



Aligned natural inflation in string theory

Cody Long,^{*} Liam McAllister,[†] and Paul McGuirk[‡]

Department of Physics, Cornell University, Ithaca, New York 14853, USA

(Received 13 May 2014; published 10 July 2014)

We propose a scenario for realizing super-Planckian axion decay constants in Calabi-Yau orientifolds of type IIB string theory, leading to large-field inflation. Our construction is a simple embedding in string theory of the mechanism of Kim, Nilles, and Peloso, in which a large effective decay constant arises from alignment of two smaller decay constants. The key ingredient is gaugino condensation on magnetized or multiply-wound D7-branes. We argue that, under very mild assumptions about the topology of the Calabi-Yau, there are controllable points in moduli space with large effective decay constants.

DOI: [10.1103/PhysRevD.90.023501](https://doi.org/10.1103/PhysRevD.90.023501)

PACS numbers: 98.80.Cq, 11.25.-w, 11.25.Uv, 98.80.-k

I. INTRODUCTION

Observations of the cosmic microwave background (CMB) [1] have provided extraordinary support for the inflationary paradigm of an early epoch of accelerated cosmic expansion [2]. Recent measurements of B-mode polarization by the BICEP2 Collaboration [3] suggest that, in addition to the scalar fluctuations that had been previously observed and well studied, the inflationary period was marked by significant tensor fluctuations. In particular, if the observed polarization is cosmological in origin, it is compatible with a tensor-to-scalar ratio of $r \approx 0.1 - 0.2$.

Detectable primordial B-mode polarization implies that the energy scale of inflation is comparable to the unification scale,

$$V_{\text{inf}}^{1/4} \approx 2.2 \times 10^{16} \text{ GeV} \left(\frac{r}{0.2} \right)^{1/4}. \quad (1)$$

In the simplest scenarios, inflation is driven by a single scalar field ϕ undergoing slow-roll evolution in a potential V . In such models, the tensor-to-scalar ratio respects the Lyth bound [4]

$$\frac{\Delta\phi}{M_{\text{pl}}} \gtrsim \left(\frac{r}{0.01} \right)^{1/2}, \quad (2)$$

in which $\Delta\phi$ is the distance in field space that the inflaton moves during inflation and $M_{\text{pl}} \approx 2.4 \times 10^{18} \text{ GeV}$ is the reduced Planck scale. The BICEP2 observation, taken at face value, implies a trans-Planckian displacement in field space for such an inflaton. Analogous bounds exist for theories with multiple fields or noncanonical kinetic terms, with the general result that the observed r requires $\Delta\phi$ to exceed the ultraviolet cutoff of the theory [5].

Inflation is famously sensitive to corrections from quantum gravity. Even if the scale of inflation were much smaller than M_{pl} , Planck-suppressed operators can make $\mathcal{O}(1)$ corrections to the slow-roll parameter $\eta = M_{\text{pl}}^2 \frac{V''}{V}$, which must remain small during inflation. In addition to this universal η problem, the large inflationary scale and the trans-Planckian excursion of the inflaton suggested by the BICEP2 results intensify the need for a consistent embedding of inflation into a theory of quantum gravity such as string theory.¹ However, despite much progress, complete and explicit models of large-field inflation in string theory remain elusive.

In this paper, we suggest a mechanism by which large-field inflation may be realized in controllable string theory constructions. We make use of the scheme of natural inflation [7] and the decay constant alignment mechanism proposed by Kim, Nilles, and Peloso (KNP) [8].² We demonstrate that compactifications of type IIB string theory allow for the large axion decay constants required to realize large-field natural inflation in this framework. However, we do not stabilize moduli in this paper, and so fall short of a complete model of inflation.

While this work was in the last stages of preparation, we learned of the independent concurrent development of closely related ideas by other authors [12].

II. NATURAL INFLATION AND DECAY CONSTANT ALIGNMENT

A natural way to control the appearance of irrelevant operators in the inflationary potential, and thus ensure that η is small, is to suppose that the inflaton enjoys a global symmetry. The natural inflation scenario [7] achieves this by using an axion to drive inflation. At the classical level, an axion enjoys a continuous shift symmetry $\phi \rightarrow \phi + c$

^{*}cel89@cornell.edu

[†]mcallister@cornell.edu

[‡]mcguirk@cornell.edu

¹See [6] for a recent comprehensive review of inflationary models in string theory.

²See [9–11] for recent field-theoretic treatments of related scenarios.

for arbitrary c . However, when nonperturbative quantum corrections are taken into account, this symmetry is broken to a discrete shift symmetry $\phi \rightarrow \phi + 2\pi f$, where f is known as the axion decay constant. An example of a four-dimensional Lagrangian that respects this symmetry is

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \Lambda^4 \left[1 - \cos\frac{\phi}{f} \right]. \quad (3)$$

When $f \gtrsim 10 M_{Pl}$ and $\Lambda \sim 10^{-3} M_{Pl}$, this potential supports realistic inflation. Indeed, for large f , the potential is well approximated by a quadratic potential, $V \approx \frac{\Lambda^4}{2f^2} \phi^2$, while higher-order terms in the potential are suppressed by f , rather than by M_{Pl} , thus ensuring the smallness of η .

It is of obvious importance to determine whether the super-Planckian decay constants required to obtain realistic natural inflation can be obtained in a theory of quantum gravity such as string theory. String theory compactifications can provide many axions descending from the various p -form potentials of ten-dimensional supergravity (see [13] for a comprehensive discussion), but there exist no complete examples in which the decay constants are suitably large and the construction is parametrically controlled.³ Indeed, there are generic arguments that suggest that it is quite difficult to obtain large decay constants in which the string construction is under good control [15]. However, it was pointed out by KNP [8] that if there are multiple axions and multiple nonperturbative effects that couple to different linear combinations of the axions, then an effectively large axion decay constant can be obtained even when the original decay constants are modest. Such an enhancement relies on the alignment of the decay constants appearing in the nonperturbative corrections to the scalar potential.

To illustrate this alignment, we follow [8] and consider a theory with two axions ϕ^1 and ϕ^2 with the nonperturbatively generated potential

$$V(\phi) = \Lambda_A^4 \left[1 - \cos\left(\frac{\phi^1}{f_{A1}} + \frac{\phi^2}{f_{A2}}\right) \right] + \Lambda_B^4 \left[1 - \cos\left(\frac{\phi^1}{f_{B1}} + \frac{\phi^2}{f_{B2}}\right) \right] \quad (4)$$

and the kinetic term

$$\mathcal{L}_{\text{kin}} = -k_{ij} \partial_\mu \phi^i \partial^\mu \phi^j. \quad (5)$$

In general, the axions may have kinetic mixing as well, with off-diagonal terms in k_{ij} .

