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Chiral gravity waves and leptogenesis in inflationary models with non-Abelian gauge fields

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We present a leptogenesis scenario associated with inflationary models involving non-Abelian gauge fields within the standard model (SM) of particle physics. We show that this class of inflationary models generates intrinsic birefringent gravitational waves that following Alexander, Peskin, and Sheikh-Jabbari [Phys. Rev. Lett. 96, 081301 (2006)], through the gravitational chiral anomaly in the SM, can naturally create a net lepton number density. The CP-violating interaction is produced by tensor fluctuations of the gauge field, while the efficiency of this process is determined by the effective background value of the gauge field. We demonstrate that this mechanism can create the observed value of baryon to photon number density in a natural range of parameters of these models.

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I. INTRODUCTION

The observable Universe is highly matter-antimatter asymmetric and, to the best of our knowledge, all of its structures consist of matter (baryons and electrons). The asymmetry between the number density of baryons, n_B, and antibaryons, $\bar{n}_B$, in the Universe can be quantified by the baryon to photon ratio as

$$\eta = \frac{n_B - \bar{n}_B}{n_\gamma}\Big|_0, \tag{1}$$

where n_γ is the number density of photons and "0" means at the present time. Observationally, η can be inferred by two independent ways: from the cosmic microwave background (CMB) (when the thermal bath temperature falls below $T \lesssim 1$ eV) [1] or big bang nucleosynthesis (BBN) ($T \lesssim 1$ MeV) [2]

$$\eta^{\text{CMB}} = (6.21 \pm 0.12) \times 10^{-10} \quad \text{and}$$
$$\eta^{\text{BBN}} = (5.80 \pm 0.27) \times 10^{-10}, \tag{2}$$

which, although they refer to epochs with 6 orders of magnitude difference in temperature, are impressively in agreement. On the other hand, various considerations suggest that the Universe started from a state with equal numbers of baryons and antibaryons. Therefore, the observed asymmetry must have been generated dynamically, "baryogenesis." For more than half a century, cosmic baryogenesis stands as one of the puzzles of astroparticles and cosmology.

In 1967, Sakharov [3] formulated the necessary and sufficient conditions under which it is possible to create a baryon-antibaryon asymmetry from symmetric initial conditions: violation of the baryon number, CP violation, and out of equilibrium state. Within the particle physics setups, it is easier to first generate the matter-antimatter asymmetry in the lepton sector and then, relying on the electroweak sphaleron processes, transform it to the baryonic sector [4,5], "baryogenesis via leptogenesis." Since the sphalerons would be activated in temperatures $T \gtrsim M_W$, these models require a reheat temperature $T_{\text{reh}} \gtrsim 100$ GeV.

First proposed by Fukugita and Yanagida [5], leptogenesis is a class of scenarios in which the cosmic baryon asymmetry originates from an initial lepton asymmetry in the early Universe. In the standard approach of leptogenesis, the "standard model is extended" by adding massive right-handed neutrinos which (provide the source of CP violation in the model) decay and generate the initial lepton asymmetry [6,7]. In this class of models, the source of CP violation is not active during inflation to compensate the washout effect caused by the (almost) exponential expansion of the Universe. Hence the standard scenarios of leptogenesis associate the matter-antimatter asymmetry of the Universe to the physics beyond the standard model (SM) and after the inflationary era. As an alternative approach, the leptogenesis mechanism can be based on the fields which are active during the inflation, i.e., (scalar and tensor parts) of metric and inflaton(s).

Introduced in Ref. [8], "gravi-leptogenesis" is a scenario of leptogenesis in which the matter-antimatter asymmetry is generated by birefringent gravitational waves during inflation. In this mechanism, the inflation is driven by a pseudoscalar field χ, while the CP-violating interaction in tensor modes is provided by adding a gravitational Chern-Simons interaction of the form $P(\chi)R\tilde{R}$ to the gravity action, where $P(\chi)$ is a generic odd function of χ. It was argued that supergravity or string theory compactifications involving axions can naturally lead to a $P(\chi) = \mathcal{N}\frac{\chi}{M_{\text{Pl}}}$ with $\mathcal{N} \sim 10^3$ [8,9]. Hence, the gravi-leptogenesis mechanism addresses the source of the CP violation to the gravitational Chern-Simons interaction added to the Einstein-Hilbert action. [Alternative inflationary baryogenesis scenarios based on using U(1) gauge fields have been introduced in [10,11].]

In this work, we demonstrate that inflationary models involving non-Abelian gauge fields (minimally coupled to gravity) generate intrinsic birefringent gravitational waves. In this class of models, the source of CP violation is generated by the non-Abelian gauge field which is active in the background, and its fluctuations contribute to the tensor perturbations during inflation. The chiral gravitational waves produced during inflation generate a nonvanishing $\langle\tilde{R}R\rangle$ which through the gravitational anomaly in the standard model leads to a net lepton number density. Hence, inflationary models with non-Abelian gauge fields provide a natural setting for leptogenesis within the standard model, "inflato-leptogenesis." Before this, the authors of Ref. [12] studied a leptogenesis scenario associated with two specific inflationary modes with non-Abelian gauge fields, chromo-natural and gauge-flation. They showed that the observed value of η can be explained naturally in this model. Here, we demonstrate that this is a generic behavior in this class of models.

This paper is organized as follows. We start in Sec. II by presenting the general setup of the inflato-leptogenesis. Section III is devoted to the inflationary models involving non-Abelian gauge fields. First, we introduce the generic setup of this family. Then, we focus on the gravitational waves and study the tensor perturbations generated in this class of models. In Sec. IV, we compute the lepton and photon number densities and compare the result with the observed data. Finally, we conclude in Sec. V. The Appendix contains some technical details of $R\tilde{R}$ calculation.

II. INFLATO-LEPTOGENESIS, A GENERAL SETUP

From the gravitational anomaly of the lepton current J^μ_l, in the standard model [13], we have

$$\nabla_\mu J^\mu_l = \frac{\mathcal{A}}{16\pi^2}\tilde{R}R, \tag{3}$$

where $\mathcal{A}$ is the difference between the number of left- and right-handed fermion degrees of freedom, $\mathcal{A} = n_L - n_R$, and $\tilde{R}R \equiv \frac{1}{2}\epsilon^{\lambda\mu\nu\xi}R_{\lambda\mu\rho\sigma}R_{\nu\xi}{}^{\rho\sigma}$. In the standard model of particle physics $\mathcal{A} = 3$, while in beyond SM with right-handed neutrinos, it can be less than 3. Integrating (3) and neglecting the surface term, we obtain the total lepton number L as

$$\mathrm{L}(\tau) = \frac{\mathcal{A}}{16\pi^2}\int_{\tau_0}^{\tau}\sqrt{-g}\langle\tilde{R}R\rangle d\tau' d^3x, \tag{4}$$

where $\langle\rangle$ denotes the quantum expectation value and τ is the conformal time ($d\tau = a^{-1}dt$). Here, we assume that at the beginning of inflation $L(\tau_0) = 0$. A nonvanishing $\langle\tilde{R}R\rangle$ can be generated by P-violating interactions which by the above anomaly leads to the imbalance of right-handed and left-handed leptons.

