{dm}{dr} \quad (12)$$

and the corresponding mass function takes the form

$$m(r) = m_0 r^3 + m_n r^{n+3} + \dots \quad (13)$$

Here  $n \geq 1$  takes integral values. As far as local visibility is concerned it is the first nonzero value of  $n$  which decides the end state of collapse [8–11,21] (see [20] for a review). If the first nonzero term near the center is  $n = 1$  or  $2$ , the singularity is always visible. At  $n = 3$  we can have a transition from visible singularity to a black hole depending on initial data (central density and inhomogeneity). For  $n \geq 4$  the collapse always ends in a black hole. However, the analysis provides a sufficient condition for the local visibility of singularity as it is limited to conditions near the center. In the remaining paper we have developed a sufficient condition for the global visibility of singularity.

In what follows it is convenient to define an auxiliary variable so that  $r = r(f)$ . The equation of outgoing null geodesics (7) can be rewritten as

$$\frac{dt}{df} = \frac{\partial R}{\partial f}. \quad (14)$$

Due to the absence of any pressure the dust particles follow geodesics, and this conserves mass within a comoving radius. Thus, instead of comoving distance  $r$ , we can also use mass function  $m(r)$  as a coordinate. Let  $f(r) = (9m(r)/2)^{\frac{1}{3}}$ , be the definition of the auxiliary variable. In terms of  $f$  the dust solution (5) can be rewritten as

$$R^{\frac{3}{2}} = r^{\frac{3}{2}} - t f^{\frac{3}{2}}. \quad (15)$$

This solution takes a particularly simple form in terms of time of formation of singularity  $t_0 = (r/f)^{\frac{3}{2}}$ ,

$$R = f(t_0 - t)^{\frac{2}{3}}. \quad (16)$$

Further, defining  $(t_0 - t) = u^3$ , we can rewrite the radial null geodesic equation (14) as

$$\frac{du}{df} = \frac{q}{3u^2} - \frac{1}{3} - \frac{2qf}{9u^3}, \quad (17)$$

where  $(dt_0/df) = q$ . We make one final transformation:

$$v = \frac{u}{f}. \quad (18)$$

The radial null geodesics in  $(v, f)$  coordinates take the following form

$$\frac{dv}{df} = \frac{1}{f} \left[ \left( -v - \frac{1}{3} \right) + \frac{q}{f^2} \left( \frac{1}{3v^2} - \frac{2}{9v^3} \right) \right]. \quad (19)$$

The proper distance  $R$ , and the comoving coordinates  $(t, r)$ , have the following functional form in the new coordinates  $(v, f)$ ,

$$\begin{aligned} R &= fu^2 = f^3 v^2, \\ r &= R(t=0) = ft_0^{\frac{2}{3}}, \\ t &= t_0 - u^3 = t_0 - f^3 v^3. \end{aligned} \quad (20)$$

Since  $f \propto m^{1/3}$ , for small  $f$ ,  $r$  is proportional to  $f$  and increases with it.  $t$  decreases with  $v$ , but lines of constant  $v$  are not lines of constant  $t$ ,

$$\left. \frac{\partial t}{\partial f} \right|_v = q - 3f^2 v^3. \quad (21)$$

The singularity is at  $t = t_0$ , i.e.,

$$u = fv = 0. \quad (22)$$

This definition is crucial in distinguishing regular and singular points along null geodesics. Along a radial null geodesic, if  $v$  attains a finite value in the limit of approach to center it implies the geodesic is singular; as  $f$  tends to zero  $u$  approaches 0 ( $t \rightarrow t_0$ ) and the singularity is approached. On the other hand,  $v$  necessarily blows up if the null geodesic approaches a regular center. In a special case, when  $v$  is proportional to  $f^{-1}$ ,  $u$  approaches a nonzero value, and a regular center is approached and  $v$  blows up.