Taking both the kinetic and potential mixings into account, the determinant of the mass matrix of the canonically normalized fields is

$$\det \partial_i \partial_j V|_{\phi_1=\phi_2=0} = \frac{(f_{A1}f_{B2} - f_{A2}f_{B1})^2}{f_{A1}^2 f_{A2}^2 f_{B1}^2 f_{B2}^2} \frac{\Lambda_A^4 \Lambda_B^4}{4 \det(k)}, \quad (6)$$

and so a flat direction emerges when the decay constants are aligned,

$$\frac{f_{A1}}{f_{A2}} = \frac{f_{B1}}{f_{B2}}. \quad (7)$$

When this alignment is achieved, both terms in (4) couple to the same linear combination of the axions, while the orthogonal direction remains a flat direction.

By slightly misaligning the decay constants, a nearly flat direction emerges and a large effective decay constant can be obtained for this direction [8]. To illustrate this, we consider an example of canonically normalized axions⁴ for which all of the decay constants are nearly equal while the dynamical scales are hierarchical,

$$f_{A1} = f_{B1} = f_{A2} = f, \quad f_{B2} = f(1 + \delta), \quad (8)$$

$$\Lambda_A^4 = \Lambda^4, \quad \Lambda_B^4 = \delta^p \Lambda^4, \quad (9)$$

where $p > 0$ and $\delta \ll 1$. Then the determinant of the Hessian at the minimum is

$$\det \partial_i \partial_j V|_{\phi=0} = \frac{\delta^{2+p} \Lambda^8}{f^4 (1 + \delta)^2}, \quad (10)$$

while the particular eigenvalues are

$$m_1^2 = \delta^{2+p} \frac{\Lambda^4}{2f^2} [1 + \mathcal{O}(\delta)] \quad m_2^2 = \frac{2\Lambda^4}{f^2} [1 + \mathcal{O}(\delta)]. \quad (11)$$

In terms of the mass eigenstates, the potential takes the form

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\sqrt{2}\psi^2}{f} + \frac{\delta^{p+1}\psi^1}{\sqrt{2}f}\right) \right] + \delta^p \Lambda^4 \left[1 - \cos\left(-\frac{\sqrt{2}\psi^2}{f} + \frac{\delta\psi^1}{\sqrt{2}f}\right) \right], \quad (12)$$

where in each cosine we have kept only the first term in which either field appears. The first condensate serves to stabilize $\psi^2 \approx 0$, while for sufficiently small ψ^1 , the potential is dominated by the second term, which exhibits an effective decay constant that is parametrically enhanced by the near alignment:

³See, however, [14] for recent proposals to obtain large decay constants in string theory.

⁴From (6), it is apparent that the effect of introducing kinetic mixing [which generally decreases $\det(k)$] is to further lift the nearly flat direction resulting from near alignment.

$$f_{\text{eff}} \sim \frac{f}{\delta}. \quad (13)$$

Interestingly, the first term in the condensate naively exhibits an even larger effective decay constant. However, its contribution to the potential of ψ^1 (for fixed ψ^2) is subdominant for small ψ^1 .

Although we illustrated the effectiveness of decay constants with a particular scaling (8), the scheme works more generally. Indeed, one of the examples of an inflationary potential that we present in Sec. VI utilizes different relationships between the decay constants and the dynamical scales.

III. AXIONS IN TYPE IIB STRING THEORY

Our objective in this paper is to propose a framework in which the success of decay constant alignment may be embedded into a string compactification. In general, the construction of inflationary models in string theory cannot be decoupled from the problem of moduli stabilization. Therefore, we focus on inflation in O3/O7 Calabi-Yau orientifold compactifications of type IIB string theory, where the understanding of moduli stabilization is presently the most mature. Although we will not stabilize moduli, the ingredients that we use to construct the inflationary potential are the same as those used for moduli stabilization in this corner of the landscape, and we have found no reason why a completely stabilized compactification could not in principle be constructed. In this section, we briefly review aspects of the effective field theories of such orientifolds, focusing on elements that are relevant for the construction of our inflationary potentials. A more detailed treatment can be found in, for example, [16].

The IIB supergravity multiplet in ten dimensions consists of the metric, the axiodilaton $\tau = C + ie^{-\Phi}$, the 2-form potentials B_2 and C_2 , and the 4-form potential C_4 . In the absence of sources, the low-energy description of a compactification on a Calabi-Yau threefold with Hodge numbers $(h^{1,1}, h^{2,1})$ is a four-dimensional $\mathcal{N} = 2$ supergravity theory with $h^{2,1}$ vector multiplets (the scalar components of which are the complex structure moduli), $h^{1,1}$ hypermultiplets (including Kähler moduli), and the universal hypermultiplet built from τ and $B_{\mu\nu} - \tau C_{\mu\nu}$.

Breaking to $\mathcal{N} = 1$ in four dimensions can be accomplished by orientifolding, in which we identify states related by an orientation reversal of the worldsheet and a holomorphic involution of the Calabi-Yau geometry. The cohomology groups split under the action of the involution

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}. \quad (14)$$

Correspondingly,

$$h^{p,q} = h_+^{p,q} + h_-^{p,q}, \quad h_{\pm}^{p,q} = \dim H_{\pm}^{(p,q)}. \quad (15)$$

The geometric involution will have fixed loci corresponding to the presence of orientifold planes. We will focus on O3/O7 orientifolds in which the fixed loci are points (O3-planes) and divisors (O7-planes). After the orientifold action, the low-energy theory is a four-dimensional $\mathcal{N} = 1$ supergravity theory with $h_+^{2,1}$ vector multiplets coming from C_4 , $h_-^{2,1}$ chiral multiplets describing complex structure deformations, a chiral multiplet with scalar component τ , $h_-^{1,1}$ chiral multiplets from the 2-form potentials, and $h_+^{1,1}$ chiral multiplets corresponding to complexified Kähler moduli. Finally, the orientifold planes carry D3- and D7-brane charge that must be canceled by the inclusion of D-branes. These give rise to additional low-energy degrees of freedom corresponding to the deformations of these branes.