Considering the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) background metric, $\langle\tilde{R}R\rangle$ vanishes in the background, while it can be sourced by the birefringent tensor modes at the perturbation level. Perturbing the metric around the FRW background, the most general perturbed metric can be parametrized as

$$ds^2 = a^2(-(1+2A)d\tau^2 + 2(\partial_i B + V_i)dx^i d\tau + ((1-2C)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}W_{j)} + h_{ij})dx^i dx^j), \tag{5}$$

where A, B, C, and E are scalar perturbations, V_i and W_i are transverse vector perturbations, and the symmetric, traceless, and divergence-free h_{ij} parametrize the tensor elements. Considering the perturbed metric (5), we obtain the second-order $\tilde{R}R$ as

$$\tilde{R}R = -\frac{2}{a^4}\epsilon^{ijk}(h''_{jl}\partial_i h'_{lk} - \partial_m h'_{jl}\partial^2_{im}h_{lk} + \partial_l h'_{jm}\partial^2_{mi}h_{kl}), \tag{6}$$

where a prime denotes a derivative with respect to the conformal time. As we see in (6), $\tilde{R}R$ is determined in terms of tensor modes h_{ij}, while scalar and vector elements make no contribution. Using the Fourier transform, we can write (6) in terms of the Fourier modes of right-handed and left-handed polarizations $h_{R,L}(\mathbf{k},\tau)$. For a wave vector $\mathbf{k} = (0,0,k)$, the right- and left-handed modes are defined as $h_{R,L} \equiv (h_{11} \pm ih_{12})/2$.

The right-handed tensor mode $\hat{h}_R(\tau,\mathbf{x})$ reads as below in terms of the creation and annihilation operators:

$$\hat{h}_R(\tau,\mathbf{x}) = \int\frac{d^3k}{(2\pi)^{3/2}}(h_R(\tau,\mathbf{k})\hat{a}_{\mathbf{k}} + h^*_L(\tau,-\mathbf{k})\hat{b}^\dagger_{-\mathbf{k}})e^{i\mathbf{k}.\mathbf{x}}. \tag{7}$$

By definition, the left-handed polarization is given as $h_L(\tau,\mathbf{x}) = h^\dagger_R(\tau,\mathbf{x})$. Using (7) in (6), and after some lengthy calculations which are presented in the Appendix, we obtain

$$\langle\tilde{R}R(\tau)\rangle = \frac{2/\pi^2}{a^4}\int_{k_{\mathrm{IR}}}^{k_{\mathrm{UV}}}k^3 dk\frac{d}{d\tau}(h'_R(\tau,k)h^{*\prime}_R(\tau,k) - k^2 h_R(\tau,k)h^*_R(\tau,k) - R\leftrightarrow L) + \mathcal{D}, \tag{8}$$

where $\mathcal{D}$ is a surface term. The integral over k runs over the momentum space from the smallest comoving momentum k_{IR}, up to the largest one k_{UV}, which are determined by IR and UV cutoffs of the physical momentum as $H \lesssim \frac{k}{a} \lesssim \Lambda$. Using the slow-roll relation $a \simeq -1/(H\tau)$, we then obtain

$$k_{\rm IR}(\tau) \simeq -\frac{1}{\tau} \quad \text{and} \quad k_{\rm UV}(\tau) \simeq -\frac{\Lambda}{H\tau}.$$

As expected, the parity violating $\langle \tilde{R}R \rangle$ is closely related to the existence of an imbalance between left and right tensor models, chiral gravitational waves, and vanishes in the special case of parity-preserving interactions [in which $h_R(\tau, k) = h_L(\tau, k)$].

Inserting (8) in (4) and omitting the surface terms, one can determine the total lepton number density n, which has been produced by the end of inflation

$$n(\tau_{\rm inf}) = \frac{\mathcal{A}/8\pi^4}{a^3(\tau_{\rm inf})} \int_{\frac{-1}{H}}^{\tau_{\rm inf}} d\tau \int_{\frac{-1}{\tau}}^{\frac{-\Lambda}{H\tau}} k^3 dk \frac{d}{d\tau} (h'_R(\tau, k) h^{*\prime}_R(\tau, k) - k^2 h_R(\tau, k) h^*_R(\tau, k) - R \leftrightarrow L), \tag{9}$$

where $n \equiv \mathrm{L}/(\int a^3 d^3x)$ and $\tau_{\rm inf}$ is the conformal time at the end of inflation. Note that, in order to determine the lepton number density, one should first (going to the Fourier space) determine $\langle \tilde{R}R(\tau) \rangle$ and then evaluate the conformal time integral [Eqs. (4) and (9)]. Because of some technical reasons which will be clear soon, it is more convenient to write the above integral in terms of τ and $\tilde{\tau} \equiv -k\tau$. Moreover, by using the standard asymptotic past normalization, $h_{R,L}(\tau, k)$ can be decomposed into a function of $\tilde{\tau}$, presented by $\bar{h}_{R,L}(\tilde{\tau})$, and a factor of k:

$$h_{R,L}(\tau, k) = \frac{H}{M_{\rm Pl}} k^{-\frac{3}{2}} \bar{h}_{R,L}(\tilde{\tau}). \tag{10}$$

Note that $h_{R,L}$ and its corresponding canonically normalized field $u_{R,L}$ are related as $u_{R,L} = \sqrt{2} a h_{R,L}$. Using the above decomposition, we can write the double integral (9) as a product of two independent single integrals in terms of τ and $\tilde{\tau}$:

$$n(\tau_{\rm inf}) \simeq -\frac{\mathcal{A}/8\pi^4}{a^3(\tau_{\rm inf})} \left(\frac{H}{M_{\rm Pl}}\right)^2 \int_{\frac{-1}{H}}^{\tau_{\rm inf}} \frac{d\tau}{\tau^4} \times \int_1^{\frac{\Lambda}{H}} \tilde{\tau}^3 \frac{d}{d\tilde{\tau}} (\partial_{\tilde{\tau}} \bar{h}_R(\tilde{\tau}) \partial_{\tilde{\tau}} \bar{h}^*_R(\tilde{\tau}) - \bar{h}_R(\tilde{\tau}) \bar{h}^*_R(\tilde{\tau}) - R \leftrightarrow L) d\tilde{\tau}. \tag{11}$$

Using the fact that $|\tau_{\rm inf}| \ll H^{-1}$ and the slow-roll condition $a(\tau) \simeq -1/(H\tau)$, we can evaluate the first integral and obtain

$$n(\tau_{\rm inf}) \simeq -\frac{\mathcal{A}H^3}{24\pi^4} \left(\frac{H}{M_{\rm Pl}}\right)^2 \int_1^{\frac{\Lambda}{H}} \tilde{\tau}^3 \frac{d}{d\tilde{\tau}} (\partial_{\tilde{\tau}} \bar{h}_R(\tilde{\tau}) \partial_{\tilde{\tau}} \bar{h}^*_R(\tilde{\tau}) - \bar{h}_R(\tilde{\tau}) \bar{h}^*_R(\tilde{\tau}) - \partial_{\tilde{\tau}} \bar{h}_L(\tilde{\tau}) \partial_{\tilde{\tau}} \bar{h}^*_L(\tilde{\tau}) + \bar{h}_L(\tilde{\tau}) \bar{h}^*_L(\tilde{\tau})) d\tilde{\tau}. \tag{12}$$

Because of its $\tilde{\tau}^3$ factor, the integrand in (12) is much larger at $\tilde{\tau} \gg 1$ than in the vicinity of the horizon crossing, $\tilde{\tau} = 1$.

Moreover, the UV cutoff scale Λ is always much larger than H in our setup. Thus, in order to calculate the net lepton number density n, we need only to determine the tensor modes on subhorizon scales, $\tilde{\tau} \gg 1$. In order to determine the net lepton number density, we need the explicit form of tensor modes. However, as a rough estimation, one may approximate the integrand in (12) as $\tilde{\tau}^3$ which leads to $n \propto (\frac{\Lambda}{H})^4$. Interestingly, this simple approximation is in agreement with the result of our direct calculations in (38).

Up to now, we performed the calculations in a general setup and showed that a nonvanishing lepton number asymmetry can be generated if the integrand in (12) is not zero. This latter is possible only if the chiral symmetry is broken and we have birefringent gravitational waves.

