$SU(5) \times SU(5)'$ unification and D_2 parity: Model for composite leptons

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We study a grand unified $SU(5) \times SU(5)'$ model supplemented by D_2 parity. The D_2 greatly reduces the number of parameters and is important for phenomenology. The model, we present, has various novel and interesting properties. Because of the specific pattern of grand unification symmetry breaking and emerged strong dynamics at low energies, the Standard Model leptons, along with right-handed/sterile neutrinos, come out as composite states. The generation of the charged fermion and neutrino masses are studied within the considered scenario. Moreover, the issues of gauge coupling unification and nucleon stability are investigated in details. Various phenomenological implications are also discussed.

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I. INTRODUCTION

The Standard Model (SM) of electroweak interactions has been a very successful theory for decades. The triumph of this celebrated model occurred thanks to the Higgs boson discovery [1] at CERN's Large Hadron Collider. In spite of this success, several phenomenological and theoretical issues motivate one to think of some physics beyond the SM. Because of renormalization running, the self-coupling of the SM Higgs boson becomes negative at a scale near $\sim 10^{10}$ GeV [2], [3] (with the Higgs mass $\simeq 126$ GeV), causing vacuum instability (becoming more severe within the inflationary setup; see the discussion in Sec. VI). Moreover, the SM fails to accommodate atmospheric and solar neutrino data [4]. The renormalizable part of the SM interactions render neutrinos to be massless. Also, Planck scale suppressed $d = 5$ lepton number violating operators do not generate neutrino mass with desirable magnitude. These are already strong motivations to think about the existence of some new physics between electroweak (EW) and Planck scales.

Among various extensions of the SM, the grand unification (GUT) [5], [6] is a leading candidate. Unifying all gauge interactions in a single group, at high energies one can deal with a single unified gauge coupling. At the same time, quantization of quark and lepton charges occurs by embedding all fermionic states in unified GUT multiplets. The striking prediction of the grand unified theory is the baryon number violating nucleon decay. This opens the prospect for probing the nature at very short distances. GUTs based on $SO(10)$ symmetry [7] [which includes $SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry [5] as a maximal subgroup] involve right-handed neutrinos (RHNs), which provide a simple and elegant way for neutrino mass generation via the seesaw mechanism [8]. In spite of these salient futures, GUT model building encounters numerous problems and phenomenological difficulties. With single scale breaking, i.e., with no new interactions and/or intermediate states between EW and GUT scales, grand unified theories [such as minimal $SU(5)$ and $SO(10)$] do not lead to successful gauge coupling unification. Besides this, building GUT with the realistic fermion sector, understanding the GUT symmetry breaking pattern, and avoiding too rapid nucleon decay remain a great challenge.

Motivated by these issues, we consider $SU(5) \times SU(5)'$ GUT augmented with D_2 parity (exchange symmetry). The latter, relating two $SU(5)$ gauge groups, reduces the number of parameters, and at and above the GUT scale, one deals with single gauge coupling. The grand unified theories with $SU(5) \times SU(5)'$ symmetry, considered in earlier works [9], in which at least one gauge factor of the SM symmetry emerges as a diagonal subgroup, have been proven to be very successful for building models with realistic phenomenology. However, to our knowledge, in such constructions the D_2 parity has not been applied before.[1] The reason could be the prejudice of remaining with extra unwanted chiral matter states in the spectrum. However, within our model due to specific construction, this does not happen, and below the few-TeV scale, surviving states are just of the Standard Model. The D_2 parity also plays a crucial role for phenomenology and has interesting implications. By the specific pattern of the $SU(5) \times SU(5)'$ symmetry breaking and spectroscopy, the successful gauge coupling unification is obtained. Interestingly, within the considered framework, the SM leptons emerge as a composite states, while the quarks are fundamental objects. Lepton mass generation occurs by a new mechanism, finding natural realization within a presented model. Since leptons and quarks have different footing, there is no problem of their mass degeneracy (unlike the minimal SO(10) and $SU(5)$ grand unified thoeries, which require

[1] In the second citation of Ref. [9], the exchange symmetry was considered; however, some terms violating this symmetry have been included.

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some extensions [10]). Moreover, along with composite SM leptons, the model involves three families of composite SM singlet fermionic states, which may be identified with RHNs or sterile neutrinos. Thus, the neutrino masses can be generated. In addition, we show that, due to the specific fermion pattern, $d=6$ nucleon decay can be adequately suppressed within the considered model. The model also has various interesting properties and implications, which we also discuss. Since two $SU(5)$ groups will be related by D_2 parity, initial states will be doubled, i.e., will be introduced in twins. Because of this, we refer to the proposed $SU(5)\times SU(5)'\times D_2$ model as twinification.

The paper is organized as follows. In the next section, first we introduce the $SU(5)\times SU(5)'\times D_2$ GUT and discuss the symmetry breaking pattern. Then, we present the spectrum of bosonic states. In Sec. III, considering the fermion sector, we give transformation properties of the GUT matter multiplets under D_2 parity and build the Yukawa interaction Lagrangian. The latter is responsible for the generation of quark masses and Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Because of the specific pattern of the symmetry breaking and strong $SU(3)'$ [originating from $SU(5)'$ gauge symmetry] dynamics, the SM leptons emerge as composite objects. We present a novel mechanism for composite lepton mass generation. Together with the SM leptons, three families of right-handed/sterile neutrinos are composite. We also discuss the neutrino mass generation within our scenario. In Sec. IV we give details of gauge coupling unification. The issue of nucleon stability is addressed in Sec. V. Although the GUT scale, within our model, comes out to be relatively low ($\simeq 5\times 10^{11}$ GeV), we show that the $d=6$ baryon number violating operators can be adequately suppressed. This happens to be possible due to the specific pattern of the fermion sector we are suggesting. In Sec. VI we summarize and discuss various phenomenological constraints and possible implications of the considered scenario. We also emphasize the model's peculiarities and novelties, which open broad prospects for further investigations. Appendix A discusses details related to the compositeness and anomaly matching conditions. In Appendix B we give details of the gauge coupling unification. In particular, the renormalization group (RG) equations and b factors at various energy intervals are presented. The short-range renormalization of baryon number violating $d=6$ operators is also performed.

II. $SU(5)\times SU(5)'\times D_2$ TWINIFICATION

Let us consider the theory based on $SU(5)\times SU(5)'$ gauge symmetry. Besides this symmetry, we postulate discrete parity D_2, which exchanges two $SU(5)$'s. Therefore, the symmetry of the model is

$$G_{\rm GUT}=SU(5)\times SU(5)'\times D_2. \tag{1}$$

As noted, the action of D_2 interchanges the gauge fields (in adjoint representations) of $SU(5)$ and $SU(5)'$,

$$D_2\colon\ (A_\mu)^a_b\to(A'_\mu)^{a'}_{b'},\qquad (A'_\mu)^{a'}_{b'}\to(A_\mu)^a_b, \tag{2}$$

with $(A_\mu)^a_b=\frac{1}{2}\sum_{i=1}^{24}A^i_\mu(\lambda^i)^a_b$ and $(A'_\mu)^{a'}_{b'}=\frac{1}{2}\sum_{i'=1}^{24}A'^{i'}_\mu(\lambda^{i'})^{a'}_{b'}$, where a,b and a',b' denote indices of $SU(5)$ and $SU(5)'$, respectively. The $\lambda^i,\lambda^{i'}$ are corresponding Gell-Mann matrices. Thanks to the D_2, at and above the GUT scale M_G, we have single gauge coupling

$$\alpha_5=\alpha_{5'}. \tag{3}$$

Grand unified theories based on product groups allow us to build simple models with realistic phenomenology [9], [11]. In our case, as we show below, the EW part [i.e., $SU(2)_w\times U(1)_Y$] of the SM gauge symmetry will belong to the diagonal subgroup of $SU(5)\times SU(5)'$.

A. Potential and symmetry breaking

For $G_{\rm GUT}$ symmetry breaking and building realistic phenomenology, we introduce the states

$$H\sim(5,1),\qquad \Sigma\sim(24,1),\qquad H'\sim(1,5),$$
$$\Sigma'\sim(1,24),\qquad \Phi\sim(5,\bar{5}), \tag{4}$$

where in brackets transformation properties under $SU(5)\times SU(5)'$ symmetry are indicated. H includes SM Higgs doublet h. The introduction of H' is required by D_2 symmetry. By the same reason, two adjoints Σ and Σ' (needed for GUT symmetry breaking) are introduced. The bifundamental state Φ will also serve for desirable symmetry breaking.

The action of D_2 parity on these fields is

$$D_2\colon\ H_a\leftrightarrows H'_{a'},\qquad \Sigma^a_b\leftrightarrows\Sigma'^{a'}_{b'},\qquad \Phi^{b'}_a\leftrightarrows(\Phi^\dagger)^b_{a'}, \tag{5}$$

where we have made explicit the indices of $SU(5)$ and $SU(5)'$. With Eqs. (5), (2), and (3), one can easily make sure that the kinetic part $|D_\mu H|^2+|D_\mu H'|^2+\frac{1}{2}{\rm tr}(D_\mu\Sigma)^2+\frac{1}{2}{\rm tr}(D_\mu\Sigma')^2+|D_\mu\Phi|^2$ of the scalar field Lagrangian is invariant.

The scalar potential, invariant under $G_{\rm GUT}$ symmetry [of Eq. (1)] is

$$V=V_{H\Sigma}+V_{H'\Sigma'}+V^{(1)}_{\rm mix}+V_\Phi+V^{(2)}_{\rm mix}, \tag{6}$$

with

$$
\begin{aligned}
V_{H\Sigma} &= -M_\Sigma^2 \mathrm{tr}\Sigma^2 + \lambda_1(\mathrm{tr}\Sigma^2)^2 + \lambda_2 \mathrm{tr}\Sigma^4 + H^\dagger(M_H^2 - h_1\Sigma^2 + h_2\mathrm{tr}\Sigma^2)H + \lambda_H(H^\dagger H)^2,\\
V_{H'\Sigma'} &= -M_\Sigma^2 \mathrm{tr}\Sigma'^2 + \lambda_1(\mathrm{tr}\Sigma'^2)^2 + \lambda_2 \mathrm{tr}\Sigma'^4 + H'^\dagger(M_H^2 - h_1\Sigma'^2 + h_2\mathrm{tr}\Sigma'^2)H' + \lambda_H(H'^\dagger H')^2,\\
V_{\mathrm{mix}}^{(1)} &= \lambda(\mathrm{tr}\Sigma^2)(\mathrm{tr}\Sigma'^2) + \tilde{h}(H^\dagger H\mathrm{tr}\Sigma'^2 + H'^\dagger H'\mathrm{tr}\Sigma^2) + \hat{h}(H^\dagger H)(H'^\dagger H'),\\
V_\Phi &= -M_\Phi^2\Phi^\dagger\Phi + \lambda_{1\Phi}(\Phi^\dagger\Phi)^2 + \lambda_{2\Phi}\Phi^\dagger\Phi\Phi^\dagger\Phi,\\
V_{\mathrm{mix}}^{(2)} &= \mu(H^\dagger\Phi H' + H\Phi^\dagger H'^\dagger) + \frac{\lambda_{1H\Phi}}{\sqrt{25}}(\Phi^\dagger\Phi)[(H^\dagger H) + (H'^\dagger H')] + \frac{\lambda_{2H\Phi}}{\sqrt{10}}(H^\dagger\Phi\Phi^\dagger H + H'^\dagger\Phi^\dagger\Phi H')\\
&\quad + \lambda_{1\Sigma\Phi}(\Phi^\dagger\Phi)(\mathrm{tr}\Sigma^2 + \mathrm{tr}\Sigma'^2) - \lambda_{2\Sigma\Phi}(\Phi^\dagger\Sigma^2\Phi + \Phi\Sigma'^2\Phi^\dagger). \qquad (7)
\end{aligned}
$$

To make analysis simpler, we have omitted terms with first powers of Σ and Σ' (such as $H^\dagger\Sigma H$, $H'^\dagger\Sigma' H'$, etc.) and also cubic terms of Σ and Σ'. This simplification can be achieved by Z_2 discrete symmetry and will not harm anything.

The potential terms and couplings in Eqs. (6) and (7) allow us to have a desirable and self-consistent pattern of symmetry breaking. First, we will sketch the symmetry breaking pattern. Then, we will analyze the potential and discuss the spectrum of bosonic states. We will stick to several stages of the GUT symmetry breaking. At the first step, the Σ develops the vacuum expectation value (VEV) $\sim M_G$ with

$$\langle\Sigma\rangle = v_\Sigma \mathrm{Diag}(2,2,2,-3,-3), \qquad v_\Sigma \sim M_G. \quad (8)$$

This causes the symmetry breaking:

$$SU(5)\overset{\langle\Sigma\rangle}{\to} SU(3)\times SU(2)\times U(1) \equiv G_{321}. \quad (9)$$

We select VEVs of Σ' and Φ much smaller than M_G. As it will turn out, the phenomenologically preferred scenario is $\langle\Sigma'\rangle \sim 4\times 10^6$ GeV and $\langle\Phi\rangle \sim 8\times 10^4$ GeV. With

$$\langle\Sigma'\rangle = v_{\Sigma'}\mathrm{Diag}(2,2,2,-3,-3), \quad (10)$$

the breaking

$$SU(5)'\overset{\langle\Sigma'\rangle}{\to} SU(3)'\times SU(2)'\times U(1)' \equiv G_{321}{}' \quad (11)$$

is achieved. The last stage of the GUT breaking is done by $\langle\Phi\rangle$ with a direction

$$\langle\Phi\rangle = v_\Phi \cdot \mathrm{Diag}(0,0,0,1,1). \quad (12)$$

This configuration of $\langle\Phi\rangle$ breaks symmetries $SU(2)\times U(1)$ [subgroup of $SU(5)$] and $SU(2)'\times U(1)'$ [subgroup of $SU(5)'$] to the diagonal symmetry group:

$$SU(2)\times U(1)\times SU(2)'\times U(1)'\overset{\langle\Phi\rangle}{\to}[SU(2)\times U(1)]_{\mathrm{diag}}. \quad (13)$$

As we see, all VEVs preserve $SU(3)$ and $SU(3)'$ groups arising from $SU(5)$ and $SU(5)'$, respectively. However, unbroken $SU(2)_{\mathrm{diag}}$ is coming (as superposition) partly from $SU(2)\subset SU(5)$ and partly from $SU(2)'\subset SU(5)'$. Similarly, $U(1)_{\mathrm{diag}}$ is superposition of two Abelian factors: $U(1)\subset SU(5)$ and $U(1)'\subset SU(5)'$.

Now, making the identifications

$$SU(3)\equiv SU(3)_c, \quad SU(2)_{\mathrm{diag}}\equiv SU(2)_w, \quad U(1)_{\mathrm{diag}}\equiv U(1)_Y \quad (14)$$

and taking into account Eqs. (9), (11), and (13), we can see that GUT symmetry is broken as

$$
\begin{aligned}
G_{\mathrm{GUT}} &\to SU(3)_c\times SU(2)_w\times U(1)_Y\times SU(3)'\\
&= G_{\mathrm{SM}}\times SU(3)', \qquad (15)
\end{aligned}
$$

where $G_{\mathrm{SM}} = SU(3)_c\times SU(2)_w\times U(1)_Y$ denotes the SM gauge symmetry. Because of these, at the intermediate scale $\mu = M_I(\sim\langle\Phi\rangle)$, we will have the matching conditions for the gauge couplings,

$$\text{at } \mu = M_I:\ \frac{1}{g_w^2} = \frac{1}{g_2^2} + \frac{1}{g_{2'}^2}, \qquad \frac{1}{g_Y^2} = \frac{1}{g_1^2} + \frac{1}{g_{1'}^2}, \quad (16)$$

where subscripts indicate to which gauge interaction the appropriate coupling corresponds [e.g., $g_{1'}$ is the coupling of $U(1)'$ symmetry, etc.].

The extra $SU(3)'$ factor has important and interesting implications, which we discuss below.

As was mentioned, while $\langle\Sigma\rangle\sim M_G$, the VEVs $\langle\Phi\rangle$ and Σ' are at intermediate scales M_I and $M_I{}'$, respectively,

$$v_\Phi \sim M_I, \qquad v_{\Sigma'} \sim M_I{}', \quad (17)$$

with the hierarchical pattern

$$M_I \ll M_I{}' \ll M_G. \quad (18)$$

Detailed analysis of the whole potential shows that there is true minimum along directions (8), (10), and (12) with $\langle H\rangle = \langle H'\rangle = 0$. With $\langle\Sigma\rangle \neq \langle\Sigma'\rangle$, the D_2 is broken spontaneously. The residual $SU(3)'$ symmetry will play an important role, and the hierarchical pattern of Eq. (18) will turn out to be crucial for successful gauge coupling unification (discussed below).

