PHYSICAL REVIEW D **90**, 023011 (2014)

Nonlinear vacuum electrodynamics birefringence effect in a pulsar's strong magnetic field

Victor I. Denisov and Vladimir A. Sokolov[*]
Physics Department, Moscow State University, Moscow 119991, Russia

Mikhail I. Vasili'ev
Tsiolkovskii Russian State Technological University, Orshanskaya Street, 3, Moscow 121552, Russia
(Received 3 April 2014; revised manuscript received 14 June 2014; published 11 July 2014)

The motion of x-ray and gamma-ray pulses in the magnetic field of the pulsar, taking into account the nonlinearity of electrodynamics in a vacuum, is calculated. It is shown that in these conditions, each pulse propagates in the form of two orthogonally polarized normal waves with different speeds. The propagation time from the common source in the pulsars' magnetosphere to a remote observer is calculated for each mode. An experiment to measure the delay time of one pulse with respect to the other is proposed.

DOI: 10.1103/PhysRevD.90.023011 PACS numbers: 97.60.Gb, 11.10.Lm, 95.30.Gv

I. INTRODUCTION

The birefringent effect is observed in crystal optics and consists in the splitting of one electromagnetic wave on two normal modes with the orthogonal polarization which propagate with the different velocities. It was shown in [1–8] that the same effect can take place without any media for the wave propagating in a strong magnetic field in vacuum.

The classical electromagnetic field theory, the Maxwell electrodynamics, does not show the ability of vacuum birefringence. At the same time the quantum field theory [1] predicts that the vacuum electrodynamics should have nonlinear corrections in the classical limit. For the parametrized post-Maxwellian approximation [9], [10] if the value of the electromagnetic field's magnitudes reaches Schwinger's critical field $B_q = 4.41 \times 10^{13}$ G, then the Lagrangian of the effective nonlinear theory may be written as

$$L = \frac{\sqrt{-g}}{32\pi}\{2I_2 + \xi[(\eta_1 - 2\eta_2)I_2^2 + 4\eta_2 I_4]\} - \frac{1}{c}j^m A_m, \tag{1}$$

where $\xi = 1/B_q^2$, $I_2 = F_{ik}F^{ki}$, $I_4 = F_{ik}F^{km}F_{mn}F^{ni}$ are the electromagnetic field tensor invariants. In Minkowski space-time these invariants take the forms $I_2 = 2(\mathbf{E}^2 - \mathbf{B}^2)$ and $I_4 = 2(\mathbf{E}^2 - \mathbf{B}^2)^2 + 4(\mathbf{EB})^2$; the value of dimensionless post-Maxwellian parameters η_1 and η_2 depends on the nonlinear vacuum electrodynamics model choice. According to QED, these parameters have values $\eta_1 = e^2/(45\pi\hbar c)$, $\eta_2 = 7e^2/(180\pi\hbar c)$, where α is the fine structure constant.

Strictly speaking this Lagrangian is valid for the constant and homogeneous electromagnetic field. A typical pulsar's electromagnetic field has a long-scale variation and change on distances much greater then the Compton wavelength. On this consideration expression (1) may be used with high accuracy [11].

There were not convincing experimental evidences for the nonlinear vacuum electrodynamics for a long time. The first experimental results proving the vacuum nonlinearity were given by the Stanford Collaboration in 1997. The experimental technic used by the authors [12] was based on the detecting of the positrons originating from vacuum pairs production. The main problem for the vacuum nonlinearity investigation consists in the creation of the electromagnetic field sources with the fields comparable to Schwinger's limit B_q. Such strong fields may be obtained in the terrestrial conditions at the small volumes and mainly in particle experiments (experiments with particles).

There are several propositions [13–23] for the vacuum nonlinear electrodynamics experiments in the terrestrial laboratory at the macroscopic conditions. But all predictions for nonlinearity resulting effects are extremely small and the detection of the effects based on nonlinear corrections to the Maxwell equations lies on the bounds of possibilities of teh contemporary experimental technic.

At the same time, the investigation of vacuum nonlinear electrodynamics basic features may be performed more effectively with the help of the natural astrophysical strong electromagnetic field sources. Such astrophysical compact objects like [24] pulsars have magnetic-dipole fields with the magnitude in ranges of $B \sim 10^{13}$ G and spread to the large distances which are comparable with radius the several pulsars. Of course, the creation of such a strong magnetic field in so large a volume is unattainable at the terrestrial conditions. The other class of recently discovered compact astrophysical objects is the magnetars which have even more stronger magnetic fields comparable to $B \sim 10^{16}$ G. All these features of the pulsars and magnetars

[*]sokolov.sev@inbox.ru

give the unique opportunities for the vacuum nonlinear electrodynamics astrophysical tests [25–27].

