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One-loop matching for parton distributions: Nonsinglet case

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We derive the one-loop matching condition for nonsinglet quark distributions in the transverse-momentum cutoff scheme, including unpolarized, helicity and transversity distributions. The matching is between the quasidistribution defined by static correlation at finite nucleon momentum and the light-cone distribution measurable in experiments. The result is useful for extracting the latter from the former in a lattice QCD calculation which uses the lattice spacing as the ultraviolet cutoff.

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I. INTRODUCTION

Parton distributions characterize the structure of nucleons in terms of partons in high-energy scattering processes. They are an essential ingredient in making physical predictions for hadron-hadron or lepton-hadron collision experiments. Although much effort has been devoted to extracting parton distributions from various experimental data [1–6], the computation of parton distributions from the underlying theory of strong interactions, quantum chromodynamics (QCD), has been a difficult task, due to their nonperturbative nature. One may wonder whether it is possible to evaluate parton distributions from lattice QCD, which is so far the only reliable framework for nonperturbative phenomena in QCD. In the field-theoretic language, the parton distribution is given in terms of nonlocal light-cone correlators, which can be viewed as the density of quarks and gluons in the infinite momentum limit (before UV cutoffs are imposed) or light-front correlations of partons at finite nucleon momentum [7]. Such correlations are time dependent and intrinsically Minkowskian, and thus cannot be readily computed on the lattice. Past attempts have been mainly focused on the evaluation of local moments of the distributions. However, the evaluation of higher moments of parton distributions on the lattice becomes considerably difficult for technical reasons [8].

Recently, a direct approach to compute parton physics on a Euclidean lattice has been proposed by one of the present authors [9]. In this approach one computes, instead of the light-cone distribution, a related quantity, which may be called quasidistribution [10]. In the case of unpolarized quark density, the quasidistribution is defined as

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{izk^z}\langle P|\bar{\psi}(0,0_\perp,z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle, \tag{1}$$

where $x = k^z/P^z$ is the longitudinal momentum fraction, V^z is the z component of four-vector V^μ, γ^μ is the Dirac matrix, ψ is the quark Dirac field, and $|P\rangle$ is the nucleon state with four-momentum $P^\mu = (P^0, 0, 0, P^z)$. All fields and coupling constant g appearing in the above expression are bare ones, and Λ is the momentum cutoff to regulate the UV divergences. The operator above is time independent and nonlocal, and its matrix element can be simulated on a lattice for any $P^z \ll 1/a \sim \Lambda$, where a is the lattice spacing. However, the result is not the light-cone distribution extracted from the experimental data, $q(x,\mu)$ (scheme dependent, usually in $\overline{\rm MS}$ and μ indicates the renormalization scale). To recover the latter from the former, one needs to find a matching condition of the type

$$\tilde{q}_{\rm NS}(x,\Lambda,P^z) = \int dy Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right)q_{\rm NS}(y,\mu) + \mathcal{O}((M/P^z)^2) \tag{2}$$

for a large P^z, where we have limited ourselves to the so-called nonsinglet quantities such as up-minus-down flavors, so that the gluon contribution can be ignored. The correction terms are in power of M/P^z, where M is a QCD scale, such as the hadron mass. Since the difference between $\tilde{q}_{\rm NS}$ and $q_{\rm NS}$ is that the former is for finite but large momentum while the latter is for infinite momentum, the IR behavior of the distribution should not change when moving from one frame to the other, and the matching factor Z captures only the UV behavior and is thus entirely perturbative. The above relation can actually be viewed as a factorization conjecture: All the soft divergences are canceled on both sides, and all the collinear divergences in $\tilde{q}(x,\Lambda,P^z)$ are the same as those in the light-cone distributions. Of course one has yet to prove that this holds to all orders in perturbation theory. However, as a first step, we will show in the present paper that it is indeed true at one loop. It is

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worthwhile to point out here that the choice of quasidistributions is not unique; one can define more than one possible quasidistribution which have similar properties as $\tilde{q}(x, \Lambda, P^z)$, for example by replacing γ^z by γ^t (the time component of the Dirac matrix). Here we will focus on the type in Eq. (1) that is simple for lattice QCD calculations.

We will concentrate on the nonsinglet quark distribution in this paper, and prove the above factorization at one-loop level. Throughout the process we obtain the one-loop matching factor Z between the time-independent quasidistribution and light-cone distribution. The result will be useful for recovering the light-cone distribution from lattice QCD simulations of the quasidistribution to the leading-logarithmic accuracy. To obtain the complete one-loop matching condition useful for lattice calculations, the calculation shall be done with the same lattice Lagrangian as for the numerical simulations. However, this is considerably more complicated, and is not needed for the purpose of demonstrating one-loop factorization. Power suppressed corrections of the type $(1/P^z)^n$ with $n \geq 1$ are also ignored and will be considered in separate publications.

Although we consider here the factorization of the bare quasidistribution only, it might be useful to study renormalized versions of the distribution in certain renormalization schemes, such as dimensional regularization. It appears that the bare distributions we define depend on the UV cutoffs in two places: one is in the wave-function renormalization associated with the quark fields, and the other is the renormalization associated with the gauge link. Although it is trivial to see that both renormalizations are multiplicative at one-loop level, it may be a nontrivial exercise to show that the bare quasidistribution can be renormalized multiplicatively to all orders in perturbation theory due to overlapping divergences.

The rest of this paper is organized as follows. In Sec. II, we present the result of one-loop calculation for unpolarized quasi quark distribution. Based on this result, we propose in Sec. III a factorization theorem valid to one-loop level and extract the matching factor between quasidistribution and light-cone distribution. In Sec. IV, the results for quark helicity and transversity distributions are given. We conclude in Sec. V. The details of one-loop computation are given in the Appendix.