III. INFLATIONARY MODELS WITH NON-ABELIAN GAUGE FIELDS

In this section, first we show that the non-Abelian gauge field theory can provide the setting for constructing isotropic and homogeneous inflationary background. Then, we focus on the tensor fluctuations which can be generated in this class of models. Dealing with non-Abelian gauge fields in inflationary models brings many new and unique features compared with the standard scalar models, among them the existence of chiral tensor modes. Because of their intrinsic birefringent gravitational waves, inflationary models involving non-Abelian gauge fields provide a natural setting for the inflato-leptogenesis mechanism.

A. Theoretical setup

The models of our interest involve some scalar and pseudoscalar fields Φ_I ($I = 1, 2, \ldots, m$) as well as a non-Abelian gauge field $A^a{}_\mu$ with a gauge group G which can be any non-Abelian compact group. As the generic model, consider a (non-Abelian) gauge-invariant action minimally coupled to the Einstein gravity in four dimensions:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R + \mathcal{L}_m(F^a{}_{\mu\nu}, \Phi_I) \right), \tag{13}$$

where $\mathcal{L}_m$ is the matter Lagrangian density and $F^a{}_{\mu\nu}$ is the strength tensor of $A^a{}_\mu$. As any non-Abelian group has a SU(2) subgroup, we choose the gauge group to be SU(2). Then, our arguments can be directly generalized to an SU(2) subgroup of a generic non-Abelian group G. The strength tensor of the gauge field is

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - g \epsilon^a{}_{bc} A^a{}_\mu A^b{}_\nu, \tag{14}$$

where g is the gauge coupling.

Consider the FRW metric and choose the temporal gauge for $A^a{}_\mu$. The following homogeneous and isotropic configuration is the solution:

$$A^a{}_\mu = \begin{cases} 0 \\ a(t)\psi(t)\delta^a_i \end{cases} \quad \text{and} \quad \Phi_I = \Phi_I(t) \quad \forall\, I = 1,\ldots,m, \tag{15}$$

where ψ is a (pseudo[1]) scalar field, which is the effective field value of the gauge field [14–16]. In other words, there exists a consistent truncation or reduction of the theory (13) to the homogeneous and isotropic configuration (15). Thus this class of models can provide the setting for constructing an isotropic and homogeneous background. For an extensive review on this topic, see [17].

Given the generic effective action (13), one can expand $\mathcal{L}_m(F^a_{\mu\nu}, \Phi_I)$ in terms of powers of the strength tensor $F^a{}_{\mu\nu}$, i.e., the Yang-Mills, (P-violating) Chern-Simon interaction $\mathrm{tr}(F\tilde{F})$, the dimension-six operator $\mathrm{tr}(FFF)$, and the (PT-violating) Weinberg operator $\mathrm{tr}(FF\tilde{F})$ [18], as

$$\mathcal{L}_m(F^a_{\mu\nu}, \Phi_I) = -\frac{1}{2}\sum_{I=1}^m (\partial_\mu \Phi_I)^2 - V(\Phi_I) - \frac{1}{4} f_1(\Phi_I) F^a{}_{\mu\nu} F_a{}^{\mu\nu} + \frac{1}{8} f_2(\Phi_I)\epsilon^{\mu\nu\lambda\sigma} F^a{}_{\mu\nu} F_{a\lambda\sigma} + \frac{1}{6} f_3(\Phi_I)\epsilon_{abc} F^a{}_{\mu\nu} F^{b\nu}{}_\lambda F^{c\lambda\mu} + \frac{1}{12} f_4(\Phi_I)\epsilon_{abc}\epsilon^{\mu\nu\lambda\sigma} F^a{}_{\mu\nu} F^b{}_\lambda{}^\xi F^c{}_{\sigma\xi} + \cdots, \tag{16}$$

where f_i's are positive definite functions of Φ_I's and $\cdots$ denotes higher dimension terms which are higher orders of the slow-roll parameter.[2] Note that $f_2(\Phi_I)$ is P violating, while $f_4(\Phi_I)$ should violate PT. Moreover, each term in (16) satisfies the weak energy condition individually (their contribution to the energy density is positive).

Plugging the homogeneous and isotropic configuration (15) into (16), we obtain the background-reduced Lagrangian

$$\mathcal{L}_m(F^a_{\mu\nu}, \Phi_I) = \frac{1}{2}\sum_{I=1}^m \dot{\Phi}_I^2 - V(\Phi_I) + \frac{3}{2} f_1(\Phi_I)((\dot\psi + H\psi)^2 - g^2\psi^4) - 3 f_2(\Phi_I) g\psi^2(\dot\psi + H\psi) + f_3(\Phi_I) g\psi^2(3(\dot\psi + H\psi)^2 - g^2\psi^4) + f_4(\Phi_I)(\dot\psi + H\psi)((\dot\psi + H\psi)^2 - 3g^2\psi^4) + \cdots, \tag{17}$$

as well as the total energy density and pressure, ρ and P:

$$\rho = \frac{1}{2}\sum_{I=1}^m \dot\Phi_I^2 + V(\Phi_I) + \frac{3}{2}\left(\left(f_1 + 2g\psi^2 f_3 + \frac{4}{3}(\dot\psi + H\psi) f_4\right)(\dot\psi + H\psi)^2 + \left(f_1 + \frac{2}{3} g\psi^2 f_3\right) g^2\psi^4\right),$$

$$P = \frac{1}{2}\sum_{I=1}^m \dot\Phi_I^2 - V(\Phi_I) + \frac{1}{2}((f_1 - 2g\psi^2 f_3)(\dot\psi + H\psi)^2 + (f_1 + 2g\psi^2 f_3 + 4(\dot\psi + H\psi) f_4) g^2\psi^4).$$

Then, demanding slow-roll inflation ($\epsilon = -\frac{\dot H}{H^2} \ll 1$), we obtain

$$V(\Phi_I) \gg \frac{1}{2}\sum_{I=1}^m \dot\Phi_I^2 + (f_1 + g\psi^2 f_3 + (\dot\psi + H\psi) f_4) \times ((\dot\psi + H\psi)^2 + g^2\psi^4), \tag{18}$$

which implies that $V(\Phi_I)$ should be much larger than the other terms in the energy density. At this point, we assume that all the fields (Φ_I's and ψ) are evolving slowly during slow-roll inflation which is a feasible assumption for most of the standard inflationary systems. Then, (18) leads to the following slow-roll conditions:

$$(f_1 + g\psi^2 f_3 + (\dot\psi + H\psi) f_4)\left(\frac{\psi}{M_{\mathrm{Pl}}}\right)^2 \ll 1 \quad \text{and} \quad \left(\frac{\dot\Phi_I}{H M_{\mathrm{Pl}}}\right)^2 \ll 1 \quad \forall\, I = 1,2,\ldots,m. \tag{19}$$

Thus, slow-roll inflation requires ψ to be a sub-Planckian field $\psi \ll M_{\mathrm{Pl}}$.

Note that, although we can effectively replace $A^a{}_\mu$ by a scalar ψ, at the background level, this system is not equivalent with a (even more complex) scalar theory. In fact, it is not possible to write this effective scalar form as a covariant quantity. Moreover, the perturbed gauge field has new scalar, vector, and tensor perturbations which make these systems very different at the perturbation level [17].

1. Two inflationary models involving non-Abelian gauge fields

(i) Among the possible forms that (16) may take, one is the "chromo-natural" model [19], with the following $\mathcal{L}_m$:

[1] In (15), one can rewrite $A^a{}_i$ as $A^a{}_i = \psi e^a{}_i$, where $\{e^a{}_i\}$ are the spatial triads of the FRW metric.