The hierarchical pattern (18), of the GUT symmetry breaking, makes it simple to minimize the potential and analyze the spectrum.

Three extremum conditions, determining v_Σ, $v_{\Sigma'}$, and v_Φ along the directions (8), (10), and (12) and obtained from whole potential, are

$$\begin{aligned}10(30\lambda_1+7\lambda_2)v_\Sigma^2+150\lambda v_{\Sigma'}^2+(10\lambda_{1\Sigma\Phi}-3\lambda_{2\Sigma\Phi})v_\Phi^2&=5M_\Sigma^2,\\ 150\lambda v_\Sigma^2+10(30\lambda_1+7\lambda_2)v_{\Sigma'}^2+(10\lambda_{1\Sigma\Phi}-3\lambda_{2\Sigma\Phi})v_\Phi^2&=5M_\Sigma^2,\\ 3(10\lambda_{1\Sigma\Phi}-3\lambda_{2\Sigma\Phi})(v_\Sigma^2+v_{\Sigma'}^2)+(4\lambda_{1\Phi}+2\lambda_{2\Phi})v_\Phi^2&=M_\Phi^2.\end{aligned} \tag{19}$$

Because of hierarchies (17) and (18), from the first equation of Eq. (19), with a good approximation we obtain

$$v_\Sigma \simeq \frac{M_\Sigma}{\sqrt{2(30\lambda_1+7\lambda_2)}}. \tag{20}$$

Thus, with $2(30\lambda_1+7\lambda_2)\sim 1$, we should have $M_\Sigma \approx M_G$. On the other hand, from the last two equations of Eq. (19), we derive

$$\begin{aligned}v_{\Sigma'}^2 &\simeq \frac{M_\Sigma^2-30\lambda v_\Sigma^2}{2(30\lambda_1+7\lambda_2)},\\ v_\Phi^2 &= \frac{M_\Phi^2-3(10\lambda_{1\Sigma\Phi}-3\lambda_{2\Sigma\Phi})(v_\Sigma^2+v_{\Sigma'}^2)}{4\lambda_{1\Phi}+2\lambda_{2\Phi}}.\end{aligned} \tag{21}$$

To obtain the scales M_I and $M_I{}'$, according to Eqs. (17) and (18), we have to arrange (by price of tunings) $M_\Sigma^2 - 30\lambda v_\Sigma^2 \approx (M_I{}')^2$ and $M_\Phi^2 - 3(10\lambda_{1\Sigma\Phi}-3\lambda_{2\Sigma\Phi})(v_\Sigma^2+v_{\Sigma'}^2) \approx M_I^2$ [with $(4\lambda_{1\Phi}+2\lambda_{2\Phi})\sim 1$].

B. The spectrum

At the first stage of symmetry breaking, the (X,Y) gauge bosons [of $SU(5)$] obtain GUT scale masses. They absorb appropriate states (with quantum numbers of leptoquarks) from the adjoint scalar Σ. The remaining physical fragments $(\Sigma_8,\Sigma_3,\Sigma_1)$ [the $SU(3)$ octet, $SU(2)$ triplet, and a singlet, respectively] receive GUT scale masses. These states are heaviest, and their mixings with other ones can be neglected. From Eq. (7), with Eq. (19) we get

$$M_{\Sigma_8}^2 \simeq 20\lambda_2 v_\Sigma^2, \qquad M_{\Sigma_3}^2 \simeq 80\lambda_2 v_\Sigma^2, \qquad M_{\Sigma_1}^2 \simeq 4M_\Sigma^2. \tag{22}$$

Further, we will not give masses of states that are singlets under all symmetry groups. The mass square of the $SU(3)'$ octet (from Σ') is

$$M_{\Sigma'_{8'}}^2 = 20\lambda_2 v_{\Sigma'}^2 + \frac{6}{5}\lambda_{2\Sigma\Phi}v_\Phi^2. \tag{23}$$

The triplet $\Sigma'_{3'}$ mixes with a real (CP even) $SU(2)_w$ triplet Φ_3 (from Φ). [Both these states are real adjoints of $SU(2)_w$.] The appropriate mass squared couplings are

$$\frac{1}{2}(\Sigma'^i{}_{3'},\Phi_3^i)\begin{pmatrix}4M_{\Sigma'_{8'}}^2-\frac{28}{5}\lambda_{2\Sigma\Phi}v_\Phi^2 & 6\sqrt{2}\lambda_{2\Sigma\Phi}v_\Phi v_{\Sigma'}\\ 6\sqrt{2}\lambda_{2\Sigma\Phi}v_\Phi v_{\Sigma'} & 4\lambda_{2\Phi}v_\Phi^2\end{pmatrix}\begin{pmatrix}\Sigma'^i{}_{3'}\\ \Phi_3^i\end{pmatrix}, \tag{24}$$

where $i=1,2,3$ labels the components of the $SU(2)_w$ adjoint. The CP-odd real $SU(2)_w$ triplet from Φ is absorbed by appropriate gauge fields after $SU(2)\times SU(2)' \to SU(2)_w$ breaking and becomes genuine Goldstone modes.

By the VEVs v_Σ and $v_{\Sigma'}$, the symmetry $SU(5)\times SU(5)'\times D_2$ is broken down to $G_{321}\times G_{321}{}'$ [see Eqs. (9) and (11)]. Thus, between the scales M_I and $M_I{}'$, we have this symmetry, and the $\Phi(5,\bar{5})$ splits into fragments

$$\Phi(5,\bar{5}) = \Phi_{DD'}\oplus\Phi_{DT'}\oplus\Phi_{TT'}\oplus\Phi_{TD'} \tag{25}$$

with transformation properties under $G_{321}\times G_{321}{}'$ given by

$$\begin{aligned}G_{321}\times G_{321}{}'\colon\ \Phi_{DD'} &\sim \left(1,2,-\frac{3}{\sqrt{60}},1,2',\frac{3}{\sqrt{60}}\right), &\quad \Phi_{DT'} &\sim \left(1,2,-\frac{3}{\sqrt{60}},\bar{3}',1,-\frac{2}{\sqrt{60}}\right),\\ \Phi_{TT'} &\sim \left(3,1,\frac{2}{\sqrt{60}},\bar{3}',1,-\frac{2}{\sqrt{60}}\right), &\quad \Phi_{TD'} &\sim \left(3,1,\frac{2}{\sqrt{60}},1,2',\frac{3}{\sqrt{60}}\right).\end{aligned} \tag{26}$$

The masses of these fragments will be denoted by $M_{DD'}, M_{DT'}, M_{TT'}$, and $M_{TD'}$, respectively. Since the breaking $G_{321}\times G_{321}{}'\to G_{\rm SM}\times SU(3)'$ is realized by the VEV of the fragment $\Phi_{DD'}$ at scale M_I, we take $M_{DD'}\simeq M_I$. The state Φ_3, participating in Eq. (24), emerges from this $\Phi_{DD'}$ fragment. The remaining three states under $G_{321}\times SU(3)'$ transform as

$$G_{321}\times SU(3)'\colon \Phi_{DT'}\sim\left(1,2,-\frac{5}{\sqrt{60}},\bar{3}'\right),\qquad \Phi_{TT'}\sim(3,1,0,\bar{3}'),\quad \Phi_{TD'}\sim\left(3,2,\frac{5}{\sqrt{60}},1\right). \tag{27}$$

The mass squares of these fields are given by

$$M^2_{DT'}=5\lambda_{2\Sigma\Phi}v^2_{\Sigma'},\qquad M^2_{TT'}=5\lambda_{2\Sigma\Phi}(v^2_\Sigma+v^2_{\Sigma'})-2\lambda_{2\Phi}v^2_\Phi,\qquad M^2_{TD'}=5\lambda_{2\Sigma\Phi}v^2_\Sigma. \tag{28}$$

With the VEVs toward the directions given in Eqs. (8), (10), and (12), and with the extremum conditions of Eq. (19), the potential's minimum is achieved with

$$\begin{aligned}&30\lambda_1+7\lambda_2>0,\qquad \lambda_2>0,\qquad \lambda>0,\\ &10\lambda_{1\Sigma\Phi}-3\lambda_{2\Sigma\Phi}>0,\qquad \lambda_{2\Sigma\Phi}>0,\\ &2\lambda_{1\Phi}+\lambda_{2\Phi}>0,\qquad \lambda_{2\Phi}>0.\end{aligned} \tag{29}$$

As far as the states H and H' are concerned, they are split as $H\to(D_H,T_H)$ and $H'\to(D_{H'},T_{H'})$, where $D_H,D_{H'}$ are doublets, while T_H and $T_{H'}$ are $SU(3)_c$ and $SU(3)'$ triplets, respectively. Mass squares of these triplets are

$$\begin{aligned}M^2_{T_H}&=M^2_H-4h_1v^2_\Sigma+30(h_2v^2_\Sigma+\tilde{h}v^2_{\Sigma'})\\&\quad+2\lambda_{1H\Phi}v^2_\Phi/\sqrt{25},\\ M^2_{T_{H'}}&=M^2_H-4h_1v^2_{\Sigma'}+30(h_2v^2_{\Sigma'}+\tilde{h}v^2_\Sigma)\\&\quad+2\lambda_{1H\Phi}v^2_\Phi/\sqrt{25}.\end{aligned} \tag{30}$$

The states D_H and $D_{H'}$, under $G_{\rm SM}$, both have quantum numbers of the SM Higgs doublet. They mix by the VEV $\langle\Phi\rangle$, and the mass squared matrix is given by

$$(D_H^\dagger,D_{H'}^\dagger)\begin{pmatrix}M^2_{T_H}-5h_1v^2_\Sigma+\lambda_{2H\Phi}v^2_\Phi/\sqrt{10} & \mu v_\Phi\\ \mu v_\Phi & M^2_{T_{H'}}-5h_1v^2_{\Sigma'}+\lambda_{2H\Phi}v^2_\Phi/\sqrt{10}\end{pmatrix}\begin{pmatrix}D_H\\ D_{H'}\end{pmatrix}. \tag{31}$$

By diagonalization of Eq. (31), we get two physical states h and D':

$$\begin{aligned}h&=\cos\theta_hD_H+\sin\theta_hD_{H'},\\ D'&=-\sin\theta_hD_H+\cos\theta_hD_{H'},\\ \tan2\theta_h&=\frac{2\mu v_\Phi}{M^2_{T_H}-M^2_{T_{H'}}-5h_1(v^2_\Sigma-v^2_{\Sigma'})}.\end{aligned} \tag{32}$$

We identify h with the SM Higgs doublet and set its mass square (by fine-tuning) $M_h^2\sim100\,{\rm GeV}^2$. We assume the second doublet D' to be heavy $M^2_{D'}\gg|M_h|^2$. For the mixing angle θ_h, we also assume $\theta_h\ll1$. Therefore, according to Eq. (32), the SM Higgs mainly resides in D_H (of the H-plet), while $D_{H'}$ (i.e., H') includes a light SM doublet with very suppressed weight.

The radiative corrections will affect obtained expressions for the masses and VEVs. However, there are enough parameters involved, and one can always get presented symmetry breaking pattern and spectrum (given in Table I). Achieving these will require some fine-tunings. Without addressing here the hierarchy problem and naturalness issues, we will proceed to study various properties and the phenomenology of the considered scenario.

III. FERMION SECTOR

A. D_2 symmetry à la P parity

We introduce three families of (Ψ,F) and three families of (Ψ',F'),

$$3\times[\Psi(10,1)+F(\bar{5},1)],\qquad 3\times[\Psi'(1,\overline{10})+F'(1,5)], \tag{33}$$

where in brackets the transformation properties under $SU(5)\times SU(5)'$ gauge symmetry are indicated. Here, each fermionic state is a two-component Weyl spinor, in $(\frac{1}{2},0)$ representation of the Lorentz group. The action of D_2 parity on these fields is determined as

$$D_2\colon\ \Psi\leftrightarrows\bar{\Psi}'\equiv(\Psi')^\dagger,\qquad \mathbf{F}\leftrightarrows\bar{F}'\equiv(F')^\dagger. \tag{34}$$

It is easy to verify that, with transformations in Eqs. (34) and (2), the kinetic part of the Lagrangian $\mathcal{L}_{\rm kin}(\Psi, F, \Psi', F')$ is invariant.[2]

We can easily write down invariant Yukawa Lagrangian

$$\mathcal{L}_Y + \mathcal{L}_{Y'} + \mathcal{L}_Y^{\rm mix} \tag{35}$$

with

$$\mathcal{L}_Y = \sum_{n=0} C_{\Psi\Psi}^{(n)} \left(\frac{\Sigma}{M_*}\right)^n \Psi\Psi H + \sum_{n=0} C_{\Psi F}^{(n)} \left(\frac{\Sigma}{M_*}\right)^n \Psi \mathbf{F} H^\dagger + \text{H.c.} \tag{36}$$

$$\mathcal{L}_{Y'} = \sum_{n=0} C_{\Psi\Psi}^{(n)*} \left(\frac{\Sigma'}{M_*}\right)^n \Psi'\Psi' H'^\dagger + \sum_{n=0} C_{\Psi F}^{(n)*} \left(\frac{\Sigma'}{M_*}\right)^n \Psi' \mathbf{F}' H' + \text{H.c.} \tag{37}$$

$$\mathcal{L}_Y^{\rm mix} = \lambda_{FF'} F \Phi F' + \lambda_{FF'} \overline{F'} \Phi^\dagger \bar{F} + \frac{\lambda_{\Psi\Psi'}}{M} \Psi (\Phi^\dagger)^2 \Psi' + \frac{\lambda_{\Psi\Psi'}}{M} \overline{\Psi'} \Phi^2 \bar{\Psi}, \tag{38}$$

where M_*, M are some cutoff scales. The coupling matrices $\lambda_{FF'}$ and $\lambda_{\Psi\Psi'}$ are Hermitian due to the D_2 symmetry. The last two higher-order operators in Eq. (38), important for phenomenology, can be generated by integrating out some heavy states with mass at or above the GUT scale. For instance, with the scalar state Ω in $(\overline{10}, 10)$ representation of $SU(5) \times SU(5)'$ and D_2 parity, $\Omega \leftrightarrows \Omega^\dagger$, the relevant terms (of fundamental Lagrangian) will be $\lambda_{\Psi\Psi'} \Omega \Psi \Psi' + \lambda_{\Psi\Psi'} \Omega^\dagger \overline{\Psi'} \cdot \bar{\Psi} + \bar{M}_\Omega (\Omega \Phi^2 + \Omega^\dagger (\Phi^\dagger)^2) + M_\Omega^2 \Omega^\dagger \Omega$. With these couplings, one can easily verify that integration of Ω generates the last two operators of Eq. (38) (with $M \approx M_\Omega^2 / \bar{M}_\Omega$). Since the Ω is rather heavy, its only low-energy implication can be the emergence of these effective operators. Thus, in our further studies, we will proceed with the consideration of Yukawa couplings given in Eqs. (36)–(38).

With obvious identifications, let us adopt the following notations for the components from Ψ, F and Ψ', F' states:

$$\Psi = \{q, u^c, e^c\}, \qquad F = \{l, d^c\},$$
$$\Psi' = \{\hat{q}, \hat{u}^c, \hat{e}^c\}, \qquad F' = \{\hat{l}, \hat{d}^c\}. \tag{39}$$

Substituting in Eqs. (36)–(38) the VEVs $\langle \Sigma \rangle$, $\langle \Sigma' \rangle$, and $\langle \Phi \rangle$, the relevant couplings we obtain are

$$\begin{aligned}\mathcal{L}_Y \to\ & q^T Y_U u^c h + q^T Y_D d^c h^\dagger + e^{cT} Y_{e^c l} l h^\dagger \\ & + (C_{qq} qq + C_{u^c e^c} u^c e^c) T_H \\ & + (C_{ql} ql + C_{u^c d^c} u^c d^c) T_H^\dagger + \text{H.c.}\end{aligned} \tag{40}$$

$$\begin{aligned}\mathcal{L}_{Y'} \to\ & C_{\Psi\Psi}^{(0)*} \left(\frac{1}{2} \hat{q}\, \hat{q} + \hat{u}^c \hat{e}^c\right) T_{H'}^\dagger \\ & + C_{\Psi F}^{(0)*} (\hat{q}\, \hat{l} + \hat{u}^c \hat{d}^c) T_{H'} + \text{H.c.} + \cdots\end{aligned} \tag{41}$$

$$\mathcal{L}_Y^{\rm mix} \to \hat{l}^T M_{\hat{l}l} l + e^{cT} M_{e^c \hat{e}^c} \hat{e}^c + \text{H.c.} \tag{42}$$

In Eq. (41) we have dropped out the couplings with the Higgs doublet because, as we have assumed, $D_{H'}$ includes the SM Higgs doublet with very suppressed weight. Also, we have ignored powers of $\langle \Sigma' \rangle / M_*$ in comparison with $\langle \Sigma \rangle / M_*$'s exponents. As we will see, the couplings of h in Eq. (40) and terms shown in Eqs. (41) and (42) are responsible for fermion masses and mixings and lead to realistic phenomenology.