II. ONE-LOOP RESULT FOR UNPOLARIZED QUASI QUARK DISTRIBUTION

In this section, we consider the one-loop correction in the case of unpolarized quasi quark distribution $\tilde{q}(x, \Lambda, P^z)$. The one-loop computation for nonsinglet quark distribution is similar to QED because the non-Abelian property does not enter in the nonsinglet case. Note that the same calculation done in Ref. [9] is incomplete and the detailed result there shall be replaced by the correct one here.

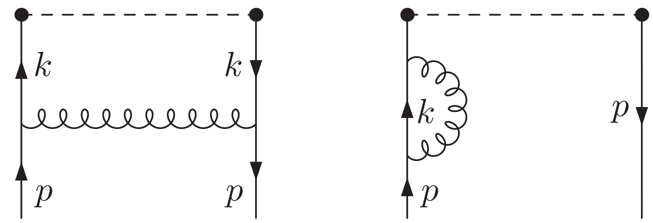

FIG. 1. One-loop corrections to quasi quark distribution.

At tree level, the quasidistribution yields the same result as the light-cone one:

$$\tilde{q}^{(0)}(x) = q^{(0)}(x) = \delta(1 - x). \tag{3}$$

The one-loop calculation can in principle be carried out in any gauge since the result is gauge invariant. We choose the axial gauge $A^z = 0$ where the gauge link in Eq. (1) becomes unity. In the axial gauge, the relevant Feynman diagrams are shown in Fig. 1, where the nonlocal operator is depicted as a dashed line. The diagrams contain UV, soft and collinear divergences. We use the quark mass m to regulate the collinear divergence. The soft divergence is expected to cancel between the diagrams. The UV divergence is regulated by a transverse-momentum cutoff Λ.

The choice of the axial gauge at one loop here is purely formal, simplifying the Feynman diagram. In fact the integrand before momentum integration is exactly the same as in the Feynman gauge. Various terms have straightforward Feynman gauge interpretations which we will use in the discussions below concerning the linear divergence.

Choosing the transverse momentum cutoff as a UV regulator needs some explanation. As we have indicated in the Introduction, we intend to find a matching condition which is useful for lattice QCD. Lattice QCD uses the short distance cutoff a, and keeps all power-divergent contributions. As we shall see, the quasidistribution has a linear divergence from the self-energy of the gauge link (as calculated in Feynman gauge). Dimensional regularization would have discarded this; using a momentum cutoff will allow us keeping track of this divergence. Clearly, the momentum cutoff scheme is difficult to generalize to more than one loop, and if a higher order calculation is needed, one shall directly do it using lattice action. At one-loop order in Abelian gauge theory, the momentum cutoff can be done without violating gauge symmetry, as for example, in the Lamb shift calculation [13]. The transverse-momentum cutoff also violates the rotational symmetry. Our intention is to show that the one-loop factorization works and to obtain the leading-logarithmic and leading-power divergent contribution in the matching condition; this can be achieved in any cutoff regularization.

The one-loop diagrams in Fig. 1 generate the following result:

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \cdots, \tag{4}$$

where we have included the tree-level result, $\tilde{q}^{(1)}(x,\Lambda,P^z)$ comes from the first diagram, and $\tilde{Z}_F^{(1)}(\Lambda,P^z)$ comes from the self-energy diagram. We defer the computational details of $\tilde{q}^{(1)}$ and $\tilde{Z}_F^{(1)}$ to the Appendix, and present their results below. The $\tilde{q}^{(1)}$ contribution can be written as

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x(\Lambda(x)-xP^z)}{(x-1)(\Lambda(1-x)+P^z(1-x))} + 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & x>1, \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x(\Lambda(x)-xP^z)}{(1-x)(\Lambda(1-x)+(1-x)P^z)} - \frac{4x}{1-x} + 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & 0<x<1, \\ \frac{1+x^2}{1-x}\ln\frac{(x-1)(\Lambda(x)-xP^z)}{x(\Lambda(1-x)+(1-x)P^z)} - 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & x<0 \end{cases} \tag{5}$$

for finite P^z, where $\Lambda(x)$ is defined in a different way from that in [9] as $\Lambda(x) = \sqrt{\Lambda^2 + x^2(P^z)^2}$, and the logarithm with collinear divergences is related to the standard Altarelli-Parisi kernel [14]. The wave function renormalization correction depends on the momentum P^z as well:

$$\begin{aligned}&\tilde{Z}_F^{(1)}(\Lambda,P^z)\\ &= \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y(\Lambda(y)-yP^z)}{(y-1)(\Lambda(1-y)+P^z(1-y))} - 1 - \frac{y\Lambda(1-y)+(1-y)\Lambda(y)}{(1-y)^2P^z} + \frac{y^2P^z}{\Lambda(y)} + \frac{y(1-y)P^z}{\Lambda(1-y)} + \frac{\Lambda(y)-\Lambda(1-y)}{P^z}, & y>1, \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y(\Lambda(y)-yP^z)}{(1-y)(\Lambda(1-y)+(1-y)P^z)} + \frac{4y^2}{1-y} + 1 - \frac{y\Lambda(1-y)+(1-y)\Lambda(y)}{(1-y)^2P^z} + \frac{y^2P^z}{\Lambda(y)} + \frac{y(1-y)P^z}{\Lambda(1-y)} + \frac{\Lambda(y)-\Lambda(1-y)}{P^z}, & 0<y<1, \\ -\frac{1+y^2}{1-y}\ln\frac{(y-1)(\Lambda(y)-yP^z)}{y(\Lambda(1-y)+(1-y)P^z)} + 1 - \frac{y\Lambda(1-y)+(1-y)\Lambda(y)}{(1-y)^2P^z} + \frac{y(1-y)P^z}{\Lambda(1-y)} + \frac{y^2P^z}{\Lambda(y)} + \frac{\Lambda(y)-\Lambda(1-y)}{P^z}, & y<0. \end{cases}\end{aligned} \tag{6}$$