[2] Recalling the slow-roll condition $-\frac{1}{4} f_1(\Phi_I) F^a{}_{\mu\nu} F_a{}^{\mu\nu} \ll V(\Phi_I)$ and assuming that the nonvanishing $f_i(\Phi_I)$'s are almost on the same order of magnitude, we find that dimension-eight and higher operators are of the order of ϵ smaller than Yang-Mills.

$$\mathcal{L}_m = -\frac{1}{2}(\partial_\mu\chi)^2 - \mu^4\left(1+\cos\frac{\chi}{f}\right) - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu} - \lambda\frac{\chi}{8f}\epsilon^{\mu\nu\lambda\sigma}F^a{}_{\mu\nu}F^a{}_{\lambda\sigma}, \quad (20)$$

where the axion field χ is the inflaton that through the Chern-Simons interaction couples to the non-Abelian gauge field $A^a{}_\mu$. This model has two dimensionless parameters, gauge coupling g and axion-gauge field coupling λ, as well as two dimensionful parameters μ and f. The slow-roll inflationary trajectories of the above model have been discussed in Ref. [20]. For these trajectories $\dot{\chi}/H\chi \sim \epsilon$, $\dot{\psi}/H\psi \lesssim \epsilon$, and during slow-roll inflation

$$3M_{\rm Pl}^2H^2 \simeq \mu^4\left(1+\cos\frac{\chi}{f}\right),$$
$$\frac{\psi}{M_{\rm Pl}} \simeq \left(\frac{\mu^4}{3g\lambda H M_{\rm Pl}}\sin\frac{\chi}{f}\right)^{1/3},$$
$$\epsilon \simeq \frac{1}{M_{\rm Pl}^2}\left(\psi^2 + \frac{3g^2\psi^4}{\mu^4(1+\cos\frac{\chi}{f})}\right). \quad (21)$$

In the absence of non-Abelian gauge fields, this model reduces to natural inflation [21]. In natural inflation, slow-roll expansion is obtained with a super-Planckian f parameter, which is not a natural scale within particle physics models. Interestingly, chromo-natural inflation fixed that problem by means of adding a non-Abelian gauge field to the model. Here, the gauge field slows down the inflaton's evolution and leads to slow-roll inflation even with the natural values of f ($f \ll M_{\rm Pl}$). Although a natural and well-motivated inflationary model at the background level, the chromo-natural model is disfavored by the resent Planck data [22–24].

(ii) Another possible inflationary model with non-Abelian gauge fields is "gauge-flation" which was also the first model in this class [14,15]. Integrating out the axion field around the minimum of its potential in the large axion region (χ/f close to π), the chromo-natural model will reduce to the gauge-flation model

$$\mathcal{L}_m = -\frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu} + \frac{\kappa}{384}(\epsilon^{\mu\nu\lambda\sigma}F^a{}_{\mu\nu}F^a{}_{\lambda\sigma})^2, \quad (22)$$

where $\kappa = \frac{3\lambda^2}{\mu^4}$ [25] and the gauge field is the inflaton. The gauge-flation and chromo-natural models are different in the scalar sector of cosmic perturbations; however, they have identical vector and tensor perturbations [17].

B. Tensor perturbations

As far as our current discussion and the gravitational anomaly are concerned, we need to study the tensor perturbations around the FRW metric, h_{ij} [Eq. (5)]. The traceless, transverse part of the Einstein equations provides the field equation of h_{ij} as

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 2a^2\pi^T_{ij}, \quad (23)$$

where $\mathcal{H} \equiv aH$ and the source term $a^2\pi^T_{ij}$ is the tensor part of the anisotropic inertia.[3] Note that the left-hand side in (23) is given by the gravity (Einstein-Hilbert) action, while the source term in the right-hand side is the contribution of the matter action. This latter vanishes in scalar field models; however, in systems involving non-Abelian gauge fields, the perturbed gauge field contributes to the anisotropic stress and $a^2\pi^T_{ij} \neq 0$.

Perturbing our fields around the homogeneous and isotropic configuration (15) and keeping only the tensor fluctuations, we have

$$\delta_T A^a{}_\mu = \begin{cases} 0 \\ a(t)M_{\rm Pl}\delta^{aj}X_{ij} \end{cases} \text{ and } \delta_T\Phi_I = 0 \quad \forall\, I = 1,2,\ldots,m, \quad (24)$$

where δ_T denotes the tensor sector of the perturbations and X_{ij} represents the tensor element of the perturbed gauge field which makes the linear order perturbed strength tensor

$$\delta_T F^a{}_{0i} = M_{\rm Pl}\delta^{aj}(aX_{ij})^{\dot{}},$$
$$\delta_T F^a{}_{ij} = 2M_{\rm Pl}(a\delta^{ak}\partial_{[i}X_{j]k} - a^2 g\psi\epsilon^{ak}{}_{[j}X_{i]k}). \quad (25)$$

Now, we are ready to determine the tensor anisotropic stress $a^2\pi^T_{ij}$. Among the five terms in (16), the Chern-Simons interaction is a topological term and makes no contribution to $T_{\mu\nu}$. Moreover, the scalar sector $\mathcal{L}_s(\Phi_I)$ has no role in the vector and tensor parts of the linear order perturbed energy-momentum tensor. In the Fourier space and in terms of the right- and left-handed polarizations (in which $a^2\pi^T_{ij}$ is diagonal) the Yang-Mills, tr(FFF), and the Weinberg operator have the following contributions to $a^2\pi^T_{R,L}$, respectively:

$$a^2\pi^T_{R,L}|_{\rm YM} \simeq 2f_1(\Phi_I)\psi\left(\frac{\psi}{2}(1-\gamma)\mathcal{H}^2h_{R,L} + (\gamma-1)\mathcal{H}^2X_{R,L} - \mathcal{H}X'_{R,L} \mp \sqrt{\gamma}k\mathcal{H}X_{R,L}\right), \quad (26)$$

[3]The tensor part of the anisotropic inertia $a^2\pi^T_{ij}$ is defined as $a^2\pi^T_{ij} = \delta_T T_{ij} - a^2\bar{P}h_{ij}$, where $\delta_T T_{ij}$ is the (traceless and divergence-free) tensor sector of the linear order perturbed energy-momentum tensor, while $\bar{P}$ is the background pressure.

$$a^2\pi^T_{R,L}|_{F^3} \simeq 2\sqrt{\gamma}f_3(\Phi_I)H\psi^2\left(\psi\mathcal{H}^2 h_{R,L} - 2\mathcal{H}^2 X_{R,L} - \mathcal{H}X'_{R,L} \pm \frac{\mathcal{H}}{\sqrt{\gamma}}kX_{R,L}\right), \tag{27}$$

$$a^2\pi^T_{R,L}|_{W} \simeq 2\gamma f_4(\Phi_I)H\psi^2\left(-\psi\mathcal{H}^2 h_{R,L} + 2\mathcal{H}^2 X_{R,L} + \mathcal{H}X'_{R,L} \mp \frac{\mathcal{H}}{\sqrt{\gamma}}kX_{R,L}\right), \tag{28}$$

where $\gamma \equiv \frac{g^2\psi^2}{H^2}$ and $\simeq$ means equality up to the first order of the slow roll ($\dot{\psi} \ll H\psi$). Some of the noteworthy features of the above anisotropic inertias are as follows:

(i) They are proportional to the effective field value of the gauge field at the background, ψ. This indicates that, to get a nonvanishing $a^2\pi^T_{R,L}$, the gauge field $A^a{}_\mu$ should be turned on at the background level.

(ii) The last terms in (26)–(28) are chiral terms that take different signs for the left and right polarizations. Hence, even the parity-preserving Yang-Mills and $\mathrm{tr}(FFF)$ have chiral anisotropic inertias $a^2\pi^T_R \neq a^2\pi^T_L$.

