Are there tetraquarks at large N_c in QCD(F)?

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Weinberg recently pointed out a flaw in the standard argument that large N_c QCD with color-fundamental quarks [QCD(F)] cannot yield narrow tetraquark states. In particular, he observed that the argument does not rule out narrow tetraquarks associated with the leading-order connected diagrams; such tetraquarks would have a width scaling as N_c^{-1}. It is shown here, however, that while the standard analysis of tetraquarks does not rule them out, a more thorough analysis rules out quantum-number exotic tetraquarks associated with the leading-order connected diagrams. This analysis is based entirely on conventional assumptions used in large N_c physics applied to the analytic properties of meson-meson scattering. Our result implies that one of three possibilities must be true: (i) quantum-number exotic tetraquarks do not exist at large N_c; (ii) quantum-number exotic tetraquarks exist, but are associated with subleading connected diagrams and have anomalously small widths that scale as N_c^{-2} or smaller; or (iii) the conventional assumptions used in large N_c analysis are inadequate.

I. INTRODUCTION

Long-accepted conventional wisdom [1,2] precludes the large N_c limit of QCD [3] from supporting narrow tetraquark states (i.e., whose widths are suppressed by powers of $1/N_c$). The original argument was formulated by Witten [1] and described in Coleman's classic lectures on large N_c physics [2]. Recently Weinberg [4] has questioned this lore by pointing out a serious loophole in the reasoning.

The argument espoused by Witten [1] and Coleman [2] states that a tetraquark source of the form $J = \bar{q}q\bar{q}q$ can always be expressed (owing to Fierz reordering) as a sum of products of two color-singlet $\bar{q}q$ operators, each of which sources single-meson states at leading order in $1/N_c$. Therefore, the two-point function of J, which is $O(N_c^2)$ due to the two closed, disconnected color loops that follow the quark and antiquark lines, is dominated by the creation and annihilation of two-meson states. Since each meson terminus supplies a decay constant $f_M = O(N_c^{1/2})$, the scaling of the full two-point function is saturated by the $O(N_c^0)$ free propagation of two noninteracting mesons and no tetraquark states.

However, as pointed out in Ref. [4], this argument only applies to the leading-order (N_c^2) disconnected diagram; the leading *connected* diagram has only a single quark color loop [$O(N_c^1)$] and therefore excludes the leading order of free two-meson propagation. For the connected contribution of the J two-point function, the propagation of single free tetraquark states with decay constants $f_T = O(N_c^{1/2})$

can contribute at leading order. Moreover, analysis [4] of the connected three-point function of a J source and two bilinear $\bar{q}q$ sources B, which is also $O(N_c^1)$, gives a tetraquark-meson-meson coupling of $O(N_c^{-1/2})$, producing a tetraquark decay width of $O(N_c^{-1})$: If tetraquarks exist as $N_c \to \infty$, they are narrow. For certain special flavor quantum numbers, the tetraquark states must be even narrower [5].

That new states should first appear in the N_c-subleading part of a correlation function is not a novel idea; in fact, precisely the same argument as above may be applied to the $BBBB$ four-point correlation function, whose leading-order (N_c^2) disconnected piece is again saturated by noninteracting two-meson states, but whose leading connected [$O(N_c^1)$] piece contains scattering through single-meson states with $O(N_c^{-1/2})$ trilinear and $O(1/N_c)$ quartic couplings. This mechanism allows, for example, the well-known resonant scattering $\pi\pi \to \rho \to \pi\pi$ to occur at large N_c. The possibility considered by Weinberg is that tetraquarks behave similarly: If a tetraquark exists and couples to two ordinary mesons with $O(N_c^{-1/2})$ strength, then one finds no contradiction with standard N_c counting rules for meson-meson scattering, nor with a leading-order (N_c^2) disconnected contribution. However, obtaining $f_T \sim N_c^{1/2}$ may require a subtle limiting mechanism [6].

We note that certain other exotic hadrons (those with neither exclusively $\bar{q}q$ meson nor qqq baryon quantum numbers) do occur in the conventional large N_c limit. Glueballs have N_c-suppressed couplings to mesons [1], leading both to a parametric suppression of glueball-meson mixing amplitudes [$O(N_c^{-1/2})$] and glueball decay widths

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to two-meson states $[O(1/N_c^2)]$. Meanwhile, hybrid mesons (those requiring at least a single "valence gluon" in addition to $\bar{q}q$ in order to obtain the quantum numbers of the state, such as $J^{PC} = 1^{-+}$) behave just like ordinary mesons at large N_c [7]. The fact that hadrons with exotic quark structure often have the same quantum numbers as nonexotic hadrons (so-called "cryptoexotic" states) and therefore can mix with them is a major contributing factor to the lack of clear evidence for exotics. The phenomenon of cryptoexotic-conventional meson mixing is just as apparent for large-N_c tetraquarks with nonexotic quantum numbers [5]. Note that, even though cleanly identifying the role of cryptoexotics in the spectrum has intrinsic difficulties, the literature has a long tradition of explaining various scalar resonances that do not fit into simple quark models as being cryptoexotics [8–16].