Both $\tilde{q}^{(1)}$ and $\tilde{Z}_F^{(1)}$ involve singularities at $x=1(y=1)$ and a linear divergent term, a detailed discussion of which will be given in the next section when we present the one-loop factorization formula. From Eqs. (5) and (6) one can check the vector current conservation condition

$$\int_{-\infty}^{+\infty} dx\tilde{q}(x,\Lambda,P^z) = 1 \tag{7}$$

to one-loop order. Since the constituent of the quark in a quasidistribution does not have a parton interpretation, the parton momentum fraction extends from $-\infty$ to $+\infty$. The y integration is logarithmically divergent; in the above result we leave it unintegrated, in order to see the match between the structures of $\tilde{Z}_F^{(1)}$ and $\tilde{q}^{(1)}$. If one chooses to perform the y integration, a momentum cutoff is also needed in the z direction. It is interesting to see that the collinear divergence exists only for $0<x<1$, which is the basis for factorization.

In field theory calculations, the ultraviolet cutoff shall be larger than any other scale in the problem. In other words, one shall take the limit $\Lambda\to\infty$ and keep only the leading contribution and ignore the power-suppressed ones. This in principle shall also be the case in lattice QCD calculations. Thus for a fixed x, the actual field-theoretical result for the quasidistribution shall be

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2P^z}, & x>1, \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2P^z}, & 0<x<1, \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2P^z}, & x<0, \end{cases} \tag{8}$$

and

$$\tilde{Z}_F^{(1)}(\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2P^z}, & y>1, \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y}{1-y} + \frac{4y^2}{1-y} + 1 - \frac{\Lambda}{(1-y)^2P^z}, & 0<y<1, \\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2P^z}, & y<0. \end{cases} \tag{9}$$

Equation (8) is valid for x well below Λ/P^z. In a nucleon with large but finite momentum P^z, the fraction of partons with momentum fraction $x \sim \Lambda/P^z$ is expected to be negligible. To ensure vector current conservation, the momentum fraction in $\tilde{Z}_F^{(1)}$ shall be understood in the same way, which means the cutoff in y is well below Λ/P^z. One shall note several interesting features of the above result: (1) There are contributions in the regions $x > 1$ and $x < 0$. The physics behind this is transparent: when the parent particle has a finite momentum P^z, the constituent parton can have momentum larger than P^z, and even negative. This is very different from the infinite momentum frame result, where the momentum fraction is restricted to $-1 < x < 1$. (Note that at one-loop level, one has just $0 < x < 1$ in a quark target; however, there are contributions for $-1 < x < 0$ at two-loop level.) (2) There is a linear divergence associated with an axial gauge singularity $1/(1-x)^2$. It comes from the last term of the axial gauge gluon propagator numerator in Eq. (A2). If one works in a covariant gauge like the Feynman gauge, this term will come from the gauge link self-energy. We will give a more detailed discussion on this issue in the next section when we present the one-loop factorization formula. (3) There is no logarithmic UV divergence in $\tilde{q}^{(1)}$. Instead there is a logarithmic dependence on P^z in the region $0 < x < 1$. We will see later on that this logarithmic dependence can be transformed into the renormalization scale dependence of the light-cone parton distribution by a matching condition. It is worthwhile to comment on the difference between the UV behavior of the light-cone distribution and quasidistribution. Unlike the light-cone distribution where the quark and gluon propagator are linear in k^- when written in light-cone coordinates, and one therefore gets a UV divergent $\vec{k}_\perp$ integral upon integration over k^-, in the quasidistribution one integrates over k^0, on which the propagator is quadratically dependent; therefore one gets a UV convergent $\vec{k}_\perp$ integral in dimensional regularization. (The reason there is a linear divergence in the above results is because we used a cutoff regulator. In dimensional regularization the linear divergence is absent due to the lack of a large scale other than P^z, but it is present in the cutoff or lattice regularization.) However, note that the momentum fraction is not restricted to [0,1] any more; the integration over y therefore has a logarithmic divergence, and this yields the usual UV divergence in the wave function renormalization constant. (4) All soft divergences are canceled. However, there are remaining collinear divergences reflected by the quark-mass dependence.

On the other hand, with the same regularization, one can calculate the light-cone parton distribution by taking the limit $P^z \to \infty$. This is done following the spirit of Ref. [15] and the result is (for details of the computation see the Appendix)

$$q(x,\Lambda) = (1 + Z_F^{(1)}(\Lambda) + \cdots)\delta(x-1) + q^{(1)}(x,\Lambda) + \cdots \tag{10}$$

with

$$q^{(1)}(x,\Lambda) = \frac{\alpha_S C_F}{2\pi}\begin{cases} 0, & x > 1 \quad \text{or} \quad x < 0, \\ \frac{1+x^2}{1-x}\ln\frac{\Lambda^2}{m^2} - \frac{1+x^2}{1-x}\ln(1-x)^2 - \frac{2x}{1-x}, & 0 < x < 1, \end{cases} \tag{11}$$

and

$$Z_F^{(1)}(\Lambda) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} 0, & y > 1 \quad \text{or} \quad y < 0, \\ -\frac{1+y^2}{1-y}\ln\frac{\Lambda^2}{m^2} + \frac{1+y^2}{1-y}\ln(1-y)^2 + \frac{2y}{1-y}, & 0 < y < 1, \end{cases} \tag{12}$$