The electromagnetic radiation is the main data source of nonlinear vacuum electrodynamics effects which take place in the magnetic-dipole field near the compact astrophysical objects. Because of the magnetosphere of the neutron stars, these effects are observable only in gamma and x-ray pulsars and magnetars radiation bands for which the magnetosphere is transparent. It is well known [28] that the accretion is one of the general mechanisms of neutron stars' gamma and x-ray radiation. A polar cape accretion model assumes that the substance from the accretion disc falls down across the lines of the stars' magnetic-dipole field to the area near the magnetic poles. It provokes the heating of this area which radiates the electromagnetic waves with spectral maximums in gamma and x-ray bands. Electromagnetic pulses emitting from this area are not polarized. This is the reason why the polar capes are the sources of the intensive hard radiation coming from the pulsars and magnetars. As the radiation has the thermal nature, the rays have an arbitrary orientation to the star's magnetic field induction vector.

The other area of the pulsar magnetosphere from where unpolarized pulses of hard emission originate is called the outer gap. Let us denote the radius of the neutron star originating the pulsar through R_0. The outer gap is located, as usual, at the distance of about $1.5R_0$–$10R_0$ above the surface of the neutron star. In particular, such pulsars like the Crab pulsar generate pulses of hard emission mainly in the outer gap.

The contemporary investigations of the gamma and x-ray radiation coming from the pulsars and magnetars are actively made [29] with the help of the equipment installed on the spacecrafts.

The x-ray and gamma pulses propagating through the pulsars' or magnetars' strong magnetic and gravitational fields undergo the nonlinear electrodynamics and gravitational influence of these fields and carry out the information about this complicated interaction. So the investigation of the gamma and x-ray radiation coming from the surrounding pulsars and magnetars may give information about the main features of the nonlinear electromagnetic field interaction. The possibilities of this approach are described, in this work, in more detail below.

II. RAY EQUATIONS IN EFFECTIVE SPACE-TIME

Let us consider a pulsar with the radius R_0 and the magnetic-dipole moment $\mathbf{m}$ placed in the origin of the cylindrical reference frame. The gamma or x-ray pulse was emitted at the time moment $t = 0$ from the point $\mathbf{r}_0 = \{r_0, \varphi_0, z_0\}$ nearby the pulsar. The z axle of the cylindrical reference frame is directed so that the gamma radiation detector's radius vector gets components $\mathbf{r}_1 = \{r_0, \varphi_0, z_1\}$. Only the rays which do not intersect the star's surface are taken into account: $\sqrt{r_0^2 + z_0^2} > R_0$.

The ray trajectories along which the gamma and x-ray pulses propagate from the emission point $\mathbf{r}_0$ to the detector position $\mathbf{r}_1$ are found out with the help of the methods developed in [9,30–33].

The equations of the electrodynamics with the Lagrangian (1) are nonlinear; therefore, their solution may be performed more effectively by the application of the eiconal method specially adopted for such kinds of problems. It was shown in [3,31,32] that the eiconal equation for the electromagnetic wave in the parametrized post-Maxwellian electrodynamics (1) depends on the wave polarization and may be written in form

$$g_{(1)}^{nk}\left(\frac{\partial S}{\partial x^n}\frac{\partial S}{\partial x^k}\right)g_{(2)}^{pm}\left(\frac{\partial S}{\partial x^p}\frac{\partial S}{\partial x^m}\right) = 0, \tag{2}$$

where the effective space-time metric tensor $g_{nm}^{(1,2)}$, along whose geodesic lines the normal waves propagate, depends on the metric tensor $g_{nm}^{(0)}$ describing the gravitational interaction and squared combination of the external electromagnetic field tensor F_{nm}:

$$g_{nm}^{(1,2)} = g_{nm}^{(0)} - 4\eta_{(1,2)}\xi F_{ni}F^{i\cdot}_{\cdot m}. \tag{3}$$

It is important to note that the indexes rising for field tensor F_{ik} should be performed by using the metric tensor $g^{(0)ik}$ in all of these equations.