In this paper, we focus on tetraquark with exotic quantum numbers, and in particular those with quantum numbers requiring at least two quarks and two antiquarks in all Fock components. Examples include isospin-2 states, doubly strange states, and strange states with isospin $\frac{3}{2}$. We do this to keep the issue as theoretically crisp as possible. The question of whether a component of a quantum state is or is not cryptoexotic has theoretical ambiguities and could depend upon the precise definition of precisely what constitutes a cryptoexotic state; such a definition may depend upon a choice of basis or of Lorentz frame. In contrast, quantum-number exotic configurations are unambiguously exotic.

It is significant that the logic of Ref. [4] does not rule out, but also does not guarantee, the existence of tetraquarks. In particular, the analysis shows that previous arguments ruling out tetraquarks at large N_c are wrong, but *not* that tetraquarks must, in fact, exist. The analysis of Ref. [4] uses the best-known $1/N_c$ expansion, in which each quark carries a single color index in the fundamental (F) representation of $SU(N_c)$, a theory we label QCD(F). However, it is worth recalling that the extrapolation from $N_c = 3$ to large N_c is not unique. For the physical case of $N_c = 3$, the color two-index antisymmetric (AS) representation of $SU(N_c)$ is isomorphic to the (anti)fundamental via the identification $q^{ij} = \epsilon^{ijk} q_k$. For larger N_c, the two representations differ [17] and therefore give rise to distinct large N_c limits and distinct $1/N_c$ expansions for the two theories. The new expansion based on quarks in the two-index antisymmetric representation is denoted as QCD(AS). While the study of QCD(AS) at large N_c was largely motivated by the appearance of elegant field theoretic dualities [18–21], one may instead develop the formalism for physical baryons [22–24] and investigate whether QCD(F) or QCD(AS) is phenomenologically superior for a given set of observables [25–27]. In the case of mesons, it was shown very recently [28] that the presence of two distinct color indices on each QCD(AS) quark field allows one to define a single-color-trace source J with

tetraquark quantum numbers. The implication of this finding is that tetraquarks contribute to the J two-point correlator in leading order in the $1/N_c$ expansion but two-meson states cannot. Therefore, tetraquarks necessarily appear in the spectrum of QCD(AS) as narrow hadrons.

The demonstration that tetraquarks exist in QCD(AS) raises the question of whether it is possible to go beyond Weinberg's argument [4] and prove the existence of tetraquarks in QCD(F). We have not been able to do so. Moreover, we suspect that the reason is likely to be that narrow tetraquark resonances do not exist in the large N_c limit of QCD(F). This suspicion is based on the demonstration reported here that the scenario of Ref. [4], in which tetraquarks associated with the leading-order connected diagrams exist and have widths of $O(1/N_c)$ (the most natural scenario if narrow tetraquarks exist), cannot be correct for quantum-number exotic channels provided the standard assumptions used in large N_c analysis hold. This demonstration is based on a careful analysis of the analytic structure of the meson-meson scattering amplitude at leading nontrivial order in the $1/N_c$ expansion. Ultimately, the result follows because all connected diagrams associated with four-point functions for quark bilinear sources at leading order in N_c possess no s-channel cuts associated with a tetraquark. The result shown here implies that one of three possibilities must be true: (i) quantum-number exotic tetraquarks do not exist at large N_c for QCD(F); (ii) tetraquarks exist, but are associated with subleading connected diagrams and have anomalously small widths that scale as N_c^{-2} or smaller; or (iii) the conventional assumptions used in large N_c analysis are inadequate.

The analysis is based on an indirect proof. We begin by supposing that the scenario proposed by Weinberg [4] is correct: A narrow quantum-number exotic tetraquark resonance exists at large N_c with a coupling to mesons of $O(N_c^{-1/2})$. Ultimately, we will show that this assumption leads to a contradiction. The process at the hadronic level is indicated in Fig. 1.