The axiodilaton, complex structure moduli, and the deformation moduli of D7-branes can be stabilized at the perturbative level by fluxes. We will assume that such fluxes have been included and that these fields are stabilized at a high scale and can be consistently integrated out.⁵ The remaining closed-string moduli are those coming from the 2-form potentials and the complexified Kähler moduli. Their scalar degrees of freedom are encoded in the Kähler coordinates [16]

$$G^a = c^a - \tau b^a, \quad (16)$$

$$T^\alpha = \tau^\alpha + i\vartheta^\alpha + \frac{1}{4} e^{\phi} \kappa^\alpha_{bc} G^b (G^c - \bar{G}^{\bar{c}}), \quad (17)$$

in which $a = 1, \dots, h_+^{1,1}$, $\alpha = 1, \dots, h_+^{1,1}$, and

$$\tau^\alpha + i\vartheta^\alpha = \frac{1}{2} \int_{\mathcal{D}^\alpha} J \wedge J + i \int_{\mathcal{D}^\alpha} C_4, \quad (18)$$

$$C_2 = c^a \omega_a, \quad B_2 = b^a \omega_a. \quad (19)$$

The scalar field τ^α is the volume of the 4-cycle $\mathcal{D}^\alpha$, where $\mathcal{D}^\alpha$ ($\mathcal{D}_a$) is a divisor that is even (odd) under the geometric involution and ω^α (ω_a) is the Poincaré dual of an even (odd) divisor. Here κ^{ijk} are triple intersection numbers in the “upstairs” Calabi-Yau, and indices on the intersection numbers are raised and lowered with the identity matrix. Note that the definition of the good Kähler coordinates is modified by the presence of open strings [17,18] or strong warping [19].

At the classical level and in the absence of localized brane sources, each of the fields ϑ^α , c^a , and b^a enjoys a continuous shift symmetry. This is reflected by the absence of a superpotential for T^α , and a Kähler potential for the Kähler moduli that depends only on 2-cycle volumes,

⁵Integrating out fields stabilized by fluxes is consistent in compactifications allowing sufficiently large hierarchies of scales, but for constructions of high-scale inflation, which allow only modest hierarchies, it would be worthwhile to relax this assumption.

$$K = -2 \log \mathcal{V}, \quad \mathcal{V} = \frac{1}{3!} \kappa^{\alpha\beta\gamma} t_\alpha t_\beta t_\gamma. \quad (20)$$

The volumes of 4-cycle volumes τ^α are related to 2-cycle volumes t_α via $\tau^\alpha = \frac{1}{2} \kappa^{\alpha\beta\gamma} t_\beta t_\gamma$ and hence the Kähler potential is an implicit function of the Kähler coordinates T^α , G^a , and their conjugates.

Nonperturbatively, the continuous shift symmetries of g^a , c^a , and b^a are broken to discrete shift symmetries. For example, a stack of D7-branes realizing an $SU(N)$ gauge theory and wrapping a divisor $\mathcal{D}$ contributes a nonperturbative correction to the superpotential via gaugino condensation

$$W_{\text{np}} = A e^{-aT}, \quad (21)$$

in which $T = \tau + \dots$ corresponds to the volume of $\mathcal{D}$, A is a function of the stabilized complex structure and brane moduli, and $a = \frac{2\pi}{N}$. In order for these superpotential terms to appear, the deformation moduli of the D7-branes must be lifted either by flux or by taking the stack to wrap a rigid divisor. The coordinate T appears because the tree-level gauge kinetic function for the gauge theory realized by the D7-branes is simply

$$\mathfrak{f}_{\text{D7}} = T. \quad (22)$$

Although the odd moduli G^a do not appear in W_{np} , the appearance of b^a in the real part of the Kähler coordinate T^a means that the b^a can be stabilized when the 4-cycle volumes are stabilized nonperturbatively [20,21]. At this level, the c^a axions remain unstabilized, though nonperturbative corrections to the Kähler potential from Euclidean D1-branes will generically induce a mass for these fields.

The situation changes when the stack of D7-branes is magnetized or is allowed to have multiple windings. We consider a restricted class of magnetizations that can be expanded in terms of pullbacks of 2-forms ω_a on the Calabi-Yau

$$F_2^{\text{int}} = \frac{1}{2\pi\alpha'} F^a \mathbf{P}[\omega_a]. \quad (23)$$

When the stack is magnetized, the gauge kinetic function is modified and depends holomorphically on the odd moduli G^a . To be more precise, let $\mathcal{D}$ be the divisor wrapped by the D7-brane stack, and let $\mathcal{D}'$ be the orientifold image. Then define the even and odd cycles,

$$\mathcal{D}^\pm = \mathcal{D} \cup (\pm \mathcal{D}'), \quad (24)$$

and the wrapping numbers,

$$\mathfrak{N}_\alpha = \int_{\mathcal{D}^+} \tilde{\omega}_\alpha, \quad \mathfrak{N}^a = \int_{\mathcal{D}^-} \tilde{\omega}^a, \quad (25)$$

where $\tilde{\omega}$ are 4-forms satisfying

$$\int \omega^\alpha \wedge \tilde{\omega}_\beta = \delta_\beta^\alpha, \quad \int \omega_a \wedge \tilde{\omega}^b = \delta_a^b, \quad (26)$$

in which the integral is taken over the Calabi-Yau. The gauge kinetic function for the D7-brane gauge theory is then [17,18,22]

$$\mathfrak{f}_{\text{D7}} = \mathfrak{N}_\alpha \left[T^\alpha + i \kappa_{bc}^a \left(G^b F^c + \frac{\tau}{2} F^b F^c \right) \right]. \quad (27)$$

The contribution to the superpotential for such a stack of magnetized D7-branes is $A e^{-2\pi \mathfrak{f}_{\text{D7}}/N}$ for an $SU(N)$ gauge theory. Including such magnetization thus breaks the continuous shift symmetry of c^a to a discrete shift symmetry at the level of the superpotential. The shift symmetry for b^a is badly broken by the appearance of $b^a b^a$ in the real part of the Kähler moduli (17).

The magnetization also contributes to the D-term for the D7-brane gauge theory [18,22]

$$D_{\text{D7}} = \frac{\alpha' t_\alpha}{2\mathcal{V}} \kappa_{bc}^\alpha (b^b - F^b) \mathfrak{N}^c. \quad (28)$$

In general, the D-term receives contributions from the matter fields living on the D7-branes, but, as with the complex structure moduli and the axiodilaton, we will assume that they have been stabilized at a very high scale by closed-string flux.

