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Higgs couplings and naturalness

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Many extensions of the standard model postulate the existence of new weakly coupled particles, the top partners, at or below the TeV scale. The role of the top partners is to cancel the quadratic divergence in the Higgs mass parameter due to top loops. We point out the generic correlation between naturalness (the degree of fine-tuning required to obtain the observed electroweak scale), and the size of top partner loop contributions to Higgs couplings to photons and gluons. If the fine-tuning is required to be at or below a certain level, a model-independent lower bound on the deviations of these Higgs couplings from the standard model can be placed (assuming no cancellations between contributions from various sources). Conversely, if a precise measurement of the Higgs couplings shows no deviation from the standard model, a certain amount of fine-tuning would be required. We quantify this connection, and argue that a measurement of the Higgs couplings at the per cent level would provide a serious and robust test of naturalness.

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I. INTRODUCTION

The recent discovery of a new particle, roughly consistent with the standard model (SM) Higgs boson, has opened a new window into physics at the electroweak scale. In the next decade, the Higgs physics will enter the precision era, in which the goal will be to measure the properties of this particle, in particular its couplings, with the highest possible accuracy. Besides the continuing experiments at the LHC, the idea of a next-generation electron-positron collider such as the International Linear Collider (ILC) is currently under active discussion, with precise measurements of the Higgs couplings as its prime motivation [1]. Such a facility would be capable of measuring several couplings at a per cent level. It is important to understand the implications that these measurements could have on our ideas about physics beyond the standard model.

Predictions of many SM extensions for the Higgs couplings have already been extensively studied. In this paper, we point out a very general, and important, feature of such predictions. In any model which stabilizes the Higgs mass against radiative corrections by postulating weakly-coupled new physics, the amount of fine-tuning required to obtain the observed electroweak scale is *inversely correlated* with the size of certain non-SM contributions to the Higgs couplings to photons and gluons. In other words, if the fine-tuning is required to be at or below a certain level, a model-independent *lower bound* on the deviations of these Higgs couplings from the SM can be placed (assuming no cancellations between contributions from various sources). Conversely, if a precise measurement of the Higgs couplings shows no deviation from the SM, a certain amount of fine-tuning would be required. We will quantify these statements, and show that per cent level Higgs coupling measurements, expected to be achievable at the next-generation experimental facilities, would provide a serious test of naturalness of the electroweak scale. This gives a clear and compelling physics motivation for such measurements.[1]

The paper is organized as follows. In Sec. II, we present the general argument for the correlation between naturalness and loop-induced Higgs couplings to gluons and photons. The key observation is that the same object, the Higgs-dependent mass of the top partner (or partners), determines the dominant radiative corrections to the Higgs mass parameter, via the Coleman-Weinberg (CW) potential, and the top partner contributions to the Higgs couplings to gluons and photons, via the well-known "low-energy theorems" [5]. In Sec. III, we study the correlation between fine-tuning and Higgs couplings quantitatively, using a simple toy model with a single top partner (scalar or fermion) as the benchmark. In Sec. IV, we explore how the picture may be affected by the presence of a second top partner, and find that excepting small regions of parameter space where accidental cancellations occur, the conclusions of the benchmark one-partner analysis remain valid. We discuss our findings and conclude in Sec. V.

II. GENERAL ARGUMENT: TOP PARTNERS, NATURALNESS, AND THE HIGGS COUPLINGS

The starting point of our analysis is a single Higgs doublet H with the SM tree-level potential

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[1]For earlier work along these lines, see Refs. [2–4].

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4. \tag{1}$$

This hypothesis is the simplest interpretation of the LHC discovery consistent with all other experimental data. In particular, there is no evidence in the data of H mixing with other scalar fields, and the constraints on such mixing are now quite stringent. In the SM, the measurements of the Higgs vacuum expectation value (vev) and mass provide precise values for the parameters in the potential:

$$\mu = 90\ \mathrm{GeV}, \qquad \lambda = 0.13. \tag{2}$$

How natural are these parameters? To address this question, we need to consider quantum corrections to the potential (1). At the one-loop order, these corrections are conveniently given by the Coleman-Weinberg (CW) formula

$$V_{\mathrm{CW}}(h) = \frac{1}{2}\sum_k g_k(-1)^{F_k}\int\frac{d^4\ell}{(2\pi)^4}\log\left(\ell^2 + m_k^2(h)\right), \tag{3}$$

where the sum runs over all particles in the model, and g_k and F_k is the multiplicity and fermion number of each particle, respectively. For example, for a gauge-singlet complex scalar, $g = 2$ and $F = 0$; for a gauge-singlet Dirac fermion, $g = 4$ and $F = 1$. Here $h/\sqrt{2}$ is the real part of the $U(1)_{\mathrm{em}}$-neutral component of H; in the SM vacuum, $\langle h\rangle = 246$ GeV. The one-loop correction to the Higgs mass parameter is given by

$$\delta\mu^2 \equiv \left.\frac{\delta^2 V_{\mathrm{CW}}}{\delta h^2}\right|_{h=0}. \tag{4}$$

In the SM, the largest contribution to the CW potential comes from the top quark, since the top Yukawa is the strongest coupling of the Higgs:

$$\delta\mu^2 = -\frac{3y_t^2}{8\pi^2}\Lambda^2 + \ldots, \tag{5}$$

where Λ is the scale at which all loop integrals in V_{CW} are cut off. Since we expect $\Lambda \gg M_{\mathrm{EW}}$, the quantum correction to μ from the top loop is unreasonably large, and would require fine-tuning if no new physics is present. If the theory is weakly coupled at the TeV scale, the only way to avoid fine-tuning is to introduce a new particle, the *top partner*, with mass at or below the TeV scale. (Multiple top partners may be involved in the divergence cancellation.) Such partners can be spin-0 scalars, as in supersymmetric (SUSY) models,[2] or vectorlike spin-1/2 fermions, as in little Higgs [9,10] or 5-dimensional composite Higgs models [11].[3] In either case, the top partner mass has the form

$$m^2(T_i) = m_{0,i}^2 + c_i h^2 + \cdots, \tag{6}$$

where we allow for the possibility of multiple top partners labeled by T_i, and drop the terms of higher order in h. By dimensional analysis, such higher-order terms need to be suppressed by powers of a mass scale; our approximation is valid if this mass scale is $\gg v$. The absence of a term linear in h in the mass is a consequence of the top partners' vectorlike $SU(2)$ charges. The combined top sector contribution to the quadratic terms in the Higgs potential is

$$\delta\mu^2 = \frac{1}{16\pi^2}\left[\left(\sum_i g_i(-1)^{F_i}c_i - 6y_t^2\right)\Lambda^2 + \sum_i g_i(-1)^{F_i}c_i m_{0,i}^2\log\frac{\Lambda^2}{m_{0,i}^2} - 6y_t^2 m_t^2\log\frac{\Lambda^2}{m_t^2} + \ldots\right]. \tag{7}$$

Cancellation of the quadratic divergence yields the sum rule

$$6y_t^2 = \sum_i g_i(-1)^{F_i}c_i. \tag{8}$$

This sum rule is imposed by the symmetry of the theory in both SUSY and little Higgs. The remaining fine-tuning can be quantified by taking the ratio of the quantum correction to μ^2 to its measured value:[4]

$$\Delta = \frac{\delta\mu^2}{\mu^2} \approx 0.78\left(\sum_i g_i(-1)^{F_i}c_i\left(\frac{m_{0,i}}{1\ \mathrm{TeV}}\right)^2\log\frac{\Lambda^2}{m_{0,i}^2} - 6y_t^2\left(\frac{m_t}{1\ \mathrm{TeV}}\right)^2\log\frac{\Lambda^2}{m_t^2}\right). \tag{9}$$

If $\Delta \gg 1$, the theory must be fine-tuned to accommodate the observed electroweak symmetry breaking. Note that Δ only measures fine-tuning in the Higgs mass parameter; we

[2]The special role played by the stops, the partners of the top quarks, in determining the degree of naturalness of the electroweak scale in SUSY models was emphasized in Refs. [6], and more recently in Refs. [7,8].