Before proceeding, it is useful to remind the reader that the scattering amplitude upon which we focus and the four-point correlation function of four-quark bilinear sources are related by the standard Lehmann-Symanzik-Zimmermann (LSZ) reduction formula. That is, the scattering amplitude is given by the appropriately normalized *amputated* four-point function. For the case in which the incident and final mesons are (pseudo)scalars, the four-point correlation function in momentum space depends upon six kinematic variables: q_A^2, q_B^2, q_C^2, and q_D^2, as well as s and t. The scattering amplitude, in contrast, fixes q_A^2, q_B^2, q_C^2, and q_D^2 to assume their on-shell values for the particular mesons of interest, and is given by

$$A(s,t) = \prod_i \lim_{q_i^2 \to m_i^2} Z_i^{-\frac{1}{2}} (q_i^2 - m_i^2) G_4^{ABCD}(\{q_i^2\}, s, t),$$

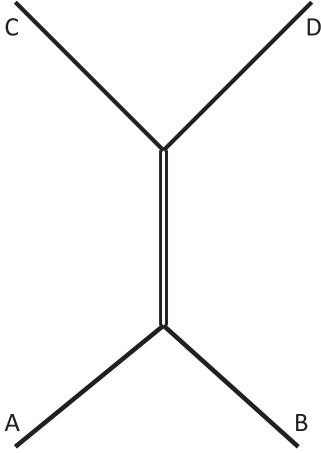$$Z_i = \lim_{q_i^2 \to m_i^2} |G_2^i(q_i^2)| \quad \text{for } i = A, B, C, D, \qquad (1)$$

FIG. 1. Diagram for the s-channel tetraquark contribution to meson-meson scattering.

where G_4^{ABCD} is the four-point correlation function in momentum space for quark bilinear sources $J_{A,B,C,D}$, and G_2^i is the diagonal two-point correlation function for linear source i. Note that this structure guarantees the external mesons are always on shell in the scattering amplitude. However, the amplitude G_4 itself is taken to be a general function of (complex) s and t, which can correspond to unphysical values that cannot be reached by the on-shell mesons. Analogous expressions hold for the case in which the external mesons carry spin. (Recall that the large-N_c world supports stable higher spin mesons.)

It is also useful to note that the scattering amplitude can be written in the form of a fixed-t dispersion relation,

$$A(s,t) = a_1(t) + a_2(t)s + \frac{1}{\pi} \int ds' \frac{\rho(s',t)s^2}{s'^2(s-s'+i\epsilon)}, \quad (2)$$

where we have assumed two subtractions. We make the standard assumption that this form continues to hold in the large N_c limit. The key to our analysis is that, if the Weinberg scenario is correct, it follows from this standard dispersion analysis that the scattering amplitude for meson-meson scattering amplitudes has s-channel spectral strength concentrated at the position of the tetraquarks over a region of width N_c^{-1} and with the integrated strength of $O(N_c^{-1})$. We focus on the s channel since the quantum numbers are given by the incident mesons, and the physical production of tetraquarks in scattering occurs only in this channel due to kinematic constraints. If one can demonstrate that the spectral function $\rho(s,t)$ for meson-meson scattering does not support s-channel spectra of this character, then the Weinberg scenario is ruled out.

The remainder of this paper is organized as follows: Sec. II focuses on Feynman diagrams at the quark-gluon level that contribute to the leading-order connected four-point correlation function for quark bilinear sources. A critical issue is the connection between a spacetime representation of the diagrams and a description in terms of color flow. In fact, the Feynman diagrams are actually identical and are merely drawn in different ways. Section III identifies all possible types of cuts of these leading-order connected diagrams and the intermediate states revealed by them. These cuts play a critical role in that they are associated with the spectral function in Eq. (2). A careful analysis of these cuts shows that no possible cut corresponds to a tetraquark intermediate state in meson-meson scattering. We summarize and discuss our conclusion in Sec. IV; here, the issue of cryptoexotics is addressed briefly. The analysis depends upon a number of standard assumptions used in large N_c, which are also discussed briefly in the concluding section.

II. SPACETIME AND COLOR-FLOW DIAGRAMS

This section focuses on the four-point function of $\bar{q}q$ bilinear sources labeled A, B, C, D. Following Weinberg [4], we consider only the leading-order connected diagrams. As a first step, we consider the diagrams rendered in spacetime so that the initial meson sources are placed at the bottom of the diagram and the final meson sources (actually sinks) are placed at the top (i.e., time flows upward). In this analysis, we focus on the N_c counting and assume the standard counting rules: We count only N_c factors arising from color traces (each closed color loop contributing N_c^1) and factors of $g_s \sim N_c^{-1/2}$ from gluon vertices.