The apparent horizon is at  $R = 2m$ , i.e.,

$$v = \frac{2}{3}. \quad (23)$$

Any region of spacetime where  $v > 2/3$  is free of trapped surfaces. If a geodesic crosses the apparent horizon it has to fall back into the singularity, i.e., after crossing the apparent horizon  $v$  remains bounded in interval  $(0, 2/3)$ . Therefore, for a singular null geodesic to escape to infinity,  $v$  has to remain greater than  $2/3$  on it throughout the collapsing cloud. In what follows we derive a sufficient condition on a radial null geodesic for which  $v$  remains above  $2/3$  until the

boundary of the cloud, and it remains finite in the  $f = 0$  limit (i.e., it is singular). Existence of such a geodesic ensures that singularity is globally visible.

From Eq. (19), if  $v > \frac{2}{3}$ ,

$$\frac{dv}{df} = \frac{1}{f} \left[ \left( -v - \frac{1}{3} \right) + \frac{q}{f^2} \left( \frac{1}{3v^2} - \frac{2}{9v^3} \right) \right] > 0, \quad (24)$$

iff

$$\frac{q}{f^2} > \frac{3v^3(3v+1)}{3v-2}. \quad (25)$$

The function  $(3v^3(3v+1))/(3v-2)$  has a local minimum  $(52 + 30\sqrt{3})/9$  at  $v = (1/3 + 1/\sqrt{3})$ . It approaches  $\infty$  as  $v \rightarrow 2/3$ , and as  $v \rightarrow \infty$ . Thus, if  $q/f^2 > (52 + 30\sqrt{3})/9$  for all  $f$ ,  $dv/df > 0$  in an interval of  $v$  containing  $(1/3 + 1/\sqrt{3})$ .

A null geodesic is characterized by parameters  $v$  and  $f$ . Consider a point with finite values  $v > 2/3$  and  $f$  on it. The geodesic is now extended from this point in both directions. If  $q(f)$  satisfies  $\frac{q}{f^2} > \frac{1}{9}(52 + 30\sqrt{3})$  for all  $f$ , the geodesic cannot cross the line  $v = 1/3 + 1/\sqrt{3}$  again as  $dv/df > 0$  on that line. Therefore,  $v$  approaches a finite value ( $2/3$ ) as  $f \rightarrow 0$ , and it remains greater than  $2/3$  as  $f \rightarrow \infty$ . This geodesic starts from the singularity at the center, and escapes boundary without getting trapped.

The sufficient condition for the global visibility of singularity (25) can also be written as

$$\frac{dt_0}{dm} > \frac{1}{3}(26 + 15\sqrt{3}) \quad (26)$$

for all  $m$ . Alternatively, using Eq. (11) the global visibility condition reads

$$\frac{dt_{ah}}{dm} > \frac{1}{3}(22 + 15\sqrt{3}) \quad (27)$$

for all  $m$ .

## V. SPECIAL CASES

In this section we provide some explicit examples of globally visible singularities based on the condition derived above (26).

### A. Non-self-similar collapse

- (i)  $\rho = \rho_0(1 - r)$ : Consider a collapsing dust cloud with central density  $\rho_0$ , and comoving boundary  $r_b = 1$ . Thus the interior region of the cloud is characterized by a range  $0 \leq r \leq 1$ . The mass as a function of radius for this density profile is

$$m(r) = 4M \left( r^3 - \frac{3}{4} r^4 \right) \quad (28)$$

Where  $M = m(1)$  is the total mass. This mass profile ensures regular mass function at the center. Another regularity condition is the requirement of no trapped surfaces on an initial hypersurface ( $2m/r < 1$ ) restricts total mass in the range  $0 < M < 0.4746$ . From Eq. (6), the singularity curve is

$$t_0 = \frac{2}{3\sqrt{2M}} \frac{1}{\sqrt{4-3r}}, \quad (29)$$

and

$$\frac{dt_0}{dm} = \frac{1}{12\sqrt{2}} \frac{1}{M^{\frac{3}{2}}} \frac{1}{r^2(1-r)(4-3r)^{\frac{3}{2}}}. \quad (30)$$