(iii) The chiral term in $a^2\pi^T_{R,L}|_{F_3}$ is of the opposite sign to the other chiral terms; hence, it can decrease the imbalance between the two tensor mode polarizations. Although not directly related to our current interest, this latter can lead to a smaller tensor to scalar ratio r, more consistent with the Planck data [1].

At this point, we need to work out the canonically normalized fields as well as the field equation of $X_{R,L}$. The second-order action of $X_{R,L}$ is determined by the tensor part of $\mathcal{L}_m$, while the second-order action of $h_{R,L}$ is given by the Einstein-Hilbert action up to the leading orders in slow roll. Thus, the canonically normalized fields are

$$u_{R,L} = \sqrt{2}ah_{R,L} \quad \text{and} \quad v_{R,L} = 2\sqrt{2}\tilde{N}aX_{R,L}, \tag{29}$$

where $\tilde{N}$ is a coefficient which will be determined by the second-order action of $X_{R,L}$. As far as our current discussion is concerned, we need the second-order action of $X_{R,L}$ in the subhorizon limit, that is,[4]

$$\delta_T S^{(2)} \simeq \frac{1}{2}\int\frac{dk^3}{(2\pi)^3}d\tau\sum_{\lambda=R,L}(v'_\lambda v^{*\prime}_\lambda - k^2 v_\lambda v^*_\lambda \pm 2\tilde{D}k\mathcal{H}(v_\lambda v^*_\lambda), \tag{30}$$

where we have[5]

$$\tilde{D} \simeq \sqrt{\gamma} + \frac{\frac{\dot{f}_2}{2H}}{(f_1 - H\psi(\sqrt{\gamma}f_3 + f_4))} \quad \text{and}$$
$$\tilde{N} = \sqrt{f_1 - H\psi(\sqrt{\gamma}f_3 + f_4)}. \tag{31}$$

Having four different gauge theories in (16), one may expect that $v_{R,L}$ has a nontrivial sound speed in (30). However, interestingly for each of $\mathrm{tr}(FF)$, $\mathrm{tr}(F\tilde{F})$, $\mathrm{tr}(FFF)$, and $\mathrm{tr}(FF\tilde{F})$ and any higher dimension combination of them, the sound speed of $v_{R,L}$ is equal to one.[6]

Using (23), (26)–(28), and (30), we obtain the field equations of $v_{R,L}$ and $u_{R,L}$ in the subhorizon region:

$$\partial^2_{\tilde{\tau}}v_{R,L} + \left(1 \mp \frac{2\tilde{D}}{\tilde{\tau}}\right)v_{R,L} \simeq 0, \tag{32}$$

$$\partial^2_{\tilde{\tau}}u_{R,L} + u_{R,L} \simeq \frac{2\psi}{\tilde{N}\tilde{\tau}}(B\partial_{\tilde{\tau}}v_{R,L} \mp \sqrt{\gamma}Dv_{R,L}), \tag{33}$$

where $\tilde{\tau} = -k\tau$,

$$B = (f_1 + \sqrt{\gamma}H\psi(f_3 - \sqrt{\gamma}f_4)), \quad \text{and}$$
$$D = \left(f_1 - \frac{H\psi}{\sqrt{\gamma}}(f_3 - \sqrt{\gamma}f_4)\right). \tag{34}$$

In both of the above field equations, the last term is parity odd and takes different signs for the right- and left-handed polarizations of modes.

Solving the field equations (32) and (33) and imposing the standard Minkowski vacuum state at the deep inside horizon limit ($k\tau \to -\infty$), we obtain the canonically normalized fields on subhorizon scales:

$$v_{R,L}(\tilde{\tau},k) \simeq \frac{1}{\sqrt{2k}\sqrt{1\mp\tilde{D}/\tilde{\tau}}}\exp(i(\tilde{\tau}\mp\tilde{D}\ln\tilde{\tau})), \tag{35}$$

$$u_{R,L}(\tilde{\tau},k) \simeq \frac{1}{\sqrt{2k}}\left(1 - \frac{\psi}{M_{\rm Pl}}\frac{(\sqrt{\gamma}D\mp iB)}{\tilde{N}\tilde{D}\sqrt{1\mp\tilde{D}/\tilde{\tau}}}\exp(\mp i\tilde{D}\ln\tilde{\tau})\right) \times \exp(i\tilde{\tau}). \tag{36}$$

[4]Note that the cross terms of v_λ and u_λ in the second-order action of v_λ have a factor of ψ which, as $\psi \ll 1$, are neglected in the dominant order action (30).

[5]The exact form of $\tilde{D}$ is $\tilde{D} = \frac{(\sqrt{\gamma}f_1 + \frac{\dot{f}_2}{2H} - g\psi^2(\sqrt{\gamma}f_3 + f_4) + \frac{(\psi Hf_3)^{\cdot}}{2H^2} - \frac{(g\psi^2 f_4)^{\cdot}}{2H})}{(f_1 - g\psi^2(f_3 + f_4/\sqrt{\gamma}))}$. However, during the slow-roll inflation, we have $\dot{f}_{3,4} \ll Hf_{3,4}$ and $\dot{\psi} \ll H\psi$; hence, we can neglect the last two terms with respect to the other terms.

[6]This is not a generic property of all of the possible gauge field theories. For instance, (although a subdominate term of the order of ϵ here) among the dimension-eight operators $F^a{}_{\mu\nu}F_a{}^\nu{}_\lambda F^{b\lambda\xi}F_{b\xi}{}^\mu$ and $F^a{}_{\mu\nu}F_b{}^\nu{}_\lambda F^{a\lambda\xi}F_{b\xi}{}^\mu$ lead to $c_s^2 = \frac{3\gamma-1}{\gamma-3}$, while the other dimension-eight terms $(\mathrm{tr}(FF))^2$, $(\mathrm{tr}(F\tilde{F}))^2$, and $(\mathrm{tr}(FF)\mathrm{tr}(F\tilde{F}))$ have a c_s^2 equal to one.

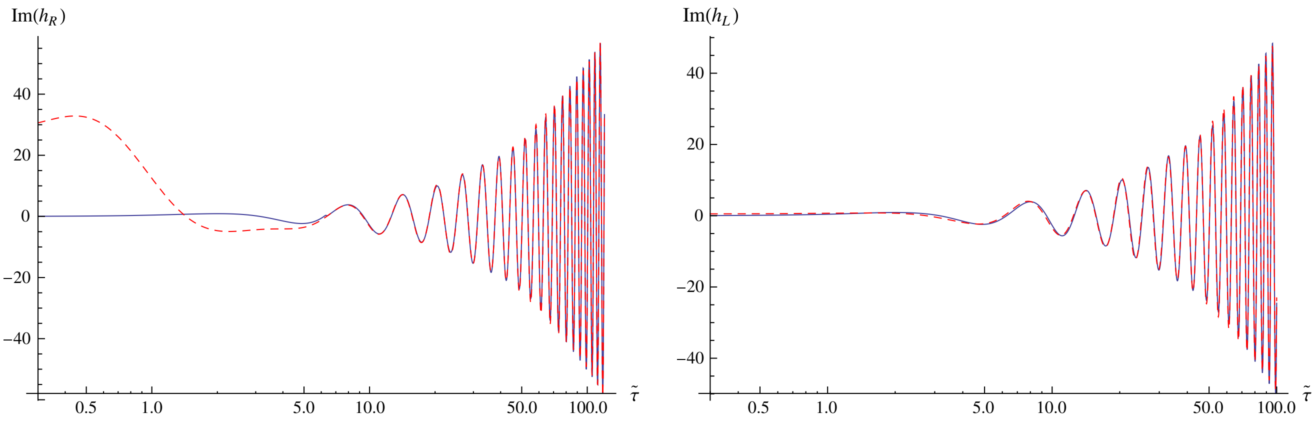

FIG. 1 (color online). Comparison of the subhorizon analytical solution of $\bar{h}_{R,L}$ (solid line) and its full numerical result (dashed) for the chromo-natural model. Here, $\psi \simeq 5\times 10^{-2}$, $H \simeq 10^{-6}$, and $\gamma = 9$ (which has highly chiral gravitational waves in this γ) [17,22]. The analytical approximation and numerical solution perfectly overlaid each other on subhorizon scales $\tilde{\tau} \gtrsim 5$, while, as we get closer to the horizon crossing point $\tilde{\tau} = 1$, they eventually start to deviate from each other. Here, we presented only the imaginary part of $\bar{h}_{R,L}$; however, the real part has the same behavior.