Typical leading-order connected diagrams are indicated in Fig. 2. It is straightforward to show, using 't Hooft double-line color-diagrams, that all of these diagrams are $O(N_c^1)$ and hence of leading order. This result might come as something of a surprise since diagrams 2(b) and 2(c) do not appear to be planar, and it is well known that all leading-order connected diagrams containing quark lines are planar and bounded by the quark loop. However, the sense in which planarity holds has to do with color flow and not with spacetime. That is, a diagram is of leading order in N_c counting if, ignoring spacetime considerations, the diagram *can* be drawn in a plane with the quark line bounding the diagram. Planarity is entirely determined by how the quarks and gluons in the diagram are connected topologically and not on how momentum flows from source to sink. This distinction is critically important since the question of s-channel spectral strength clearly depends upon the spacetime flow of momentum in the diagram.

The leading-order connected diagrams in Fig. 2 can easily be seen to be planar and bounded by a quark loop if one ignores the momentum flow and unfolds them, as depicted in Fig. 3. In fact, the diagrams in Fig. 2 are identical to the corresponding ones in Fig. 3, in which it is apparent that any leading-order connected diagram associated with a four-point function can be represented by a square with the four quark bilinear sources (sinks) on the vertices. One can then return to the momentum-flow form by considering how the sources inject momentum into the system. Again, the issue is
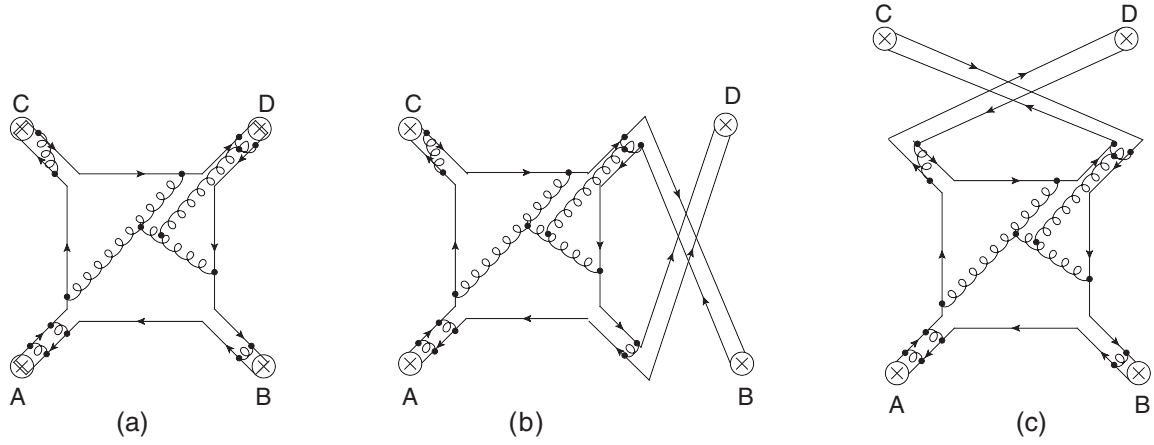
FIG. 2. Typical leading-order (N_c^1) Feynman diagrams for the connected four-point function. In each case, the diagram follows a spacetime ordering, in that the bilinear sources A, B represent initial states and the bilinear sources C, D represent final states.
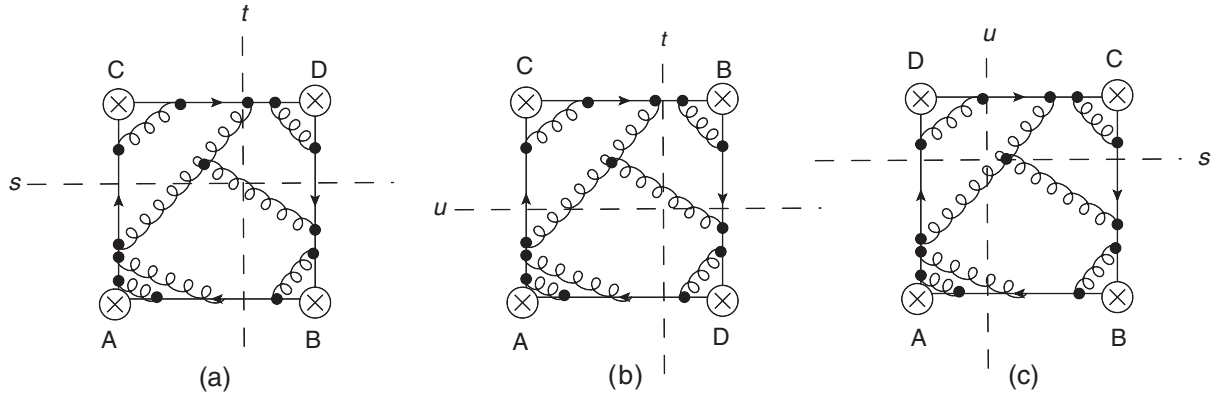

FIG. 3. The diagrams of Fig. 2, recast as leading-order (N_c^1), manifestly planar diagrams in terms of color structure. Cuts giving intermediate states in the indicated Mandelstam channels are denoted by dashed lines.