The function  $r^2(1-r)(4-3r)^{\frac{3}{2}}$  goes to 0 as  $r$  goes to 0 or 1, it has a maximum of 0.0618508 at  $r = 0.514191$ . Thus

$$\frac{dt_0}{dm} \geq \frac{1}{8.3971} M^{-\frac{3}{2}}, \quad (31)$$

and the sufficient condition for global visibility (26) is satisfied if

$$M < 0.036149. \quad (32)$$

(ii)  $\rho = \rho_0(1-r^2)$ : The mass function in this case becomes

$$m(r) = \frac{5}{2} M \left( r^3 - \frac{3}{5} r^5 \right). \quad (33)$$

The condition for absence of trapped surfaces on initial hypersurface gives the following condition on initial data,  $0 < M < 0.48$ . The singularity curve for this profile is

$$t_0 = \frac{2}{3\sqrt{M}} \frac{1}{\sqrt{5-3r^2}}, \quad (34)$$

and

$$\frac{dt_0}{dm} = \frac{4}{15} \frac{1}{M^{\frac{3}{2}}} \frac{1}{(5-3r^2)^{\frac{3}{2}} r(1-r^2)}. \quad (35)$$

The function  $r(1-r^2)(5-3r^2)^{\frac{3}{2}}$  goes to 0 as  $r$  goes to 0 or 1, and has a maximum value 0.206295 at  $r = 0.403231$ . Thus,

$$\frac{dt_0}{dm} \geq \frac{1}{8.64918} M^{-\frac{3}{2}}, \quad (36)$$

and the sufficient condition for global visibility (26) is satisfied if

$$M < 0.035443. \quad (37)$$

The global visibility of singularity for various values of initial mass functions is shown in Fig. 1 below. We start with some value  $v > 2/3$  on the boundary on a radial null geodesic Eq. (19) and integrate it toward the center for different values of total mass of the cloud. When the value of total mass violates the global visibility condition (37),  $v$  blows up as we move toward the center showing that such geodesics can only originate from a regular center.

(iii)  $\rho = \rho_0(1-r^3)$ : The mass function corresponding to this density profile is

$$m = M(2r^3 - r^6). \quad (38)$$

Absence of trapped surfaces on a regular initial hypersurface gives the following bound on total mass  $0 < M < 0.4835$ . The singularity curve and its derivative with respect to mass function is given by, respectively,

$$t_0 = \frac{2^{1/2}}{3} \frac{1}{(2-r^3)^{1/2}}, \quad (39)$$

and

$$\frac{dt_0}{dm} = \frac{2^{1/2}}{12} \frac{1}{M^{\frac{3}{2}}} \frac{1}{(1-r^3)(2-r^3)^{\frac{3}{2}}}. \quad (40)$$

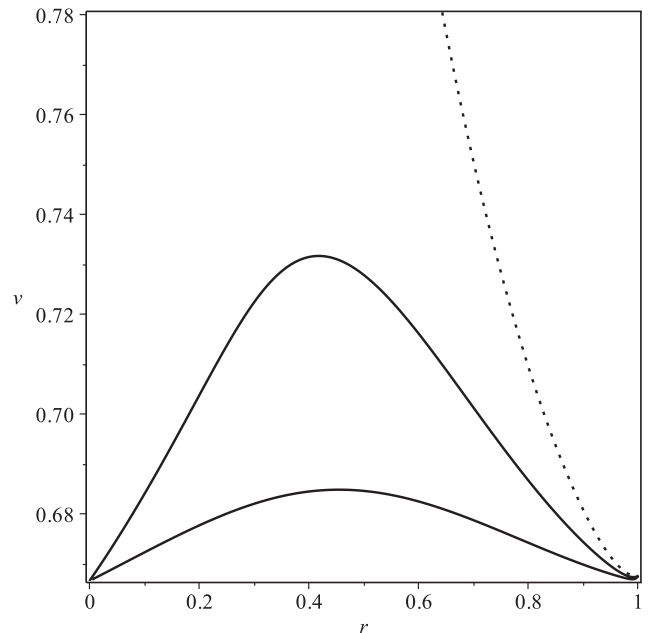


FIG. 1. Figure showing regular (dotted) and singular (bold) curves, respectively for  $M = 0.05, 0.02, \text{ and } 0.01$ .