Equation (36) indicates that the chiral term in $u_{R,L}$ is proportional to ψ and is related to D and $\tilde{D}$. In the case that $D = \tilde{D} = 0$, we have $u_{R,L}(\tilde{\tau},k) \simeq \frac{1}{\sqrt{2k}}(1 + \frac{\psi}{M_{\rm Pl}}\frac{B}{\tilde{N}}\ln\tilde{\tau})\exp(i\tilde{\tau})$. That is expected, because D and $\tilde{D}$ are coefficients of parity-odd terms, and, if they vanish, then $u_R = u_L$.

Numerical solution vs analytical subhorizon approximation.—Let us now compare the analytical subhorizon approximation (36) with the full numerical solution of a specific model, chromo-natural [Eq. (20)]. Field equations of the chromo-natural model (and gauge-flation) are specified by these parameters: $B = 1$, $D = 1$, $\tilde{N} = 1$, and $\tilde{D} = (1 + 2\gamma)/\sqrt{\gamma}$ [12]. Figure 1 presents the analytical approximation of $\bar{h}_{R,L}$ (solid line) and its full numerical solution (dashed line) with respect to $\tilde{\tau} = -k\tau$. Analytical and numerical solutions perfectly overlaid each other on subhorizon scales $\tilde{\tau} \gtrsim 5$, which confirms the validity of our approximations (36). As we get closer to the horizon crossing point $\tilde{\tau} = 1$, analytical and numerical solutions eventually start to deviate from each other. It is noteworthy to mention that the system which is presented here (with $\gamma = 9$) leads to highly chiral tensor modes [17,22]. Let us quantify the enlargement of chirality in the system by $\Theta \equiv \frac{P_R - P_L}{P_R + P_L}$, where $P_{R,L}$ is the superhorizon power spectrum of right- or left-handed polarization. Then, $\Theta = 0$ represents a system with parity symmetry ($P_R = P_L$), while a Θ close to one parametrizes a case with highly chiral gravitational waves. Even in this highly chiral system, due to its $\tilde{\tau}^3$ factor, the integrand in (12) is much larger in $\tilde{\tau} \gg 1$ than at the vicinity of the horizon crossing point.

IV. CONFRONTING WITH THE OBSERVATION

As for their intrinsic chiral gravitational waves, inflationary models with non-Abelian gauge fields naturally generate a nonvanishing $\langle \tilde{R}R \rangle$, which makes them perfect for the inflato-leptogenesis mechanism. To complete our leptogenesis model, now we need to determine the net lepton and photon number densities predicted by these models.

A. Lepton number density

At this point, we are ready to compute the net lepton number density n, which through the gravitational anomaly is generated during inflation. From (10) and (36), one can read $\bar{h}_{R,L}(\tilde{\tau})$ as

$$\bar{h}_{R,L}(\tilde{\tau}) \simeq \frac{\tilde{\tau}}{2}\left(1 - \frac{\psi}{M_{\rm Pl}}\frac{\sqrt{\gamma}D \mp iB}{\tilde{N}\tilde{D}\sqrt{1 \mp \tilde{D}/\tilde{\tau}}}\exp(\mp i\tilde{D}\ln\tilde{\tau})\right) \times \exp(i\tilde{\tau}). \tag{37}$$

Similarly to $u_{R,L}$, the chiral term in $\bar{h}_{R,L}$ is proportional to ψ and is related to D and $\tilde{D}$. Inserting the above solution in (12) and performing the integral in the $\Lambda \gg H$ limit, we obtain the lepton number density by the end of inflation as

$$n(\tau_{\rm inf}) \simeq \frac{\mathcal{C}\mathcal{A}}{24\pi^4}\frac{\psi}{M_{\rm Pl}}H^3\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\Lambda}{H}\right)^4, \tag{38}$$

where $\mathcal{C}$ is given as

$$\mathcal{C} \simeq \frac{4\alpha}{\tilde{N}(16 + \tilde{D}^2)}\left(\tilde{\alpha}\cos\left(\tilde{D}\ln\left(\frac{\Lambda}{H}\right)\right) - \sin\left(\tilde{D}\ln\left(\frac{\Lambda}{H}\right)\right)\right), \tag{39}$$

where $\alpha = ((1 + \tilde{D}^2/4)B + \frac{3}{4}\sqrt{\gamma}D\tilde{D})$ and $\tilde{\alpha} = (\frac{3}{4}B\tilde{D} - \sqrt{\gamma}(1 + \tilde{D}^2/4)D)/((1 + \tilde{D}^2/4)B + \frac{3}{4}\sqrt{\gamma}D\tilde{D})$.

Equation (38) is the generic form of the net lepton number density predicted by inflationary models with non-Abelian gauge field (16). Some noteworthy features of n are as follows:

(i) The net lepton density is proportional to $\psi/M_{\rm Pl}$ (the effective gauge field value on the background) as well as $\mathcal{A}$, which is the difference between the number of left- and right-handed fermions. Thus, CP-violating sources and the birefringent gravitons originated from the gauge field in the background and a nonvanishing $\mathcal{A}$.

(ii) The factor H^3 is the inverse of the volume (horizon) size during inflation, which has the same unit as n.

(iii) n is proportional to the scale of inflation as $(\frac{H}{M_{\rm Pl}})^2$. We emphasize that one cannot directly relate $(\frac{H}{M_{\rm Pl}})^2$ to the power spectrum of the tensor modes after horizon crossing, because (i) n is mainly generated by subhorizon gravitational waves, and (ii) compared with the standard scalar models, the field equation of $h_{R,L}$ is modified by tensor perturbations of the gauge field $X_{R,L}$. That leads to right- and left-handed superhorizon power spectrums which are different from the standard prediction of scalar inflationary models [17,22].

(iv) n is related to the UV cutoff scale Λ, by a factor of $(\frac{\Lambda}{H})^4$, in agreement with our rough approximation in Sec. II. The Λ^4 term is intriguingly similar to the zero-point energy of corresponding gravity waves $\rho_{\rm vac}=\frac{\Lambda^4}{16\pi^2}$.

(v) $\mathcal{C}$ is determined by the specific form of the matter content $\mathcal{L}_m$ and in terms of B, D, $\tilde{D}$, $\tilde{N}$, and $\frac{\Lambda}{H}$ in (39). If D and $\tilde{D}$ (the coefficient of the parity-odd terms) vanish, then $\mathcal{C}=0$, as expected. Typical values of f_i's, B, D, $\tilde{D}$, and $\tilde{N}$ are of the order of one, which leads to $\mathcal{C}\sim 1$, e.g., in the chromo-natural and gauge-flation models [12].

(vi) Altogether, $\mathcal{C}\mathcal{A}\frac{\psi}{M_{\rm Pl}}$ is the coefficient that parametrizes the efficiency of the CP-violating process in the system.

(vii) $n(\tau)$ scales as a^{-3}; hence, the number density by the end of inflation $n(\tau_{\rm inf})$ and $n(\tau)$ for a given time τ are related as $a^3(\tau_{\rm inf})n(\tau_{\rm inf})=a^3(\tau)n(\tau)$.