where the integrand of $Z_F^{(1)}(\Lambda)$ is exactly opposite to that of $q^{(1)}(x,\Lambda)$, indicating the quark number conservation at one loop. If dimensional regularization is used for the UV divergence, the results $q^{(1)}(x,\mu)$ and $Z_F^{(1)}(\mu)$ (with μ the renormalization scale) are slightly different, and can be obtained from the above ones by making the replacement $\ln\Lambda^2 \to 1/\epsilon_{\mathrm{UV}} - \gamma_E + \ln 4\pi\mu^2$. This result agrees with that derived from the light-cone definition of parton distribution. Also the collinear or mass singularity is the same as in the quasi parton distribution. This shows that at one-loop level, the quasi parton distribution captures all the collinear physics in the infinite momentum frame. Moreover, the contribution comes only from the diagram in which the intermediate gluon has a cut, which has a parton interpretation.

III. ONE-LOOP FACTORIZATION

Now we are ready to construct a factorization formula at one-loop order. In the infinite momentum frame or on the light cone, the momentum fraction in parton distributions and splitting functions is limited to $[-1, 1]$. However, in the present case, the splitting in the quasidistribution is not constrained to this region; it can be in $[-\infty, \infty]$. Thus the connection of the two distributions is reflected through the following factorization theorem up to power corrections in the large P^z limit

$$\tilde{q}(x,\Lambda,P^z) = \int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right)q(y,\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2/(P^z)^2, M^2/(P^z)^2), \quad (13)$$

where the integration range is determined by the support of the quark distribution $q(y)$ on the light cone (at one loop one has just $0<y<1$ for a quark target), and the momentum fraction x is defined in the finite momentum frame. We define the light-cone distribution $q(y,\mu)$ in the $\overline{\mathrm{MS}}$ subtraction scheme with μ the renormalization scale.

The Z factor has a perturbative expansion in α_s:

$$Z\left(\xi,\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) = \delta(\xi-1) + \frac{\alpha_s}{2\pi}Z^{(1)}\left(\xi,\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) + \cdots \quad (14)$$

Before we present the results of the Z factor, it is worthwhile to comment on the linear divergence coupled to the double pole $1/(1-\xi)^2$ in the quasidistribution, as can be seen from Eqs. (8) and (9) above. In the previous section we worked in the axial gauge, and the linear divergence comes from the last term in the numerator of the axial gauge gluon propagator Eq. (A2) (for the treatment of this double pole in axial gauge computations see e.g. Ref. [16]). If one chooses a covariant gauge like the Feynman gauge, one has, in addition to those in Fig. 1, extra diagrams involving the gauge link, and the linear divergence will come from the gauge link self-energy diagram. In dimensional regularization, the linear divergence is absent due to the lack of a cutoff scale, and the term leading to the linear divergence becomes, after k^0 and $\vec{k}_\perp$ integration, a term linear in P^z which reduces the double pole coupled to it to a single pole. The combination of the two contributions in Fig. 1 then leads to the usual plus distribution. However, the lattice simulations of quasidistributions require a momentum cutoff. From our one-loop computation a linear divergence associated with the double pole cannot be avoided in the cutoff scheme. The prescription for the double pole is given as $1/((1-\xi)^2+\epsilon^2)$, which can be clearly seen in a nonaxial gauge like Feynman gauge. In Feynman gauge, this prescription follows from the requirement of well-defined Wilson line propagators. Then after combining the real and virtual contributions, the double pole reduces to a single one prescribed by its principal value. From the renormalization point of view, a revised definition of the quasidistribution is preferred such that the gauge link self-energy is subtracted in a gauge invariant way (for a similar case in transverse-momentum-dependent parton distribution see discussions e.g. in [7]). This is also preferred by lattice simulations of the quasidistribution. We will explore this possibility in a forthcoming paper.

Now we are ready to write down the matching factor connecting the quasi quark distribution to the light-cone quark distribution. For $\xi>1$, one has

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \quad (15)$$

whereas for $0<\xi<1$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln[4\xi(1-\xi)] - \frac{2\xi}{1-\xi} + 1 + \frac{\Lambda}{(1-\xi)^2P^z}, \quad (16)$$

and for $\xi<0$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi} - 1 + \frac{\Lambda}{(1-\xi)^2P^z}. \quad (17)$$

Near $\xi=1$, one has an additional term coming from the self-energy correction

$$Z^{(1)}(\xi) = \delta Z^{(1)}(2\pi/\alpha_s)\delta(\xi-1) \quad (18)$$

with

$$\delta Z^{(1)} = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2P^z}, & y>1, \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{\mu^2} - \frac{1+y^2}{1-y}\ln[4y(1-y)] + \frac{2y(2y-1)}{1-y} + 1 - \frac{\Lambda}{(1-y)^2P^z}, & 0<y<1, \\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2P^z}, & y<0, \end{cases} \quad (19)$$

which is extracted from Eqs. (9) and (12) above, and provides a plus distribution for the singularity in the single pole term $1/(1-\xi)$ at $\xi=1$, as well as a principal value prescription for the double pole. The large logarithmic dependence on P^z in $\tilde{q}(x,\Lambda,P^z)$ can be transformed into the renormalization scale dependence through the above matching condition. On the lattice, the matching can be recalculated up to a constant accuracy using the standard approach, where the longitudinal and transverse momentum cutoffs are done in a way consistent with lattice symmetry [17].