The general equations (2) are not easy for the application to our problem and it is more effective to rewrite them in a different form using the Lagrange-Sharpie method [34]:

$$\frac{dk^n}{d\sigma} + \Gamma_{mp}^n k^m k^p = 0, \tag{4}$$

where σ, the affine parameter along the ray line, $k^m = dx^m/d\sigma$, the wave four-vector and Christophel symbols, Γ_{mp}^n, which are constructed with the help of the metric tensor $g_{nm}^{(1)}$ for the first normal wave mode and with the $g_{nm}^{(2)}$ for the second mode.

The massless condition for the photon which is a consequence of Eq. (4) should be taken into account as an addition to these equations:

$$g_{nm}^{(1,2)}k^n k^m = 0. \tag{5}$$

Pulsar's gravitational field may be estimated with dimensionless potential $U = GM/(c^2R_0) \sim 10^{-2}$ which is more than 4 orders of magnitude and exceeds the potential on the Sun's surface. That is why electromagnetic pulse propagation in the pulsar magnetosphere will take influence both from nonlinear electrodynamics and gravitation. It was shown in numerous calculations [35–37] that in the field of the gravitating center, the light ray is bending and its propagation law does not depend on light polarization. Since, first of all, we are interested in effects depending on electromagnetic wave polarization, for the simplification of the resulting equations, we will neglect all additive terms derived from the gravitational action on wave propagation. Then $g^{(0)}_{nm} = \mathrm{diag}\{1, -1, -r^2, -1\}$.

In the nearest pulsars' and magnetars' neighborhood ($\sqrt{r^2 + z^2} < 10R_0$), the estimate $\eta_{1,2}\xi\mathbf{B}^2(r, z) \sim 10^{-5}$ can be made so the following calculations will be written only with the terms proportional to $\eta_{1,2}\xi\mathbf{B}^2(r, z)$ in Eqs. (4) and (5).

The pulsar's magnetic-dipole moment in cylindrical coordinates has only two components $\mathbf{m} = \{m_r, 0, m_z\}$ which are used for the calculation of the electromagnetic field tensor F_{ik}. Nonvanishing components of this tensor for the pulsars magnetic field calculated with the appropriate accuracy in cylindrical coordinates are

$$
\begin{aligned}
F_{13} &= -F_{31} = \frac{m_r \sin(\varphi - \psi)}{\sqrt{[r^2 + z^2]^3}},\\
F_{21} &= -F_{12} = \frac{r[3rzm_r \cos(\varphi - \psi) - (r^2 - 2z^2)m_z]}{\sqrt{[r^2 + z^2]^5}},\\
F_{32} &= -F_{23} = \frac{r[3rzm_z - (z^2 - 2r^2)m_r \cos(\varphi - \psi)]}{\sqrt{[r^2 + z^2]^5}},
\end{aligned} \tag{6}
$$

where ψ is the polar angle of $\mathbf{m}$.

Pulsars are rotated around the axle not coinciding with their magnetic-dipole moment direction. The rotating period for the most pulsars is much greater then the time required for the electromagnetic pulse to propagate on a distance equal to several pulsars' radius, where the magnetic field is much smaller than on the pulsar's surface and so where the action of nonlinear electrodynamics may be omitted. That is why angle ψ and components m_z, m_r are so slowly varying functions with the time, so that in future calculations their dependence on time may be neglected.

In accordance with Eqs. (3) and (6), the nonvanishing components of the effective space-time metric tensor $g^{(1,2)}_{nm}$ take the following form:

$$
\begin{aligned}
g^{(1,2)}_{00} &= 1,\\
g^{(1,2)}_{11} &= -1 - 4\xi\eta_{1,2}\frac{(r^2F^2_{13} + F^2_{12})}{r^2},\\
g^{(1,2)}_{22} &= -r^2 - 4\xi\eta_{1,2}[F^2_{12} + F^2_{23}],\\
g^{(1,2)}_{12} &= -4\xi\eta_{1,2}F_{13}F_{23},\\
g^{(1,2)}_{13} &= \frac{4\xi\eta_{1,2}F_{12}F_{23}}{r^2},\\
g^{(1,2)}_{23} &= -4\xi\eta_{1,2}F_{12}F_{13},\\
g^{(1,2)}_{33} &= -1 - 4\xi\eta_{1,2}\frac{(r^2F^2_{13} + F^2_{23})}{r^2}.
\end{aligned} \tag{7}
$$