topological: how the various momenta inserted into the system are related. One can divide such diagrams into topological classes. Superficially there appear to be 24 classes of leading-order connected diagrams since each of the four bilinears can attach to any of the four vertices. However, since the value of the diagram does not change if the square is rotated through any multiple of $\frac{\pi}{2}$ rad, the number of independent classes reduces to six. One can label the class of diagram by the order of the vertices, with A and B representing the sources of the two incoming mesons and C and D the outgoing ones, ordered counterclockwise starting at the bottom left, so that diagram (a) of Fig. 3 is labeled $ABDC$. Cyclic combinations of the square are identified because they correspond to the same diagram: $ABDC$ is equivalent to $BDCA$. Finally, because of crossing symmetry, if one reverses the order of the sources and flips all internal gluons along the diagonal, the value of the diagram changes by at most a phase. Thus one can reverse the ordering so that, e.g., $ABDC$ is in the same class as $ACDB$, leaving three classes. Figures 2 and 3 give one representative of each class.

It is worth noting that not all of these classes of leading-order connected diagrams contribute to processes in the quantum-number exotic channels of interest here. Consider for concreteness the case of processes that carry $I = 2$ in the s channel. It is easy to show that classes $ABDC$ and $ABCD$ [corresponding to diagrams (a) and (c) in Fig. 3] do not contribute to $I = 2$ processes in the s channel, while class $ADBC$ [corresponding to diagram (b) in Fig. 3] does. To see why, note that the s channel carries the quantum numbers injected at sources A and B. Since the process carries $I = 2$ in the s channel, the sources A and B together must provide $I = 2$. Moreover, since the gluons are isosinglets, isospin can be injected into the diagram only at sources A, B, C, and D. In diagrams of classes $ABDC$ and $ABCD$, vertices A and B are adjacent. The quark line connecting the adjacent A and B vertices, treated as a $\bar{q}q$ pair, can be considered an isosinglet source (since deep in the diagram, it turns into pure glue); the two remaining quarks in the s channel can carry no more than $I = 1$ and therefore give a vanishing matrix element with an $I = 2$ operator.

III. CUTS AND INTERMEDIATE STATES

This section describes the role in the analysis of "cuts," for which two related meanings are relevant. The first meaning is related to the analytic structure of the scattering amplitude [$A(s,t)$ in Eq. (1)], or more generally, n-point correlation functions G_n. Consider for example Eq. (1), which relates the real and imaginary parts of $A(s,t)$. As an analytic function of s, $A(s,t)$ has a cut over the full region in which the spectral function $\rho(s,t)$ is a nonzero and continuous function. Note that the cut is associated with accessible physical states: It corresponds to all values of s for which the system can go on shell.

At the level of perturbation theory for the underlying theory of quarks and gluons, "cut" can also be used to describe the process of dividing a diagram into two distinct regions. Typically, this separation can be done in more than one way. The region along the boundary between the regions can be referred to as the cut. The combination of quarks and gluons along the cut corresponds to a particular quark-gluon intermediate state contributing in perturbation theory.

These two meanings are strongly related: When the hadronic intermediate states revealed by cutting a diagram go on shell, they contribute to the cut in the sense of the analytic function (to the extent that such states are calculable in perturbation theory).

One class of cuts is easily depicted by using the color-flow diagrams of Fig. 3: A diagram in each of the three classes can be cut in two natural ways. In the class $ABDC$ of 3(a), one can generate an s-channel cut (cutting through both quark lines separating the A and B vertices from the C and D vertices) or a t-channel cut (separating vertices A, C from B, D). Of course, more than one particular cut is available, depending upon which specific gluon lines are

cut, but these two basic cut structures naturally arise. Similarly, Fig. 3 shows u- and t-channel cuts for class $ADBC$ and s- and u-channel cuts for class $ABCD$.

The types of cuts shown in Fig. 3 clearly do not contain intermediate quark-gluon states that contribute to tetraquark states. All of these cuts contain precisely one $q\bar{q}$ pair in the intermediate states; thus, they contribute to mesons, not tetraquarks. Recall that we seek only s-channel singularities connected with the physical intermediate states created in the scattering process. It is significant that Fig. 3 has s-channel cuts only in classes $ABDC$ and $ABCD$ but not $ADBC$, but this result is not surprising: As noted in Sec. II, s-channel quantum-number exotic channels correspond only to class $ADBC$ among the leading-order connected diagrams. If class $ADBC$ had meson-type s-channel contributions, one would have a contradiction: The exotic channels associated with this class by definition lack ordinary meson contributions.