So far, we have considered only the quark contribution. One can start with an antiquark to do the one-loop calculation. In this case, one also has a contribution to $\tilde{q}(x,\Lambda,P^z)$ from $\bar{q}(x)$. However, the antiquark distribution has the property

$$\bar{q}(x) = -q(-x), \tag{20}$$

which is related to quark distribution at negative x. Moreover, the Z factor has the same property. After including both quark and antiquark contributions, the factorization Eq. (13) still applies, but now the quantities on the rhs include also the antiquark contribution reflected by the negative y region. The above is the complete one-loop factorization theorem, which replaces Eq. (11) and the Z factor in Ref. [9].

We have constructed at one-loop level a factorization formula connecting the quasi parton distribution to the light-cone parton distribution. Of course it remains to be shown that there exists such a formula to all-loop orders. The factorization formula then allows one to extract the parton distribution $q(x,\mu)$ from calculating the quasi parton distribution on the lattice by measuring the time-independent, nonlocal quark correlator $\bar{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)$ in a state with increasingly large P^z (maximum $\sim 1/a$ with a denoting lattice spacing).

IV. HELICITY AND TRANSVERSITY DISTRIBUTIONS

In previous sections, we have considered the matching condition for unpolarized quark distribution. Here we present the results for the quark helicity and transversity distribution. For the quark helicity distribution in a longitudinally polarized quark, the quasidistribution $\Delta\tilde{q}^{(1)}(x)$ can be obtained by replacing γ^z with $\gamma^z\gamma^5$ in Eq. (1). The one-loop result then reads

$$\Delta\tilde{q}^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x(\Lambda(x)-xP^z)}{(x-1)(\Lambda(1-x)+P^z(1-x))} + 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & x>1,\\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x(\Lambda(x)-xP^z)}{(1-x)(\Lambda(1-x)+(1-x)P^z)} - \frac{4}{1-x} + 2x + 3 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & 0<x<1.\\ \frac{1+x^2}{1-x}\ln\frac{(x-1)(\Lambda(x)-xP^z)}{x(\Lambda(1-x)+(1-x)P^z)} - 1 - \frac{xP^z}{\Lambda(x)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & x<0.\end{cases} \tag{21}$$

Taking the limit $\Lambda\to\infty$ yields

$$\Delta\tilde{q}^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2P^z}, & x>1,\\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4}{1-x} + 2x + 3 + \frac{\Lambda}{(1-x)^2P^z}, & 0<x<1,\\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2P^z}, & x<0.\end{cases} \tag{22}$$

The result for the light-cone distribution is again given by taking $P^z\to\infty$

$$\Delta q^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} 0, & x>1 \quad \text{or} \quad x<0,\\ \frac{1+x^2}{1-x}\ln\frac{\Lambda^2}{m^2} - \frac{1+x^2}{1-x}\ln(1-x)^2 - \frac{2}{1-x} + 2x, & 0<x<1.\end{cases} \tag{23}$$

Note that as in the unpolarized case, the collinear singularity in the quasi quark helicity distribution is exactly the same as in the light-cone distribution.

Similarly, for the transversity distribution in a transversely polarized quark, the quasidistribution $\delta\tilde{q}^{(1)}(x)$ is obtained by replacing γ^z with $\gamma^z\gamma^\perp\gamma^5$ in Eq. (1). The one-loop result is

$$\delta\tilde{q}^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{2x}{1-x}\ln\frac{x(\Lambda(x)-xP^z)}{(x-1)(\Lambda(1-x)+P^z(1-x))} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & x>1,\\ \frac{2x}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{2x}{1-x}\ln\frac{4x(\Lambda(x)-xP^z)}{(1-x)(\Lambda(1-x)+(1-x)P^z)} - \frac{4x}{1-x} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & 0<x<1,\\ \frac{2x}{1-x}\ln\frac{(x-1)(\Lambda(x)-xP^z)}{x(\Lambda(1-x)+(1-x)P^z)} + \frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}, & x<0.\end{cases} \tag{24}$$

The limit $\Lambda\to\infty$ gives

$$\delta\tilde{q}^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{2x}{1-x}\ln\frac{x}{x-1} + \frac{\Lambda}{(1-x)^2P^z}, & x>1,\\ \frac{2x}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{2x}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + \frac{\Lambda}{(1-x)^2P^z}, & 0<x<1,\\ \frac{2x}{1-x}\ln\frac{x-1}{x} + \frac{\Lambda}{(1-x)^2P^z}, & x<0,\end{cases} \tag{25}$$

and the result in the infinite momentum frame is

$$\delta q^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} 0, & x>1 \quad \text{or} \quad x<0, \\ \frac{2x}{1-x}\ln\frac{\Lambda^2}{m^2} - \frac{2x}{1-x}\ln(1-x)^2 - \frac{2x}{1-x}, & 0<x<1. \end{cases} \tag{26}$$

One can construct similar matching conditions as in Eq. (12) for the helicity and transversity distributions. We just list the results for the matching factors here, noting that the quark self-energy is the same. For the quark helicity distribution, one has for $\xi > 1$

$$\Delta Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \tag{27}$$

whereas for $0 < \xi < 1$

$$\Delta Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln[4\xi(1-\xi)] - \frac{2}{1-\xi} + 3 + \frac{\Lambda}{(1-\xi)^2 P^z}, \tag{28}$$

and for $\xi < 0$

$$\Delta Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi} - 1 + \frac{\Lambda}{(1-\xi)^2 P^z}. \tag{29}$$

The linearly divergent term is the same as in the unpolarized case.