So, the affine parameter σ changes in the coordinate z in Eqs. (4) and (5) with the help of the differential relation $d/d\sigma = k^3 d/dz$. Then this equations take the following form:

$$
\begin{aligned}
\frac{d^2ct}{dz^2} + \left\{\Gamma^0_{ni} - \frac{dct}{dz}\Gamma^3_{ni}\right\}\frac{dx^i}{dz}\frac{dx^n}{dz} &= 0,\\
\frac{d^2r}{dz^2} + \left\{\Gamma^1_{ni} - \frac{dr}{dz}\Gamma^3_{ni}\right\}\frac{dx^i}{dz}\frac{dx^n}{dz} &= 0,\\
\frac{d^2\varphi}{dz^2} + \left\{\Gamma^2_{ni} - \frac{d\varphi}{dz}\Gamma^3_{ni}\right\}\frac{dx^i}{dz}\frac{dx^n}{dz} &= 0,\\
g^{(1,2)}_{ip}\frac{dx^i}{dz}\frac{dx^p}{dz} &= 0.
\end{aligned} \tag{8}
$$

Obtained equations may be used for describing photon trajectories in effective space-time.

III. PHOTON TRAJECTORIES IN THE MAGNETIC FIELD OF A PULSAR

The solution for Eq. (8) may be found with the help of the successive approximation method. On this approach r, φ, and ct are decomposed in small parameter powers, holding on only to the first nonvanishing terms with the respect of nonlinear vacuum electrodynamics corrections:

$$
\begin{aligned}
ct &= ct_0(z) + \eta_{1,2}\xi ct_1(z),\\
r &= r_0(z) + \eta_{1,2}\xi r_1(z),\\
\varphi &= \varphi_0(z) + \eta_{1,2}\xi\varphi_1(z).
\end{aligned}
$$

The integration constants for Eq. (8) may be found from the initial and bound conditions for the ray starting at the moment $t = 0$ in the point $\mathbf{r}_0 = \{r_0, \varphi_0, z_0\}$ and transiting through the gamma radiation detectors which are located in point $\mathbf{r}_1 = \{r_0, \varphi_0, z_1\}$,. These initial conditions give several relations for the integration constants' definition in any successive approximation:

$$
\begin{aligned}
&r_0(z_0) = r_0(z_1) = r_0, \qquad \varphi_0(z_0) = \varphi_0(z_1) = \varphi_0,\\
&t_0(z_0) = t_1(z_0) = r_1(z_0) = r_1(z_1) = \varphi_1(z_0) = \varphi_1(z_1) = 0.
\end{aligned} \tag{9}
$$

These equations in the initial approximation take a form:

$$
\begin{aligned}
&\frac{d^2ct_0(z)}{dz^2} = 0, \qquad \frac{d^2r_0(z)}{dz^2} = r\left(\frac{d\varphi_0(z)}{dz}\right)^2,\\
&\frac{d^2\varphi_0(z)}{dz^2} = -\frac{2}{r}\frac{d\varphi_0(z)}{dz}\frac{dr_0(z)}{dz},\\
&\left(\frac{dct_0(z)}{dz}\right)^2 - \left(\frac{dr_0(z)}{dz}\right)^2 - r^2\left(\frac{d\varphi_0(z)}{dz}\right)^2 = 1.
\end{aligned}
$$

The solution for these equations with the initial condition (9) has a form of a straight line passing through the source and detector:

$$
ct_0(z) = z - z_0, \qquad r_0(z) = r_0, \qquad \varphi_0(z) = \varphi_0.
$$

For the next nonlinear electrodynamics approximation Eq. (8) gives

$$\frac{d^2ct_1(z)}{dz^2}=\frac{180r_0^4}{R(z)^{12}}[zm_z^2+2r_0m_rm_z\cos(\varphi_0-\psi)-zm_r^2\cos^2(\varphi_0-\psi)]$$
$$+\frac{12r_0^2}{R(z)^{10}}[8zm_r^2\cos^2(\varphi_0-\psi)-35r_0m_rm_z\cos(\varphi_0-\psi)-12zm_z^2]+\frac{12m_r}{R(z)^8}[7r_0m_z\cos(\varphi_0-\psi)-zm_r],$$
$$\frac{d^2r_1(z)}{dz^2}=\frac{180r_0^4}{R(z)^{12}}[r_0m_r^2\cos^2(\varphi_0-\psi)+2zm_rm_z\cos(\varphi_0-\psi)-r_0m_z^2]$$
$$+\frac{12r_0^2}{R(z)^{10}}[25r_0m_z^2-31zm_rm_z\cos(\varphi_0-\psi)-21r_0m_r^2\cos^2(\varphi_0-\psi)]$$
$$+\frac{12}{R(z)^8}[5r_0m_r^2\cos^2(\varphi_0-\psi)+3zm_rm_z\cos(\varphi_0-\psi)-r_0m_r^2-11r_0m_z^2],$$
$$\frac{d^2\varphi_1(z)}{dz^2}=\frac{30r_0m_r}{R(z)^{10}}[2zm_z\sin(\varphi_0-\psi)+r_0m_r\sin 2(\varphi_0-\psi)]-\frac{6m_r}{R(z)^8r_0}[6zm_z\sin(\varphi_0-\psi)+5r_0m_r\sin 2(\varphi_0-\psi)].$$
$$\frac{18r_0^3}{R(z)^{10}}[r_0m_z^2-2zm_rm_z\cos(\varphi_0-\psi)-r_0m_r^2\cos^2(\varphi_0-\psi)]$$
$$+\frac{6r_0}{R(z)^8}[2r_0m_r^2\cos^2(\varphi_0-\psi)+2zm_rm_z\cos(\varphi_0-\psi)-3r_0m_z^2]-\frac{2m_r^2}{R(z)^6}+\frac{dct_1(z)}{dz}=0,$$

where the following notation is used: $R(z)=\sqrt{z^2+r_0^2}$.

Solving these equations with the account of the initial conditions (9), after elementary but tedious calculations, the relation for $ct=ct(z)$ is finally received:

$$ct=z-z_0+\eta_{1,2}\xi\bigg\{\frac{9r_0^2}{4(z_0^2+r_0^2)^4}[z_0m_z^2-z_0m_r^2\cos^2(\varphi_0-\psi)+2r_0m_zm_r\cos(\varphi_0-\psi)]$$
$$-\frac{1}{8(z_0^2+r_0^2)^3}[3z_0m_z^2+5z_0m_r^2\cos^2(\varphi_0-\psi)+16r_0m_zm_r\cos(\varphi_0-\psi)]$$
$$-\left[\frac{z_0(5r_0^2+3z_0^2)}{64r_0^4(z_0^2+r_0^2)^2}+\frac{3}{64r_0^5}\,\mathrm{arctg}\left(\frac{z_0}{r_0}\right)\right][16m_r^2+15m_z^2+25m_r^2\cos(\varphi_0-\psi)]$$
$$-\frac{9r_0^2}{4R(z)^8}[zm_z^2-zm_r^2\cos^2(\varphi_0-\psi)+2r_0m_zm_r\cos(\varphi_0-\psi)]$$
$$+\frac{1}{8R(z)^6}[3zm_z^2+5zm_r^2\cos^2(\varphi_0-\psi)+16r_0m_zm_r\cos(\varphi_0-\psi)]$$
$$+\left[\frac{z(5r_0^2+3z^2)}{64R(z)^4r_0^4}+\frac{3}{64r_0^5}\,\mathrm{arctg}\left(\frac{z}{r_0}\right)\right][16m_r^2+15m_z^2+25m_r^2\cos^2(\varphi_0-\psi)]\bigg\}. \tag{10}$$

The solutions for $r=r(z)$ may be found to be quite similar:

$$r=r_0+\eta_{1,2}\xi\left\{\frac{(z-z_0)}{(z_1-z_0)}f(z_1)-f(z)-\frac{(z-z_1)}{(z_1-z_0)}f(z_0)\right\}, \tag{11}$$

where for brevity the following notation is used:

$$
\begin{aligned}
f(z) = &\frac{9r_0^2}{4R(z)^8}[2zm_zm_r\cos(\varphi_0-\psi) - m_z^2 r_0 + r_0 m_r^2\cos^2(\varphi_0-\psi)] \\
&+\frac{1}{8R(z)^6}[23r_0m_z^2 - 15r_0m_r^2\cos^2(\varphi_0-\psi) - 20zm_zm_r\cos(\varphi_0-\psi)] \\
&-\frac{1}{32R(z)^4 r_0^2}[25r_0m_r^2\cos^2(\varphi_0-\psi) + 52zm_zm_r\cos(\varphi_0-\psi) + r_0(16m_r^2+15m_z^2)] \\
&-\frac{1}{64R(z)^2 r_0^4}[156zm_zm_r\cos(\varphi_0-\psi) + 125r_0m_r^2\cos^2(\varphi_0-\psi) + r_0(80m_r^2+75m_z^2)] \\
&+\frac{3}{64r_0^6}[80zm_r^2 + 125zm_r^2\cos^2(\varphi_0-\psi) - 52r_0m_zm_r\cos(\varphi_0-\psi) + 75zm_z^2]\,\mathrm{arctg}\left(\frac{z}{r_0}\right).
\end{aligned}
$$

And, finally, we will obtain the relation $\varphi = \varphi(z)$:

$$
\begin{aligned}
\varphi = \varphi_0 + \eta_{1,2}\xi\frac{m_r\sin(\varphi_0-\psi)}{32r_0^7} \\
\times\left\{\Phi(z) + \frac{(z-z_1)}{(z_1-z_0)}\Phi(z_0) - \frac{(z-z_0)}{(z_1-z_0)}\Phi(z_1)\right\},
\end{aligned} \quad (12)
$$

where

$$
\begin{aligned}
\Phi(z) = &\frac{2r_0^4}{R(z)^4}[5r_0m_r\cos(\varphi_0-\psi) - zm_z] \\
&-\frac{40r_0^6}{R(z)^6}[r_0m_r\cos(\varphi_0-\psi) + zm_z] \\
&+\frac{r_0^2}{R(z)^2}[25r_0m_r\cos(\varphi_0-\psi) - 3zm_z] \\
&-3[25zm_r\cos(\varphi_0-\psi) + r_0m_z]\,\mathrm{arctg}\left(\frac{z}{r_0}\right).
\end{aligned}
$$

Equations (10)–(12) give the low of propagation for the first and the second normal modes coming from the common source to the detector in the parametrized form.

IV. DISCUSSION

The expressions $ct = ct(z)$, $r = r(z)$ and $\varphi = \varphi(z)$ give a possibility to analyze the nonlinear electrodynamics interaction between the pulsar's magnetic field and the electromagnetic wave propagating through it. These relations show that the electromagnetic waves rays bend in the magnetic field of the pulsar. The ray bending angles are different [38] for the first and the second normal modes of the wave, when the post-Maxwellian parameters of nonlinear electrodynamics are not equal $\eta_1 \neq \eta_2$.

However, this effect cannot be measured in contemporary experimental conditions as the distance between the Earth and the nearest pulsar $\sim 10^{17}$ km is much greater than the pulsars typical radius.

The other effect coupled to condition $\eta_1 \neq \eta_2$ and originating from the nonlinear interaction between the pulsar's magnetic field and the electromagnetic wave is the difference for the propagation time (10) of the first and the second normal modes coming from the common source to the detector. The time delay ΔT between the moments when the normal modes with the orthogonal polarizations come to the detector is calculated with the help of the expression (10):

$$
\Delta T = \frac{(\eta_2-\eta_1)\xi}{c}ct_1(z_1). \quad (13)
$$

When the nearest pulsar disposes from the Earth at the distance of more than 10 kpc $\approx 3\times 10^{17}$ km, then only the asymptotically main part with $z_1 \to \infty$ should be taken into account. Let us assume that the source of the gamma and x ray is located in the pulsar's magnetosphere at the outer gap.

An analysis of the expression (13) shows that the value of ΔT increases with the decreasing of the z_0 coordinate. Since our calculation is just a kind of exploratory study of what kinds of effects to expect in principle, let us assume that a considerable hard emission pulse was emitted from the point with the coordinate $z_0 = -r_0$ located in the outer gap. As the result of these assumptions from (13), the final expression for the time delay is

$$
\begin{aligned}
\Delta T = &\frac{(\eta_2-\eta_1)\xi}{256cr_0^5} \\
&\times\{(9\pi+8)[16m_r^2 + 15m_z^2 + 25m_r^2\cos^2(\varphi_0-\psi)] \\
&+8[7m_r^2\cos^2(\varphi_0-\psi) - 3m_z^2 + m_zm_r\cos(\varphi_0-\psi)]\}.
\end{aligned} \quad (14)
$$

From this relation it follows that in Heisenberg-Euler electrodynamics, unlike that of Born-Infeld electrodynamics, $\Delta T \neq 0$.