IV. LARGE DECAY CONSTANTS FROM MAGNETIZED BRANES

We will consider two different, though closely related, mechanisms by which large axion decay constants can be realized via an implementation of the alignment scenario reviewed in Sec. II. In this section, we will show how such an alignment can be arranged by the magnetization of homologous D7-branes.⁶

As discussed in the previous section, in the presence of condensing magnetized branes, g^a and c^a enjoy discrete shift symmetries, while τ^a and b^a do not. Therefore, g^a and c^a are candidates for natural inflation. Unless we allow the D7-branes to have multiple windings on the 4-cycles (as we do in the next section), we will not have the freedom to arrange for the near alignment of decay constants for the even axions g^a . We will therefore focus first on the odd axions c^a .⁷

In order to obtain alignment in the odd sector, we need $h_{\perp}^{1,1} \geq 2$. For simplicity, we will assume that the overall

⁶The wrapping of homologous but distinct cycles was utilized in related constructions of axion monodromy inflation [20].

⁷See, for example, [20,23–25] for other models where these particular axions drive inflation.

volume of the Calabi-Yau takes the “strong Swiss cheese” form,

$$\mathcal{V} = (\tau^1)^{3/2} - \gamma(\tau^2)^{3/2}. \quad (29)$$

We will further assume that $h_{-}^{1,1} = 2$ and that the odd cycles intersect only with $\mathcal{D}^2$, the volume of which is controlled by τ^2 . (In fact our construction allows for intersections with other cycles, as long as these cycles are not wrapped by branes that would contribute to the superpotential for c^a .)

The other crucial ingredient in the alignment scenario is the existence of two nonperturbative potentials coupling to different combinations of the axions c^a . Using the results of the previous section, this can be arranged by taking two stacks of magnetized D7-branes wrapping distinct representatives of $\mathcal{D}^2$. Configurations of this form can arise if the cycle $\mathcal{D}^2$ is not rigid, so that the D7-branes have deformation moduli in the absence of flux, but are stabilized by flux on distinct representatives. An alternative possibility is that at the geometric level, before the inclusion of flux, $\mathcal{D}^2$ has more than one isolated locally volume-minimizing representative.

By an appropriate choice of world volume flux on the two stacks, we will be able to arrange for approximate alignment of the decay constants, as the gauge kinetic function for the magnetized D7-brane will depend linearly on the odd moduli (27). However, $\hat{\vartheta}^\alpha = \text{Im}T^\alpha$ remains an unstabilized axion that appears in the same nonperturbative effect. Since the appearance of $\hat{\vartheta}^\alpha$ in the gauge kinetic function is independent of the magnetization, it is difficult to utilize the C_4 axions in the scheme of decay constant alignment via magnetization. We therefore consider a third stack, wrapping a third distinct representative (see Fig. 1). This unmagnetized stack will allow us to stabilize the τ^α , $\hat{\vartheta}^\alpha$, and b^a . In addition, b^a and τ^α will receive contributions to their potentials from D-terms (28). These ingredients can be used together to stabilize the saxions τ^α and b^a , as well as the additional axion $\hat{\vartheta}^\alpha$, at a higher scale than c^a .

With these ingredients in hand, the nonperturbative superpotential from gaugino condensation on the three different stacks takes the form

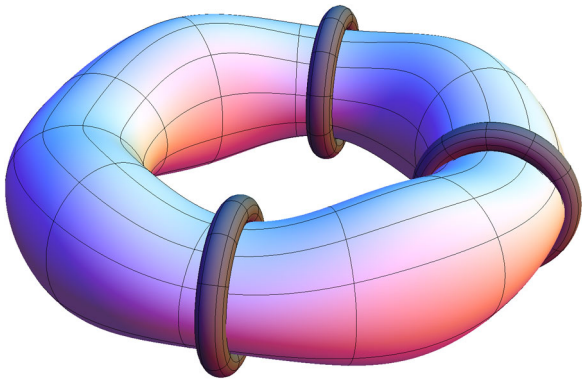


FIG. 1 (color online). Three branes wrapping locally volume minimizing representatives of the same homology class.

$$W_{\text{np}} = \sum_{\xi=A,B,C} A_\xi e^{-\frac{2\pi}{N_\xi} \mathfrak{f}_\xi}, \quad (30)$$

in which

$$\mathfrak{f}_\xi = T^2 + i\kappa^2_{bc} F_\xi^b \left(G^c + \frac{\tau}{2} F_\xi^c \right), \quad (31)$$

where F_ξ^a are the world volume flux quanta (23). The unmagnetized stack has $F_C^a = 0$ and so $\mathfrak{f}_C = T^2$. These nonperturbative superpotential terms supplement the classical flux contribution [26]

$$W_0 = \int G_3 \wedge \Omega. \quad (32)$$

The F -term contribution to the scalar potential is

$$V_F = e^K (K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2), \quad (33)$$

in which $D_i W = W_{,i} + K_{,i} W$ is the Kähler-covariant derivative of the superpotential, $K^{i\bar{j}}$ is the inverse of the Kähler metric $K_{i\bar{j}} = K_{,i\bar{j}}$, and the sum in (33) runs over all chiral fields. The complex structure moduli z are assumed to be supersymmetrically stabilized at a high scale and so we take $D_z W = 0$. We assume also that τ^α , ϑ^α , and b^a are all stabilized by D terms and by the unmagnetized condensate. We further assume that $b^a = 0$ at the minimum, so that the Kähler metric for the even and odd moduli becomes block diagonal, $K_{\alpha\bar{a}} = 0$. Using the no-scale result (which applies even when $b^a \neq 0$),

$$K^{I\bar{J}} K_{I\bar{J}} = 3, \quad (34)$$

where the sum is over all Kähler moduli, and assuming that W_0 is large compared to W_{np} , the scalar potential takes the form

$$V = -\frac{2}{\mathcal{V}^2} (\tau^\alpha \bar{W}_0 \partial_\alpha W_{\text{np}} + \text{c.c.}) + V_{\text{uplift}}, \quad (35)$$

in which we have included an uplift potential coming from ingredients such as anti-D3-branes or frustrated D terms. By an appropriate tuning of V_{uplift} , the potential for the c^a axions takes the form necessary for decay constant alignment (4)

$$V = \sum_\xi \Lambda_\xi^4 \left[1 - \cos \left(\frac{2\pi}{N_\xi} \kappa^2_{bc} F_\xi^b c^c \right) \right], \quad (36)$$

in which

$$\Lambda_\xi^4 = \frac{8\pi |W_0| A_\xi}{\mathcal{V}^2 N_\xi} \tau^2 e^{-\frac{2\pi}{N_\xi} (\tau^2 - \frac{1}{2\phi^2} \kappa^2_{bc} F_\xi^b F_\xi^c)}, \quad (37)$$

where we have assumed that $W_0 < 0$ and $A_\xi > 0$ and have set the stabilized axiodilaton to $\tau = \frac{i}{g_s}$.