[3]In principle, a spin-1 top partner is also a possibility [12]; we will not consider this case here.

[4]Our fine-tuning measure is essentially equivalent to taking a logarithmic derivative of the Higgs mass parameter prediction at the weak scale with respect to the value of this parameter at scale Λ: $\Delta \approx \frac{d\log\mu^2(m_{\mathrm{weak}})}{d\log\mu^2(\Lambda)}$, up to corrections of order $1/\Delta$. In this sense, it is close to the familiar "Barbieri-Giudice" (BG) fine-tuning measure [13] used in many SUSY analyses. In the BG approach, sensitivity with respect to all parameters of the UV theory is taken into account; in our framework, we aim for a high degree of model-independence, and do not specify a complete UV theory. Nevertheless, our approach applied to SUSY theories would produce results similar to BG, except in the special situation where nongeneric "focusing" behavior of RG evolution of μ^2 is taking place [14]. In that case, the fine-tuning indicated by our measure would exceed the BG tuning.

assume that the observed quartic coupling can be generated with no additional fine-tuning. In general, this assumption is justified by the absence of quadratic divergences in the renormalization of λ: the SM top loop contribution is $\delta\lambda \sim (y^4/16\pi^2)\log\Lambda/m_t\lambda$ even for a Planck-scale cutoff. In certain models, λ may need to be fine-tuned for specific model-dependent reasons: for example, in λ-SUSY models, there are two tree-level contributions to λ of different physical origin, which are individually too large and need to cancel to reproduce the observed Higgs mass. In such models, fine-tuning in λ needs to be considered [15], and the discussion becomes highly model dependent. However, since the required cancellation in μ and λ have very different physical origins, one should regard them as uncorrelated, additive effects, so that the Δ defined in this paper effectively provides at least a lower bound on the fine-tuning in all situations that we are aware of.

The effects of the top partners on the Higgs couplings first appear at the one-loop level. The best place to look for such effects is in the couplings which vanish in the SM at the tree level. We focus on the couplings of the Higgs to gluons and photons. At the one-loop order, the contributions of particles with masses $\gg m_h$ to these couplings are described by effective operators,

$$\mathcal{L}_{h\gamma\gamma} = \frac{2\alpha}{9\pi v} C_\gamma h F_{\mu\nu}F^{\mu\nu}, \qquad \mathcal{L}_{hgg} = \frac{\alpha_s}{12\pi v} C_g h G_{\mu\nu}G^{\mu\nu}. \tag{10}$$

The Wilson coefficients can be found using the well-known "low-energy theorems" [5]:

$$\begin{aligned} C_\gamma &= 1 + \frac{3}{8}\sum_f^{\text{Dirac fermions}} N_{c,f}Q_f^2 \frac{\partial \ln m_f^2(v)}{\partial \ln v} \\ &\quad + \frac{3}{32}\sum_s^{\text{scalars}} N_{c,s}Q_s^2 \frac{\partial \ln m_s^2(v)}{\partial \ln v}, \\ C_g &= 1 + \sum_f^{\text{Dirac fermions}} C(r_f)\frac{\partial \ln m_f^2(v)}{\partial \ln v} \\ &\quad + \frac{1}{4}\sum_s^{\text{scalars}} C(r_s)\frac{\partial \ln m_s^2(v)}{\partial \ln v}, \end{aligned} \tag{11}$$

where the first term is the contribution of the SM top loops, the sum runs over the top partners, and $N_{c,i}$ and Q_i are the dimension of the $SU(3)_c$ representation and the electric charge (in units of electron charge) of the particle i. Note that the exact same objects, the Higgs-dependent masses of top partners $m_i(h)$, enter the CW potential and the Higgs couplings, providing a very general and robust connection between these quantities. In the approximation of Eq. (6), we obtain

$$\begin{aligned} C_\gamma &\approx 1 + \frac{3}{4}\sum_f \frac{N_{c,f}Q_f^2 c_f v^2}{m_{0,f}^2 + c_f v^2} + \frac{3}{16}\sum_s \frac{N_{c,s}Q_s^2 c_s v^2}{m_{0,s}^2 + c_s v^2}, \\ C_g &\approx 1 + 2\sum_f \frac{C(r_f)c_f v^2}{m_{0,f}^2 + c_f v^2} + \frac{1}{2}\sum_s \frac{C(r_s)c_s v^2}{m_{0,s}^2 + c_s v^2}. \end{aligned} \tag{12}$$

The set of coefficients $\{m_{0,i}, c_i\}$ determines both the fine-tuning Δ and the Wilson coefficients, generically resulting in a correlation between these quantities. Assuming that there are no other non-SM contributions to the Higgs couplings to photons and gluons, the deviations of these couplings from the SM in the presence of top partners are given by

$$\begin{aligned} R_g &\equiv \frac{g(hgg)}{g(hgg)|_{\text{SM}}} = C_g, \\ R_\gamma &\equiv \frac{g(h\gamma\gamma)}{g(h\gamma\gamma)|_{\text{SM}}} \approx 1 - 0.27(C_\gamma - 1), \end{aligned} \tag{13}$$

where the contribution of the W loop has been taken into account in the photon coupling.

It should be noted that in the above discussion, we assumed that the top loop contribution to the Higgs couplings is exactly equal to its value in the SM. This assumption may break down due to deviations of the top Yukawa from its SM value: this situation is generic in models with extended Higgs sectors, such as the minimal supersymmetric standard model (MSSM) away from the decoupling limit, many composite Higgs and little Higgs models, etc. In such models, the deviation of the $hgg/h\gamma\gamma$ couplings from the SM would be due to a combination of the top-partner loops that we focus on, and the effect of top Yukawa shift in the SM diagram. It should be emphasized that the second effect is highly model dependent, and cannot be quantitatively correlated with naturalness in a broad framework. However, the top Yukawa coupling can be independently measured, for example in the $t\bar{t}h$ final state at the ILC running at $\sqrt{s} = 1$ TeV, where an accuracy of about 2–3% is projected for this coupling [16]. If a deviation of the top Yukawa from the SM is observed, it should be taken into account in the computation of the top loop to $hgg/h\gamma\gamma$. Once that correction is implemented, our analysis would apply equally well in the situation with SM and non-SM top Yukawa. Of course, our ability to probe top partners up to a certain mass scale would then be limited by the accuracy of the direct top Yukawa measurement as well as the measurements of $hgg/h\gamma\gamma$ couplings. This should be kept in mind when evaluating the potential of future Higgs precision program to probe naturalness.