However, as it turns out, the kinds of cuts shown in Fig. 3 that seem natural when the diagrams are drawn in planar form are not the only ones appearing in the four-point functions at leading order in N_c. The key point is that the diagrams of Fig. 3 are drawn in a form to emphasize color-flow connectivity and not the flow in spacetime. However, the spacetime description is the most relevant one for understanding the cuts. To make this point clear, consider Fig. 4. The diagram is of class $ADBC$ and hence can arise in an exotic s-channel scattering process. This class is also analogous to diagrams considered by Weinberg in Ref. [4]. The critical point is that the diagram viewed in its spacetime form clearly has an s-channel cut passing through two quarks and two antiquarks. That is, it has the correct quantum-number content to be a tetraquark. In fact, if one considers quantum-number exotic channels and
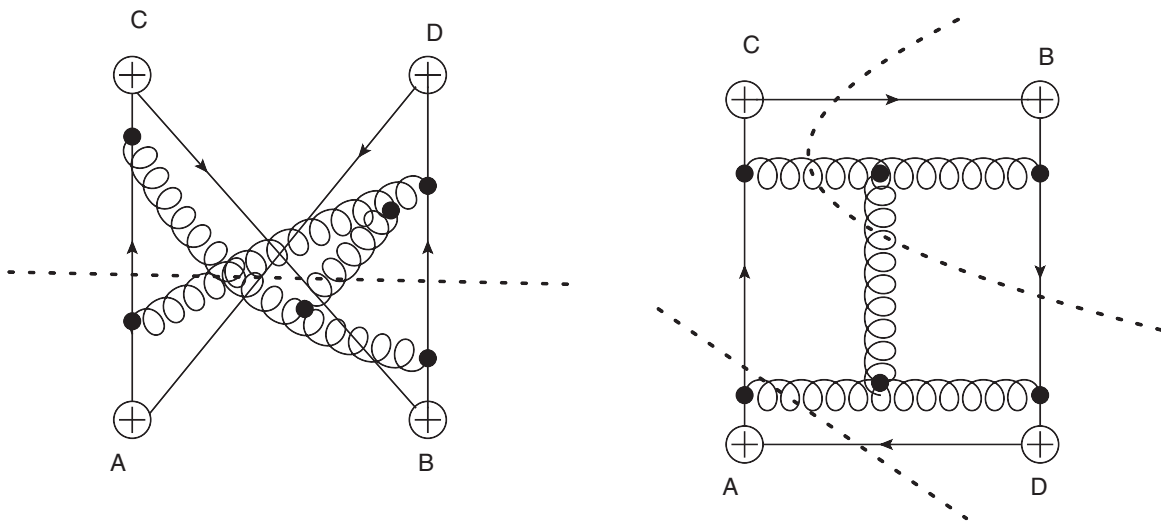

FIG. 4. An s-channel cut in leading-order (N_c^1) drawn both as a spacetime diagram and as a manifestly planar diagram emphasizing the color structure. The cut in the second inset is actually continuous, with the cut termini at the top and bottom of the diagram being identified.

restricts attention to leading-order connected diagrams, all s-channel cuts clearly have this character: The intermediate states revealed by the cut always contain two quarks and two antiquarks.

The critical question is whether these cuts reveal resonant tetraquark states or merely two noninteracting (or weakly interacting) mesons. It is possible to definitively answer this question if one accepts the standard assumptions used in large N_c analysis. To see how, note that if one unfolds the diagram of Fig. 4 given in spacetime form (the first inset) to write it in manifestly planar color-flow form (the second inset), the line denoting the cut breaks into two distinct parts. In the case indicated here, one part cuts off vertex A and the other cuts off vertex B; that is, it literally cuts the corners off the diagram. (In the literal transcription of this cut, the cut termini at the top and bottom of the color-flow diagram of Fig. 4 are identified.) It is guaranteed that *any* s-channel cut for all diagrams in class $ADBC$ (and hence for all leading-order connected diagrams in quantum-number exotic channels) cut corners in the diagram written in planar form, either the corners A and B as the preceding example, or the corners C and D. The reason is simple topology: In class $ADBC$, corners A and B are opposite from each other, as are corners C and D. However, an s-channel cut by definition cuts the region associated with the incoming particles (which contains vertices A, B) from the region associated with the outgoing particles (which contains vertices C, D). In a spacetime description, a single line across the diagram always accomplishes the cut. However, since A and B lie diagonally across from each other in planar form (as do C and D), no single line across the diagram can represent the cut. The only resolution is to make such a cut remove two diagonally separated corners and associate the disconnected regions of the two corners to be part of the same "side" of the cut.