B. Fermion masses and mixings: Composite leptons

Let us first indicate transformation properties of all matter states, given in Eq. (39), under the unbroken $G_{\rm SM} \times SU(3)' = SU(3)_c \times SU(2)_w \times U(1)_Y \times SU(3)'$ gauge symmetry. Fragments from Ψ, F transform as

$$\begin{aligned} & q \sim \left(3, 2, -\frac{1}{\sqrt{60}}, 1\right), \qquad u^c \sim \left(\bar{3}, 1, \frac{4}{\sqrt{60}}, 1\right), \\ & e^c \sim \left(1, 1, -\frac{6}{\sqrt{60}}, 1\right), \qquad l \sim \left(1, 2, \frac{3}{\sqrt{60}}, 1\right), \\ & d^c \sim \left(\bar{3}, 1, -\frac{2}{\sqrt{60}}, 1\right), \end{aligned} \tag{43}$$

while the states from Ψ', F' have the following transformation properties:

[2]The D_2 transformation of Eq. (34) resembles usual P parity, acting between the electron and positron, within QED. Unlike the QED, the states (Ψ, F) and $(\bar{\Psi}', F')$ transform under different gauge groups.

$$\hat{q}\sim\left(1,2,\frac{1}{\sqrt{60}},\bar{3}'\right),\qquad \hat{u}^c\sim\left(1,1,-\frac{4}{\sqrt{60}},3'\right),$$
$$\hat{e}^c\sim\left(1,1,\frac{6}{\sqrt{60}},1\right),\qquad \hat{l}\sim\left(1,2,-\frac{3}{\sqrt{60}},1\right),$$
$$\hat{d}^c\sim\left(1,1,\frac{2}{\sqrt{60}},3'\right). \tag{44}$$

In transformation properties of Eq. (44), by primes we have indicated triplets and antitriplets of $SU(3)'$. As we see, transformation properties of quark states in Eq. (43) coincide with those of the SM. Therefore, for quark masses and CKM mixings, the first two couplings of Eq. (40) are relevant. Since in $Y_{U,D}$ and $Y_{e^c l}$ contribute also higher-dimensional operators, the Y_U is not symmetric and $Y_D\neq Y_{e^c l}$. Thus, quark Yukawa matrices can be diagonalized by biunitary transformations

$$L_u^\dagger Y_U R_u=Y_U^{\rm Diag},\qquad L_d^\dagger Y_D R_d=Y_D^{\rm Diag}. \tag{45}$$

With these, the CKM matrix (in standard parametrization) is

$$V_{\rm CKM}=P_1L_u^TL_d^*P_2\qquad \text{with}\quad P_1={\rm Diag}(e^{i\omega_1},e^{i\omega_2},e^{i\omega_3}),$$
$$P_2={\rm Diag}(e^{i\rho_1},e^{i\rho_2},1). \tag{46}$$

1. Composite leptons

Turning to the lepton sector, we note that $\hat{l}$ and $\hat{e}^c$ have opposite/conjugate transformation properties with respect to l and e^c, respectively. From couplings in Eq. (42), we see that these vectorlike states acquire masses $M_{\hat{l}l}$ and $M_{e^c\hat{e}^c}$ and decouple. However, within this scenario, composite leptons emerge. The $SU(3)'$ becomes strongly coupled and confines at scale $\Lambda'\sim$TeV (for details, see Sec. IV). Because of confinement, $SU(3)'$ singlet composite states—baryons (B') and/or mesons (M')—can emerge. The elegant idea of fermion emergence through the strong dynamics as bound states of more fundamental constituents was suggested and developed in Refs. [12–22]. Within our scenario, this idea finds an interesting realization for the lepton states. Formation of composite fermions should satisfy 't Hooft anomaly matching conditions[3] [14]. These give a severe constraint on building models with composite fermions [16–18], [20–22].

Let us focus on the sector of (three-family) $\hat{q},\hat{u}^c$ and $\hat{d}^c$ states, which have $SU(3)'$ strong interactions. Ignoring local EW and Yukawa interactions, the Lagrangian of these states possesses global $G_f^{(6)}=SU(6)_L\times SU(6)_R\times U(1)_{B'}$ chiral symmetry. Under the $SU(6)_L$, three families of $\hat{q}=(\hat{u},\hat{d})$ transform as sextet 6_L, while three families of $(\hat{u}^c,\hat{d}^c)\equiv\hat{q}^c$ form sextet 6_R of $SU(6)_R$. The $U(1)_{B'}$ (B') charges of $\hat{q}$ and $\hat{q}^c$ are, respectively, $1/3$ and $-1/3$. Thus, transformation properties of these states under

$$G_f^{(6)}=SU(6)_L\times SU(6)_R\times U(1)_{B'} \tag{47}$$

chiral symmetry are

$$\hat{q}_\alpha=(\hat{u},\hat{d})_\alpha\sim\left(6_L,1,\frac{1}{3}\right),$$
$$\hat{q}^c_\alpha=(\hat{u}^c,\hat{d}^c)_\alpha\sim\left(1,6_R,-\frac{1}{3}\right), \tag{48}$$

where $\alpha=1,2,3$ is the family index. Because of the strong $SU(3)'$ attractive force, condensates that will break the $G_f^{(6)}$ chiral symmetry can form. The breaking can occur by several steps, and at each step the formed composite states should satisfy anomaly matching conditions.

In Appendix A, we give a detailed account of these issues and demonstrate that within our scenario three families of l_0,e_0^c,ν_0^c composite states,

$$(\hat{q}\,\hat{q})\hat{q}\sim l_{0\alpha}=\begin{pmatrix}\nu_0\\ e_0\end{pmatrix}_\alpha,\qquad (\hat{q}^c\hat{q}^c)\hat{q}^c=((\hat{u}^c\hat{d}^c)\hat{d}^c,(\hat{u}^c\hat{d}^c)\hat{u}^c)\sim l^c_{0\alpha}\equiv(\nu_0^c,e_0^c)_\alpha,\qquad \alpha=1,2,3, \tag{49}$$

emerge. In Eq. (49), for combinations $(\hat{q}\,\hat{q})\hat{q}$ and $(\hat{q}^c\hat{q}^c)\hat{q}^c$, the spin-1/2 states are assumed with suppressed gauge and/or flavor indices. For instance, under $(\hat{q}\,\hat{q})\hat{q}$ we mean $\epsilon^{a'b'c'}\epsilon_{ij}(\hat{q}_{a'i}\hat{q}_{b'j})\hat{q}_{c'k}$, where $a',b',c'=1,2,3$ are $SU(3)'$ indices and $i,j,k=1,2$ stand for $SU(2)_w$ (or $SU(2)_L$) indices. Thus, $(\hat{q}\,\hat{q})\hat{q}$ and $(\hat{q}^c\hat{q}^c)\hat{q}^c$ are singlets of $SU(3)'$. From these, taking into account Eqs. (44) and (49), it is easy to verify that the quantum numbers of composite states under SM gauge group $G_{\rm SM}=SU(3)_c\times SU(2)_w\times U(1)_Y$ are

$$G_{\rm SM}:\ l_0\sim\left(1,2,\frac{3}{\sqrt{60}}\right),\qquad e_0^c\sim\left(1,1,-\frac{6}{\sqrt{60}}\right),$$
$$\nu_0^c\sim(1,1,0). \tag{50}$$

[3] In case the chiral symmetry remains unbroken (at least partially) at the composite level. The models avoiding anomaly conditions were suggested in Ref. [19].

As we see, along with SM leptons (l_0 and e_0^c), we get three families of composite SM singlets fermions-ν_0^c. The latter can be treated as composite right-handed/sterile neutrinos in the spirit of Ref. [23]. Note that, with this composition, as was expected, the gauge anomalies also vanish (together with the chiral anomaly matching; for details, see Appendix A). Interestingly, the $SU(3)'$ [originating from $SU(5)'$] triplet and antitriplets $\hat{u}^c, \hat{d}^c$, and $\hat{q}$ play the role of "preon" constituents for the bound-state leptons and right-handed/sterile neutrinos. Moreover, in our scheme the lepton number L is related to the $U(1)_{B'}$ charge as $L = 3B'$. Therefore, "primed baryon number" B' [of the $SU(5)'$] is the origin of the lepton number.

2. Charged lepton masses

Now, we turn to the masses of the charged leptons, which are composite within our scenario. As it turns out, their mass generation does not require additional extension. It happens via integration of the states that are present in the model. As we see from Eq. (41), the $SU(5)'$ matter couples with the $SU(3)'$ triplet scalar $T_{H'}$ with mass $M_{T_{H'}}$. Relevant 4-fermion operators, emerging from the couplings of Eq. (41) and by integration of $T_{H'}$, are

$$\mathcal{L}_{Y'}^{\rm eff} = \frac{C_{\Psi\Psi}^{(0)*} C_{\Psi F}^{(0)*}}{M_{T_{H'}}^2} \left[\frac{1}{2} (\hat{q}\,\hat{q})(\hat{q}\,\hat{l}) + (\hat{u}^c \hat{e}^c)(\hat{u}^c \hat{d}^c) \right] + \text{H.c.} \tag{51}$$

As we see, here appear the combinations $(\hat{q}\,\hat{q})\hat{q}$ and $(\hat{u}^c \hat{d}^c)\hat{u}^c$, which according to Eq. (49) form composite charged lepton states. We will use the parametrizations

$$\frac{1}{2}(\hat{q}_\alpha \hat{q}_\beta)\hat{q}_\gamma = \Lambda'^3 c_{\alpha\beta\gamma\delta} l_{0\delta}, \qquad (\hat{u}_\alpha^c \hat{d}_\beta^c)\hat{u}_\gamma^c = \Lambda'^3 \bar{c}_{\alpha\beta\gamma\delta} e_{0\delta}^c, \tag{52}$$

where Greek indices denote family indices and $c, \bar{c}$ are dimensionless couplings—four index tensors in a family space. The $(l_0, e_0^c)_\delta$ denote three families of composite leptons. Using Eq. (52) in Eq. (51), we obtain

$$\mathcal{L}_{Y'}^{\rm eff} \to \hat{l}\,\hat{\mu}\, l_0 + e_0^c \tilde{\mu} \hat{e}^c + \text{H.c.}$$
$$\text{with} \quad \hat{\mu}_{\delta'\delta} \equiv \frac{\Lambda'^3}{M_{T_{H'}}^2} (C_{\Psi\Psi}^{(0)*})_{\alpha\beta} (C_{\Psi F}^{(0)*})_{\gamma\delta'} c_{\alpha\beta\gamma\delta},$$
$$\tilde{\mu}_{\delta\delta'} \equiv \frac{\Lambda'^3}{M_{T_{H'}}^2} (C_{\Psi\Psi}^{(0)*})_{\gamma\delta'} (C_{\Psi F}^{(0)*})_{\alpha\beta} \bar{c}_{\alpha\beta\gamma\delta}. \tag{53}$$

At the next stage, we integrate out the vectorlike states $\hat{l}, l$ and $e^c, \hat{e}^c$, which, respectively, receive masses $M_{\hat{l}l}$ and $M_{e^c\hat{e}^c}$ through the coupling in Eq. (42). Integrating out these heavy states, from Eqs. (42) and (53), we get

$$l \simeq -\frac{1}{M_{\hat{l}l}} \hat{\mu} l_0, \qquad e^{cT} \simeq -e_0^{cT} \tilde{\mu} \frac{1}{M_{e^c\hat{e}^c}}. \tag{54}$$

Substituting these in the $e^{cT} Y_{e^c l} l h^\dagger$ coupling of Eq. (40), we see that the effective Yukawa couplings for the leptons are generated:

$$l_0^T Y_E e_0^c h^\dagger + \text{H.c.} \quad \text{with} \quad Y_E^T \simeq \tilde{\mu} \frac{1}{M_{e^c\hat{e}^c}} Y_{e^c l} \frac{1}{M_{\hat{l}l}} \hat{\mu}. \tag{55}$$

The diagram corresponding to the generation of this effective Yukawa operator is shown in Fig. 1. This mechanism is novel and differs from those suggested earlier for the mass generation of composite fermions [22]. From the observed values of the Yukawa couplings, we have $|\text{Det} Y_E| = \lambda_e \lambda_\mu \lambda_\tau \approx 1.8 \times 10^{-11}$. On the other hand, natural values of the eigenvalues of $Y_{e^c l}$ can be ~ 0.1. Thus, $|\text{Det} Y_{e^c l}| \sim 10^{-3}$. From these and the expression given in Eq. (55), we obtain

$$\left| \text{Det}\left(\tilde{\mu} \frac{1}{M_{e^c\hat{e}^c}} \right) \right| \cdot \left| \text{Det}\left(\frac{1}{M_{\hat{l}l}} \hat{\mu} \right) \right| \sim 10^{-8}, \tag{56}$$

the constraint that should be satisfied by two matrices $\tilde{\mu} \frac{1}{M_{e^c\hat{e}^c}}$ and $\frac{1}{M_{\hat{l}l}} \hat{\mu}$.

3. Neutrino masses

Now, we discuss the neutrino mass generation. To accommodate the neutrino data [4], one can use SM singlet fermionic states in order to generate either Majorana- or Dirac-type masses for the neutrinos. Within our model, among the composite fermions, we have SM singlets ν_0^c [see Eqs. (49) and (50)]. Here, we stick to the possibility of the Dirac-type neutrino masses, which can be naturally suppressed [23]. Because of compositeness, there is no direct Dirac couplings Y_ν of ν_0^c's with lepton doublets l_0. Similar to the charged lepton Yukawa couplings, we need to generate Y_ν. For this purpose, we introduce the $SU(5) \times SU(5)'$ singlet (two-component) fermionic states N.[4] Assigning the D_2 parity transformations $N \leftrightarrows \bar{N}$ and taking into account Eqs. (5) and (34), relevant couplings, allowed by $SU(5) \times SU(5)' \times D_2$ symmetry, will be

$$\mathcal{L}_N = C_{FN} F N H + C_{FN}^* F' N H'^\dagger - \frac{1}{2} N^T M_N N + \text{H.c.}$$
$$\text{with} \quad M_N = M_N^*. \tag{57}$$

These give the following interaction terms:

$$\mathcal{L}_N \to C_{FN} l N h + C_{FN}^* \hat{d}^c N T_{H'}^\dagger - \frac{1}{2} N^T M_N N + \text{H.c.} \tag{58}$$

[4]The number of N states is not limited, but for simplicity we can assume that they are not more than 3.

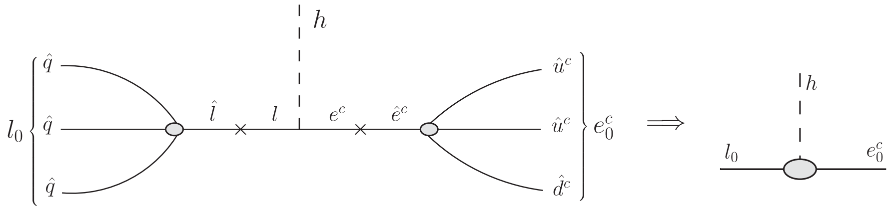

FIG. 1. Diagram responsible for the generation of the charged lepton effective Yukawa matrix.

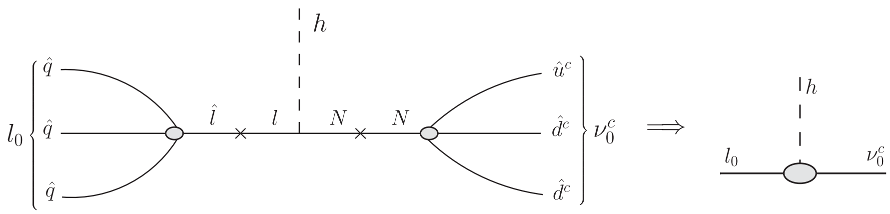

FIG. 2. Diagram responsible for the generation of the effective Dirac Yukawa matrix for the neutrinos.