With b^a stabilized at zero, the metric for the odd moduli is

$$K_{a\bar{b}} = -\frac{g_s}{\mathcal{V}} \kappa^\alpha_{ab} t_\alpha. \quad (38)$$

If we consider an example with $\kappa^2_{bc} = -\delta_{bc}$, then the kinetic term for the odd axions is

$$\mathcal{L}^{\text{kin}} = -\frac{g_s t_2}{\mathcal{V}} \partial_\mu c^a \partial^\mu c^a. \quad (39)$$

The determinant of the Hessian at the minimum is

$$\det \partial_a \partial_b V|_{c=0} = \frac{(f_1^A f_2^B - f_1^B f_2^A)^2}{N_A^2 N_B^2} \frac{2\pi^4 \mathcal{V} \Lambda_A^4 \Lambda_B^4}{t_2 g_s}. \quad (40)$$

The form of (40) exhibits the effect of decay constant alignment (6). If we relax the assumption $\kappa^2_{bc} = -\delta_{bc}$, then the resulting kinetic mixing will generally make the task of arranging a suitably flat potential more delicate.

V. LARGE DECAY CONSTANTS FROM MULTIPLE WINDINGS

In the example of the previous section, we made use of the C_2 axions c^a since their decay constants could be aligned by adjusting the magnetic flux on the D7-branes. While the decay constants for the C_4 axions ϑ^a cannot be altered by magnetization, they can be aligned by winding numbers if we allow multiply-wrapped D7-branes.

To realize alignment for ϑ^a , we need $h_+^{1,1} \geq 2$. For simplicity, we consider examples with $h_-^{1,1} = 0$ (in which case $\hat{\vartheta}^a = \vartheta^a$). We furthermore need two separate condensates to generate two distinct nonperturbative terms in the superpotential. As in the previous section, this can be arranged by taking two stacks of D7-branes, each of which wraps a different representative of the two homology classes. The nonperturbative superpotential realized on these condensates is

$$W_{\text{np}} = A_A e^{-\frac{2\pi}{N_A}(T^1+T^2)} + A_B e^{-\frac{2\pi}{N_B}(T^1+T^2)}. \quad (41)$$

The fact that both T^1 and T^2 appear in the same linear combination eliminates our ability to arrange for near alignment of the decay constants ϑ^a . However, we gain additional freedom in aligning decay constants by allowing the D7-branes to wrap multiple times.⁸ Including such wrapping numbers modifies the nonperturbative superpotential (27) to

⁸A critical assumption is that the separate windings are spaced apart, as illustrated in Fig. 2, so that there are no new light non-Abelian gauge bosons from the multiwrappings. Arranging for spiraling wrappings of this form may be possible given suitable fluxes, as we discuss below.

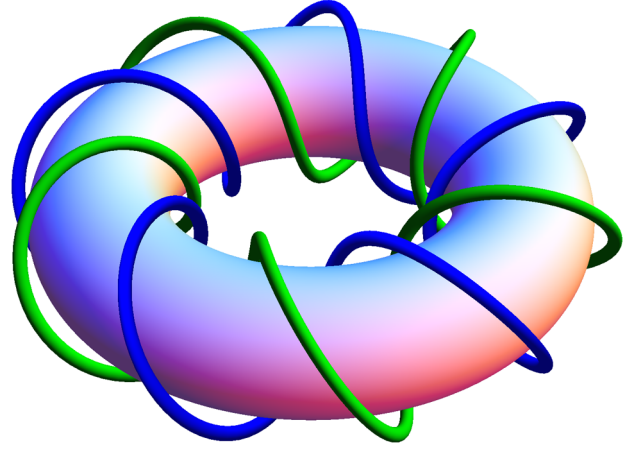


FIG. 2 (color online). Two branes multiply wrapping the same cycle the same number of times.

$$W_{\text{np}} = A_A e^{-\frac{2\pi}{N_A}(\mathbf{s}_1^A T^1 + \mathbf{s}_2^A T^2)} + A_B e^{-\frac{2\pi}{N_B}(\mathbf{s}_1^B T^1 + \mathbf{s}_2^B T^2)}. \quad (42)$$

These multiple windings effectively increase the volume of the D7-branes while keeping the rank of the low-energy gauge group unchanged. Again, by a suitable tuning of V_{uplift} , we have a potential of the form (4)

$$V = \sum_{\xi=a,b} \Lambda_\xi^4 \left\{ 1 - \cos \left[\frac{2\pi}{N_\xi} (\mathbf{s}_1^\xi \vartheta^1 + \mathbf{s}_2^\xi \vartheta^2) \right] \right\}, \quad (43)$$

in which

$$\Lambda_\xi^4 = \frac{8\pi |W_0| A_\xi}{N_\xi \mathcal{V}^2} \left(\mathbf{s}_1^\xi \tau^1 + \mathbf{s}_2^\xi \tau^2 \right) e^{-\frac{2\pi}{N_\xi} (\mathbf{s}_1^\xi \tau^1 + \mathbf{s}_2^\xi \tau^2)}, \quad (44)$$

where we have taken $-W_0$ and A_ξ to be real and positive.