The physical significance of the fact that the s-channel cuts always isolate opposite corners for leading-order connected diagrams in exotic channels (and for class $ADBC$ generally) is clear: The individual cuts across the two corners carry precisely the four-momentum and color structure (singlet) injected at each of the two sources. The four-momentum entering at sources A and B is conserved at every vertex and must flow out of the corner, but the only way out of the corner is across the cut. This result is crucial because the object of interest is the scattering amplitude $A(s,t)$—i.e., the appropriately normalized *amputated* four-point correlation function—rather than the four-point correlation function G_4^{ABCD} itself; as noted in the Introduction, the scattering amplitude *must* contain a tetraquark if the mesons couple to tetraquarks. Moreover, if one accepts the standard assumptions underlying the large N_c analysis of hadrons and Weinberg's scenario that tetraquarks couple to two meson with a strength of $\sim N_c^{-1/2}$, then it must be true that the scattering amplitude computed using the leading-order connected diagrams contains analytic

singularities in the s channel associated with the tetraquark intermediate state. However, the LSZ reduction rule removes all spectral strength from the incoming and outgoing mesons to produce the scattering amplitude. In $A(s,t)$, the only contributions to the spectral strength $\rho(s,t)$ come from intermediate states that are distinct from the incident and outgoing ones; the factors of $(q_i^2 - m_i^2)$ in Eq. (1) ensure this. In particular, when one cuts the corner of a diagram, the contribution associated with the cut corner to the scattering gives no spectral strength and thus no cut (in the sense of analytic functions); it has been removed from the full four-point function via LSZ reduction. Physically, the spectral strength in the four-point function associated with cut corners corresponds to the external meson states only and not to any intermediate state.

The preceding argument implies that an exotic diagram such as Fig. 4 produces no spectral strength at leading N_c order in the s channel, and hence no leading-order contribution to the scattering amplitude. In terms of Eq. (2), $\rho(s,t)$ vanishes at $O(N_c^{-1})$. However, in the scenario in which tetraquarks exist and couple to two mesons with a strength $\sim N_c^{-1/2}$, tetraquarks must be present in the scattering amplitude if the standard assumptions for large N_c hadrons hold. Thus we are forced to conclude that, for the case of exotic channels, either the scenario in which quantum-number exotic tetraquarks exist and couple with a strength $\sim N_c^{-1/2}$ is incorrect or the standard large N_c assumptions are inadequate.

We should note that the vanishing of the leading-order spectral strength in Eq. (2) for exotic channels does *not* imply the absence of scattering at this order. Rather, it means that leading-order scattering comes through $a_1(t)$ and $a_2(t)$, which can arise through t-channel exchange.

IV. CONCLUSIONS

The argument we have presented implies that either the scenario of Ref. [4], in which quantum-number exotic tetraquarks exist and couple with a strength $\sim N_c^{-1/2}$, is incorrect or the standard assumptions about hadrons in large N_c are inadequate.

Let us first consider the possibility that some of the standard assumptions used in large N_c hadronic analysis might be incorrect. One should note that a variety of such assumptions occur—some explicitly made in the large N_c literature and others that are implicit. We believe that these assumptions are highly plausible, and we have no reason to doubt any of them.

As a question of logic, however, it remains possible that one or more of these assumptions is not valid and that as a result, quantum-number exotic tetraquarks are not excluded from arising through leading-order connected diagrams. Let us consider an example: It is always implicitly assumed that the standard dispersion relations and spectral representations of field theory hold in the large N_c limit. Now of course, they are supposed to hold for any legitimate field

theory, and hence should hold for QCD at any N_c. However, this assertion does not necessarily imply that these relations are valid for the large N_c limit of the theory. Such issues could arise, for example, due to some types of nonanalytic behavior in $1/N_c$ for correlation functions as $N_c \to \infty$. As a concrete model in which such behavior could allow quantum-number exotic tetraquarks, consider the example in Ref. [6]. There, it is assumed that the four-point function in position space has a coupling to the tetraquark scaling as $\exp[-N_c^{1/3}(x_1 - x_2)^2]$, where x_1 and x_2 are the positions of the two quark bilinears. Such nonanalytic behavior has the consequence that widely separated quark bilinear sources—the type associated with external mesons—do not couple to tetraquarks, while overlapping sources do. Thus, even though tetraquarks by construction exist in the theory and couple to mesons, at large N_c they have no spectral strength in the scattering amplitude, in contradiction to what one expects from a spectral representation.