This has a minimum at  $r = 0$ . This is unlike the previous two cases where the denominator had a maximum value in the interval  $r \in [0, 1]$ . Thus,

$$\frac{dt_0}{dm} \geq \frac{1}{24} M^{-\frac{3}{2}} \quad (41)$$

and the condition (26) is satisfied if

$$M < \frac{1}{4} (26 - 15\sqrt{3})^{\frac{2}{3}} = 0.017949, \quad (42)$$

and a black hole forms if  $M > 0.017949$ .

First, this is the same condition as obtained by [21]. Assuming  $2m = r^3(F_0 + F_3 r^3 + \dots)$  the singularity curve is given by:

$$t_0 = \frac{2}{3} \frac{r^{\frac{3}{2}}}{\sqrt{2m}} = \frac{2}{3\sqrt{F_0}} - \frac{2F_3}{3F_0^{\frac{5}{2}}} m + \dots$$

At  $m = 0$ ,

$$\left. \frac{dt_0}{dm} \right|_{m=0} = \frac{2 - F_3}{3 F_0^{\frac{5}{2}}}.$$

So the condition for local visibility in the  $n = 3$  case is

$$\left. \frac{dt_0}{dm} \right|_{m=0} > \frac{1}{3} (26 + 15\sqrt{3}),$$

i.e.,

$$\frac{-F_3}{F_0^{\frac{5}{2}}} > \frac{1}{2} (26 + 15\sqrt{3}) = 25.9904.$$

For the  $\rho(1 - r^3)$  profile,  $dt_0/dm$  has a minimum at  $m = 0$ . If the singularity is locally visible,  $\left. \frac{dt_0}{dm} \right|_{m=0} > \frac{1}{3} (26 + 15\sqrt{3})$ , this condition holds for all  $m$ , and the singularity is also globally visible.

Therefore, in this case, we have either a black hole or a globally naked singularity, there is no mass such that the singularity is locally but not globally naked. The special nature of this profile comes out in the plot of null geodesics. The singular geodesics in Fig. 2 meet the central singularity with different values of  $v$  unlike previous two cases where  $v = 2/3$  at the center on singular geodesics.

### B. Self-similar collapse

The assumption of self-similarity restricts mass function to have a linear form

$$m = \kappa r. \quad (43)$$

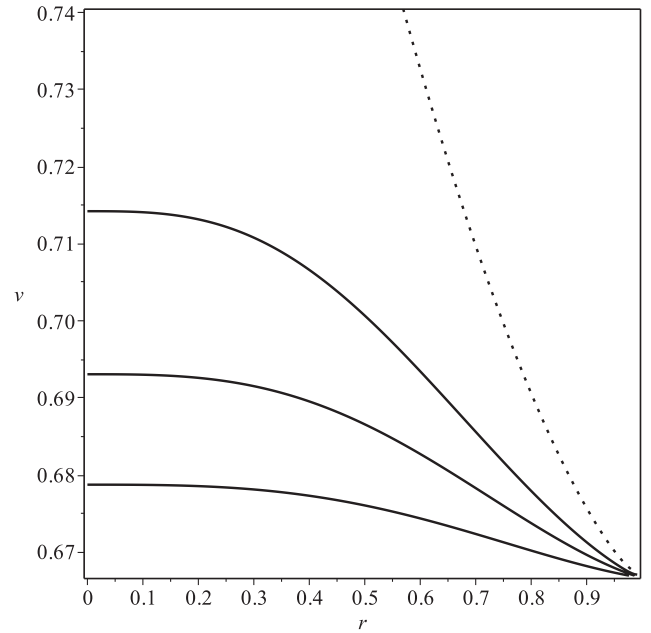


FIG. 2. Figure showing a regular (dotted) and singular (bold) curves, respectively, for  $M = 0.019, 0.011, 0.008$ , and  $0.005$ .