Although the number of D7-branes N_ξ and the winding numbers $\mathbf{s}_\alpha^\xi$ are integers, they provide enough freedom to realize the near alignment of the axion decay constants. However, the story is complicated by the fact that the scalar fields T^a do not have canonical kinetic terms and will be subject to kinetic mixing. As a simple example, we assume that the volume takes the form

$$\mathcal{V} = (\tau^1)^{3/2} - \gamma (\tau^2)^{3/2}, \quad (45)$$

for some $\gamma > 0$. The kinetic term for the axions is then

$$\mathcal{L}^{\text{kin}} = -g_{\alpha\beta} \partial_\mu \vartheta^\alpha \partial^\mu \vartheta^\beta, \quad (46)$$

with

$$g_{ij} = \frac{1}{\mathcal{V}^2} \begin{pmatrix} \frac{6(\tau^1)^{3/2} + 3\gamma(\tau^2)^{3/2}}{8\sqrt{\tau^1}} & -\frac{9\gamma\sqrt{\tau^1\tau^2}}{8} \\ -\frac{9\gamma\sqrt{\tau^1\tau^2}}{8} & \frac{3\gamma(\tau^1)^{3/2} + 6\gamma^2(\tau^2)^{3/2}}{8\sqrt{\tau^2}} \end{pmatrix}. \quad (47)$$

The determinant of the Hessian at the minimum of the potential is (after canonically normalizing the fields)

$$\det \partial_\alpha \partial_\beta V|_{\theta=0} = \frac{(\mathfrak{N}_1^A \mathfrak{N}_2^B - \mathfrak{N}_1^B \mathfrak{N}_2^A)^2}{N_A^2 N_B^2} \frac{128\pi^4 \mathcal{V}^2 \sqrt{\tau^1 \tau^2} \Lambda_B^4 \Lambda_B^4}{9\gamma}. \quad (48)$$

Again, this form exhibits the flat direction that appears upon alignment of the decay constants (6).

VI. EXAMPLES

We can illustrate the parametric success of alignment in these constructions by considering some particular examples. Although we provide only a few toy cases here, the mechanism to arrange for decay constant alignment with our ingredients is more general. Further control could be obtained by using additional stacks and axions to align multiple decay constants as in the field-theoretic treatment of [27].

A. Alignment from multiply-wrapped branes

We first present an example of alignment resulting from multiply-wrapped branes discussed in Sec. V. We consider a strong Swiss cheese Calabi-Yau such that the volume can be written as

$$\mathcal{V} = (\tau^1)^{3/2} - (\tau^2)^{3/2}. \quad (49)$$

We take this toy example just to illustrate the success of alignment, and the scheme will work more generally in real Calabi-Yaus provided that multiple windings can be accommodated. We wrap two stacks of D7-branes with the following data for stack A,

$$N_A = 35, \quad \mathfrak{N}_1^A = 1, \quad \mathfrak{N}_2^A = 19, \quad (50)$$

while for stack B,

$$N_B = 30, \quad \mathfrak{N}_1^B = 1, \quad \mathfrak{N}_2^B = 20. \quad (51)$$

Taking

$$-W_0 = A_A = A_B = .1, \quad (52)$$

and considering the point $\tau^1 = 15$, $\tau^2 = 2$, the masses at the minimum are

$$m_1 = 4 \times 10^{-3} M_{\text{pl}}, \quad m_2 = 4 \times 10^{-6} M_{\text{pl}}. \quad (53)$$

In terms of these mass eigenstates, the potential takes the form

$$\begin{aligned} \frac{V}{M_{\text{pl}}^4} &\approx (1.1 \times 10^{-8}) \\ &- (9.2 \times 10^{-9}) \cos \left[-\frac{37\psi^1}{M_{\text{pl}}} + \frac{0.02\psi^2}{M_{\text{pl}}} \right] \\ &- (1.5 \times 10^{-9}) \cos \left[\frac{46\psi^1}{M_{\text{pl}}} + \frac{.099\psi^2}{M_{\text{pl}}} \right]. \end{aligned} \quad (54)$$

Thus, ψ^1 can be consistently integrated out, and what remains is a potential for ψ^2 that can accommodate natural inflation (see Fig. 3). Note that the mass for ψ^2 in (53) is the appropriate scale for realizing $m^2 \phi^2$ chaotic inflation.

The large ranks of the D7-branes and the relatively small τ^2 also lead to parametric increases in the decay constant, so we should ensure that the enhancements in the decay constants are in fact a result of the alignment and not of the large rank or small 4-cycle volume. To see this, we could repeat the analysis of the above example but remove the possibility for alignment by taking $\mathfrak{N}_1^A = \mathfrak{N}_2^B = 1$ and $\mathfrak{N}_1^B = \mathfrak{N}_2^A = 0$ (note that increasing the winding numbers decreases the decay constants). In this reference case, the potential in the mass eigenbasis is

$$\begin{aligned} \frac{V}{M_{\text{pl}}^4} &\approx (6 \times 10^{-6}) \\ &- (2 \times 10^{-6}) \cos \left[-\frac{1.5\psi^1}{M_{\text{pl}}} + \frac{1.7\psi^2}{M_{\text{pl}}} \right] \\ &- (4 \times 10^{-6}) \cos \left[\frac{2.1\psi^1}{M_{\text{pl}}} + \frac{.6\psi^2}{M_{\text{pl}}} \right]. \end{aligned} \quad (55)$$

Although the decay constants are indeed relatively large, they are only $\mathcal{O}(M_{\text{pl}})$, and natural inflation could not be sustained. Further increasing the number of D7-branes so that the total tension of the D7-branes is comparable to the total tension of the multiply wound branes would increase the decay constants, but would also suppress the dynamical

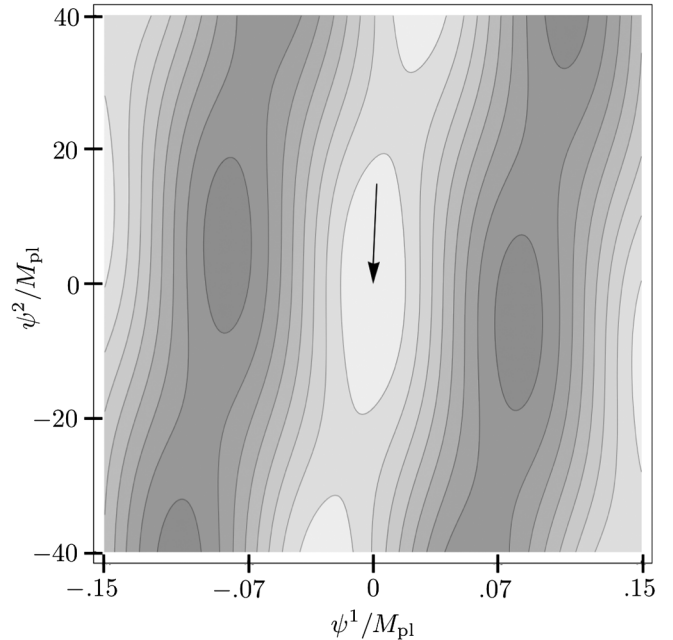


FIG. 3. The 2-axion potential (54). Darker regions correspond to higher points in the potential. Inflation can be accommodated along the super-Planckian trajectory indicated by the arrow. The very slight tilt of the trajectory is a consequence of a very slight shift in the position of the minimum for ψ^1 as ψ^2 evolves.

scales, making it difficult to match the COBE normalization of the scalar power spectrum.