Finally, in the factorization theorem for transversity distribution, one has the matching factor for $\xi > 1$,

$$\delta Z^{(1)}(\xi)/C_F = \left(\frac{2\xi}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \tag{30}$$

whereas for $0 < \xi < 1$

$$\delta Z^{(1)}(\xi)/C_F = \left(\frac{2\xi}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{2\xi}{1-\xi}\right)\ln[4\xi(1-\xi)] - \frac{2\xi}{1-\xi} + \frac{\Lambda}{(1-\xi)^2 P^z}, \tag{31}$$

and for $\xi < 0$

$$\delta Z^{(1)}(\xi)/C_F = \left(\frac{2\xi}{1-\xi}\right)\ln\frac{\xi-1}{\xi} + \frac{\Lambda}{(1-\xi)^2 P^z}. \tag{32}$$

One again has a linearly divergent contribution. Near $\xi = 1$, one needs to include an extra contribution from self-energy as before.

V. CONCLUSION

We have derived one-loop matching conditions for nonsinglet quark distributions, including unpolarized, helicity and transversity distributions. The matching condition, which can also be viewed as a factorization, connects the quasi quark distribution to the light-cone distribution measurable in experiments, and thereby allows an extraction of the latter from the former, which is defined as a time-independent correlation at finite nucleon momentum and therefore can be evaluated on the lattice. To carry out a lattice simulation of the quasi quark distribution, one employs the momentum cutoff regulator for UV divergences. This UV regulator generates a linear divergence multiplied by a singular factor, as can be seen from our one-loop calculation in the axial gauge $A^z = 0$. In a nonaxial gauge like Feynman gauge, this structure arises from the self-energy of the Wilson line. Although in our one-loop factorization formula this divergence can be treated with a suitable prescription, to eventually achieve a factorization to all-loop orders, it is important to investigate how this divergence structure affects the matching between quasidistribution and light-cone distribution beyond one-loop level.

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APPENDIX: ONE-LOOP COMPUTATION IN THE AXIAL GAUGE

In this Appendix we give the details of the one-loop computation in the axial gauge. Let us start with the self-energy diagram in Fig. 1, which can be written as

$$\begin{aligned} -i\Sigma(p) &= \int \frac{d^4k}{(2\pi)^4}(-igt^a\gamma^\mu)\frac{i}{k-m}(-igt^b\gamma^\nu)\frac{-iD^{ab}_{\mu\nu}(p-k)}{(p-k)^2} \\ &= -g^2 C_F \int \frac{d^4k}{(2\pi)^4}\frac{\gamma^\mu(k+m)\gamma^\nu D_{\mu\nu}(p-k)}{(k^2-m^2)(p-k)^2}, \end{aligned} \tag{A1}$$

where

$$D_{\mu\nu}(k) = g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n \cdot k} + n^2 \frac{k_\mu k_\nu}{(n \cdot k)^2} \tag{A2}$$

and $n \cdot k = k^z$, $n^2 = -1$.

The numerator of Eq. (A1) gives

$$-2\not{k} + 4m - \frac{2n \cdot k\not{p} - 2n \cdot p\not{k} + 2k \cdot (p-k)\not{n} + 2mn \cdot (p-k)}{n \cdot (p-k)} + n^2 \frac{2k \cdot (p-k)(\not{p} - \not{k}) - \not{k}(p-k)^2 + m(p-k)^2}{[n \cdot (p-k)]^2}. \tag{A3}$$

We now calculate these contributions separately. The contribution of the first part is

$$\begin{aligned} I_1 &= \int \frac{d^4k}{(2\pi)^4} \frac{-2\not{k} + 4m}{(k^2 - m^2)(p-k)^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \frac{-2\not{k} + 4m}{[(k - yp)^2 + y(1-y)p^2 - (1-y)m^2]^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dy \frac{-2(yp^0\gamma^0 - k^z\gamma^z) + 4m}{[(k^0)^2 - (k^z - yp^z)^2 - \vec{k}_\perp^2 + y(1-y)p^2 - (1-y)m^2]^2} \\ &= \int_0^1 dy \int \frac{dk^z}{2\pi} \frac{d^2k_\perp}{(2\pi)^2} \frac{i}{2} \frac{-yp^0\gamma^0 + k^z\gamma^z + 2m}{[\vec{k}_\perp^2 + (k^z - yp^z)^2 - y(1-y)p^2 + (1-y)m^2]^{\frac{3}{2}}} \\ &= \frac{i}{8\pi^2} \int_0^1 dy \int dk^z \frac{-yp^0\gamma^0 + k^z\gamma^z + 2m}{\sqrt{(k^z - yp^z)^2 - y(1-y)p^2 + (1-y)m^2}} \\ &= \frac{i}{8\pi^2} \int dx p^z \int_0^1 dy \frac{-y\not{p} + (x-y)p^z\gamma^z + 2m}{\sqrt{((y-x)p^z)^2 - y(1-y)p^2 + (1-y)m^2}}, \end{aligned} \tag{A4}$$

where we have used the fact that the external momentum p is along the z direction, and $k^z = xp^z$. Note that the integration of x is not limited to [0, 1].

The second part gives

$$\begin{aligned} I_2 &= \int \frac{d^4k}{(2\pi)^4} \frac{-2k^z\not{p} + 2p^z\not{k} - 2k \cdot (p-k)\gamma^z - 2m(p^z - k^z)}{(k^2 - m^2)(p-k)^2(p^z - k^z)} \\ &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{-2k^z\not{p} + 2p^z\not{k} - 2k \cdot p\gamma^z + 2m^2\gamma^z - 2m(p^z - k^z)}{(k^2 - m^2)(p-k)^2(p^z - k^z)} + \frac{2\gamma^z}{(p-k)^2(p^z - k^z)}\right] \\ &= \frac{i}{8\pi^2} \int dx p^z \int_0^1 dy \frac{(y-x)p^z\not{p} - yp^2\gamma^z - m(1-x)p^z + m^2\gamma^z}{\sqrt{((y-x)p^z)^2 - y(1-y)p^2 + (1-y)m^2}(1-x)p^z} \\ &\quad - \frac{i}{4\pi^2} \int dx p^z\gamma^z \frac{\sqrt{\Lambda^2 + (1-x)^2(p^z)^2} - p^z\sqrt{(1-x)^2}}{(1-x)p^z}, \end{aligned} \tag{A5}$$

where Λ is the ultraviolet cutoff on the $\vec{k}_\perp$ integration.