Let us discuss this effect in more detail. For observations of this effect, there are more suitable unpolarized pulses with the duration $\tau > \Delta T$ or pulses with the elliptical polarization. As is well known, there are several sources for the unpolarized hard emission of the pulsar. First of all, it is emission coming from the polar cape and the outer gap. As it follows from the expressions (3)–(5), each pulse emitted in some point $\mathbf{r}_0$ in the pulsar magnetic field will split on two eigenmodes with orthogonal polarization. Propagation of these modes from their common source to a gamma or x-ray detector will occur on different ray lines and with the different velocity will depend on an external magnetic field. The faster mode will arrive to detect on a time ΔT earlier than the slower mode.

That is why polarization of the detected electromagnetic pulse will be linear within time ΔT and will coincide to the faster eigenmode polarization.

After the time ΔT, the slower eigenmode, whose polarization is orthogonal to the faster mode, will reach the detector. As a result of the superposition of both modes, the radiation registered by the detector within time $\tau - \Delta T$ will not be linearly polarized.

Let us estimate the value of ΔT which can be obtained in the most favorable conditions. It is more convenient for this calculation to represent the expression (14) in the following form:

$$\Delta T = \frac{(\eta_2 - \eta_1) r_0}{256c}\left(\frac{B(r_0)}{B_q}\right)^2 \times \{(9\pi + 8)[15 + \sin^2\alpha + 25\sin^2\alpha\cos^2(\varphi_0 - \psi)] + 4[14\sin^2\alpha\cos^2(\varphi_0 - \psi) - 6\cos^2\alpha + \sin 2\alpha\cos(\varphi_0 - \psi)]\}, \quad (15)$$

where $B(r_0)$ is the pulsar's magnetic field at the distance r_0 from its center, α—the angle between the magnetic-dipole moment vector and axis z.

As the pulsar example, the soft gamma repeater SGR 0526-66 may be considered. According to Ref. [39] this pulsar has a radius $R_0 = 10$ km and its hard emission pulse's duration is $\tau \sim 0.15$ s. The magnitude of the magnetic field on its surface was estimated [39] at the level of $B(R_0) \approx 6 \times 10^{14}$ G.

Let us assume that the hard emission source is located in the outer gap of the pulsar and its coordinates are $r_0 = 30$ km and $z_0 = -r_0$. At this case, the ratio of $B(r_0)/B_q$, incoming as a multiplier in the expression (15), may be estimated as $B(r_0)/B_q \sim 0.5 < 1$. Because of the pulsar rotation, the angles α and ψ are slowly varying functions of time. That is why for some electromagnetic pulses these angles may take values for which $\sin^2\alpha = 1$ and $\cos^2(\varphi_0 - \psi) = 1$. For such pulses the eigenmodes' delay (15) will reach the maximum value $\Delta T \approx 10^{-8}$ s.

Advances of contemporary high energy polarimetry reveal new possibilities for the precise measuring of polarization for gamma and x-ray radiation coming from astrophysical sources and it gives an opportunity for measuring the time delay ΔT. A realization of such measurements may be performed in the future at most promising missions such as MEGA [40] and GRIPS [41].

V. CONCLUSION

The time duration of the pulse leading part may be assessed at several nanoseconds and it is sufficient for contemporary registering devices. In cases when the pulsar rotates around the axle not coinciding with the magnetic-dipole moment direction, the polarization plane of the pulse leading part rotates with the same frequency as the pulsar. It is well known that the small magnitudes' effects modulated with the determined frequency are easier for detecting. Therefore, the pulsars with the well-determined rotating frequency are more suitable for the registration of the nonlinear vacuum electrodynamics birefringence in the magnetic field. So describing the above effect of the wave modes' time delay gives good prospects for astrophysical tests of nonlinear vacuum electrodynamics in the extreme conditions of the pulsars' neighborhood, and also permits us to calculate directly the difference $\eta_1 - \eta_2$ of the post-Maxwellian parameters. The time duration of the pulse leading part may be assessed at several nanoseconds and it is sufficient for contemporary registering devices.