Here  $\kappa > 0$ , and  $r = 1$  characterizes boundary of the cloud. The condition for absence of trapped surfaces ( $2m/r < 1$ ) implies  $0 < \kappa < 0.5$ . The singularity curve is

$$t_0 = \frac{2}{3} \frac{r}{\sqrt{2\kappa}}, \quad (44)$$

and its derivative with respect to mass

$$\frac{dt_0}{dm} = \frac{\sqrt{2}}{3} \frac{1}{\kappa^{\frac{3}{2}}}. \quad (45)$$

By Eq. (26) the singularity is globally naked if

$$\kappa < (26 - 15\sqrt{3})^{\frac{2}{3}} 2^{\frac{1}{2}} = 0.090458. \quad (46)$$

A black hole forms if  $\kappa > 0.090458$  [24]. In this case also, our sufficient condition is also a necessary condition. Moreover, the nature of singular geodesics, Fig. 3, is very similar to the previous case where they met central singularity with finite but different values of  $v$ .

### C. A special case

The analysis of the local visibility of the singularity has been limited to density functions which are Taylor expandable near the center. The visibility or otherwise of the singularity is then shown to be related to the first nonzero coefficient in the expansion of density near the center. Clearly, during late stages of collapse, it difficult to justify mathematical properties like smoothness of functions which allows expansion inside the matter cloud. In what

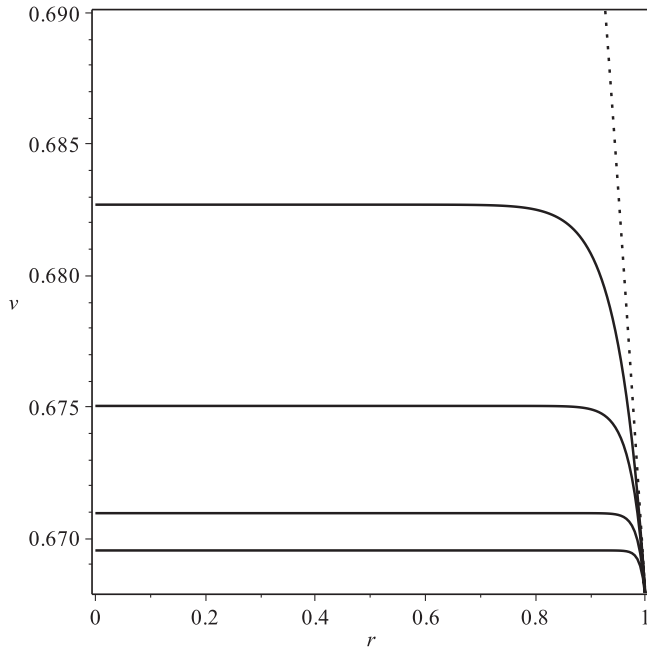


FIG. 3. Figure showing a regular (dotted) and singular (bold) curves, respectively, for  $\kappa = 0.1, 0.03, 0.02, 0.013$ , and  $0.01$ .

follows we propose as an example a density profile which is not Taylor expandable near the center. This function has a reasonable behavior as density falls away from the center, and mass function vanishes at the center.