B. Lepton to photon density ratio

At this point, we should determine the number density of photons at the present time, for which we need a reheating model. If the energy density at the reheating time $\rho_{\rm reh}$ is rapidly converted into radiation, we have

$$\rho_{\rm reh}=\frac{\pi^2}{30}g_*T^4_{\rm reh}, \tag{40}$$

where g_* is the number of relativistic degrees of freedom at the time of reheating and $T_{\rm reh}$ is the reheating temperature. Consider that the energy densities at the reheating time and during inflation ($\rho_{\rm inf}=3M^2_{\rm Pl}H^2$) are related as

$$a^4(\tau_{\rm reh})\rho_{\rm reh}=\sigma a^4(\tau_{\rm inf})\rho_{\rm inf}, \tag{41}$$

where, in the phenomenological reheating model above, σ parametrize the "efficiency" of the reheating process. Moreover, as ρ scales as a^{-4} at the end of the reheating era, $a^4\rho$ in (41) is a constant at that period.

It is interesting to note that, within the supersymmetric extension of the SM, gravitino production gives an upper bound on the reheating temperature $T_{\rm reh}<10^4$ TeV [26]. On the other hand, by relying on SM sphalerons to convert the generated asymmetry in the lepton sector into baryon asymmetry, this mechanism requires a reheat temperature $T_{\rm reh}\gtrsim 100$ GeV.

Having the reheating temperature from (40) as

$$\left(\frac{T_{\rm reh}}{M_{\rm Pl}}\right)=\left(\frac{90\sigma}{\pi g_*}\right)^{\frac{1}{4}}\left(\frac{a(\tau_{\rm inf})}{a(\tau_{\rm reh})}\right)\left(\frac{H}{M_{\rm Pl}}\right)^{\frac{1}{2}}, \tag{42}$$

one obtains the reheating entropy density

$$s_{\rm reh}=\frac{2\pi^2}{45}g_*T^3_{\rm reh}=2.3g_*^{\frac{1}{4}}\sigma^{\frac{3}{4}}(HM_{\rm Pl})^{\frac{3}{2}}\left(\frac{a(\tau_{\rm inf})}{a(\tau_{\rm reh})}\right)^3, \tag{43}$$

which, after using the standard assumption that the comoving entropy density of the Universe is constant since the end of reheating ($a^3s=cst$) and the relation $s_0\simeq 7.04n_{\gamma 0}$, determines the photon number density at the present time, $n_{\gamma 0}$.

Finally, we can compute the desired $\eta=n_0/n_{\gamma 0}$ [Eq. (1)]:

$$\eta\simeq 1.3\times 10^{-3}\frac{\mathcal{A}\mathcal{C}}{g_*^{\frac{1}{4}}\sigma^{\frac{3}{4}}}\frac{\psi}{M_{\rm Pl}}\left(\frac{H}{M_{\rm Pl}}\right)^{\frac{7}{2}}\left(\frac{\Lambda}{H}\right)^4, \tag{44}$$

which should be compared with the observed value $\eta\simeq 6\times 10^{-10}$ [1].

For typical values of $g_*\sim 10^2$ and $\psi\sim 10^{-1}$, a successful leptogenesis model requires

$$\frac{\mathcal{A}\mathcal{C}}{\sigma^{\frac{3}{4}}}\left(\frac{\Lambda}{H}\right)^4\left(\frac{H}{M_{\rm Pl}}\right)^{\frac{7}{2}}\sim 10^{-5}. \tag{45}$$

This relation can be fulfilled for typically reasonable values of the reheating temperature and UV cutoff Λ. For instance, consider the standard model with $\mathcal{A}=3$, and suppose $\mathcal{C}\sim 1$ and $H\sim 10^{-6}M_{\rm Pl}$. Then, for $\Lambda\sim 10$–$100H$, a reheating efficiency $\sigma\sim 10^{-10}$–10^{-16} leads to a successful leptogenesis mechanism. In order to determine the reheating temperature corresponding to the above values, we need more details about the reheating model, i.e., $a(\tau_{\rm inf})/a(\tau_{\rm reh})$. However, we have an upper value, which leads to $T_{\rm reh}\lesssim 10^{10}$ GeV.

V. SUMMARY AND CONCLUSIONS

We present a scenario of leptogenesis associated with inflationary models involving non-Abelian gauge fields within the SM, inflato-leptogenesis. The idea of using non-Abelian gauge fields in an inflationary setting was put forward in Refs. [14,15], in which it is shown that the non-Abelian gauge field theory can provide the setting for constructing isotropic and homogeneous inflationary background. Dealing with gauge fields in inflationary models brings many new and unique features compared with the standard scalar models, among them tensor fluctuations of the non-Abelian gauge field [17]. In this work, we demonstrated that almost all inflationary models with non-Abelian gauge fields produce intrinsic birefringent tensor modes.

Compared with the standard scalar models, tensor fluctuations of the non-Abelian gauge field interact with the metric tensor mode and modify its field equation. These new interactions involve some parity-odd terms, which take different signs for different (left- and right- handed) polarizations of tensor modes and lead to chiral tensor modes. Because of their intrinsic birefringent gravitational waves, inflationary models involving non-Abelian gauge fields provide natural settings for the leptogenesis mechanism, inflato-leptogenesis. Following Ref. [8] and using the gravitational chiral anomaly in the standard model, we showed that these chiral tensor fluctuations produced during inflation can generate a net lepton number.

These models predict a nonvanishing net lepton number density n, proportional to ψ and related to the UV cutoff of the physical momentum Λ, as $(\frac{\Lambda}{H})^4$. The factor $\psi/M_{\rm Pl}$ in n indicates that the demanding P-violating interactions originated from the non-Abelian gauge field in the background. Moreover, the factor Λ^4 is intriguingly similar to the zero-point energy of corresponding gravity waves ($\rho_{\rm vac} = \frac{\Lambda^4}{16\pi^2}$ [27]).

In order to complete our inflato-leptogenesis mechanism, we then considered a phenomenological reheating model with the efficiency parameter σ and determined the photon number density at the present time, n_γ. Finally, we compared n/n_γ predicted by our scenario with the observational data $\eta \sim 6\times 10^{-10}$. We argued that this scenario can explain the observed value of baryon to photon number density with a natural range of parameters, e.g., $H \simeq 10^{-6} M_{\rm Pl}$, $\Lambda \sim 10$–$100H$, and a reheating temperature of the order of $T_{\rm reh} \lesssim 10^{10}$ GeV (these values correspond to $\sigma \sim 10^{-10}$–10^{-16}). In Ref. [12], the inflato-leptogenesis scenario has been studied in two specific inflationary models of this class, the chromo-natural and gauge-flation models.

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APPENDIX: DETAILS OF $\tilde{R}R$ CALCULATION

$\tilde{R}R$ has the following explicit form:

$$\tilde{R}R \equiv \frac{1}{2}\epsilon^{\lambda\mu\nu\xi} R_{\lambda\mu\rho\sigma} R_{\nu\xi}{}^{\rho\sigma}, \tag{A1}$$

where $\epsilon^{\lambda\mu\nu\xi}$ is the totally antisymmetric tensor and $R^{\mu}{}_{\nu\lambda\sigma}$ is the Riemann tensor. This parity-odd term vanishes in the unperturbed homogeneous and isotropic FRW background, while the perturbations of the metric sources the second-order $\tilde{R}R$. By perturbing the metric around the FRW background, the most general perturbed metric can be parametrized as

$$ds^2 = -(1+2A)dt^2 + 2a(\partial_i B + V_i)dx^i dt + a^2((1-2C)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}W_{j)} + h_{ij})dx^i dx^j, \tag{A2}$$

where A, B, C, and E are scalar perturbations, V_i and W_i parametrize divergence-free vector perturbations, and h_{ij}, which is symmetric, traceless, and divergence-free, is the tensor mode.