[1] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936).
[2] S. L. Adler, J. N. Bahcal, C. G. Callan, and M. N. Rosenbluth, Phys. Rev. Lett. **25**, 1061 (1970).
[3] V. I. Denisov, Theor. Math. Phys. **132**, 1071 (2002).
[4] R. Baier and P. Breitenlohner, Nuovo Cimento B **47**, 117 (1967).
[5] W. Y. Tsai and T. Erber, Phys. Rev. D **12**, 1132 (1975).
[6] T. Erber, Nature (London) **25**, 4770 (1961).
[7] D. B. Melrose and R. J. Stoneham, Nuovo Cimento A **32**, 435 (1976).
[8] A. E. Shabad, Ann. Phys. (N.Y.) **90**, 166 (1975).
[9] V. I. Denisov and I. P. Denisova, Dokl. Phys. **46**, 377 (2001).
[10] V. I. Denisov and I. P. Denisova, Opt. Spectrosc. **90**, 282 (2001).
[11] V. B. Berestetskii, L. P. Pitaevskii, and E. M. Lifshitz, *Quantum Electrodynamics* (Elsevier, Oxford, 1982).
[12] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).
[13] T. Heinzl, B. Liesfeld, K.-U. Amthor, H. Schwoerer, R. Sauerbrey, and A. Wipf, Opt. Commun. **267**, 318 (2006).
[14] V. I. Denisov, J. Opt. A **2**, 372 (2000).
[15] N. N. Rozanov, J. Exp. Theor. Phys. **86**, 284 (1998).
[16] E. Zavattini *et al.*, Phys. Rev. D **77**, 032006 (2008).
[17] V. I. Denisov and I. P. Denisova, Opt. Spectrosc. **90**, 928 (2001).

[18] G. Brodin, M. Marklund, and L. Stenflo, Phys. Rev. Lett. **87**, 171801 (2001).
[19] P. Berceau, R. Battesti, M. Fouché, and C. Rizzo, Can. J. Phys. **89**, 153 (2011).
[20] V. I. Denisov, I. V. Krivchenkov, and N. V. Kravtsov, Phys. Rev. D **69**, 066008 (2004).
[21] V. A. Sokolov, Moscow Univ. Phys. Bull. **64**, 349 (2009).
[22] R. Battesti and C. Rizzo, Rep. Prog. Phys. **76**, 016401 (2013).
[23] V. I. Denisov, N. V. Kravtsov, and I. V. Krivchenkov, Opt. Spectrosc. **100**, 641 (2006).
[24] A. K. Harding and D. Lai, Rep. Prog. Phys. **69**, 2631 (2006).
[25] V. A. Sokolov, Moscow Univ. Phys. Bull. **63**, 279 (2008).
[26] V. I. Denisov and V. A. Sokolov, J. Exp. Theor. Phys. **113**, 926 (2011).
[27] J. Y. Kim and T. Lee, J. Cosmol. Astropart. Phys. 11 (2011) 017.
[28] A. K. Harding and A. G. Muslimov, Astrophys. J. **508**, 328 (1998).
[29] M. S. Strickman, A. K. Harding, and O. C. D. Jager, Astrophys. J. **524**, 373 (1999).
[30] V. I. Denisov, I. V. Krivchenkov, and I. P. Denisova, J. Exp. Theor. Phys. **95**, 194 (2002).
[31] V. I. Denisov, I. P. Denisova, and S. I. Svertilov, Theor. Math. Phys. **135**, 720 (2003).
[32] P. A. Vshivtseva and M. M. Denisov, Comput. Math. Math. Phys. **49**, 2092 (2009).
[33] V. I. Denisov, I. P. Denisova, and I. V. Krivchenkov, Dokl. Math. **67**, 90 (2003).
[34] L. E. El'sgol'ts, *Differential Equations* (Gordon and Breach, New York, 1961).
[35] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, 1981).
[36] R. Epstein and I. I. Shapiro, Phys. Rev. D **22**, 2947 (1980).
[37] A. P. Lightman *et al.*, *Problem Book in Relativity and Gravitation* (Princeton University Press, Princeton, 1975).
[38] J. Y. Kim, J. Cosmol. Astropart. Phys. 10 (2011) 056.
[39] C. Thomson and R. C. Duncan, Astrophys. J. **473**, 322 (1996).
[40] P. Bloser, R. Andritschke, G. Kanbach, V. Schönfelder, F. Schopper, and A. Zoglauer, New Astron. Rev. **46**, 611 (2002).
[41] J. Greiner *et al.*, Exp. Astron. **23**, 91 (2009).