The third part yields

$$\begin{aligned} I_3 &= -\int \frac{d^4k}{(2\pi)^4} \frac{2k \cdot (p-k)(\not{p} - \not{k}) - (\not{k} - m)(p-k)^2}{(k^2 - m^2)(p-k)^2(p^z - k^z)^2} \\ &= \frac{i}{8\pi^2} \int dx p^z\gamma^z \frac{\sqrt{\Lambda^2 + (1-x)^2(p^z)^2} - p^z\sqrt{(1-x)^2}}{(1-x)p^z} \\ &\quad - \frac{i}{16\pi^2} \int dx p^z \int_0^1 dy \frac{(p^2 - m^2)\not{p} - (p^2 - m^2)(y\not{p} + (y-x)p^z\gamma^z)}{\sqrt{((y-x)p^z)^2 - y(1-y)p^2 + (1-y)m^2}(1-x)^2(p^z)^2} \\ &\quad - \frac{i}{8\pi^2} \int dx p^z(\not{p} - m) \frac{\sqrt{\Lambda^2 + x^2(p^z)^2} - p^z\sqrt{x^2}}{(1-x)^2(p^z)^2}. \end{aligned} \tag{A6}$$

The quark wave function renormalization factor can be extracted by making use of the Ward identity [16]. To this end, let us consider the limit

$$n^\mu \Gamma_\mu = \lim_{\lambda\to 0}\frac{1}{\lambda} q^\mu \Gamma_\mu|_{q^\mu=\lambda n^\mu}. \tag{A7}$$

At tree level, this gives

$$\begin{aligned} n^\mu \Gamma_\mu^{(0)} &= n^\mu \gamma_\mu = \lim_{\lambda\to 0}\frac{1}{\lambda} q^\mu \Gamma_\mu^{(0)}|_{q^\mu=\lambda n^\mu} \\ &= \lim_{\lambda\to 0}\frac{1}{\lambda}[(\not p + \not q - m) - (\not p - m)] = \not n. \end{aligned} \tag{A8}$$

At one-loop level, one has

$$n^\mu \Gamma_\mu^{(1)} = \lim_{\lambda\to 0}\frac{1}{\lambda} q^\mu \Gamma_\mu^{(1)}|_{q^\mu=\lambda n^\mu} = -\lim_{\lambda\to 0}\frac{1}{\lambda}[\Sigma(p+q) - \Sigma(p))], \tag{A9}$$

or

$$n^\mu \Gamma_\mu^{(1)} = -n^\mu \frac{\partial \Sigma}{\partial p^\mu}. \tag{A10}$$

We therefore have

$$n^\mu \bar u(p) \Gamma_\mu^{(1)} u(p) = -n^\mu \bar u(p) \frac{\partial \Sigma}{\partial p^\mu} u(p). \tag{A11}$$

The general structure of $\bar u \Gamma_\mu^{(1)} u$ in the axial gauge is

$$\bar u \Gamma_\mu^{(1)} u = \bar u(\Gamma_1 \gamma_\mu + \Gamma_2 n_\mu + \Gamma_3 \sigma_{\mu\nu} n^\nu) u. \tag{A12}$$

Clearly Γ_3 does not contribute when dotted with n^μ. The second structure will be proportional to the quark mass m when sandwiched between spinors, and can be neglected if we keep m as a regulator for collinear divergence only. This can be seen from taking a spin sum in Eq. (A11). Consequently, only the first structure contributes, and the one-loop correction is proportional to the tree-level structure. One can see this from the result of vertex correction below. From the self-energy computation, the rhs of Eq. (A11) is proportional to the tree-level structure as well.

Now the lhs of Eq. (A11) gives the vertex renormalization factor; we can find from the rhs the wave function renormalization factor, which is given by

$$Z_F^{(1)} = n^\mu \bar u \frac{\partial \Sigma}{\partial p^\mu} u / \bar u \not n u. \tag{A13}$$

Actually Eq. (A11) can be easily shown to hold even before the momentum integration, thereby ensuring the cancellation between vertex and self-energy correction, if the UV regulator preserves shift invariance of the integral. The shift invariance is preserved by dimensional regularization, but not by the momentum cutoff, which is exactly what we are using here. This might lead to a problem in the exact cancellation between vertex and self-energy corrections. In general, one can add extra terms to restore gauge invariance. However it turns out that, in the present one-loop computation the vector current is conserved without introducing such terms.

Applying Eq. (A13) to the quark self-energy above, we obtain the wave function renormalization constant. Note that we leave the momentum fraction x unintegrated; taking the derivative on the integrand ignores a surface term, which is momentum independent and can therefore be absorbed by the mass counterterm. However, in our calculation we need the wave function renormalization constant only. Moreover, since we keep the quark mass as a collinear regulator only, the mass counterterm simply drops out because it is proportional to the quark mass.