Even if no direct measurement of the top Yukawa is available, our framework would still be relevant, in the following sense. If the $hgg/h\gamma\gamma$ couplings are measured to be consistent with the SM to a certain precision, our analysis would provide a lower bound on the top partner mass, and therefore on fine-tuning. This bound could be relaxed if there is a cancellation (full or partial) between the top-partner loop and the shift in the top loop from the SM. However, in most models, these two effects are controlled by independent parameters: for example, in SUSY, stop-sector and Higgs-sector soft SUSY-breaking terms, respectively. If this is the case, any cancellation between the two must be regarded as accidental, and, if precise, fine-tuned. So, a fine-tuning bound obtained without including the top Yukawa shifts is still applicable, at least qualitatively. The only exception that we are aware of occurs in some composite Higgs models. In these models, the shift in the top loop contribution to hgg and $h\gamma\gamma$ couplings is of the same order as the top partner loop contributions to these couplings [17]. The effect of the additional shift is model dependent. In some simple models, a cancellation between the top-Yukawa and top partner loop effects may occur, due to the specific structure of the top mass matrix [18,19] (although it should be emphasized that this cancellation is *not* a direct consequence of the shift symmetry responsible for keeping the Higgs light in these models). In this case,

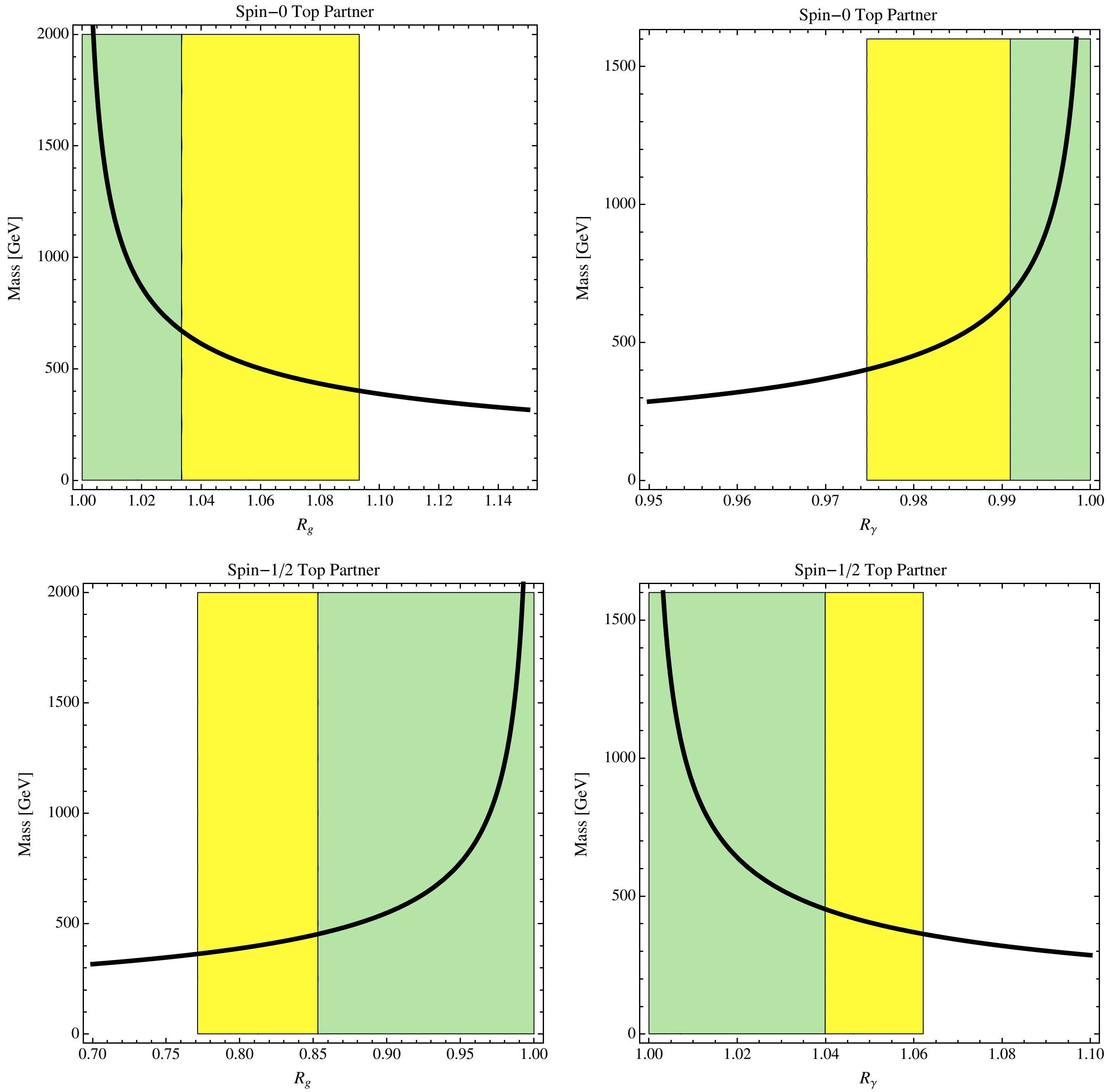

FIG. 1 (color online). Fractional deviation of the Higgs coupling to gluons (left panel) and photons (right panel) from the SM value, as a function of the top partner mass. Top row: Spin-0 top partner. Bottom row: Spin-1/2 top partner. Regions currently allowed by the LHC and Tevatron data are shown in dark-gray/green (68% c.l.) and light-gray/yellow (95% c.l.).

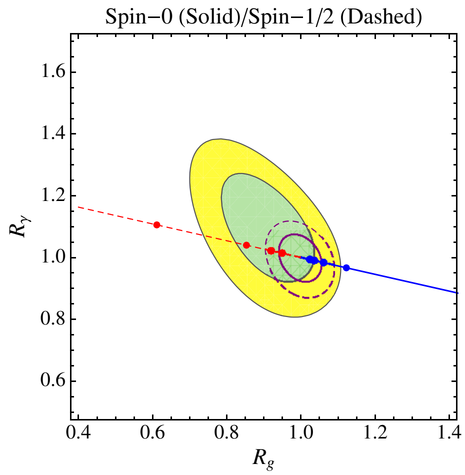

FIG. 2 (color online). Regions allowed by the LHC and Tevatron measurements of the Higgs rates in the $R_\gamma - R_g$ plane, at the 68% c.l. (dark-gray/green) and 95% c.l. (light-gray/yellow). The spin-0 top partner model predicts deviations along the solid/blue line, while the spin-1/2 top partner induces deviations along the dashed/red line. The points on both lines correspond to the partner masses of 350, 500, 650, and 800 GeV. For comparison, projected constraints from the LHC-14 [1] are shown by dashed/purple ellipses.

our analysis would not apply. Note, however, that in all theoretically motivated examples that we are aware of, the shift in the top Yukawa is due to Higgs compositeness, at a scale not far above the electroweak scale. Such models also predict large, tree-level deviations of the Higgs couplings to W and Z bosons, which will be probed with high precision by any experiment capable of precise measurements of gluon and photon couplings.