From these and Eq. (41), integration of $T_{H'}$ state gives the additional affective four-fermion operator

$$\frac{C^{(0)*}_{\Psi F}C^{*}_{FN}}{M^2_{T_{H'}}}(\hat{u}^c\hat{d}^c)(\hat{d}^cN)+\text{H.c.} \qquad (59)$$

By the parametrization

$$(\hat{u}^c_\alpha\hat{d}^c_\beta)\hat{d}^c_\gamma=\Lambda'^3\tilde{c}_{\alpha\beta\gamma\delta}\nu^c_{0\delta}, \qquad (60)$$

operators in Eq. (59) are given by

$$\mathcal{L}^{\text{eff}}_{N\nu^c}=N\mu_\nu\nu^c_0+\text{H.c.}$$
$$\text{with}\quad (\mu_\nu)_{\delta'\delta}\equiv\frac{\Lambda'^3}{M^2_{T_{H'}}}(C^{(0)*}_{\Psi F})_{\alpha\beta}(C^{*}_{FN})_{\gamma\delta'}\tilde{c}_{\alpha\beta\gamma\delta}. \qquad (61)$$

Subsequent integration of N states, from Eq. (61) and the last term of Eq. (58) gives

$$N\simeq\frac{1}{M_N}\mu_\nu\nu^c_0. \qquad (62)$$

Substituting this, and the expression of l from Eq. (54), in the first term of Eq. (58), we arrive at

$$l^T_0Y_\nu\nu^c_0h+\text{H.c.}\quad\text{with}\quad Y_\nu\simeq-\hat{\mu}^T\frac{1}{M^T_{\hat{l}l}}C_{FN}\frac{1}{M_N}\mu_\nu. \qquad (63)$$

The relevant diagram generating this effective Dirac Yukawa couplings is given in Fig. 2. With $\frac{1}{M_{\hat{l}l}}\hat{\mu}\sim10^{-2}$ and $C_{FN}\sim M_N\sim\frac{1}{M_N}\mu_\nu\sim10^{-5}$, we can get the Dirac neutrino mass $M^D_\nu=Y_\nu\langle h^{(0)}\rangle\sim0.1$ eV, which is the right scale to explain neutrino anomalies. Note that using Eq. (62) in the last term of Eq. (58) we also obtain the term $-\frac{1}{2}\nu^{cT}_0M_{\nu^c}\nu^c_0$ with $M_{\nu^c}\simeq\mu^T_\nu\frac{1}{M_N}\mu_\nu$. By proper selection of the couplings C_{FN} and eigenvalues of M_N, the M_{ν^c} can be strongly suppressed. In this case, the neutrinos will be (quasi) Dirac. However, it is possible that some of the species of light neutrinos to be (quasi) Dirac and some of them Majoranas. Detailed studies of such scenarios and their compatibilities with current experiments [24] are beyond the scope of this paper.

IV. GAUGE COUPLING UNIFICATION

In this section we will study the gauge coupling unification within our model. We show that the symmetry breaking pattern gives the possibility for successful unification.[5] As it turns out, the $SU(3)'$ gauge interaction becomes strongly coupled at scale Λ'(∼few TeV). Thus, below this scale, $SU(3)'$ confines, and all states (including composite ones) are $SU(3)'$ singlets. Therefore, with the masses $M^{(\alpha)}_{\hat{l}l}$ and $M^{(\alpha)}_{e^c\hat{e}^c}$ ($\alpha=1,2,3$) of vectorlike states $l,\hat{l}$ and $e^c,\hat{e}^c$ being above the scale Λ', in the energy interval $\mu=M_Z-\Lambda'$, the states are just those of SM (plus possibly right-handed/sterile neutrinos having no impact on gauge coupling running), and corresponding one-loop β-function coefficients are $(b_Y,b_w,b_c)=(\frac{41}{10},-\frac{19}{6},-7)$. Since Λ' is the characteristic scale of the strong dynamics, it is clear that pseudo-Goldstone and composite states (besides SM leptons) emerging through chiral symmetry breaking and strong dynamics can have masses below Λ' (in a certain range). Instead, investigating their spectrum and dealing with corresponding threshold effects, we parametrize all

[5]Possibilities of gauge coupling unification, with the intermediate symmetry breaking pattern and without invoking low-scale supersymmetry, have been studied in Ref. [25].

TABLE I. Particle spectroscopy.

M_a	GeV	M_a	GeV	M_a	GeV	M_a	GeV	M_a	GeV
$M^{(1)}_{\hat{l}l}$	7.54×10^4	$M^{(2)}_{e^c\hat{e}^c}$	7.54×10^4	$M_{D'}$	4.16×10^6	$M_{TD'}$	3.92×10^6	$M_{X'}$	2.08×10^6
$M^{(2)}_{\hat{l}l}$	7.54×10^4	$M^{(3)}_{e^c\hat{e}^c}$	1.2×10^5	$M_{TT'}$	1874.7	$M_{\Sigma'_{8'}}$	9277	M_{T_H}	5×10^{11}
$M^{(3)}_{\hat{l}l}$	1.2×10^5	Λ'	1851	$M_{DD'}$	8.25×10^4	$M_{\Sigma'_{3'}}$	$2M_{\Sigma'_{8'}}$	M_X	4.95×10^{11}
$M^{(1)}_{e^c\hat{e}^c}$	7.54×10^4	$M_{T_{H'}}$	1851	$M_{DT'}$	8250	$M_{\Sigma'_{1'}}$	4.16×10^6	M_Σ	5×10^{11}

these as a single effective Λ' scale, below which theory is the SM. This phenomenological simplification allows us to proceed with RG analysis. Note, however, that even with taking those kinds of thresholds into account should not harm the success of coupling unification with the price of proper adjustment of the mass scales (given in Table I and discussed later on).

In the energy interval $\Lambda' - M_I$, we have the symmetry $SU(3)_c \times SU(2)_w \times U(1)_Y \times SU(3)'$, and $SU(3)'$ non-singlet states (i.e., $\hat{q}, \hat{u}^c, \hat{d}^c$, $T_{H'}$, etc.) must be taken into account. As was noted in Sec. II, we consider hierarchical breaking: $M_I \ll M_I{}' \ll M_G$ [see Eqs. (17) and (18)]. This choice allows us to have successful unification with confining scale $\Lambda' \sim$ few TeV.[6] Thus, between the scales M_I and $M_I{}'$, the symmetry is $G_{321} \times G_{321}{}'$ [see Eqs. (9) and (11)], and states should be decomposed under these groups [see, for instance, Eqs. (25) and (26)]. Since the breaking $G_{321} \times G_{321}{}' \to G_{\rm SM} \times SU(3)'$ is realized by the VEV of the fragment $\Phi_{DD'}$ at scale M_I, we take $M_{DD'} \simeq M_I$. The remaining three masses, of the fragments coming from Φ, can be in a range $\Lambda' - M_G$. Giving more detailed account to these issues in Appendix B, below we sketch the main details.

Above the scale M_I, all matter states should be included in the RG. Above the scale $M_I{}'$, we have the $SU(5)'$ symmetry, and the fragments $\Phi_{DD'}, \Phi_{DT'}$ form the unified $(2,\bar{5})$-plet of $G_{321} \times SU(5)'$: $(\Phi_{DD'}, \Phi_{DT'}) \subset \Phi_{D\bar{5}'}$, while $\Phi_{TT'}$ and $\Phi_{TD'}$ states unify in $(3,\bar{5})$-plet: $(\Phi_{TT'}, \Phi_{TD'}) \subset \Phi_{T\bar{5}'}$. These states, together with the Σ'-plet, should be included in the RG above the scale $M_I{}'$.

According to Eq. (16), at scale M_I, for the EW gauge couplings, we have the boundary conditions

$$\alpha_Y^{-1}(M_I) = \alpha_1^{-1}(M_I) + \alpha_{1'}^{-1}(M_I),$$
$$\alpha_w^{-1}(M_I) = \alpha_2^{-1}(M_I) + \alpha_{2'}^{-1}(M_I). \qquad (64)$$

The couplings of $G_{321}{}'$ gauge interactions unify and form single $SU(5)'$ coupling at scale $M_I{}'$:

$$\alpha_{1'}(M_I{}') = \alpha_{2'}(M_I{}') = \alpha_{3'}(M_I{}') = \alpha_{5'}(M_I{}'). \qquad (65)$$

Finally, at the GUT scale M_G, the coupling of G_{321} and $SU(5)'$ unifies:

$$\alpha_1(M_G) = \alpha_2(M_G) = \alpha_3(M_G) = \alpha_{5'}(M_G) \equiv \alpha_G. \qquad (66)$$

With solutions (B5) and (B6) of RG equations at corresponding energy scales, and taking into account the boundary conditions (64)–(66), we derive

$$\begin{pmatrix} (b_1^{IG} - b_Y^{ZI} + b_{3'}^{\Lambda' I}), & -b_1^{IG}, & (b_{3'}^{II'} - b_{1'}^{II'}), & -2\pi \\ (b_2^{IG} - b_w^{ZI} + b_{3'}^{\Lambda' I}), & -b_2^{IG}, & (b_{3'}^{II'} - b_{2'}^{II'}), & -2\pi \\ (b_3^{IG} - b_c^{ZI}), & -b_3^{IG}, & 0, & -2\pi \\ (b_{5'}^{I'G} - b_{3'}^{\Lambda' I}), & -b_{5'}^{I'G}, & (b_{5'}^{I'G} - b_{3'}^{II'}), & -2\pi \end{pmatrix} \begin{pmatrix} \ln\frac{M_I}{M_Z} \\ \ln\frac{M_G}{M_Z} \\ \ln\frac{M_I{}'}{M_I} \\ \alpha_G^{-1} \end{pmatrix} = \begin{pmatrix} 2\pi(\alpha_{3'}^{-1}(\Lambda') - \alpha_Y^{-1}) + b_{3'}^{\Lambda' I}\ln\frac{\Lambda'}{M_Z} \\ 2\pi(\alpha_{3'}^{-1}(\Lambda') - \alpha_w^{-1}) + b_{3'}^{\Lambda' I}\ln\frac{\Lambda'}{M_Z} \\ -2\pi\alpha_c^{-1} \\ -2\pi\alpha_{3'}^{-1}(\Lambda') - b_{3'}^{\Lambda' I}\ln\frac{\Lambda'}{M_Z} \end{pmatrix}, \qquad (67)$$

where on the right-hand side of this equation the couplings $\alpha_{Y,w,c}$ are taken at scale M_Z. The factors $b_i^{\mu_a\mu_b}$ (like b_1^{IG}, $b_{3'}^{\Lambda' I}$, etc.) stand for effective b factors corresponding to the energy interval $\mu_a - \mu_b$ and can also include two-loop effects. All expressions and details are given in Appendix B.

[6]One can have unification with $\langle\Sigma'\rangle = 0$, (i.e., $M_I = M_I{}'$) and with a modified spectrum. However, with such a choice, the value of Λ' comes out rather large ($\gtrsim 10^5$ GeV). This would also imply the breaking of EW symmetry at a high scale and thus should be discarded from the phenomenological viewpoint. More discussion about this issue is given in Sec. VI.

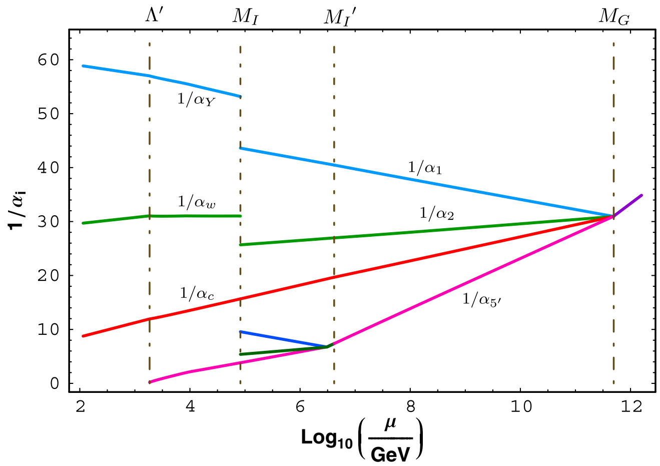

FIG. 3 (color online). Gauge coupling unification. $\{\Lambda', M_I, M_I', M_G\} \simeq \{1800, 8.25\times10^4, 4.16\times10^6, 4.95\times10^{11}\}$ GeV and $\alpha_G(M_G) \simeq 1/31$.

From Eq. (67) we can calculate $\{M_I, M_G, M_I', \alpha_G\}$ in terms of the remaining inputs. For instance, a phenomenologically viable scenario is obtained when $SU(3)'$ confines at scale $\Lambda' \sim 1$ TeV. Thus, we will take $\Lambda' \sim 1$ TeV and $\alpha_{3'}^{-1}(\Lambda') \ll 1$. In Table I we give selected input mass scales, leading to successful unification with

$$\{M_I, M_I', M_G\} \simeq \{8.25\times10^4, 4.16\times10^6, 4.95\times10^{11}\}\ \mathrm{GeV},\quad \alpha_G \simeq 1/31. \tag{68}$$

The corresponding picture of gauge coupling running is given in Fig. 3. This result is obtained by solving RGs in the two-loop approximation. More details, including one- and two-loop RG factors at each relevant mass scale, are given in Appendix B.

V. NUCLEON STABILITY

In this section we show that, although the GUT scale M_G is relatively low (close to 5×10^{11} GeV), the nucleon's lifetime can be compatible with current experimental bounds. In achieving this, a crucial role is played by lepton compositeness, because leptons have no direct couplings with X, Y gauge bosons of $SU(5)$. The baryon number violating $d=6$ operators, induced by integrating out of the X, Y bosons, are

$$\frac{g_X^2}{M_X^2}(\overline{u^c}_a\gamma_\mu q_b^i)(\overline{d^c}_c\gamma^\mu l^j)\epsilon^{abc}\epsilon_{ij},$$
$$\frac{g_X^2}{M_X^2}(\overline{u^c}_a\gamma_\mu q_b^i)(\overline{e^c}\gamma^\mu q_c^j)\epsilon^{abc}\epsilon_{ij}, \tag{69}$$

where g_X is the $SU(5)$ gauge coupling at scale M_X (the mass of the X, Y states). According to Eq. (54), the states l, e^c contain light leptons l_0, e_0^c. Using this and going to the mass eigenstate basis [with Eqs. (45) and (46)], from Eq. (69), we get operators

$$\mathcal{O}_{d6}^{(e^c)} = \frac{g_X^2}{M_X^2}C_{\alpha\beta}^{(e^c)}(\overline{u^c}\gamma_\mu u)(\overline{e^c_\alpha}\gamma^\mu d_\beta),$$
$$\mathcal{O}_{d6}^{(e)} = \frac{g_X^2}{M_X^2}C_{\alpha\beta}^{(e)}(\overline{u^c}\gamma_\mu u)(\overline{d^c}_\beta\gamma^\mu e_\alpha),$$
$$\mathcal{O}_{d6}^{(\nu)} = \frac{g_X^2}{M_X^2}C_{\alpha\beta\gamma}^{(\nu)}(\overline{u^c}\gamma_\mu d_\alpha)(\overline{d^c}_\beta\gamma^\mu \nu_\gamma), \tag{70}$$

with

$$C_{\alpha\beta}^{(e^c)} = (R_u^\dagger L_u^*)_{11}\left(R_e^\dagger\tilde{\mu}^*\frac{1}{M^*_{e^c\hat{e}^c}}L_u^*P_1^*V_{\mathrm{CKM}}\right)_{\alpha\beta} + (R_u^\dagger L_u^* P_1^* V_{\mathrm{CKM}})_{1\beta}\left(R_e^\dagger\tilde{\mu}^*\frac{1}{M^*_{e^c\hat{e}^c}}L_u^*\right)_{\alpha1},$$
$$C_{\alpha\beta}^{(e)} = (R_u^\dagger L_u^*)_{11}\left(R_d^\dagger\frac{1}{M_{\hat{l}l}}\hat{\mu}L_e^*\right)_{\beta\alpha},$$
$$C_{\alpha\beta\gamma}^{(\nu)} = (R_u^\dagger L_u^* P_1^* V_{\mathrm{CKM}})_{1\alpha}\left(R_d^\dagger\frac{1}{M_{\hat{l}l}}\hat{\mu}L_e^*\right)_{\beta\gamma}, \tag{71}$$

where in Eq. (70) we have suppressed the color indices. Similar to quark Yukawa matrices, the charged lepton Yukawa matrix has been diagonalized by transformation $L_e^\dagger Y_E R_e = Y_E^{\mathrm{Diag}}$. All fields in Eq. (70), are assumed to denote mass eigenstates. We have ignored the neutrino masses (having no relevance for the nucleon decay) and rotated the neutrino flavors $\nu_0 = L_e^*\nu$ similar to the left-handed charged leptons $e_0 = L_e^* e$.