Consider an initial density profile of the form

$$\rho = \rho_0(1 + r \ln r - r).$$

In this distribution, the density cannot be expanded as a Taylor series about the center. The corresponding mass function takes a form

$$m = M(16r^3 - 15r^4 + 12r^4 \ln r). \quad (47)$$

And,

$$t_0 = \frac{2}{3\sqrt{2M}} \frac{1}{\sqrt{16 - 15r + 12r \ln r}}, \quad (48)$$

$$\frac{dt_0}{dm} = \frac{1}{48\sqrt{2}} \frac{1}{M^{\frac{3}{2}} r^2 (1 + r \ln r - r) (16 - 15r + 12r \ln r)^{\frac{3}{2}}}, \quad (49)$$

$$\frac{dt_0}{dm} \geq 0.13777M^{-\frac{3}{2}}. \quad (50)$$

Condition (26) is satisfied if

$$M < 0.039837. \quad (51)$$

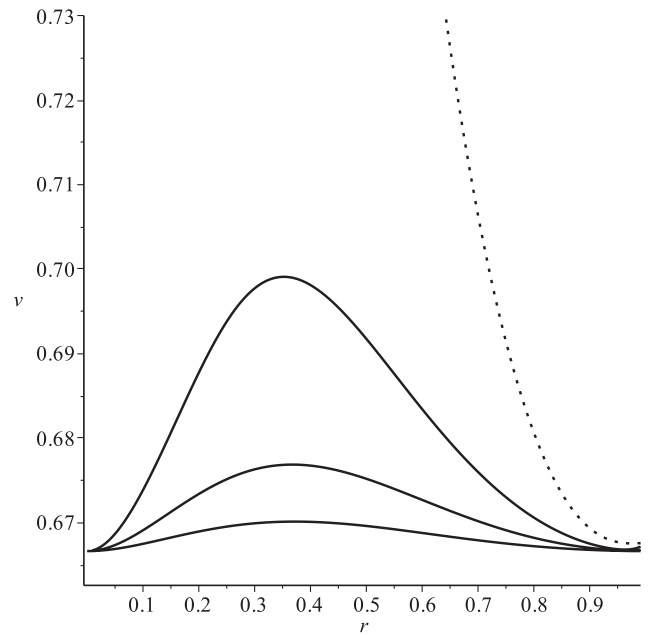


FIG. 4. Figure showing a regular (dotted) and singular (bold) curves, respectively, for  $M = 0.1, 0.02, 0.01$ , and  $0.005$ .

A naked singularity forms if the radius of the cloud is less than 12.55 times the Schwarzschild radius (see Fig. 4).

## VI. DISCUSSION AND CONCLUSIONS

The cosmic censorship conjecture remains one of the most important unsolved problem in classical general relativity. The visibility of a singularity to an observer falling onto a black hole after it crosses the horizon is benign compared to singular geodesics escaping to far away observers. Though such a violation of the censorship conjecture should have serious consequences for physics, it also allows us the possibility to model violent high energy phenomenon in astrophysics using naked singularities. Recently, Jhingan *et al.* [25] proposed a possibility to quantify energy coming out from such singularities.

In this paper we have studied the collapse of spherically symmetric dust models. These models have provided some of the most serious counterexamples to the cosmic censorship conjecture. The studies in dust models have been focused on developing local nakedness criterion bases on expansion of initial data near the center. This restricts the analysis to a special class of initial data space, i.e., expandable near the center. The analysis of causal structure has been limited to analyzing the apparent horizon curve. Moreover, astrophysically interesting scenarios of globally visible singularities have been limited to numerical studies (see, for example, [23]).

To see the causal structure of a spacetime we need to trace the null geodesics in it. However, this can be done only in homogeneous dust models. In inhomogeneous

models it is not possible to integrate null geodesics analytically and our understanding is based on numerical studies [23]. In this paper we have given a sufficient criterion for existence of a globally naked singularity (26). It is shown how the known examples of initial data which lead to formation locally visible singularities can be extended to global visible cases. This condition for global visibility does not depend on the expansion of functions near the center. Therefore, it allows us to probe end states of gravitational collapse for a wider class of functions representing initial data. It would be interesting to see if

this condition for global visibility can yield results in the case of perfect fluid collapse where local analysis near singularity relating initial data with visibility of singularity is not known.

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