The example above exists at relatively small volume, and α' corrections are of concern. We can move to a point of larger volume, but this acts to suppress the size of the nonperturbative effects, which can be compensated by an increase in the ranks of the condensing gauge groups. For example, if we consider the point $\tau^1 = 25$, $\tau_2 = 10$, $N_A = 100$, $N_B = 90$, $\aleph_1^A = \aleph_1^B = 1$, $\aleph_2^B = 20$, and $\aleph_2^A = 19$ with $-W_0 = 10$, $A_A = A_B = 1$, we find the potential

$$\begin{aligned} \frac{V}{M_{\text{pl}}^4} &\approx (9.5 \times 10^{-8}) \\ &- (8.4 \times 10^{-8}) \cos \left[-\frac{30\psi^1}{M_{\text{pl}}} + \frac{.0065\psi^2}{M_{\text{pl}}} \right] \\ &- (1.1 \times 10^{-8}) \cos \left[-\frac{35\psi^1}{M_{\text{pl}}} + \frac{.04\psi^2}{M_{\text{pl}}} \right]. \end{aligned} \quad (56)$$

This again supports natural inflation. Once again, the large decay constants are not a result of the large ranks, but instead a manifestation of the power of decay constant alignment. Setting $\aleph_1^A = \aleph_2^B = 1$ and $\aleph_1^B = \aleph_2^A = 0$, we again find $\mathcal{O}(M_{\text{pl}})$ decay constants (which are again larger than one expects at generic points, but are not large enough to sustain inflation).

In either of these cases, obtaining realistic inflation using just the nonperturbative superpotentials requires a nonperturbative effect that is not too small. Since the additional windings increase the action of the D7-branes, arranging for alignment quickly pulls down the dynamical scale unless the rank of the gauge groups is increased or the volume of the cycles is shrunk. This is an important constraint on realizing inflation via decay constant alignment that cannot be seen in field theoretic treatments where the decay constants and dynamical scales are independently adjustable. If we are not concerned with realizing the COBE normalization, then we can obtain large decay constants with more moderate volumes and ranks. For example, if we take $\tau^1 = 25$, $\tau_2 = 10$, $N_A = N_B = 15$, $\aleph_1^A = \aleph_1^B = 1$, $\aleph_2^B = 30$, and $\aleph_2^A = 29$ with $-W_0 = A_A = A_B = 1.0$ we find a decay constant of $\sim 5M_{\text{pl}}$ (compared to $.2M_{\text{pl}}$ without the mixing), but the mass of the would-be inflaton is $\mathcal{O}(10^{-31}M_{\text{pl}})$, which is far too small to produce the observed CMB anisotropies. This could still be useful for constructing models of axion monodromy inflation [20,28] in which the shift symmetry of the axion plays an important role, but inflation is driven by explicit breaking terms.

B. Alignment from magnetized branes

As a final example we present a realization of the magnetized brane scenario supporting natural inflation. We write the volume of the Calabi-Yau in a form similar to that of the previous examples (49),

$$\mathcal{V} = \frac{1}{6}(t_1^3 - t_2^3), \quad (57)$$

where t_1 and t_2 control the volumes of 2-cycles and are constrained to be positive. The D7-branes are taken to wrap a divisor whose volume is $\tau^2 = \frac{1}{2}t_2^2$. For simplicity, we assume that the only nonvanishing even-odd-odd intersection numbers are of the form

$$\kappa_{ab}^2 = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}.$$

As in the previous case, the intersection numbers that we have chosen in this example are just for illustrative purposes and the mechanism will work more generally. We wrap two stacks of magnetized D7-branes, with the following data for stack A,

$$N_A = 35, \quad F_1^A = 1, \quad F_2^A = 6, \quad (58)$$

while for stack B,

$$N_B = 40, \quad F_1^B = 1, \quad F_2^B = 7. \quad (59)$$

Taking

$$-W_0 = A_A = A_B = 1, \quad g_s = 0.5, \quad (60)$$

and considering the point $t_1 = 7$, $t_2 = 3$, the masses at the minimum are

$$m_1 = 4 \times 10^{-4}M_{\text{pl}}, \quad m_2 = 2 \times 10^{-6}M_{\text{pl}}. \quad (61)$$

In terms of these mass eigenstates, the potential takes the form

$$\begin{aligned} \frac{V}{M_{\text{pl}}^4} &\approx (8.3 \times 10^{-9}) \\ &- (7.6 \times 10^{-9}) \cos \left[\frac{4\psi^1}{M_{\text{pl}}} + \frac{0.006\psi^2}{M_{\text{pl}}} \right] \\ &- (6 \times 10^{-10}) \cos \left[-\frac{4\psi^1}{M_{\text{pl}}} + \frac{.066\psi^2}{M_{\text{pl}}} \right], \end{aligned} \quad (62)$$

which again supports natural inflation.

VII. CONCLUSIONS

In this paper, we have presented an embedding into type IIB string theory of the field-theoretic axion decay constant alignment mechanism proposed by Kim, Nilles, and Peloso [8]. Our primary tool is gaugino condensation on multiple stacks of D7-branes wrapping homologous cycles in a Calabi-Yau orientifold. When the branes are magnetized, gaugino condensation leads to nonperturbative superpotentials that give the leading breaking of the shift symmetry

of the C_2 axions. Approximate alignment of the decay constants can then be achieved by an appropriate choice of the magnetic fluxes. Alternatively, if the D7-branes are multiply wound, their couplings to C_4 axions can be aligned by adjusting the winding numbers.

Although our constructions require only ingredients that are commonplace in stabilized flux vacua of type IIB string theory compactified on O3/O7 orientifolds of Calabi-Yau manifolds, further care must be taken to ensure compatibility of moduli stabilization with axion alignment. Consistently stabilizing all moduli, leading to a fully realized model of large-field inflation in string theory, remains a significant challenge in our construction, just as in all alternative scenarios for inflation in string theory. In the closed-string sector, this may be particularly delicate in the magnetized case of Sec. IV where the B_2 saxions b^a must have small VEVs to ensure that mixing between the even and odd moduli can be neglected and that the dynamical scales (which are exponentially sensitive to $b^a b^b/g_s$) are not too suppressed. Although the case of winding branes of Sec. V does not suffer from such a dramatic saxion problem (though of course the 4-cycle volumes must be stabilized), it may be difficult to arrange for the required winding numbers without the D7-branes intersecting themselves or each other. Such intersections will introduce additional vector-like matter fields that must be made massive in order for the nonperturbative effects that our constructions invoke to be present. Even if such intersections can be avoided, finding volume-minimizing cycles allowing for multiple windings, or fluxes that stabilize such windings, may be difficult. However, once arranged, the masses of the D7-brane moduli will be comparable to the masses of the complex structure moduli and so these fields will be inert during inflation. An additional constraint is that the D7-brane charge (and induced lower brane charge in the magnetized case) must be canceled. Although such cancellation can be achieved by orientifold planes, it requires more detailed constructions than those that we provide here.