As we will show now, with proper selection of appropriate parameters (such as $\tilde{\mu}\frac{1}{M_{e^c\hat{e}^c}}$, $\frac{1}{M_{\hat{l}l}}\hat{\mu}$ and/or corresponding entries in some of unitary matrices), appearing in Eq. (71), we can adequately suppress nucleon decays within our model.[7] Upon the selection of parameters, the constraint

[7]The importance of flavor dependence in $d=6$ nucleon decay was discussed in Refs. [26] and [27]. As was shown [27], in specific circumstances, within GUTs one can suppress or even completely rotate away the $d=6$ nucleon decays.

(56) must be satisfied in order to obtain observed values of charged fermion masses. Introducing the notations

$$R_u^\dagger L_u^* \equiv \mathcal{U}, \qquad R_d^\dagger \frac{1}{M_{\hat{l}l}} \hat{\mu} L_e^* \equiv \mathcal{L}, \qquad R_e^\dagger \tilde{\mu}^* \frac{1}{M^*_{e^c \hat{e}^c}} L_u^* \equiv \mathcal{R}, \tag{72}$$

the couplings in Eq. (71) can be rewritten as

$$\mathcal{C}^{(e^c)}_{\alpha\beta} = \mathcal{U}_{11}(\mathcal{R}P_1^* V_{\rm CKM})_{\alpha\beta} + (\mathcal{U}P_1^* V_{\rm CKM})_{1\beta}(\mathcal{R})_{\alpha 1},$$
$$\mathcal{C}^{(e)}_{\alpha\beta} = \mathcal{U}_{11}\mathcal{L}_{\beta\alpha}, \qquad \mathcal{C}^{(\nu)}_{\alpha\beta\gamma} = (\mathcal{U}P_1^* V_{\rm CKM})_{1\alpha}\mathcal{L}_{\beta\gamma}. \tag{73}$$

Since the matrices $\mathcal{U}$, $\mathcal{L}$, and $\mathcal{R}$ are not fixed yet, for their structures we will make the selection

$$\mathcal{U}_{11} = 0, \qquad \mathcal{L} = \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \qquad \mathcal{R} = \begin{pmatrix} 0 & \times & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \tag{74}$$

where $\times$ stands for some nonzero entry. With this structure we see that for $\alpha, \beta = 1, 2$ we have $\mathcal{C}^{(e^c)}_{\alpha\beta} = \mathcal{C}^{(e)}_{\alpha\beta} = 0$, and therefore nucleon decays with emission of the charged leptons do not take place. With one more selection, we will be able to eliminate some nucleon decay modes (but not all) with neutrino emissions. We can impose one more condition, involving $\mathcal{U}_{12}$ and $\mathcal{U}_{13}$ entries of $\mathcal{U}$, in such a way as to have $(\mathcal{U}P_1^* V_{\rm CKM})_{11} = 0$. The latter, in expanded form, reads

$$(\mathcal{U}P_1^* V_{\rm CKM})_{11} = \mathcal{U}_{12}e^{-i\omega_2}V_{cd} + \mathcal{U}_{13}e^{-i\omega_3}V_{td} = 0,$$
$$\Rightarrow \mathcal{U}_{12}e^{-i\omega_2} = -\frac{V_{td}}{V_{cd}}\mathcal{U}_{13}e^{-i\omega_3} \tag{75}$$

and leads to $\mathcal{C}^{(\nu)}_{12\gamma} = \mathcal{C}^{(\nu)}_{11\gamma} = 0$. Thus, the decays $p \to \bar{\nu}\pi^+, n \to \bar{\nu}\pi^0, n \to \bar{\nu}\eta$ do not take place. Nonvanishing relevant $\mathcal{C}^{(\nu)}$ couplings are $\mathcal{C}^{(\nu)}_{21\gamma}$, which, taking into account Eqs. (74) and (75), are

$$\mathcal{C}^{(\nu)}_{21\gamma} = (\mathcal{U}P_1^* V_{\rm CKM})_{12}\epsilon_\gamma = \epsilon_\gamma \mathcal{U}_{13}e^{-i\omega_3}\frac{V_{ts}V_{cd} - V_{td}V_{cs}}{V_{cd}} \simeq \epsilon_\gamma \mathcal{U}_{13}e^{-i\omega_3}\frac{s_{13}e^{i\delta}}{V_{cd}}, \tag{76}$$

where in last step we have used standard parametrization of the CKM matrix. Since the matrix $\mathcal{U}$ is unitary, due to selection $\mathcal{U}_{11} = 0$ and the unitarity condition, we will have $|\mathcal{U}_{12}|^2 + |\mathcal{U}_{13}|^2 = 1$. With this, by Eq. (75) and using central values [28] of CKM matrix elements, we obtain $|\mathcal{U}_{12}| \simeq 0.038, |\mathcal{U}_{13}| \simeq 1$ and $|\frac{s_{13}}{V_{cd}}| = |\frac{V_{ub}}{V_{cd}}| \simeq 1.56 \times 10^{-2}$. These give $|\mathcal{C}^{(\nu)}_{21\gamma}| \simeq 1.56 \times 10^{-2}|\epsilon_\gamma|$. Taking into account all this, for expressions of $p \to \bar{\nu}K^+$ and $n \to \bar{\nu}K^0$ decay widths, we obtain [29]

$$\Gamma(p \to \bar{\nu}K^+) \simeq \Gamma(n \to \bar{\nu}K^0) = \frac{(m_p^2 - m_K^2)^2}{32\pi f_\pi^2 m_p^3}\left(1 + \frac{m_p}{3m_B}(D + 3F)\right)^2 \left(\frac{g_X}{M_X^2}A_R|\alpha_H|\right)^2 \cdot 2.43 \times 10^{-4}\sum_{\gamma=1}^{3}|\epsilon_\gamma|^2, \tag{77}$$

where $|\alpha_H| = 0.012\ \mathrm{GeV}^3$ is a hadronic matrix element and $A_R = A_L A_S^l \simeq 1.48$ takes into account long- ($A_L \simeq 1.25$) and short-distance ($A_S^l \simeq 1.18$) renormalization effects (see Refs. [30] and [31], respectively. Some details of the calculation of A_S^l, within our model, are given in Appendix 1). To satisfy current experimental bound $\tau_p^{\rm exp}(p \to \bar{\nu}K^+) \lesssim 5.9 \times 10^{33}$ years [32], for $M_X \simeq 5 \times 10^{11}$ GeV and $\alpha_X \simeq 1/31$, we need to have $\sqrt{|\epsilon_1|^2 + |\epsilon_3|^2 + |\epsilon_3|^2} \lesssim 4.8 \times 10^{-6}$. This selection of parameters is fully consistent with the charged fermion masses. Note, that with Eq. (74) there is no conflict with the constraint of Eq. (56). We can lower values of $|\epsilon_\gamma|$; however, there is a low bound dictated from this constraint. With $|\mathrm{Det}(\tilde{\mu}\frac{1}{M_{e^c\hat{e}^c}})| \cdot |\mathrm{Det}(\frac{1}{M_{\hat{l}l}}\hat{\mu})| = |\mathrm{Det}(\mathcal{L})| \cdot |\mathrm{Det}(\mathcal{R})| \sim 10^{-8}$, the lowest value can be $|\epsilon_\gamma| \sim 10^{-8}$, obtained with $|\mathrm{Det}(\mathcal{R})| \sim 1$. More natural would be to have $|\mathrm{Det}(\mathcal{R})| \lesssim 10^{-2}$, which suggests $|\mathrm{Det}(\mathcal{L})| \lesssim 10^{-6}$, and therefore $\sqrt{|\epsilon_1|^2 + |\epsilon_3|^2 + |\epsilon_3|^2} \gtrsim \sqrt{3} \times 10^{-6}$. This dictates an upper bound for the proton lifetime $\tau_p = \tau(p \to \bar{\nu}K^+) \lesssim 5 \times 10^{34}$ years and will allow us to test the model in the future [32].

Besides X, Y gauge boson mediated operators, there are $d = 6$ operators generated by the exchange of colored triplet scalar T_H. From the couplings of Eq. (40), we can see that the integration of T_H induces baryon number violating $\frac{1}{M_{T_H}^2}(q^T C_{qq} q)(q^T C_{ql} l)$ and $\frac{1}{M_{T_H}^2}(u^c C_{u^c e^c} e^c)(u^c C_{u^c d^c} d^c)$ operators, which lead to the couplings $\frac{1}{M_{T_H}^2}(q^T C_{qq} q)$ $(q^T C_{ql}\frac{1}{M_{\hat{l}l}}\hat{\mu} l_0)$ and $\frac{1}{M_{T_H}^2}(u^c C_{u^c e^c}\frac{1}{M^T_{e^c\hat{e}^c}}\tilde{\mu}^T e_0^c)(u^c C_{u^c d^c} d^c)$. Couplings C_{ab} appearing in these operators are independent from Yukawa matrices, and proper suppression of relevant terms is possible [similar to the case of couplings in Eq. (73)], leaving fermion masses and a mixing pattern consistent with experiments. To make a more definite statement about the nucleon lifetime, one has to study in detail the structure of Yukawa matrices. In this respect, extension with flavor symmetries is a motivated framework and can play a crucial role in generating the desirable Yukawa textures [guaranteeing the forms given in Eq. (74)]. Preserving these issues for being addressed elsewhere, let us move to the next section.

VI. VARIOUS PHENOMENOLOGICAL CONSTRAINTS AND IMPLICATIONS

In this section we discuss and summarize some peculiarities, phenomenological implications of our model, and constraints needed to be satisfied in order to be consistent with experiments. Also, we list issues opening prospects for further investigations within presented scenario:

(i) The discovery of the Higgs boson [1], with mass $\approx$126 GeV, revealed that the Standard Model suffers from vacuum instability. Detailed analysis has shown [2] that, due to RG, the Higgs self-coupling becomes negative near the scale $\sim 10^{10}$ GeV. If the Higgs field is sure to remain in the EW vacuum, the problem perhaps is not as severe. However, with an inflationary universe with the Hubble parameter $\gg 10^{10}$ GeV (preferred by the recent BICEP2 measurement [33]), the EW vacuum can be easily destabilized by the Higgs's move/tunneling to the "true" anti-de Sitter (AdS) vacuum [34]. Whether AdS domains take over or crunch depends on the details of inflation, the reheating process, nonminimal Higgs/inflaton couplings, etc. (a detailed overview of these questions can be found in Refs. [35] and [34]). While these and related issues need more investigation, to be on the safe side, it is desirable to have a model with positive λ_h at all energy scales (up to the $M_{\rm Pl}$).

Since within our model above the Λ' scale new states appear, this problem can be avoided. As was mentioned in Sec. II, in our model a light SM doublet h dominantly comes from the H-plet. The coupling $\lambda_H(H^\dagger H)^2$ gives the self-interaction term $\lambda_h(h^\dagger h)^2$ (with $\lambda_h \approx \lambda_H$ at the GUT scale). The running of λ_h will be given by

$$16\pi^2\frac{d}{dt}\lambda_h = \beta^{\rm SM}_{\lambda_h} + \Delta\beta_{\lambda_h},$$

where $\beta^{\rm SM}_{\lambda_h}$ corresponds to the SM part, while $\Delta\beta_{\lambda_h}$ accounts for new contributions. Since the H-plet in the potential (7) has additional interaction terms, some of those couplings can help to increase λ_h. For instance, the couplings $\lambda_{1H\Phi}$, $\lambda_{2H\Phi}$, $\hat{h}$, etc., contribute as

$$\Delta\beta_{\lambda_h} \approx \frac{(\lambda_{1H\Phi})^2}{25}[9\theta(\mu - M_{TT'}) + 6\theta(\mu - M_{DT'}) + 6\theta(\mu - M_{TD'}) + 4\theta(\mu - M_{DD'})] \times \frac{(\lambda_{2H\Phi})^2}{10}[3\theta(\mu - M_{DT'}) + 2\theta(\mu - M_{DD'})] + 3\hat{h}^2\theta(\mu - M_{T_{H'}}) + \cdots \quad (78)$$

Detailed analysis requires numerical studies by solving the system of coupled RG equations (involving multiple couplings[8]). While this is beyond the scope of this work, we see that, due to positive contributions (see above) into the β function, there is potential to prevent λ_h becoming negative all the way up to the Planck scale.

(ii) Since in our model leptons are composite, there will be additional contributions to their anomalous magnetic moment, given by [15]

$$\delta a_\alpha \sim \left(\frac{m_{e_\alpha}}{\Lambda'}\right)^2. \quad (79)$$

Current experimental measurements [28] of the muon anomalous magnetic moment give $\Delta a_\mu^{\rm exp} \approx 6\times 10^{-10}$. This, having in mind a possible range $\sim(1/5 - 1)$ of an undetermined prefactor in the expression of Eq. (79), constrains the scale Λ' from below: $\Lambda' \gtrsim (1.8 - 4.3)$ TeV. The selected value of Λ', within our model ($\Lambda' = 1851$ GeV), fits well with this bound.[9] The value of δa_e is more suppressed (for $\Lambda' \simeq 1.8$ TeV, we get $\delta a_e \sim 10^{-13}$) and is compatible with experiments ($\Delta a_e^{\rm exp} \approx 2.7\times 10^{-13}$). Planned measurements [38] with reduced uncertainties will provide severe constraints and test the viability of the proposed scenario.

Similarly, having flavor violating couplings at the level of constituents (i.e., in the sector of $SU(3)'$ fermions $\hat{q}, \hat{u}^c, \hat{d}^c$), the new contribution in $e_\alpha \to e_\beta\gamma$ rare decay processes will emerge. For instance, the contribution in the $\mu \to e\gamma$ transition amplitude will be $\sim\lambda_{12}\frac{m_\mu}{(\Lambda')^2}$, where λ_{12} is the (unknown) flavor violating coupling coming from the Yukawa sector of $\hat{q}, \hat{u}^c, \hat{d}^c$. This gives $Br(\mu\to e\gamma) \sim \lambda_{12}^2(\frac{M_W}{\Lambda'})^4$, and for $\Lambda' \simeq$ 1.8 TeV the constraint $\lambda_{12} \lesssim 4\times 10^{-4}$ should be satisfied in order to be consistent with the latest experimental limit ${\rm Br}^{\rm exp}(\mu\to e\gamma) < 5.7\times 10^{-13}$ [39].

[8] For methods studying the stability of multifield potentials, see Refs. [3] and [36] and references therein.

[9] In fact, this new contribution to a_μ has the potential of resolving a 3–4σ discrepancy [28] (if it will persist in the future) between the theory and experiment [37].