III. BENCHMARK MODEL: A SINGLE TOP PARTNER

The simplest possibility is that there is a single top partner, in fundamental rep of $SU(3)$ and with electric charge $2/3$, just like the SM top. (The single partner model is applicable to models with multiple top partners if they have the same $m_{0,i}$ parameters: for example, the MSSM with two degenerate stops.) This simple model can be used as a benchmark for evaluating the potential of precision Higgs couplings to probe naturalness. In this case, c_1 is fixed by the sum rule (8), and $m_{0,1}$ is the only free parameter in the predictions:

$$C_\gamma = C_g = 1 + \frac{1}{4}\frac{y_t^2 v^2}{m_{0,1}^2 + y_t^2 v^2} \quad (\text{spin 0 partner});$$
$$C_\gamma = C_g = 1 - \frac{y_t^2 v^2}{2m_{0,1}^2 - y_t^2 v^2} \quad (\text{spin 1/2 partner}). \qquad (14)$$

The correlation between the Higgs coupling deviations from the SM and the mass of the top partner is shown in Fig. 1. For reference, we also show constraints obtained from a fit to the current LHC-7, LHC-8 [20,30], and Tevatron [31] data, assuming that top-partner loops are the only non-SM contribution to Higgs couplings. To obtain these constraints, we fit to the published Higgs event rates observed in various channels, assuming no correlation between any of the data points, and include the theoretical uncertainties provided by the Higgs cross section working group [32]. (For details of the fit, see the Appendix.) Our results are roughly consistent with the more detailed fits performed by the LHC collaborations [33,34]: for example, our one-sigma error bar on R_g

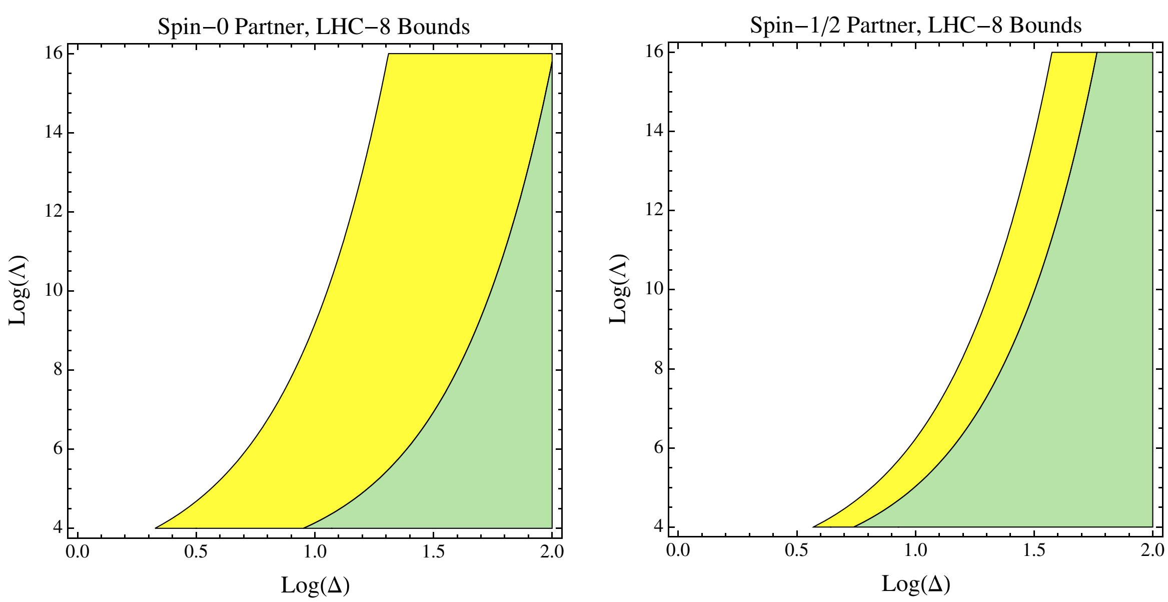

FIG. 3 (color online). Regions allowed by the LHC and Tevatron data in the $\Delta - \log\Lambda$ plane, at the 68% c.l. (dark-gray/green) and 95% c.l. (light-gray/yellow). Here, Λ is the scale (in GeV) where the logarithmic divergence in the Higgs mass renormalization is cut off. Left panel: Spin-0 top partner. Right panel: Spin-1/2 top partner.

is about ± 0.1, compared to 0.14 reported by the ATLAS collaboration [33] in a two-parameter fit where R_g and R_γ were assumed to be the only non-SM contributions to the Higgs rates. A broad range of top partner masses in the region motivated by naturalness are currently allowed by data: the 95% c.l. limit on the top partner mass is about 320 GeV for a spin-0 top partner, and 400 GeV for a spin-1/2 partner. (Note that our best-fit value for R_g is about 0.7σ below the SM expectation of 1.0, resulting in a slightly stronger bound on the spin-0 partners and a slightly weaker bound on the spin-1/2 case.) However, future precise measurements of the Higgs coupling at the LHC-14 and a future e^+e^- facility would probe much of the interesting parameter space. For example, a 1% measurement of the gluon coupling will probe the top partner masses in excess of 1 TeV, for both spin-0 and spin-1/2 top partners.

Since the one-partner model has only one free parameter, the deviations in gluon and photon couplings are correlated. This is shown in Fig. 2, along with the current and future LHC constraints on the two couplings. (We used the information provided in Ref. [1] to estimate the LHC-14 contours.) It is clear that the constraints are strongly dominated by the gluon coupling measurement, due to both the slope of the trajectory and the stronger