The example above is hardly unique. Many assumptions used in large N_c analysis are believed to be true but have never been rigorously proven and, if false, would invalidate the argument of the preceding section. For example, a basic tenet of the analysis to assume the leading $1/N_c$ scaling of a correlation function is the same as that of the leading class of contributing Feynman diagrams. This assertion is highly plausible and is the basis for almost all analyses of hadrons at large N_c. It is nevertheless not guaranteed to be true in a mathematical sense, since the perturbative expansion embodied in the Feynman diagrams is not convergent. If it is false, one can easily evade the argument given above: Only the *leading*-order diagrams lack spectral strength in the s channel for the scattering amplitude in exotic channels, and the tetraquark could be built up from formally subleading perturbative orders. Overall, we view such possibilities (while being logically not excluded) as being highly unlikely. The standard large N_c analysis has produced many useful insights into hadronic physics, and it seems unlikely that exceptions should first become apparent in the tetraquark sector.

If one accepts the standard large N_c assumptions, then the analysis here rules out the possibility that quantum-number exotic channels follow the scenario of Ref. [4], in which tetraquarks exist and arise due to the leading-order connected diagrams, and accordingly couple to two mesons with a strength of $O(N_c^{-1/2})$. Then two possibilities remain: Either narrow tetraquarks do not exist or they arise from subleading diagrams and couple to mesons at $O(N_c^{-1})$ or less (making them even narrower).

Consideration of the second possibility actually follows the basic insight of Ref. [4], where Weinberg observed that just because the leading $[O(N_c^2)]$ contribution to the two-point correlation function of a source with tetraquark quantum numbers is a pure two-meson state with no resonant tetraquark contribution does not mean that no

tetraquark contribution arises in the first subleading contribution, which happens to be the leading contribution for the connected correlator. Here we have shown (modulo the assumptions discussed above) that in exotic channels the leading connected diagrams also contain no tetraquark contributions. However, a simple extension of Weinberg's logic leads one to the observation that subleading contributions could contain a tetraquark.

How plausible is this? We believe it unlikely to be correct. To see why, consider the analogous situation of mesons at large N_c, in which mesons arise from the leading-order $[O(N_c^1)]$ correlation functions of quark bilinears. One deduces that mesons, for reasons of self-consistency, have widths that scale as $1/N_c$. Using the standard analysis, can one falsify the existence of an additional class of special mesons with the same quantum numbers as the ordinary ones but that arise from subleading diagrams and have anomalously small $[O(1/N_c^2)]$ widths? At a strict formal level, the answer is "no." However, the idea seems quite farfetched. Indeed, Occam's razor argues against it: We know of no reason why such a peculiar behavior ought to emerge, and in the absence of a compelling reason, it appears to be unlikely. In much the same way, it seems quite farfetched that ultranarrow tetraquarks should arise from subleading diagrams. As discussed in Ref. [5], one finds channels with special flavor content that would require tetraquarks (were they to exist) to have particularly narrow $[O(N_c^{-2})]$ widths. However, the case considered here is whether "ordinary" tetraquarks with, e.g., $I = 2$ are anomalously narrow.

Thus, we conclude that quantum-number exotic narrow tetraquarks are unlikely to exist as resonant states at large N_c in QCD(F). This behavior is qualitatively very different from that of QCD(AS), in which such states are known to exist [28]. Such a difference is both awkward and interesting, since one hopes to use large N_c behavior as a guide for our $N_c = 3$ world. Since the two distinct large N_c limits are qualitatively different for tetraquarks, the presence or absence of observably narrow tetraquarks at $N_c = 3$ provides an exciting opportunity to distinguish which limit is superior in this case.

Finally, the analysis here is limited to quantum-number exotic channels. What would change for nonexotic channels? As noted above, the situation becomes more ambiguous, because it is difficult to separate cryptoexotic components from those of conventional mesons. One thing is clear: Our analysis shows that the diagrams in class $ADBC$ cannot contribute to the spectral strength in the scattering amplitude for tetraquark channels. On the other hand, as seen in Fig. 3, classes $ABDC$ and $ABCD$ do have s-channel spectral strength, but these are of a $\bar{q}q$ nature and can be ascribed to mesons rather than tetraquarks. Thus it appears likely that any sensible definition of "tetraquark" in cryptoexotic channels yields no spectral strength that is specifically of a tetraquark nature.

Since the large N_c limit of QCD(F) has produced so many useful phenomenological insights, the exciting possibility of narrow tetraquarks gives hope for the eventual observation of such a state. However, we have found that no mechanism relying upon the conventional counting of N_c factors, even ones with unusual diagrammatic and cut structures, give rise to parametrically narrow tetraquarks. Even so, the alternative and phenomenologically viable QCD(AS) large N_c limit remains a possibility for producing naturally narrow tetraquarks. Last, other suppressions having nothing to do with N_c counting [e.g., by heavy quark masses, as may be the case for the $X(3872)$ [29]] might still make an observably narrow tetraquark state viable.

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