The contribution of the first diagram in Fig. 1 can be written as

$$\begin{aligned} \Gamma_1 &= \int \frac{d^4k}{(2\pi)^4} \bar u(p)(-igt^a\gamma^\mu) \frac{i(\not k + m)\gamma^z i(\not k + m)}{(k^2-m^2)^2} (-ig\tau^a\gamma^\nu) u(p) \frac{-iD_{\mu\nu}(p-k)}{(p-k)^2} \delta\left(x - \frac{k^z}{p^z}\right) \\ &= -ig^2 C_F \int \frac{d^4k}{(2\pi)^4} \frac{\bar u(p)\gamma^\mu(\not k + m)\gamma^z(\not k + m)\gamma^\nu u(p) D_{\mu\nu}(p-k)}{(k^2-m^2)^2(p-k)^2} \delta\left(x - \frac{k^z}{p^z}\right) \\ &= -ig^2 C_F \int \frac{d^4k}{(2\pi)^4} \bar u(p)\left[\frac{2\gamma^z}{(k^2-m^2)(p-k)^2} + \frac{8mk^z - 4k^z \not k}{(k^2-m^2)^2(p-k)^2}\right. \\ &\quad \left. + \frac{2(2k^z\gamma^z + \not k - m)}{(k^2-m^2)(p-k)^2(p^z-k^z)} - \frac{\gamma^z}{(p-k)^2(p^z-k^z)^2}\right] u(p)\delta\left(x - \frac{k^z}{p^z}\right). \end{aligned} \tag{A14}$$

The above integrals can be computed in the same way as in Eqs. (A4), (A5) and (A6), and lead to Eq. (8). From Eqs. (8) and (9), we find

$$\int dx(\tilde q^{(1)}(x) + \tilde Z_F^{(1)}\delta(1-x)) = \frac{\alpha_S C_F}{2\pi}\int_0^1 dx(2-4x) = 0, \tag{A15}$$

which indicates the quark number conservation at one-loop level.

Now let us look at the result in the infinite momentum frame. Following the spirit of old-fashioned perturbation theory, the result in the infinite momentum frame can be obtained by first integrating over the zero component of loop momentum, and then taking the limit $p^z \to \infty$ before integrating over the transverse momentum. To illustrate the procedure, let us look at the first integral in the last two lines of Eq. (A14), which is

$$\Gamma_{11} = \int \frac{d^4k}{(2\pi)^4} \frac{2\gamma^z}{(k^2 - m^2)(p-k)^2} \delta\left(x - \frac{k^z}{p^z}\right). \tag{A16}$$

By integrating over k^0, one picks up residues in the upper- or lower-half k^0 plane by enclosing the contour in the same half plane. If we enclose the contour in the upper-half plane, the residues at the following poles contribute to the integral:

$$k^0 = p^0 - \sqrt{\vec{k}_\perp^2 + (k^z - p^z)^2},$$
$$k^0 = -\sqrt{\vec{k}_\perp^2 + (k^z)^2 + m^2}. \tag{A17}$$

Summing over the residues at these poles, we then let $p^z \to \infty$ and keep only terms that are not suppressed by p^z; this gives the following contribution

$$\delta\Gamma_{11} = \frac{i}{2\pi}\begin{cases} 0, & x > 1 \quad \text{or} \quad x < 0, \\ \int \frac{d^2k_\perp}{(2\pi)^2} \frac{-1}{\vec{k}_\perp^2 + m^2(1-x)^2}, & 0 < x < 1. \end{cases} \tag{A18}$$

Similarly, the remaining three terms yield

$$\delta\Gamma_{12} = \frac{i}{2\pi}\begin{cases} 0, & x > 1 \quad \text{or} \quad x < 0, \\ \int \frac{d^2k_\perp}{(2\pi)^2} \frac{x(\vec{k}_\perp^2 + m^2(3-4x+x^2))}{(\vec{k}_\perp^2 + m^2(1-x)^2)^2}, & 0 < x < 1, \end{cases}$$
$$\delta\Gamma_{13} = \frac{i}{2\pi}\begin{cases} 0, & x > 1 \quad \text{or} \quad x < 0, \\ \int \frac{d^2k_\perp}{(2\pi)^2} \frac{-2x}{(\vec{k}_\perp^2 + m^2(1-x)^2)(1-x)}, & 0 < x < 1, \end{cases}$$
$$\delta\Gamma_{14} = 0, \quad \text{for all } x. \tag{A19}$$

The sum of these contributions gives Eq. (11) upon integration over $\vec{k}_\perp$. The computation of quark self-energy is similar, and gives the wave function renormalization factor Eq. (12).

In the above results, we have made different expansions in finite and infinite momentum frames ($\Lambda \to \infty$ in the former and $p^z \to \infty$ in the latter); therefore one cannot get the correct infinite momentum result by simply setting $p^z \to \infty$ in the finite momentum one. If we keep all Λ and p^z dependence in the calculation of the finite momentum result, we have Eqs. (5) and (6). We can then take $\Lambda \to \infty$ or $p^z \to \infty$ to reproduce the correct finite and infinite momentum results in previous sections. We still have quark number conservation for the complete results Eqs. (5) and (6)

$$\begin{aligned} &\int dx(\tilde{q}^{(1)}(x) + \tilde{Z}_F^{(1)}\delta(1-x)) \\ &= \frac{\alpha_S C_F}{2\pi}\left\{\int_0^1 dx(2-4x) \right. \\ &\quad + \int dx\left[\frac{(x^2-x)P^z}{\Lambda(x)} + \frac{x(1-x)P^z}{\Lambda(1-x)} \right. \\ &\quad \left.\left. + \frac{\Lambda(x) - \Lambda(1-x)}{P^z}\right]\right\} = 0, \end{aligned} \tag{A20}$$

since the two integrals on the rhs vanish, respectively.

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