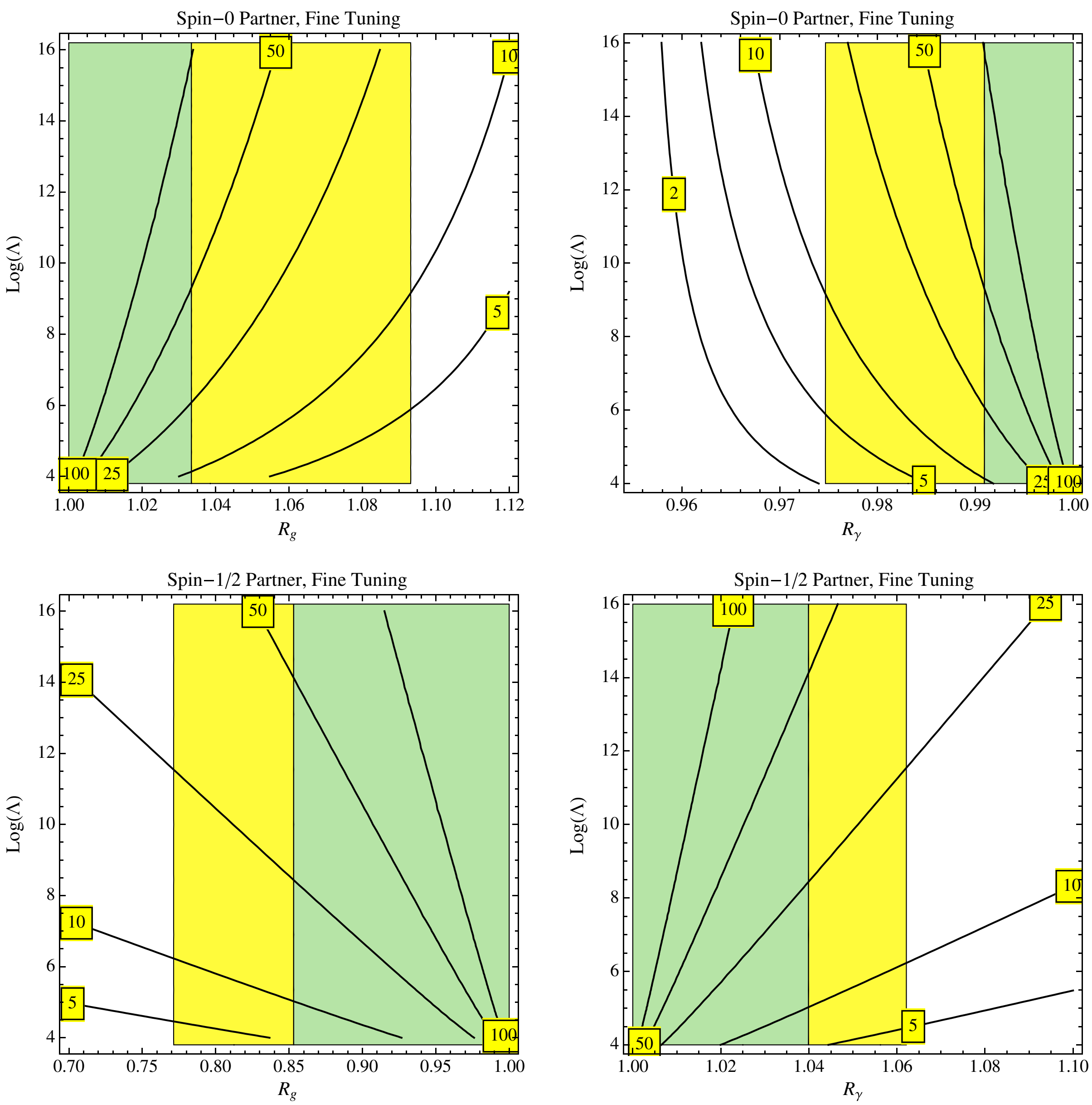

FIG. 4 (color online). Fine-tuning as a function of the fractional deviation of the Higgs coupling to gluons (left panel) and photons (right panel) from the SM value, and the energy scale Λ (in GeV) where the logarithmic divergence in the Higgs mass renormalization is cut off. Top row: Spin-0 top partner. Bottom row: Spin-1/2 top partner. Regions currently allowed by the LHC and Tevatron data are shown in dark-gray/green (68% c.l.) and light-gray/yellow (95% c.l.).

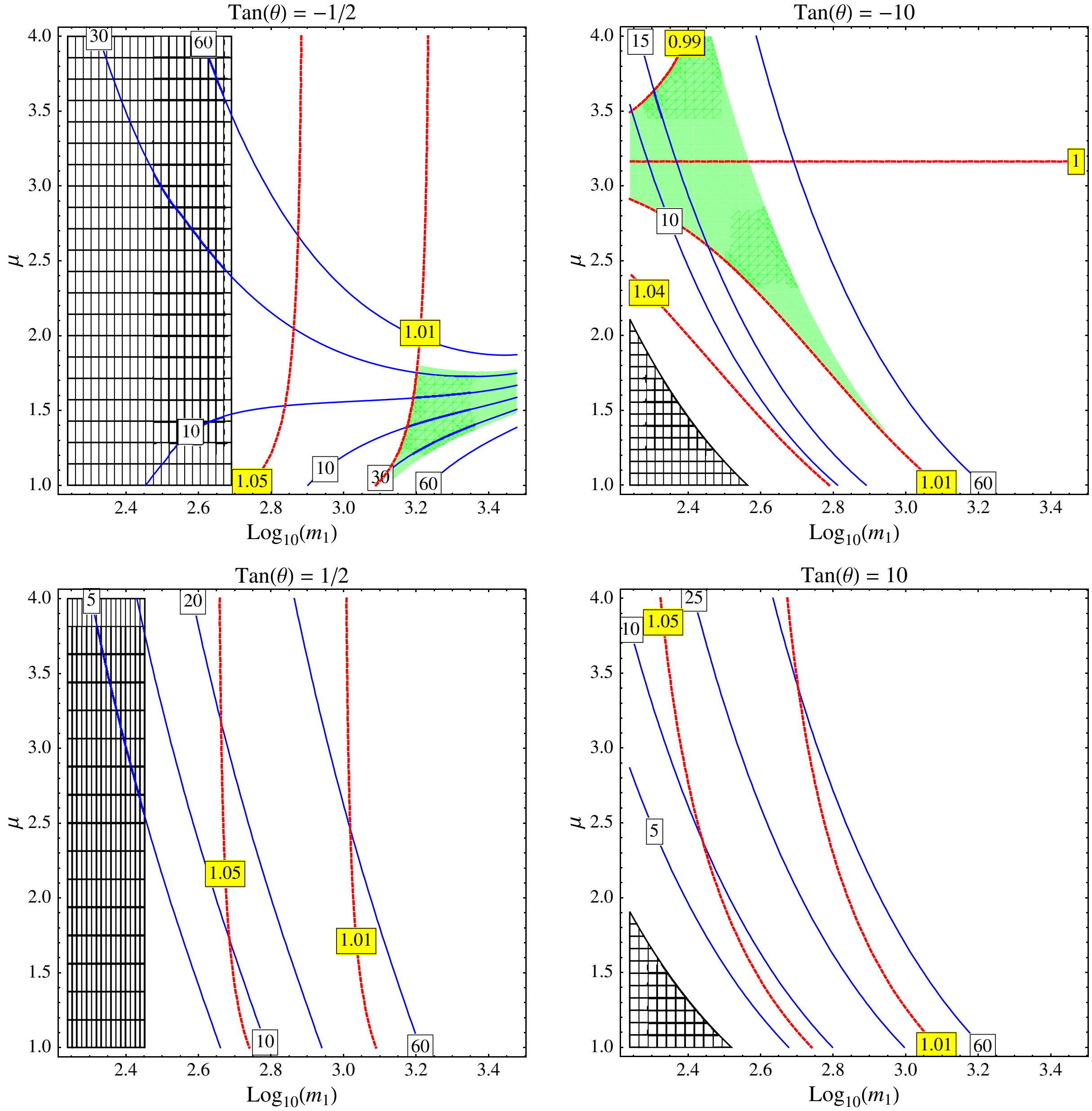

FIG. 5 (color online). Contours of fine tuning (solid/blue) and R_g (dashed/red) for fixed values of θ, with $\Lambda = 20$ TeV. The shaded regions correspond to points where $|R_g - 1| < 0.01$ but for which the amount of fine tuning is less than what is predicted for a one scalar partner model with $R_g = 0.01$. The cross-hatched regions corresponds to points where $|c_i v^2/m_{0,i}^2| > 1$. The top partner mass m_1 is in units of GeV.

experimental bound on R_g. If a deviation from the SM is observed, it would be straightforward to check whether it can be interpreted within the one-partner framework by simply checking whether the trajectories shown here intersect with the experimentally determined region. If the answer is positive, these measurements will also allow us to unambiguously determine the top partner spin.