(iii) As was mentioned in Sec. III B (and will be discussed also in Appendix A), the matter sector of $SU(3)'$ symmetry (ignoring EW and Yukawa interactions) possesses $G_f^{(6)}$ chiral symmetry with sextets $6_L \sim \hat{q}_\alpha$ and $6_R \sim \hat{q}^c_\alpha$ [see Eqs. (47) and (48)]. The breaking of this chiral symmetry proceeds by several steps. At the first stage, at scale $\Lambda' \approx 1.8$ TeV, the condensates $\langle 6_L 6_L T^\dagger_{H'}\rangle \sim \langle 6_R 6_R T_{H'}\rangle \sim \Lambda'$ break the $G_f^{(6)}$. However, these condensates preserve SM gauge symmetry. At the next stage (of chiral symmetry breaking), the condensate $\langle 6_L 6_R\rangle \equiv F_{\pi'}$, together with the Higgs VEV $\langle h\rangle \equiv v_h$, contributes to the EW symmetry breaking. The $F_{\pi'}$ denotes the decay constant of the (techni) π' meson and should satisfy $v_h^2 + F_{\pi'}^2 = (246.2\ \text{GeV})^2$. With the light (very SM-like) Higgs boson mainly residing in h and with $F_{\pi'} \lesssim 0.2 v_h$, the h's signal will be very compatible with LHC data [40]. Since the low-energy potential would involve VEVs $\langle 6_L 6_L T^\dagger_{H'}\rangle$, $\langle 6_R 6_R T_{H'}\rangle$, $F_{\pi'}$, and v_h, obtaining mild hierarchy $\frac{F_{\pi'}}{\Lambda'} \lesssim 1/40$ will be possible by proper selection (not by severe fine-tunings) of parameters from perturbative and nonperturbative (effective) potentials. The situation here (i.e., the symmetry breaking pattern, potential (being quite involved because of these VEVs), etc.) will differ from case obtained within QCD with $SU(n)_L \times SU(n)_R$ chiral symmetry and with the $\langle n_L \times n_R\rangle$ condensate only [41]. Moreover, the hierarchy between the confinement scale and the decay constant can have some dynamical origin (see, e.g., Refs. [10][43]). Without addressing these details, our approach is rather phenomenological, with the assumption $F_{\pi'}/v_h \lesssim 0.2$ and h being the Higgs boson (with mass ≈ 126 GeV), such that there is allowed a window for a heavier π' state and the model is compatible with current experiments [44]. Models with partially composite Higgs, in which the light Higgs doublet (dominantly coming from H) has some ed-mixture of a composite (technipion π') state (i.e., $h = h_H + c_{\pi'}\pi'$, with $c_{\pi'} \ll 1$) with various interesting implications (including necessary constraints, limits, and compatibility with LHC data), were studied in Ref. [40]. As mentioned in Sec. IV, it is possible to have unification with the symmetry breaking pattern and the spectrum of intermediate states that give larger values of Λ' (even with $\Lambda' \sim 10^5$ GeV). However, in such a case, the value of $F_{\pi'}$ would be also large, and it would be impossible to bring $F_{\pi'}$ to the low value even with fine-tuning. This would mean that the EW symmetry breaking scale would be also large. That is why such a possibility has not been considered.
In addition, it is rather generic that the model with composite leptons will be accompanied with excited massive leptons (lepton resonances). Current experiments have placed low bounds on masses of the excited electron and muon to be heavier than ~ 1.8 TeV. This scale is close to the value of Λ' we have chosen within our model and will allow us to test the lepton substructure [45] hopefully in the not-far future. Details, related to these issues, deserve separate investigations.

(iv) Since the condensate $\langle 6_L 6_R\rangle = F_{\pi'}$, by some amount, can contribute to the chiral [of the $SU(3)'$ strong sector] and EW symmetry breaking, the scenario shares some properties of hybrid technicolor models with fundamental Higgs states. Moreover, together with technipion π', near the Λ' scale, there will be technimeson states ρ_T, ω_T, etc., with peculiar signatures [46], [47], which can be probed by collider experiments.

(v) Because of the new states around and above the $\Lambda' \approx 1.8$ TeV scale, there will be additional corrections to the EW precision parameters T, S, U, etc. While because of strong dynamics near the Λ' scale, the accurate calculations require some effort, the symmetry arguments provide a good estimate of the additional corrections—ΔT, ΔS, etc. One can easily notice that the isospin breaking effects are suppressed in the sector of additional states. Therefore, the mass splittings between doublet components of the additional states will be suppressed (i.e., $\Delta M \ll M$), and pieces $\Delta T_f, \Delta T_s$ of $\Delta T = \Delta T_f + \Delta T_s$ will be given as [48]

$$\Delta T_f \simeq \frac{N_f}{12\pi s_W^2}\left(\frac{\Delta M_f}{m_W}\right)^2, \quad \Delta T_s \simeq \frac{N_s}{24\pi s_W^2}\left(\frac{\Delta M_s}{m_W}\right)^2, \tag{80}$$

where subscripts f and s stand for fermions and scalars, respectively, and N_f, N_s account for the multiplicity [or dimension with respect to the group different from $SU(2)_w$] of the corresponding doublet state. One can easily verify that within our model in the sector of extra vectorlike $(\hat{l} + l)_\alpha$ states the mass splitting between doublet components is suppressed as $\Delta M^{(\alpha)}_{\hat{l}l} \lesssim \frac{v_h^2}{M^{(\alpha)}_{\hat{l}l}}$. This, according to Eq. (80) and Table I, gives the negligible contribution: $\Delta T_{\hat{l}l} \lesssim \frac{2\cdot 2}{12\pi s_W^2} v_h^4/(m_W M^{(1)}_{\hat{l}l})^2 \sim 10^{-5}$. Within the fragments of the scalar Φ, the lightest is $\Phi_{DT'}$ with mass $M_{DT'} \simeq 8.3$ TeV. Splitting between the doublet components comes from the potential term $\frac{\lambda_{2H\Phi}}{\sqrt{10}} H^\dagger \Phi \Phi^\dagger H$, giving $\Delta M_{DT'} \simeq \lambda_{2H\Phi} v_h^2/(4\sqrt{10} M_{DT'})$. This, according to Eq. (80),

[10]If a conformal window is realized, the value of $F_{\pi'}$ can be more reduced [42].

causes enough suppression: $\Delta T_{DT'}\simeq\frac{3}{24\pi s_W^2}\times\lambda_{4H\Phi}^2 v_h^4/(2\sqrt{10}M_{DT'}m_W)^2\lesssim 2\times 10^{-5}$ (for $\lambda_{2H\Phi}\lesssim 1.5$). As pointed out above, besides the fundamental Higgs doublet (h), which dominantly includes SM Higgs, there is a composite doublet (π'—similar to technicolor models) with suppressed VEV—$F_{\pi'}$. Contribution of this extra doublet, into the T parameter, is estimated to be

$$\Delta T_{\pi'}\approx\frac{1}{24\pi s_W^2}\left(\frac{\Delta M_{\pi'}}{m_W}\right)^2-\frac{c_W^2}{4\pi}c_{\pi'}^2\ln\frac{M_{\pi'}^2}{m_Z^2},\qquad(81)$$

where the first term is due to the mass splitting $\Delta M_{\pi'}(\sim v_h^2/(4M_{\pi'}))$ between doublet components of π', while second term emerges due to the VEV $\langle\pi'\rangle=F_{\pi'}$ with $c_{\pi'}\approx 2m_Z^2F_{\pi'}/(M_{\pi'}^2v_h)$ (where $F_{\pi'}\lesssim 0.2v_h$). This contribution is also small ($\Delta T_{\pi'}\approx 2\times 10^{-3}$) for $M_{\pi'}\sim 1$ TeV. Since π' is a composite state, due to the strong dynamics, special care is needed to derive a more accurate result (as was done in Ref. [49] for models with a single composite Higgs performing proper matching at different energy scales). However, since $\Delta T_{\pi'}$ is protected by isospin symmetry, we limit ourselves to the estimates performed here. Moreover, the source of the isospin breaking in the strong $SU(3)'$ sector is $F_{\pi'}\lesssim 0.2v_h$, causing the mass splitting between composite "technihadrons" (denoted collectively as $\{\rho'\}$) of $\Delta M_{\rho'}\sim F_{\pi'}^2/M_{\rho'}$. This, for $M_{\rho'}\sim\Lambda'$, would give the correction $\Delta T_{\rho'}\sim\frac{1}{12\pi s_W^2}\times F_{\pi'}^4/(m_WM_{\rho'})^2\lesssim 10^{-5}$. Note that the direct isospin (custodial symmetry) breaking within $\hat{q}_\alpha$ states is much more suppressed (we have no direct EW symmetry breaking in the Yukawa sector of $\hat{q}$, $\hat{u}^c$, and $\hat{d}^c$ states) and thus conclude that within the considered scenario extra corrections to the T parameter are under control.

Let us now give the estimate of the additional contributions into the S parameter. Contributions to this parameter from the additional vectorlike $(\hat{l}+l)_\alpha$, $(\hat{e}^c+e^c)_\alpha$ states decouple [50] and are estimated to be $\Delta S_{\hat{l}l}\sim\Delta S_{\hat{e}^ce^c}\lesssim\frac{1}{4\pi}\frac{v_h^2}{(M_{\hat{l}l}^{(1)})^2}\ln\frac{M_{\hat{l}l}^{(1)}}{m_\tau}\sim 10^{-5}$. The contribution from the scalar $\Phi_{DT'}$ is $\Delta S_{DT'}\simeq\frac{3}{6\pi}\Delta M_{DT'}/M_{DT'}\simeq\lambda_{2H\Phi}v_h^2/(8\pi\sqrt{10}M_{DT'}^2)\lesssim 2\times 10^{-5}$, also suppressed, as expected. The contribution of extra (heavy π') composite doublet is

$$\Delta S_{\pi'}\approx\frac{1}{6\pi}\frac{\Delta M_{\pi'}}{M_{\pi'}}+\frac{1}{6\pi}c_{\pi'}^2\ln\frac{M_{\pi'}}{m_h},\qquad(82)$$

where first term is due to the splitting of the doublet components, while second term comes from the VEV $\langle\pi'\rangle=F_{\pi'}$. With $\Delta M_{\pi'}\sim v_h^2/(4M_{\pi'})$ and $M_{\pi'}\gtrsim 1$ TeV, Eq. (82) gives $\Delta S_{\pi'}\lesssim 10^{-3}$. Similarly suppressed contributions would arise from the techni-ρ' hadrons: $\Delta S_{\rho'}\sim\frac{1}{6\pi}\Delta M_{\rho'}/M_{\rho'}\sim\frac{1}{6\pi}F_{\pi'}^2/M_{\rho'}^2\lesssim 4\times 10^{-5}$ (for $M_{\rho'}\sim\Lambda'$).

As far as the contribution from the matter states $\hat{q},\hat{u}^c,\hat{d}^c$ are concerned, since their masses are too suppressed, in the chiral limit $\frac{m_f}{m_Z}\to 0$, we can use the expression [48]

$$\Delta S_f\to\frac{N_fY_f}{6\pi}\left(-2\ln\frac{x_1}{x_2}+G(x_1)-G(x_2)\right),$$

$$\text{with}\quad G(x)=-4\,\text{arc tanh}\frac{1}{\sqrt{1-4x}},\qquad x_i=\frac{m_{fi}^2}{m_Z^2},\qquad(83)$$

where $m_{f1,2}$ are masses of the components of the f fermion with hypercharge Y_f. Verifying that in the limit $x\to 0$ the function $G(x)$ goes to $2\ln x$, we see that expression for ΔS_f in Eq. (83) vanishes. Moreover, new contributions to the U parameter are more suppressed. For instance, the contribution due to the π' is

$$\Delta U_{\pi'}\approx\frac{1}{15\pi}\left(\frac{\Delta M_{\pi'}}{M_{\pi'}}\right)^2-\frac{1}{12\pi}c_\pi^2\frac{\Delta M_{\pi'}}{M_{\pi'}},\qquad(84)$$

which for $M_{\pi'}\sim 1$ TeV, $F_{\pi'}\lesssim 0.2v_h$ becomes $\Delta U_{\pi'}\lesssim 5\times 10^{-6}$. All other new contributions to the U are also more suppressed than the corresponding ΔS and ΔT. This is understandable since U is related to the effective operator with a dimension higher than those of S and T. All these allow us to conclude that new contributions to the EW precision parameters are well below the current experimental bounds [51].

(vi) Within the proposed model, spontaneous breaking of two non-Abelian groups $SU(5)\times SU(5)'$ and discrete D_2 parity will give monopole and domain wall solutions, respectively. Since the symmetry breaking scales are relatively low ($\lesssim 5\times 10^{11}$ GeV), the inflation would not dilute number densities of these topological defects in a straightforward way. Thus, one can think of alternative solutions. For instance, as it was shown in Refs. [52], within models with a certain field content and couplings, it is possible that symmetry restoration cannot happen for arbitrary high temperatures. This would avoid the phase transitions (which usually cause the formation of topological defects). Moreover, by proper selection of the model parameters, it is possible to suppress the thermal production rates of the topological defects (for detailed discussions, see the last two works of Ref. [52]). From this viewpoint, our model with a multiscalar sector and various couplings has potential to avoid domain wall and monopole problems. Thus, it is inviting to investigate the parameter space and see how desirable ranges are compatible with those needed values appearing in Eq. (78) (for "improving" the running of λ_h).

To cure problems related with topological defects, also other different noninflationary solutions have been proposed [53], and one (if not all) of them could be invoked as well.

Certainly, these and other cosmological implications, of the presented scenario, deserve separate investigations.

At the end let us note that it would be interesting to build a supersymmetric extension of the considered $SU(5)\times SU(5)'\times D_2$ GUT and study related phenomenology. These and related issues will be addressed elsewhere.

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APPENDIX A: COMPOSITE LEPTONS AND ANOMALY MATCHING

Here we demonstrate how the composite leptons emerge within our scenario and also discuss anomaly matching conditions. As was noted in Sec. III B, the sector of $\hat{q}$, $\hat{u}^c$, and $\hat{d}^c$ states have $G_f^{(6)}$ chiral symmetry [see Eq. (47)] with the transformation properties of these states given in Eq. (48). At scale $SU(3)'$ interaction becomes strong, and the $G_f^{(6)}$ symmetry breaking condensates can be formed. The chiral symmetry breaking can proceed through several steps, and at each level the formed composite states should satisfy anomaly matching conditions [14].

The bilinear [$SU(3)'$-invariant] condensate can be $\langle 6_L\times 6_R\rangle = F_{\pi'}$, with corresponding breaking scale $F_{\pi'}$. As was shown in Ref. [41], with only fundamental states, the chiral symmetry $SU(n)_L\times SU(n)_R$ will be broken down to the diagonal $SU(n)_{L+R}$ symmetry. Since in our case $F_{\pi'}$ also contributes to EW symmetry breaking, we have a bound $F_{\pi'}\lesssim 100$ GeV. This scale, in comparison with $\Lambda'\sim \text{few}\times\text{TeV}$, can be ignored at the first stage. Moreover, in our case, light $SU(3)'$ nonsinglet field content is reacher (including light scalars), and the chiral symmetry breaking pattern is also different. Other $SU(3)'$ invariant condensates, including matter bilinears, are

$$\langle 6_L 6_L T^\dagger_{H'}\rangle \quad \text{and} \quad \langle 6_R 6_R T_{H'}\rangle. \tag{A1}$$

Note, that the product of $SU(6)$ sextets gives either symmetric or antisymmetric representations ($6\times 6=15_A+21_S$), but due to $SU(3)'$ contractions, in Eq. (A1) the antisymmetric 15-plets (i.e., 15_L and 15_R) participate. The condensates (A1) transform as 15_L and 15_R under $SU(6)_L$ and $SU(6)_R$, respectively, and therefore break these symmetries. A possible breaking channel is

$$SU(6)_L\to SU(4)_L\times SU(2)'_L\equiv G_L^{(4,2)},$$
$$SU(6)_R\to SU(4)_R\times SU(2)'_R\equiv G_R^{(4,2)}. \tag{A2}$$

Indeed, with respect to $G_L^{(4,2)}$ and $G_R^{(4,2)}$, the 15_L and 15_R decompose as

$$SU(6)_L\to G_L^{(4,2)}: 15_L=(1,1)_L+(6,1)_L+(4,2)_L,$$
$$SU(6)_R\to G_R^{(4,2)}: 15_R=(1,1)_R+(6,1)_R+(4,2)_R, \tag{A3}$$

and the VEVs $\langle(1,1)_L\rangle$ and $\langle(1,1)_R\rangle$ leave $G_L^{(4,2)}\times G_R^{(4,2)}$ chiral symmetry unbroken. The singlet components ($\langle(1,1)_L\rangle$ and $\langle(1,1)_R\rangle$) from Eq. (A1) are $\frac{1}{2}\langle\hat{q}\,\hat{q}\,T^\dagger_{H'}\rangle = \langle\hat{u}\,\hat{d}\,T^\dagger_{H'}\rangle$ and $\langle\hat{u}^c\hat{d}^c T_{H'}\rangle$ combinations, which *leave* G_{SM} gauge symmetry unbroken. Therefore, the values of these condensates can be ~few TeV(~Λ') without causing any phenomenological difficulties. Thus, as the first stage of the chiral symmetry breaking, we stick to the channel

$$G_f^{(6)}\xrightarrow{\Lambda'} G_L^{(4,2)}\times G_R^{(4,2)}\times U(1)_{B'}, \tag{A4}$$

with

$$\langle 6_L 6_L T^\dagger_{H'}\rangle = \langle\hat{u}\,\hat{d}\,T^\dagger_{H'}\rangle\sim\Lambda',$$
$$\langle 6_R 6_R T_{H'}\rangle = \langle\hat{u}^c\hat{d}^c T_{H'}\rangle\sim\Lambda'. \tag{A5}$$

The $SU(6)_{L,R}$ sextets under $G_{L,R}^{(4,2)}$ are decomposed as $6_L=(4,1)_L+(1,2)_L$ and $6_R=(4,1)_R+(1,2)_R$, respectively. If composite objects are picked up as $(4',1)_{L,R}\subset[(4,1)_{L,R}]^3$ and $(1,2')_{L,R}\subset[(1,2)_{L,R}]^3$, then one can easily check out that the anomalies (of initial and composite states) indeed match and $(4',1)_{L,R}$ and $(1,2')_{L,R}$ can be identified with three families of leptons plus three states of right-handed/sterile neutrinos. For demonstrating all these, it is more convenient to work in a different basis. That would also make it simpler to identify composite states.