Finally, in schemes of decay constant alignment, there is some tension between the arrangement of a large effective decay constant and the scale needed to match the normalization of the scalar power spectrum. This can be seen in (4),

where although the decay constant is indeed enhanced by the misalignment δ , it is at a cost of a decreased inflaton mass. In string-theoretic implementations, this problem is exacerbated as the dynamical scale will often decrease exponentially as the misalignment is obtained. This tension is not a fatal flaw of our proposal, and indeed it is easy to find examples with properly normalized scalar power spectra.

Some of these difficulties can be ameliorated by combining our scheme with other recent proposals for producing large effective decay constants. In particular, in [27] it was shown that the success of decay constant alignment in field-theoretic models can be extended by increasing the number of axions that mix in the scalar potential. Chaining together alignment effects would be useful in our construction, because although alignment does provide for a parametric enhancement, the points in moduli space that exhibit natural inflation are often near the edge of control. Using a combination of alignments would allow us to obtain parametrically large axion decay constants within a region of robust control. In addition, it was recently demonstrated [30] that the kinematic extension of field range resulting from the combination of many axions *without* aligned decay constants can be more efficient than suggested previously in the N-flation literature [29]. The kinetic alignment effect of [30] in concert with decay constant alignment may be particularly powerful. Moreover, one could use axion alignment to build a broader range of scenarios for axion monodromy [20,28]: see in particular the two-axion monodromy constructions of [24].

Constructing a fully stabilized compactification that implements our proposal is an important problem for the future.

ACKNOWLEDGMENTS

We thank Thomas Bachlechner, John Stout, and Timm Wrase for related discussions. We are also grateful to Ido Ben-Dayan, Francisco Pedro, and Alexander Westphal for sharing a draft of their forthcoming related work. We are especially indebted to Julia Goodrich for technical assistance. This work was supported by NSF Grant No. PHY-0757868.

-
- [1] G. Hinshaw *et al.* (WMAP), *Astrophys. J. Suppl. Ser.* **208**, 19 (2013); P. A. R. Ade *et al.* (Planck Collaboration), arXiv:1303.5082.
 - [2] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981); A. D. Linde, *Phys. Lett.* **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
 - [3] P. A. R. Ade *et al.* (BICEP2 Collaboration), *Phys. Rev. Lett.* **112**, 241101 (2014).
 - [4] D. H. Lyth, *Phys. Rev. Lett.* **78**, 1861 (1997).
 - [5] D. Baumann and D. Green, *J. Cosmol. Astropart. Phys.* **05** (2012) 017.
 - [6] D. Baumann and L. McAllister, arXiv:1404.2601.

- [7] K. Freese, J. A. Frieman, and A. V. Olinto, *Phys. Rev. Lett.* **65**, 3233 (1990).
- [8] J. E. Kim, H. P. Nilles, and M. Peloso, *J. Cosmol. Astropart. Phys.* **01** (2005) 005.
- [9] R. Kappl, S. Krippendorff, and H. P. Nilles, *arXiv:1404.7127*.
- [10] S. H. H. Tye and S. S. C. Wong, *arXiv:1404.6988*.
- [11] M. Czerny, T. Higaki, and F. Takahashi, *J. High Energy Phys.* **05** (2014) 144; *Phys. Lett. B* **734**, 167 (2014); T. Higaki and F. Takahashi, *arXiv:1404.6923*.
- [12] I. Ben-Dayana, F. Pedro, and A. Westphal (to be published).
- [13] P. Svrček and E. Witten, *J. High Energy Phys.* **06** (2006) 051.
- [14] R. Blumenhagen and E. Plauschinn, *arXiv:1404.3542*; T. W. Grimm, *arXiv:1404.4268*.
- [15] T. Banks, M. Dine, P. J. Fox, and E. Gorbatov, *J. Cosmol. Astropart. Phys.* **06** (2003) 001.
- [16] T. W. Grimm and J. Louis, *Nucl. Phys.* **B699**, 387 (2004).
- [17] H. Jockers and J. Louis, *Nucl. Phys.* **B705**, 167 (2005); H. Jockers, *Fortschr. Phys.* **53**, 1087 (2005).
- [18] H. Jockers and J. Louis, *Nucl. Phys.* **B718**, 203 (2005).
- [19] A. R. Frey, G. Torroba, B. Underwood, and M. R. Douglas, *J. High Energy Phys.* **01** (2009) 036; F. Marchesano, P. McGuirk, and G. Shiu, *J. High Energy Phys.* **04** (2009) 095; L. Martucci, *J. High Energy Phys.* **05** (2009) 027.
- [20] L. McAllister, E. Silverstein, and A. Westphal, *Phys. Rev. D* **82**, 046003 (2010).
- [21] R. Flauger, L. McAllister, E. Pajer, A. Westphal, and G. Xu, *J. Cosmol. Astropart. Phys.* **06** (2010) 009.
- [22] T. W. Grimm, M. Kerstan, E. Palti, and T. Weigand, *Phys. Rev. D* **84**, 066001 (2011).
- [23] T. W. Grimm, *Phys. Rev. D* **77**, 126007 (2008).
- [24] M. Berg, E. Pajer, and S. Sjors, *Phys. Rev. D* **81**, 103535 (2010).
- [25] I. Ben-Dayana, F. G. Pedro, and A. Westphal, *arXiv:1404.7773*.
- [26] S. Gukov, C. Vafa, and E. Witten, *Nucl. Phys.* **B584**, 69 (2000).
- [27] K. Choi, H. Kim, and S. Yun, *arXiv:1404.6209*.
- [28] E. Silverstein and A. Westphal, *Phys. Rev. D* **78**, 106003 (2008).
- [29] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker, *J. Cosmol. Astropart. Phys.* **08** (2008) 003.
- [30] T. C. Bachlechner, M. Dias, J. Frazer, and L. McAllister, *arXiv:1404.7496*.