The connection between Higgs couplings and fine-tuning is illustrated more directly in Figs. 3 and 4. Since the top partners only cut off the quadratic divergence in the top loop, leaving the logarithmic divergence uncanceled, the value of the fine-tuning measure Δ depends logarithmically on the scale Λ where the logarithmic divergence is cut off. The value of Λ is very model dependent. To demonstrate its effect, we vary Λ between the "low" 10 TeV scale, representing a rough lower bound on this scale in realistic models, and the "high" 10^{16} GeV, motivated by grand unification. In the case of a spin-0 partner, the 95% c.l. lower bound on the fine-tuning from the current Higgs data varies between ~1/2 for a low-scale model and ~1/20 for a high-scale model. The bounds for the spin-1/2 partner are slightly stronger, between ~1/3 and ~1/30. Of course, these bounds can be dramatically improved by the future precise measurements of Higgs couplings. For example, if the gluon

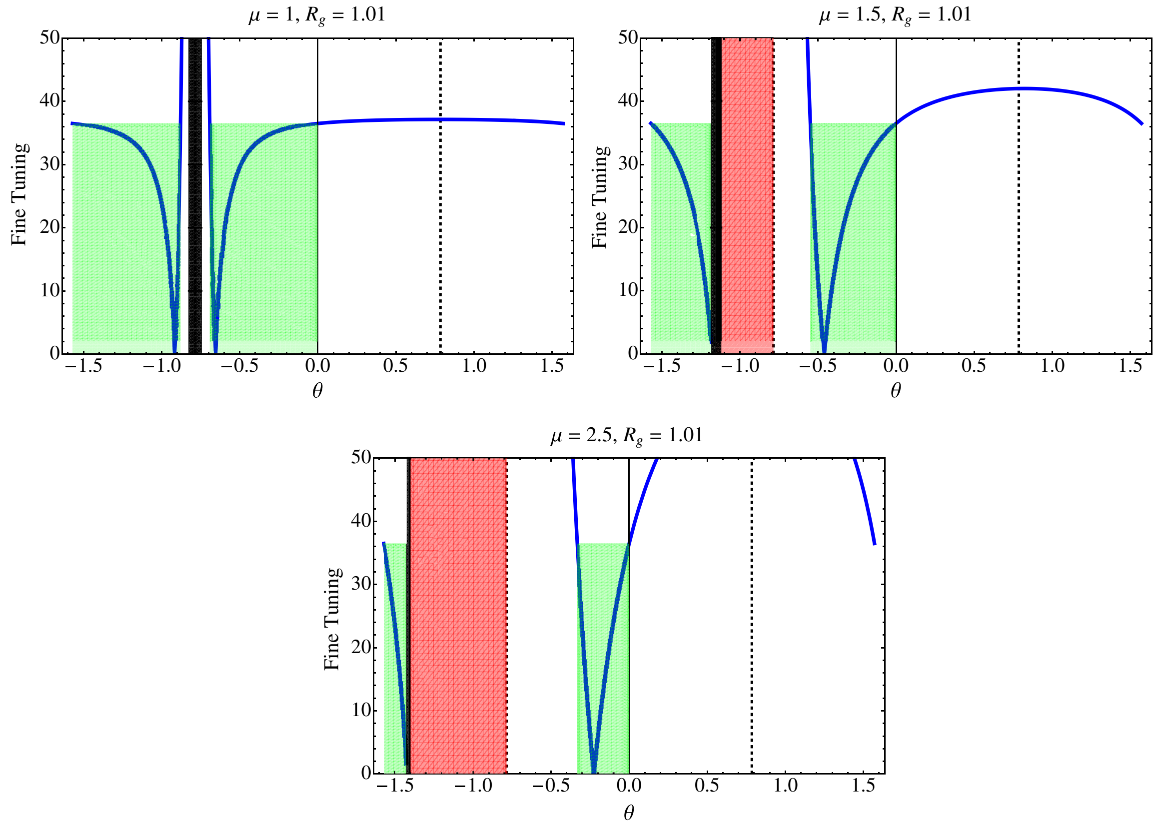

FIG. 6 (color online). Fine tuning (thick lines) as a function of θ for fixed values of μ and R_g, with $\Lambda = 20$ TeV. The regions shaded in black indicate values of θ where $|c_i v^2/m_{0,i}^2| > 1$; the regions shaded in red/dark-gray are unphysical due to $m_1^2 < 0$. Light-gray/green regions indicate values of θ for which the fine-tuning is less than what is predicted for a one scalar partner model with $R_g = 0.01$.

coupling is found to agree with the SM at a 1% level, the minimal amount of fine-tuning required would be $\sim 1/25$ for the low-scale model, and $\sim 1/400$ for a high-scale model.

We emphasize that the probe of naturalness advocated here is complementary to direct searches for top partners. Sensitivity of the direct searches, especially at hadron colliders such as the LHC, depends on details of the spectrum and the decay patterns of the produced top partners. For example, while the LHC "headline" direct bounds on stops are already about 600–700 GeV [35], these bounds can be evaded in a variety of ways, e.g. "stealthy" [36] or compressed stop spectra, or R-parity violation. In contrast, the nature of the Higgs coupling deviations discussed here is very tightly connected to the restoration of naturalness, and the connection is essentially model independent. Of course, the simple correlation exhibited in the benchmark one-partner models may be violated in more complicated setups, where for example cancellation among various loop contributions is in principle possible. We will investigate an example of this in the next section. Still, it is worth emphasizing that the "loopholes" inherent in the test of naturalness proposed here are completely different from the ones plaguing direct searches. Together, these techniques should provide an extremely powerful and robust test of naturalness.

IV. TWO TOP PARTNERS

Cancellation of the top loop divergence does not have to be achieved with a single new particle. For example, in the MSSM, there are two spin-0 top partners, $\tilde{t}_1$ and $\tilde{t}_2$, generically with different masses, both of which participate in divergence cancellation. Models with multiple top partners are characterized by multidimensional parameter spaces, even after the divergence cancellation sum rule is imposed. We expect that throughout most of the parameter space of a given model, the correlation between Higgs couplings and fine-tuning studied in Sec. III continues to hold. However, there could be special regions of parameter space where it can fail, due to cancellations between contributions of the two top partners, either to the CW potential or to the Higgs couplings. To illustrate this, in this section we will consider a toy model with two spin-0 partners, both in fundamental rep of $SU(3)$ and with electric charge $2/3$. (These are the quantum number assignments of the MSSM stops, so the results of this section will approximately apply in that model; the correspondence becomes exact in the limit of soft masses large compared to v.[5]) The model has four free parameters, $\{m_{0,i}, c_i\}$, $i = 1, 2$; after the

[5]Many authors examined the stop loop contributions to Higgs couplings in the MSSM; see, for example, Refs. [37].

sum rule (8) is imposed, the number is reduced to 3. We choose to work in terms of

$$m_1; \quad \mu = \frac{m_2}{m_1}; \quad \theta = \tan^{-1}\frac{c_2}{c_1}, \tag{15}$$

where $m_i = \sqrt{m_{0,i}^2 + c_i v^2}$ are the physical masses of the top partners, and $m_1 < m_2$. Note that in the limits ($\mu \to 1$, any θ) and ($\theta \to 0$, any μ), the model reduces to the one-partner model with the same $m_{0,1}$, considered in Sec. III above.