As it is well known (and in our case turns out more useful), one can describe the $SU(6)$ symmetry (and its representations as well) by its special subgroup ("S subgroup" [54]) $SU(3)_f\otimes SU(2)\subset SU(6)$. In our case,

$$SU(6)_L \supset SU(3)_{fL}\otimes SU(2)_L,\qquad SU(6)_R \supset SU(3)_{fR}\otimes SU(2)_R. \tag{A6}$$

Under these S subgroups, the sextets decompose as[11]

$$\hat q(6_L)=\hat q(3,2)_L,\qquad \hat q^c(6_R)=\hat q^c(3,2)_R. \tag{A7}$$

In these decompositions, $\hat q$ and $\hat q^c$ can be written as matrices,

$$\hat q=\overset{\leftarrow SU(3)_{fL}\rightarrow}{\begin{pmatrix}\hat u & \hat c & \hat t\\ \hat d & \hat s & \hat b\end{pmatrix}}\ \updownarrow SU(2)_L,\qquad \hat q^c=\overset{\leftarrow SU(3)_{fR}\rightarrow}{\begin{pmatrix}\hat u^c & \hat c^c & \hat t^c\\ \hat d^c & \hat s^c & \hat b^c\end{pmatrix}}\ \updownarrow SU(2)_R, \tag{A8}$$

where schematically actions of $SU(3)$ and $SU(2)$ rotations are depicted. Therefore, transformation properties under the chiral group

$$G_f^{(3,2)}=SU(3)_{fL}\otimes SU(2)_L\times SU(3)_{fR}\otimes SU(2)_R\times U(1)_{B'} \tag{A9}$$

are

$$G_f^{(3,2)}:\hat q\sim\left(3_{fL},2_L,1,1,\frac{1}{3}\right),\qquad \hat q^c\sim\left(1,1,3_{fR},2_R,-\frac{1}{3}\right). \tag{A10}$$

Relevant anomalies that do not vanish are

$$A([SU(3)_{fL}]^2\cdot U(1)_{B'})=-A([SU(3)_{fR}]^2\cdot U(1)_{B'})=1,$$
$$A([SU(2)_L]^2\cdot U(1)_{B'})=-A([SU(2)_R]^2\cdot U(1)_{B'})=\frac{3}{2}. \tag{A11}$$

The anomaly matching condition can be satisfied with the spontaneous breaking of the symmetries $SU(3)_{fL}$ and $SU(3)_{fR}$ down to $SU(2)_{fL}$ and $SU(2)_{fR}$, respectively. [This happens by condensates (A5) discussed above.] Thus, the chiral symmetry $G_f^{(3,2)}$ is broken down to $G_f^{(2,2)}$, where

$$G_f^{(2,2)}=SU(2)_{fL}\otimes SU(2)_L\times SU(2)_{fR}\otimes SU(2)_R\times U(1)_{B'}. \tag{A12}$$

This breaking is realized, for instance, by the condensates $\langle\hat u_3\hat d_3T^\dagger_{H'}\rangle$ and $\langle\hat u_3^c\hat d_3^cT_{H'}\rangle$. Note that with $SU(3)_{fL}\to SU(2)_{fL}$ and $SU(3)_{fR}\to SU(2)_{fR}$ we will have decompositions $3_{fL}=2_{fL}+1_{fL}$ and $3_{fR}=2_{fR}+1_{fR}$. At the composite level, the spin-1/2 and $SU(3)'$ singlet combinations $(\hat q\,\hat q)\hat q$ and $(\hat q^c\hat q^c)\hat q^c$ picked up as $[2'_{fL}+1'_{fL}]$ from $[2_{fL}+1_{fL}]^3$ and $[2'_{fR}+1'_{fR}]$ from $[2_{fR}+1_{fR}]^3$. Thus, transformations of $(\hat q\,\hat q)\hat q$ and $(\hat q^c\hat q^c)\hat q^c$ composites under $G_f^{(2,2)}$ are[12]

$$G_f^{(2,2)}:(\hat q\,\hat q)\hat q\sim([2_{fL}+1_{fL}],2_L,1,1,1),\qquad (\hat q^c\hat q^c)\hat q^c\sim(1,1,[2_{fR}+1_{fR}],2_R,-1). \tag{A13}$$

These representations will have anomalies that precisely match with those given in Eq. (A11). Thus, we have three families of l_0,e_0^c,ν_0^c composite states represented in Eq. (49), with transformation properties under G_{SM} given in Eq. (50).

APPENDIX B: RG EQUATIONS AND b FACTORS

In this appendix we discuss details of gauge coupling unification and present one- and two-loop RG coefficients at each relevant energy scale. At the end we calculate short-range renormalization factors A_S^l and $A_S^{e^c}$ for baryon number violating $d=6$ operators.

The two-loop RG equation, for gauge coupling α_i, has the form [55]

$$\frac{d}{d\ln\mu}\alpha_i^{-1}=-\frac{b_i}{2\pi}-\frac{1}{8\pi^2}\sum_j b_{ij}\alpha_j+\frac{1}{32\pi^3}\sum_f a_i^f\lambda_f^2, \tag{B1}$$

where b_i and b_{ij} account for one- and two-loop gauge contributions, respectively, and c_i^f represents the two-loop correction via Yukawa coupling λ_f. For consistency, it is enough to consider the Yukawa coupling RG at the one-loop approximation:

$$16\pi^2\frac{d}{d\ln\mu}\lambda_f=c_f\lambda_f^3+\lambda_f\left(\sum_{f'}d_f^{f'}\lambda_{f'}^2-4\pi\sum_i c_f^i\alpha_i\right). \tag{B2}$$

RG factors can be calculated using general formulas [55]. Since at different energy scales different states appear, these

[11]Similar to the description of three-flavor QCD with (u,d,s) spin-1/2 states, either by the sextet of $SU(6)$ or by (3,2) of $SU(3)_f\times SU(2)_s$—the Wigner–Weyl realization of the $SU(6)$ chiral symmetry. Here, however, $SU(2)_s$ stands for the spin group and $SU(3)_f$ for the flavor. In our case of Eq. (A6), $SU(2)$ factors act like isospin rotations relating $\hat u_\alpha$ and $\hat d_\alpha$ and $\hat u_\alpha^c$ with $\hat d_\alpha^c$, respectively ($\alpha=1,2,3$).

[12]Under combination $(\hat q\,\hat q)\hat q$ (suppressed gauge/chiral indices), we mean $\epsilon^{a'b'c'}\epsilon_{ij}(\hat q_{a'i}\hat q_{b'j})\hat q_{c'k}$, where $a',b',c'=1,2,3$ are $SU(3)'$ indices and $i,j,k=1,2$ stand for $SU(2)_L/SU(2)_w$ indices.

factors also change with energy. For instance, at scale μ, the b_i and b_{ij} can be written as $b_i(\mu)=\sum_a\theta(\mu-M_a)b_i^a$ and $b_{ij}(\mu)=\sum_a\theta(\mu-M_a)b_{ij}^a$, where a stands for the state with mass M_a and step function $\theta(x)=0$ for $x\le 0$, and $\theta(x)=1$ for $x>0$.

Integration of Eq. (B1), in energy interval $\mu_1-\mu_2$, gives

$$\alpha_i^{-1}(\mu_2)=\alpha_i^{-1}(\mu_1)-\frac{b_i^{\mu_1\mu_2}}{2\pi}\ \ln\ \frac{\mu_2}{\mu_1}, \tag{B3}$$

where an effective $b_i^{\mu_1\mu_2}$ factor is given by

$$b_i^{\mu_1\mu_2}=\left(\sum_a\theta(\mu_2-M_a)b_i^a\ \ln\ \frac{\mu_2}{M_a}+\frac{1}{4\pi}\sum_a\int_{\mu_1}^{\mu_2}\theta(\mu-M_a)b_{ij}^a\alpha_j d\ \ln\ \mu-\frac{1}{8\pi^2}\int_{\mu_1}^{\mu_2}c_i^f\lambda_f^2 d\ \ln\ \mu\right)\frac{1}{\ln\frac{\mu_2}{\mu_1}}. \tag{B4}$$

The second and third terms in Eq. (B4) can be evaluated iteratively [56]. Although Eq. (B1) can be solved numerically (which we do perform for obtaining final results), expressions (B3) and (B4) are useful for understanding how unification works.

In the energy interval $M_Z-\Lambda'$, we have just SM, while between Λ' and M_I scales, we have $G_{SM}\times SU(3)'$ gauge interactions plus additional states. Applying Eq. (B3) for the couplings α_Y, α_w, α_c, and $\alpha_{3'}$, we will have

$$\alpha_i^{-1}(M_I)=\alpha_i^{-1}(M_Z)-\frac{b_i^{ZI}}{2\pi}\ \ln\ \frac{M_I}{M_Z},\qquad i=Y,w,c,\qquad \alpha_{3'}^{-1}(M_I)=\alpha_{3'}^{-1}(\Lambda')-\frac{b_{3'}^{\Lambda' I}}{2\pi}\ \ln\ \frac{M_I}{\Lambda'}, \tag{B5}$$

where $b_i^{ZI}, b_{3'}^{\Lambda' I}$ can be calculated via Eq. (B4) having appropriate RG factors.

Above the scale M_I, we have gauge interactions G_{321} going all the way up to the GUT scale. The $G_{321}{}'$ gauge symmetry appears between scales M_I and $M_I{}'$, while $SU(5)'$ appears above the $M_I{}'$ scale. Therefore, we will have

$$\begin{aligned}
&\alpha_i^{-1}(M_G)=\alpha_i^{-1}(M_I)-\frac{b_i^{IG}}{2\pi}\ \ln\ \frac{M_G}{M_I}, && i=1,2,3,\\
&\alpha_{i'}^{-1}(M_I{}')=\alpha_{i'}^{-1}(M_I)-\frac{b_{i'}^{II'}}{2\pi}\ \ln\ \frac{M_I{}'}{M_I}, && i'=1',2',3',\\
&\alpha_{5'}^{-1}(M_G)=\alpha_{5'}^{-1}(M_I{}')-\frac{b_{5'}^{I'G}}{2\pi}\ \ln\ \frac{M_G}{M_I{}'}.
\end{aligned} \tag{B6}$$

From Eqs. (B5) and (B6) and taking into account the boundary conditions (64)–(66), we arrive at relations given in Eq. (67). The four equations in Eq. (67) allow us to determine M_I, $M_I{}'$, M_G, and α_G in terms of other input mass scales. The latter must be selected in such a way as to get successful unification. This has been done numerically, and results are given in Table I, Eq. (68), and Fig. 3.

Now, we present all RG b factors needed for writing down RG equations. In the energy interval $\mu=M_Z-\Lambda'$, the RG factors are just those of the SM:

$$\mu=M_Z-\Lambda':b_i=\left(\frac{41}{10},-\frac{19}{6},-7\right),\qquad b_{ij}=\begin{pmatrix}\frac{199}{50} & \frac{27}{10} & \frac{44}{5}\\ \frac{9}{10} & \frac{35}{6} & 12\\ \frac{11}{10} & \frac{9}{2} & -26\end{pmatrix},\qquad (i=Y,w,c). \tag{B7}$$

In the energy interval $\Lambda'-M_I$, we have the symmetry $SU(3)_c\times SU(2)_w\times U(1)_Y\times SU(3)'$. Also, instead of composite leptons, we have three families of $SU(3)'$ triplets $\hat{q}$, $\hat{u}^c$, $\hat{d}^c$, and vectorlike states $(l,\hat{l})_\alpha$ and $(e^c,\hat{e}^c)_\alpha$ $(\alpha=1,2,3)$ with masses $M_{l\hat{l}}^{(\alpha)}$ and $M_{e^c\hat{e}^c}^{(\alpha)}$, respectively. Moreover, some fragments of $\Phi(5,\bar{5})$ [see Eq. (25)] and $\Sigma'_{8'}$ (of Σ') can appear below M_I. Thus, the corresponding b factors in this energy interval are given by

$$\mu=\Lambda'-M_I:$$
$$b_Y=\frac{9}{2}+\frac{1}{15}\theta(\mu-M_{T_{H'}})+\frac{2}{5}\sum_{\alpha=1}^{3}\theta(\mu-M_{\hat{l}l}^{(\alpha)})+\frac{4}{5}\sum_{\alpha=1}^{3}\theta(\mu-M_{e^c\hat{e}^c}^{(\alpha)})+\frac{5}{6}\theta(\mu-M_{DT'})+\frac{5}{6}\theta(\mu-M_{TD'})$$
$$b_w=-\frac{7}{6}+\frac{2}{3}\sum_{\alpha=1}^{3}\theta(\mu-M_{\hat{l}l}^{(\alpha)})+\frac{1}{2}\theta(\mu-M_{DT'})+\frac{1}{2}\theta(\mu-M_{TD'}),$$
$$b_c=-7+\frac{1}{3}\theta(\mu-M_{TD'})+\frac{1}{2}\theta(\mu-M_{TT'}),$$
$$b_{3'}=-7+\frac{1}{6}\theta(\mu-M_{T_{H'}})+\frac{1}{3}\theta(\mu-M_{DT'})+\frac{1}{2}\theta(\mu-M_{TT'})+\frac{1}{2}\theta(\mu-M_{8'}),\tag{B8}$$

$$\mu=\Lambda'-M_I:b_{ij}=\begin{pmatrix}\frac{13709}{50}&\frac{9}{5}&\frac{44}{5}&\frac{44}{5}\\ \frac{3}{5}&\frac{91}{3}&12&12\\ \frac{11}{10}&\frac{9}{2}&-26&0\\ \frac{11}{10}&\frac{9}{2}&0&-26\end{pmatrix}+\sum_a\theta(\mu-M_a)b_{ij}^a,\qquad(i,j=Y,w,c,3')\qquad\text{with }:$$
$$b_{ij}^{T_{H'}}=\begin{pmatrix}\frac{4}{75}&0&0&\frac{16}{15}\\0&0&0&0\\0&0&0&0\\ \frac{2}{15}&0&0&\frac{11}{3}\end{pmatrix},\quad b_{ij}^{DT'}=\begin{pmatrix}\frac{25}{6}&\frac{15}{2}&0&\frac{40}{3}\\ \frac{5}{2}&\frac{13}{2}&0&8\\0&0&0&0\\ \frac{5}{3}&3&0&\frac{22}{3}\end{pmatrix},\quad b_{ij}^{TT'}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&11&8\\0&0&8&11\end{pmatrix},\quad b_{ij}^{TD'}=\begin{pmatrix}\frac{25}{6}&\frac{15}{2}&\frac{40}{3}&0\\ \frac{5}{2}&\frac{13}{2}&8&0\\ \frac{5}{2}&3&\frac{22}{3}&0\\0&0&0&0\end{pmatrix},$$
$$b_{ij}^{(l,\hat{l})_\alpha}=\begin{pmatrix}\frac{9}{50}&\frac{9}{10}&0&0\\ \frac{3}{10}&\frac{49}{6}&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\quad b_{ij}^{(e^c,\hat{e}^c)_\alpha}=\mathrm{Diag}\left(\frac{36}{25},0,0,0\right),\quad b_{ij}^{\Sigma'_{8'}}=\mathrm{Diag}(0,0,0,21).\tag{B9}$$