Plugging (A2) into (A1), we obtain the second-order $\tilde{R}R$:

$$\tilde{R}R = -\frac{2}{a^4}\epsilon^{ijk}(h''_{jl}\partial_i h'_{lk} - \partial_m h'_{jl}\partial^2_{im}h_{lk} + \partial_l h'_{jm}\partial^2_{mi}h_{kl}), \tag{A3}$$

where a prime denotes a derivative with respect to the conformal time ($d\tau = a^{-1}dt$). Note that $\tilde{R}R$ contains only tensor perturbations h_{ij}, and the scalar and vector fluctuations do not contribute.

It is convenient to use Fourier modes in the linear theory of a flat universe, as they evolve independently. The real space perturbation $h_{ij}(\tau,\mathbf{x})$ can be written as below in terms of its Fourier components:

$$h_{ij}(\tau,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \mathsf{h}_{ij}(\tau,\mathbf{k}) e^{i\mathbf{x}.\mathbf{k}}.$$

Using the above, we can write $\tilde{R}R$ in terms of the Fourier modes $\mathsf{h}_{ij}(\tau,\mathbf{k})$:

$$\tilde{R}R(\tau,\mathbf{x}) = -\frac{2i\epsilon^{ijk}}{a^4}\iint \frac{d^3k d^3k'}{(2\pi)^3} k'^i (\mathsf{h}''_{jl}(\tau,\mathbf{k})\mathsf{h}'_{lk}(\tau,\mathbf{k}') + \mathbf{k}.\mathbf{k}'\mathsf{h}'_{jl}(\tau,\mathbf{k})\mathsf{h}_{lk}(\tau,\mathbf{k}'))e^{i(\mathbf{k}+\mathbf{k}').\mathbf{x}} + \mathcal{D}, \tag{A4}$$

where $\mathcal{D}$ is a total derivative term. This quantity is most simplified in terms of right- and left-handed polarizations in the Fourier space, $\mathsf{h}_{R,L}(\tau,\mathbf{k})$[7]:

$$\tilde{R}R(\tau,\mathbf{x}) = -\frac{8i}{a^4}\iint \frac{d^3k d^3k'}{(2\pi)^3} k'(\mathsf{h}''_R(\tau,\mathbf{k})\mathsf{h}'_L(\tau,\mathbf{k}') + \mathbf{k}.\mathbf{k}'\mathsf{h}'_R(\tau,\mathbf{k})\mathsf{h}_L(\tau,\mathbf{k}') - \text{c.c.})e^{i(\mathbf{k}+\mathbf{k}').\mathbf{x}} + \mathcal{D}. \tag{A6}$$

For a wave vector $\mathbf{k} = (0,0,k)$, h_{ij} and $h_{R,L}$ are related as follows:

$$\mathsf{h}_{ij}(\tau,\mathbf{k}) = \begin{pmatrix} \mathsf{h}_R + \mathsf{h}_L & -i(\mathsf{h}_R - \mathsf{h}_L) & 0 \\ -i(\mathsf{h}_R - \mathsf{h}_L) & -(\mathsf{h}_R + \mathsf{h}_L) & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{A7}$$

Expanding $\hat{h}_{R,L}$ in terms of the creation and annihilation operations, we have

$$\hat{h}_R(\tau,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{\mathsf{h}}_R(\tau,\mathbf{k})e^{i\mathbf{k}.\mathbf{x}} = \int \frac{d^3k}{(2\pi)^{3/2}} (h_R(\tau,\mathbf{k})\hat{a}_\mathbf{k} + h^*_L(\tau,-\mathbf{k})\hat{b}^\dagger_{-\mathbf{k}})e^{i\mathbf{k}.\mathbf{x}}, \tag{A8}$$

[7] Upon naively writing (A4) in terms of $\mathsf{h}_{R,L}$, one obtains

$$\tilde{R}R(\tau,\mathbf{x}) = -\frac{8i}{a^4}\iint \frac{d^3k d^3k'}{(2\pi)^3} k'(\mathsf{h}''_R(\tau,\mathbf{k})\mathsf{h}'_L(\tau,\mathbf{k}') + \mathbf{k}.\mathbf{k}'\mathsf{h}'_R(\tau,\mathbf{k})\mathsf{h}_L(\tau,\mathbf{k}') - R\leftrightarrow L)e^{i(\mathbf{k}+\mathbf{k}').\mathbf{x}} + \mathcal{D}, \tag{A5}$$

which is not a Hermitian operator. In order to write $\tilde{R}R$ in the form of a Hermitian operator, one has to not only exchange R and L ($R\leftrightarrow L$) in the last terms, but also change the order of operators.

$$\hat{h}_L(\tau,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \hat{\mathsf{h}}_L(\tau,\mathbf{k})e^{i\mathbf{k}.\mathbf{x}} = \int \frac{d^3k}{(2\pi)^{3/2}} (h_L(\tau,\mathbf{k})\hat{b}_\mathbf{k} + h^*_R(\tau,-\mathbf{k})\hat{a}^\dagger_{-\mathbf{k}})e^{i\mathbf{k}.\mathbf{x}}, \tag{A9}$$

where the creation and annihilation operators $\hat{a}_\mathbf{k}$ and $\hat{b}_\mathbf{k}$ satisfy the standard canonical relations [e.g., $[\hat{a}_\mathbf{k}, \hat{a}^\dagger_{\mathbf{k}'}] = \delta^{(3)}(\mathbf{k}-\mathbf{k}')$]. Moreover, the left and right polarizations are related as $\hat{h}_L(\tau,\mathbf{x}) = \hat{h}^\dagger_R(\tau,\mathbf{x})$, which implies that the Fourier operator components are related as[8] $\hat{\mathsf{h}}_R(\tau,\mathbf{k}) = \hat{\mathsf{h}}^\dagger_L(\tau,-\mathbf{k})$. Note the difference between Fourier operator components $\hat{\mathsf{h}}_{R,L}(\tau,\mathbf{k})$ and Fourier mode functions $h_{R,L}(\tau,\mathbf{k})$.

Using (A8) and (A9) in (A6), we determine the vacuum expectation value of $\tilde{R}R$:

$$\langle\tilde{R}R(\tau)\rangle = \frac{4}{a^4}\int \frac{kd^3k}{(2\pi)^3}\frac{d}{d\tau}(h'_R(\tau,\mathbf{k})h^{*\prime}_R(\tau,\mathbf{k}) - k^2 h_R(\tau,\mathbf{k})h^*_R(\tau,\mathbf{k}) - R\leftrightarrow L) + \mathcal{D}, \tag{A10}$$

which, assuming the statistical isotropy of the primordial fluctuations, leads to

$$\langle\tilde{R}R(\tau)\rangle = \frac{2/\pi^2}{a^4}\int k^3 dk \frac{d}{d\tau}(h'_R(\tau,k)h^{*\prime}_R(\tau,k) - k^2 h_R(\tau,k)h^*_R(\tau,k) - R\leftrightarrow L) + \mathcal{D}. \tag{A11}$$

The above equation indicates that the parity-odd $\langle\tilde{R}R\rangle$ is tightly related to birefringent gravitational waves, and in the special case of parity symmetry [in which $h_R(\tau,\mathbf{k}) = h_L(\tau,\mathbf{k})$], it vanishes. Thus, a nonzero $\langle\tilde{R}R\rangle$ requires a mechanism to generate chiral tensor modes.

[8] In general, the Fourier mode functions $h_R(\tau,\mathbf{k})$ and $h_L(\tau,\mathbf{k})$ are two independent solutions of two different field equations. In the special case with parity-preserving action, then we have $h_R(\tau,\mathbf{k}) = h_L(\tau,\mathbf{k})$.

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