The main conclusion of our analysis of this model is that the correlation of the Higgs coupling deviations and fine-tuning, observed in the benchmark one-partner models of Sec. III, is rather robust. This is illustrated in Fig. 5. For example, suppose that the gluon coupling is found experimentally to agree with the SM prediction at a level of 1%. Interpreting this bound within a one-partner model places a lower bound on fine-tuning of about $1/35$ assuming $\Lambda = 20$ TeV. The two-partner model *can* produce the gluon coupling within 1% of the SM value with *smaller* fine-tuning; however, the regions of parameter space where this occurs (shaded in green in Fig. 5) are rather small, so an accidental cancellation is clearly involved. Note also that such accidental reduction in fine-tuning can only occur when c_1 and c_2 have opposite signs ($\theta < 0$). The accidental nature of the fine-tuning reduction is further illustrated in Fig. 6: once the masses are fixed, fine-tuning drops significantly below the value inferred from the one-partner model only for a narrow range of the couplings.

In principle, cancellation of the top quadratic divergence may involve > 2 new particles, although we are not aware of any explicit model in which this is the case. It seems reasonable to conjecture that if this were the case, the correlation of Higgs coupling deviation and fine-tunings would persist, modulo possible accidental cancellations.

V. CONCLUSIONS

In this paper, we pointed out and quantified a correlation between the level of fine-tuning of electroweak symmetry breaking and the deviations of the Higgs couplings to photons and gluons from their SM values. The connection holds in a very large class of well-motivated models: the basic assumptions are that the physics at the weak/TeV scale is weakly coupled, and that the quadratic divergence in the Higgs mass from the SM top loop is canceled by loops of new particles, the top partners. The top partners' contributions to the Higgs mass parameter and to the Higgs couplings to photons and gluons are determined by the same objects, their Higgs-dependent masses, resulting in a simple relationship between them. Thus, measuring Higgs couplings precisely provides a robust, model-independent test of naturalness. We showed that a measurement of Higgs couplings to gluons and photons at a per cent level will either result in a discovery of a deviation from the SM, or imply that electroweak symmetry breaking is significantly tuned. This test of naturalness should be within the power of the proposed next-generation electron-positron collider such as the ILC.

A potential "loophole" in our argument is that the top partner contributions to the hgg and $h\gamma\gamma$ couplings may be canceled by other non-SM contributions to these vertices. In the case of multiple top partners, there is also the possibility of cancellations of the top partners' contributions to hgg and $h\gamma\gamma$ couplings among themselves. Typically, such cancellations should be regarded as accidental, and therefore unlikely. This was illustrated with an example of a two top partner model in Sec. IV. The only example that we are aware of where the cancellation of the top partner contributions to hgg and $h\gamma\gamma$ happens for a reason that seems inherent to the structure of the theory and not accidental is the model studied in Ref. [18]. However, the composite nature of the Higgs in that model implies large tree-level deviations of the Higgs couplings to W and Z bosons, and therefore it will still not escape detection via measurements of Higgs couplings.

So far, naturalness of the electroweak scale has been mainly probed through direct searches for the top partners, which will of course continue in the next decade. We emphasize the complementarity between this program and the test of naturalness proposed here. The Higgs couplings test does not suffer from the well-known loopholes which plague direct searches (e.g. special spectra or R-parity violation). At the same time, there seems to be no reason for models where the deviations of hgg and $h\gamma\gamma$ are suppressed, for whatever reason, to pose unusual difficulties for direct searches. Taken together, the two programs will provide a powerful and robust test of naturalness.

In summary, we believe that the test of naturalness proposed here provides a compelling motivation for the future program of precision Higgs coupling measurements. We hope that this program will be realized in the coming years.

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Note added.—As we were completing this manuscript, Ref. [38] appeared on the arXiv.org in which similar ideas were explored.

APPENDIX: FIT PROCEDURE

The fit to the experimental results has been performed by employing the 2013 Moriond data released by ATLAS, CMS, and the Tevatron experiments, summarized in Tables I–III. The data is presented in the form of the ratios

TABLE I. ATLAS data used in fits. We list best fits to signal strength μ and relative errors, as well as weights ζ (when provided) corresponding to gluon-gluon fusion (G), vector initiated (V), and top associated (T) production. In the $\gamma\gamma$ channels first U (C) corresponds to mostly unconverted (converted) photons, c (t) if detected centrally (or not), and H or L if at high or low p_T. The label "comb" indicates that the result is a combination of 7 and 8 TeV data sets.