Between the scales M_I and $M_I{}'$, the symmetry is $G_{321}\times G_{321}{}'$, and all matter states are massless. Also, above the scale M_I, we should include the states $T_{H'}$ and $\Phi_{DD'}$ as massless and remaining fragments above their mass thresholds. Since G_{321} goes all the way up to the M_G, its one-loop b factors can be determined in the interval M_I-M_G and are given by

$$\mu=M_I-M_G:b_1=\frac{43}{10}+\frac{3}{10}\theta(\mu-M_{DT'})+\frac{1}{5}\theta(\mu-M_{TT'})+\frac{2}{15}\theta(\mu-M_{TD'}),$$
$$b_2=-\frac{17}{6}+\frac{1}{2}\theta(\mu-M_{DT'}),$$
$$b_3=-7+\frac{1}{2}\theta(\mu-M_{TT'})+\frac{1}{3}\theta(\mu-M_{TD'}).\tag{B10}$$

The gauge group $G_{321}{}'$ appears in the interval $M_I-M_I{}'$, and corresponding one-loop b factors are

$$\mu=M_I-M_I{}':b_{1'}=\frac{64}{15}+\frac{1}{10}\theta(\mu-M_{D'})+\frac{2}{15}\theta(\mu-M_{DT'})+\frac{1}{5}\theta(\mu-M_{TT'})$$
$$+\frac{3}{10}\theta(\mu-M_{TD'})-\frac{55}{3}\theta(\mu-M_{X'}),$$
$$b_{2'}=-3+\frac{1}{6}\theta(\mu-M_{D'})+\frac{1}{2}\theta(\mu-M_{TD'})-11\theta(\mu-M_{X'}),$$
$$b_{3'}=-\frac{41}{6}+\frac{1}{2}\theta(\mu-M_{TT'})+\frac{1}{3}\theta(\mu-M_{DT'}),\tag{B11}$$

where terms with $\theta(\mu - M_{X'})$ account for the threshold of (X', Y') gauge bosons of $SU(5)'$, in case their masses $M_{X'}$ lie slightly below the $M_I{}'$ scale. We will take this effect into account at the one-loop level. The two-loop b_{ij} factors of $G_{321} \times G_{321}{}'$ form 6×6 matrices and are determined in the interval $M_I - M_I{}'$:

$$\mu = M_I - M_I{}' : b_{ij} = \left((b^f + b^h + b^g + b^{T_{H'}} + b^{DD'})_{ij} + \sum_a \theta(\mu - M_a) b^a_{ij} \right), \qquad (i, j = 1, 2, 3, 1', 2', 3')$$

$$\text{with} \; : b^f_{ij} = 3 \begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} & 0 & 0 & 0 \\ \frac{1}{5} & \frac{49}{3} & 4 & 0 & 0 & 0 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ 0 & 0 & 0 & \frac{1}{5} & \frac{49}{3} & 4 \\ 0 & 0 & 0 & \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{pmatrix}, \qquad b^h_{ij} = \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 & 0 & 0 & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$b^{T_{H'}}_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{75} & 0 & \frac{16}{15} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{15} & 0 & \frac{11}{3} \end{pmatrix}, \qquad b^{DD'}_{ij} = \begin{pmatrix} \frac{9}{25} & \frac{9}{5} & 0 & \frac{9}{25} & \frac{9}{5} & 0 \\ \frac{3}{5} & \frac{13}{3} & 0 & \frac{3}{5} & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{9}{25} & \frac{9}{5} & 0 & \frac{9}{25} & \frac{9}{5} & 0 \\ \frac{3}{5} & 3 & 0 & \frac{3}{5} & \frac{13}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$b^{DT'}_{ij} = \begin{pmatrix} \frac{27}{50} & \frac{27}{10} & 0 & \frac{6}{25} & 0 & \frac{24}{5} \\ \frac{9}{10} & \frac{13}{2} & 0 & \frac{2}{5} & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{6}{25} & \frac{6}{5} & 0 & \frac{8}{75} & 0 & \frac{32}{15} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{9}{15} & 3 & 0 & \frac{4}{15} & 0 & \frac{22}{3} \end{pmatrix}, \qquad b^{TT'}_{ij} = \begin{pmatrix} \frac{4}{25} & 0 & \frac{16}{5} & \frac{4}{25} & 0 & \frac{16}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 11 & \frac{2}{5} & 0 & 8 \\ \frac{4}{25} & 0 & \frac{16}{5} & \frac{4}{25} & 0 & \frac{16}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{5} & 0 & 8 & \frac{2}{5} & 0 & 11 \end{pmatrix},$$

$$b^{TD'}_{ij} = \begin{pmatrix} \frac{8}{75} & 0 & \frac{32}{15} & \frac{6}{25} & \frac{6}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4}{15} & 0 & \frac{22}{3} & \frac{3}{5} & 3 & 0 \\ \frac{6}{25} & 0 & \frac{24}{5} & \frac{27}{50} & \frac{27}{10} & 0 \\ \frac{2}{5} & 0 & 8 & \frac{9}{10} & \frac{13}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad b^{D'}_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{9}{50} & \frac{9}{10} & 0 \\ 0 & 0 & 0 & \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$b^g_{ij} = \mathrm{Diag}\left(0, -\frac{136}{3}, -102, 0, -\frac{136}{3}, -102\right), \qquad b^{\Sigma'_{8'}}_{ij} = \mathrm{Diag}(0, 0, 0, 0, 0, 21). \tag{B12}$$

In this $M_I - M_I{}'$ energy interval, we have two Abelian factors $U(1)$ and $U(1)'$ and states Φ_i (the fragments of Φ) charged under both gauge symmetries. Because of this, the gauge kinetic mixing will be induced [57], [58]. Parametrizing the latter as $-\frac{\sin\chi}{2} F_1^{\mu\nu} F_{1'\mu\nu}$, and bringing whole gauge kinetic part to the canonical form, one can obtain Φ_i's covariant derivative as [58] $[\partial^\mu + \frac{i}{2} g_1 Q_i A_1^\mu + \frac{i}{2}(\bar{g}_{1'} Q_i{}' + g_{11'} Q_i) A_{1'}^\mu]\Phi_i$. In this basis Q_i charges are unshifted, and g_1 and its RG are unchanged. On the other hand, $\bar{g}_{1'} = g_{1'}/\cos\chi$ and $g_{11'} = -g_1 \tan\chi$. Introducing the ratio $\delta = g_{11'}/\bar{g}_{1'}$, the RGs for $\bar{\alpha}_{1'}$ and δ will be [58]

$$\frac{d}{d\ln\mu}(\bar{\alpha}_{1'})^{-1} = \cdots - \frac{b_1}{2\pi}\delta^2 - \frac{B_{11'}}{\pi}\delta,$$
$$\frac{d}{d\ln\mu}\delta = \frac{b_1}{2\pi}\alpha_1\delta + \frac{B_{11'}}{8\pi^2}, \tag{B13}$$

where "…" denote standard one- and two-loop contributions [with form of Eq. (B1)] and $B_{11'} = \sum_i Q_i Q_i'$ is given by

$$B_{11'} = \frac{1}{5}[\theta(\mu - M_{DT'}) - \theta(\mu - M_{DD'}) - \theta(\mu - M_{TT'}) + \theta(\mu - M_{TD'})]. \tag{B14}$$

Because of the mass splitting between Φ's fragments, $B_{11'} \neq 0$ in the interval $M_I - M_{TD'}$, and therefore $\delta \neq 0$; i.e., the kinetic mixing is generated. This causes the shift $\alpha_{1'}^{-1} \to \alpha_{1'}^{-1} + \mathcal{O}(\delta)$. However, as it turns out, within our model this effect is negligible. We have taken these into account upon numerical studies and got $\delta(M_I) \simeq 9.5\cdot 10^{-3}$, $\sin\chi(M_I) \simeq -2\cdot 10^{-2}$, causing the change of $\alpha_{1'}^{-1}(M_I)$ by 0.01%. This has no practical impact on the matching conditions of Eq. (64) and does not affect the picture of gauge coupling unification and therefore can be safely ignored.

Since at and above the scale $M_I{}'$ the $G_{321}{}'$ is embedded in $SU(5)'$, we will deal with b factors of $G_{321}\times SU(5)'$ symmetry, and one-loop b factors of G_{321} are given in Eq. (B10). At energies corresponding to unbroken $SU(5)'$, the fragments $(\Phi_{DD'}, \Phi_{DT'})$ form the unified $(2,\bar{5}) \equiv \Phi_{D\bar{5}'}$-plet of $G_{321}\times SU(5)'$. Similarly, $(T_{H'}, D') \subset H'$. Above the scale $M_I{}'$, these states (together with all fragments of the Σ′-plet) should be included as massless states. Thus, the one-loop b factor of $SU(5)'$ is given as

$$\mu = M_I{}' - M_G : b_{5'} = -13 + \frac{1}{2}\theta(\mu - M_{T\bar{5}'}), \tag{B15}$$

where $M_{T\bar{5}'} = \max(M_{TT'}, M_{TD'})$ denotes the mass of the $(3,\bar{5})$-plet, which includes $\Phi_{TT'}$ and $\Phi_{TD'}$ states: $(\Phi_{TT'}, \Phi_{TD'}) \subset \Phi_{T\bar{5}'}$. The two-loop b_{ij} factors, above the scale $M_I{}'$, form 4×4 matrices and are

$$\mu = M_I{}' - M_G : b_{ij} = (b^f + b^h + b^g + b^{H'} + b^{\Sigma'} + b^{D\bar{5}'})_{ij} + \theta(\mu - M_{T\bar{5}'})b_{ij}^{T\bar{5}'}, \qquad (i,j = 1,2,3,5')\text{with:}$$
$$b_{ij}^f = 3\begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} & 0 \\ \frac{1}{5} & \frac{49}{3} & 4 & 0 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} & 0 \\ 0 & 0 & 0 & \frac{698}{15} \end{pmatrix}, \qquad b_{ij}^h = \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$
$$b_{ij}^g = \text{Diag}\left(0, -\frac{136}{3}, -102, -\frac{850}{3}\right), \qquad b_{ij}^{H'} = \frac{97}{15}\delta_{i5'}\delta_{j5'}, \qquad b_{ij}^{\Sigma'} = \frac{175}{3}\delta_{i5'}\delta_{j5'},$$
$$b_{ij}^{D\bar{5}'} = \begin{pmatrix} \frac{9}{10} & \frac{9}{2} & 0 & \frac{72}{5} \\ \frac{3}{2} & \frac{65}{6} & 0 & 24 \\ 0 & 0 & 0 & 0 \\ \frac{3}{5} & 3 & 0 & \frac{194}{15} \end{pmatrix}, \qquad b_{ij}^{T\bar{5}'} = \begin{pmatrix} \frac{4}{15} & 0 & \frac{16}{3} & \frac{48}{5} \\ 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & \frac{55}{3} & 24 \\ \frac{2}{5} & 0 & 8 & \frac{97}{5} \end{pmatrix}. \tag{B16}$$

As far as the Yukawa coupling involving RG factors, a_i^f, c_f, $d_f^{f'}$, and c_f^i [see Eqs. (B1) and (B2)], are concerned, within our model only top and "mirror-top" Yukawa couplings are large. All other Yukawa interactions are small and can be ignored. Thus, the Yukawa terms $\lambda_t q_3 t^c h$, $(\lambda_{\hat{t}\hat{b}}\hat{t}\hat{b} + \lambda_{\hat{t}^c\hat{\tau}^c}\hat{t}^c\hat{\tau}^c)T_{H'}$, and $\lambda_{\hat{t}}\hat{q}_3\hat{t}^c D'$ are relevant. All these four couplings unify at M_G due to gauge symmetry and D_2 parity. For the top Yukawa involved RG factors, in the energy interval $M_Z - M_I$, we have

$$a_i^t = \left(\frac{17}{10}, \frac{3}{2}, 2\right), \qquad c_t^i = \left(\frac{17}{20}, \frac{9}{4}, 8\right), \qquad (i = Y, w, c), \qquad c_t = \frac{9}{2}, \qquad d_t^{f'} = 0. \tag{B17}$$

In energy interval $M_I - M_G$, with replacement of the indices $(Y, w, c) \to (1, 2, 3)$, the corresponding RG factors will be the same. Since the mass of the state D' is $\sim M_I{}'$, the RG with $\lambda_{\hat{t}}$ will be relevant above the scale $M_I{}'$. Within our model, $M_{T_{H'}} \sim \Lambda'$, and in the RG, the couplings $\lambda_{\hat{t}\hat{b}}$ and $\lambda_{\hat{t}^c\hat{\tau}^c}$ will be relevant above the scale Λ'. Between the scales Λ' and M_I, the mirror matter has EW and $SU(3)'$ interactions. Therefore, we have

$$\mu = \Lambda' - M_I : (a_Y, a_w, a_{3'})^{\hat{\iota}\hat{b}} = \left(\frac{1}{15}, 2, \frac{4}{3}\right), \qquad (a_Y, a_w, a_{3'})^{\hat{\tau}^c\hat{\tau}^c} = \theta(\mu - M^{(3)}_{e^c\hat{e}^c})\left(\frac{13}{15}, 0, \frac{1}{3}\right),$$
$$(c^Y, c^w, c^{3'})_{\hat{\iota}\hat{b}} = \left(\frac{1}{10}, \frac{9}{2}, 8\right), \qquad (c^Y, c^w, c^{3'})_{\hat{\tau}^c\hat{\tau}^c} = \theta(\mu - M^{(3)}_{e^c\hat{e}^c})\left(\frac{13}{5}, 0, 4\right), \qquad c_{\hat{\iota}\hat{b}} = 4,$$
$$d^{\hat{\tau}^c\hat{\tau}^c}_{\hat{\iota}\hat{b}} = \theta(\mu - M^{(3)}_{e^c\hat{e}^c}), \qquad c_{\hat{\tau}^c\hat{\tau}^c} = 3\theta(\mu - M^{(3)}_{e^c\hat{e}^c}), \qquad d^{\hat{\iota}\hat{b}}_{\hat{\tau}^c\hat{\tau}^c} = 2\theta(\mu - M^{(3)}_{e^c\hat{e}^c}). \tag{B18}$$

Between M_I and $M_I{}'$ scales, with replacements $(Y, w) \to (1', 2')$, the corresponding factors will be the same. At and above the scale M_I, the $G_{321}{}'$ is unified in the $SU(5)'$ group, D' should be included in the RG, and three Yukawas unify $\lambda_{\hat{\iota}\hat{b}} = \lambda_{\hat{\tau}^c\hat{\tau}^c} = \lambda_{\hat{\iota}}$. Thus, dealing with $\lambda_{\hat{\iota}}$, we will have

$$\mu = M_I{}' - M_G : a^{\hat{\iota}}_{5'} = \frac{9}{2}, \qquad c_{\hat{\iota}} = 9, \qquad c^{5'}_{\hat{\iota}} = \frac{108}{5}, \qquad d^{f'}_{\hat{\iota}} = 0. \tag{B19}$$

1. Short-range RG factors for $d = 6$ operators

The baryon number violating $d = 6$ operators of Eq. (70) involve couplings $\mathcal{C}^{(e^c)}$ and $\mathcal{C}^{(l)}$, respectively. These couplings run, and in nucleon decay amplitudes, the short-range RG factors

$$A^l_S = \frac{\mathcal{C}^{(l)}(M_Z)}{\mathcal{C}^{(l)}(M_X)}, \qquad A^{e^c}_S = \frac{\mathcal{C}^{(e^c)}(M_Z)}{\mathcal{C}^{(e^c)}(M_X)} \tag{B20}$$

emerge. These factors, having SM gauge interactions and states below the GUT scale, were calculated in Ref. [31]. Within our model, calculation can be done similarly. The RG equations for $\mathcal{C}^{(l)}$ and $\mathcal{C}^{(e^c)}$, in one-loop approximation, are given by

$$4\pi\frac{d}{dt}\mathcal{C}^{(l)} = -\mathcal{C}^{(l)}\left[\theta(M_I - \mu)\left(\frac{23}{20}\alpha_Y + \frac{9}{4}\alpha_w\right) + 2\alpha_c + \theta(\mu - M_I)\left(\frac{23}{20}\alpha_1 + \frac{9}{4}\alpha_2\right)\right],$$
$$4\pi\frac{d}{dt}\mathcal{C}^{(e^c)} = -\mathcal{C}^{(e^c)}\left[\theta(M_I - \mu)\left(\frac{11}{20}\alpha_Y + \frac{9}{4}\alpha_w\right) + 2\alpha_c + \theta(\mu - M_I)\left(\frac{11}{20}\alpha_1 + \frac{9}{4}\alpha_2\right)\right]. \tag{B21}$$

Having numerical solutions for the gauge couplings, Eqs. (B21) can be integrated. Doing so and taking into account Eqs. (B20), within our model we obtain $A^l_S = 1.18$ and $A^{e^c}_S = 1.17$.

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