Channel	$\hat{\mu}$ (7 TeV)	$\zeta^{(\mathrm{G,V,T})}$ (%)	$\hat{\mu}$ (8 TeV)	$\zeta^{(\mathrm{G,V,T})}$ (%)	Refs.
$b\bar{b}$	-2.7 ± 1.55	(0, 100, 0)	1.0 ± 1.4	(0, 100, 0)	[20]
$\tau\tau$	Comb.	—	0.7 ± 0.7	(20, 80, 0)	[21]
$WW(0j)$	0.06 ± 0.60	Inclusive	$0.92^{+0.63}_{-0.49}$	Inclusive	[22]
$WW(1j)$	$2.04^{+1.88}_{-1.30}$	Inclusive	$1.11^{+1.20}_{-0.82}$	Inclusive	
$WW(2j)$	—	—	$1.79^{+0.94}_{-0.75}$	(20, 80, 0)	
ZZ	Comb.	—	$1.7^{+0.5}_{-0.4}$	Inclusive	[23]
$\gamma\gamma$ (U-c-L)	$0.53^{+1.37}_{-1.44}$	(93, 7, 0)	$0.87^{+0.73}_{-0.7}$	(93.7,6.2,0.2)	
$\gamma\gamma$ (U-c-H)	$0.22^{+1.97}_{-1.91}$	(67, 31, 2)	$0.96^{+1.07}_{-0.95}$	(79.3, 19.2, 1.4)	
$\gamma\gamma$ (U-r-L)	$2.53^{+1.69}_{-1.69}$	(93, 7, 0)	$2.50^{+0.92}_{-0.77}$	(93.2, 6.6, 0.1)	
$\gamma\gamma$ (U-r-H)	$10.45^{+3.65}_{-3.73}$	(65, 33, 2)	$2.69^{+1.35}_{-1.17}$	(78.1, 20.8, 1.1)	
$\gamma\gamma$ (C-c-L)	$6.1^{+2.6}_{-2.66}$	(93, 7, 0)	$1.39^{+1.01}_{-0.95}$	(93.6, 6.2, 0.2)	
$\gamma\gamma$ (C-c-H)	$-4.38^{+1.82}_{-1.74}$	(67, 31, 2)	$1.98^{+1.54}_{-1.26}$	(78.9, 19.6, 1.5)	
$\gamma\gamma$ (C-r-L)	$2.72^{+1.99}_{-2.02}$	(93, 7, 0)	$2.23^{+1.14}_{-0.95}$	(93.2, 6.7, 0.1)	
$\gamma\gamma$ (C-r-H)	$-1.7^{+2.99}_{-2.81}$	(65, 33, 2)	$1.27^{+1.32}_{-1.23}$	(77.7, 21.2, 1.1)	[24,25]
$\gamma\gamma$ (C-trans)	$0.37^{+3.6}_{-3.63}$	(89, 11, 0)	$2.78^{+1.72}_{-1.57}$	(90.7, 9.0, 0.2)	
$\gamma\gamma$ (dijet)	$2.72^{+1.9}_{-1.88}$	(23, 77, 0)	—	—	
$\gamma\gamma$ (loose high mass jj)	—	—	$2.75^{+1.78}_{-1.38}$	(45, 54.9, 0.1)	
$\gamma\gamma$ (tight high mass jj)	—	—	$1.61^{+0.83}_{-0.67}$	(23.8, 76.2, 0)	
$\gamma\gamma$ (low mass jj)	—	—	$0.32^{+1.72}_{-1.44}$	(48.1, 49.9, 1.9)	
$\gamma\gamma$ ($E_{\mathrm{T}}^{\mathrm{miss}}$ significance)	—	—	$2.97^{+2.71}_{-2.15}$	(4.1, 83.8, 12.1)	
$\gamma\gamma$ (One-lepton)	—	—	$2.69^{+1.97}_{-1.66}$	(2.2, 79.2, 18.6)	

TABLE II. CMS data used in fits. The diphoton channels notation matches that of [30].

Channel	$\hat{\mu}$ (7 TeV)	$\zeta_i^{(\mathrm{G,V,T})}$ (%)	$\hat{\mu}$ (8 TeV)	$\zeta_i^{(\mathrm{G,V,T})}$ (%)	Refs.
$b\bar{b}$	0.59 ± 1.17	(0, 100, 0)	0.41 ± 0.94	(0, 100, 0)	[26]
$\tau\tau$ (0/1j)	Comb.	—	$0.74^{+0.49}_{-0.52}$	Inclusive	
$\tau\tau$ (VBF)	Comb.	—	1.39 ± 0.59	(0, 100, 0)	[27]
$\tau\tau$ (VH)	Comb.	—	0.76 ± 1.48	(0, 100, 0)	
$WW(0/1j)$	Comb.	—	0.76 ± 0.21	Inclusive	[28]
WW (VH)	Comb.	—	0.3 ± 1.5	(0, 100, 0)	
ZZ (untagged)	Comb.	—	$0.84^{+0.32}_{-0.26}$	(95, 5, 0)	[29]
ZZ (dijet tag)	—	—	$1.22^{+0.84}_{-0.57}$	(80, 20, 0)	
$\gamma\gamma$ (untagged 0)	$3.85^{+2.01}_{-1.66}$	(61.4, 35.5, 3.1)	$2.19^{+0.95}_{-0.79}$	(72.9, 24.6, 2.6)	
$\gamma\gamma$ (untagged 1)	$0.19^{+1.01}_{-0.95}$	(87.6, 11.8, 0.5)	$0.05^{+0.68}_{-0.67}$	(83.5, 15.5, 1.0)	
$\gamma\gamma$ (untagged 2)	$0.05^{+1.26}_{-1.15}$	(91.3, 8.3, 0.3)	$0.32^{+0.51}_{-0.50}$	(91.7, 7.9, 0.4)	
$\gamma\gamma$ (untagged 3)	$1.48^{+1.66}_{-1.6}$	(91.3, 8.5, 0.2)	$-0.36^{+0.89}_{-0.85}$	(92.5, 7.2, 0.2)	
$\gamma\gamma$ (dijet)	$4.19^{+2.35}_{-1.76}$	(26.8, 73.1, 0.0)	—	—	[30]
$\gamma\gamma$ (dijet loose)	—	—	$0.8^{+1.12}_{-0.99}$	(46.8, 52.8, 0.5)	
$\gamma\gamma$ (dijet tight)	—	—	$0.29^{+0.68}_{-0.58}$	(20.7, 79.2, 0.1)	
$\gamma\gamma$ (MET)	—	—	$1.92^{+2.61}_{-2.31}$	(0.0, 79.3, 20.8)	
$\gamma\gamma$ (Electron)	—	—	$-0.63^{+2.75}_{-1.97}$	(1.1, 79.3, 19.7)	
$\gamma\gamma$ (Muon)	—	—	$0.42^{+1.8}_{-1.39}$	(21.1, 67.0, 11.8)	

TABLE III. Tevatron data used in fits.

Channel	$\hat{\mu}$ (2 TeV)	$\zeta^{(\mathrm{G,V,T})}$ (%)	Ref.
$b\bar{b}$	$1.59^{+0.69}_{-0.72}$	(0, 100, 0)	
$\tau\tau$	$1.68^{+2.28}_{-1.68}$	(50, 50, 0)	[31]
WW	$0.94^{+0.85}_{-0.83}$	(77.5, 22.5, 0)	
$\gamma\gamma$	$5.97^{+3.39}_{-3.12}$	(77.5, 22.5, 0)	

between the observed signal strength in a certain channel and the standard model prediction:

$$\mu = \frac{(\sigma_{\mathrm{prod}} \times \mathrm{BR})^{\mathrm{obs}}}{(\sigma_{\mathrm{prod}} \times \mathrm{BR})^{\mathrm{SM}}}. \tag{A1}$$

The production cross section is defined as

$$\sigma_{\mathrm{prod}} = \sum_{\alpha} \xi^{\alpha} \sigma^{\alpha}, \tag{A2}$$

where σ^{α} represents the cross section for a particular production mechanism, while ξ^{α} are the corresponding efficiencies (which vary depending on the production channel, due to different kinematics of the Higgs boson, trigger efficiencies, etc.) The three production channels included in this analysis are: gluon gluon fusion (G), vector associated production and vector boson fusion (V), top associated production (T). We reconstruct the efficiencies ξ^{α} from the weights ζ^{α} provided by the experiments.

The fit is obtained with the minimum χ^2 procedure, with the χ^2 function defined as

$$\chi^2(R_j) = \sum_i \frac{(\hat{\mu}_i - \mu_i(R_j))^2}{\delta\mu_i^2}, \tag{A3}$$

where $\hat{\mu}_i$ is the experimental central value, $\mu_i(R_j)$ is the model prediction as function of the parameters R_i, and $\delta\mu_i$ is the total error. The theoretical error is estimated by propagating the single production cross sections errors as given in Ref. [32]. It is then summed in quadrature with the experimental errors listed in Tables I–III. We assume that all measurements are uncorrelated.

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