Vacuum polarization corrections to low energy quark effective couplings

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In this work corrections to low energy punctual effective quark couplings up to the eighth order are calculated by considering vacuum polarization effects with the scalar quark-antiquark condensate. The departing point is a QCD-based Nambu–Jona-Lasinio model. By separating the quark field into two components, one that condenses and another one for interacting quarks, the former is integrated out with the help of usual auxiliary fields and an effective action in terms of interacting quark fields is found. The scalar auxiliary field reduces to the quark-antiquark condensate in the vacuum and the determinant is expanded in powers of the quark-antiquark bilinears generating chiral invariant effective 2N-quark interactions ($N = 2, 3\ldots$). The corresponding coupling constants and effective masses are estimated, and the general trend is that for increasing the effective gluon mass the values of the effective coupling constants decrease. All the values are in good agreement with phenomenological fits.

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I. INTRODUCTION AND EXTENDED NAMBU-JONA-LASINIO MODEL FROM QCD

In spite of spectacular progress in lattice calculations it still is extremely important to rely on the description of hadron, and more generally nuclear, processes on QCD-based hadron effective models. These models are expected to incorporate the most important symmetries and properties of the fundamental theory (QCD) such as the chiral symmetry and its spontaneous breakdown. Many effective models have been proposed to describe the low energy regime of QCD phase diagram and the quark Nambu–Jona–Lasinio (NJL) model [1–4] is one of the most emblematic. The motivation for that is due to its relative simplicity and power to describe some aspects of low energy hadron physics due to the spontaneous chiral symmetry breakdown by means of the chiral condensate $\langle\bar{\psi}\psi\rangle$. In spite of the recent controversy on the formation of the scalar quark condensate [5–7], it is widely recognized for its contribution, for example, to the nucleon and quark masses [8–10], even if gluon dressing might also be important [11]. It is, however, very important to extend its validity and refine its predictions by including further effective quark interactions [12–19]. For the high energy limit of the phase diagram, Polyakov loops were included to incorporate the deconfinement phase transition [18], thereby extending the validity of the model. In spite of being suitable for describing low energy physics, this punctual effective interaction has also been envisaged for high energies [20]. Osipov *et al*. have found that an eighth order quark interaction term restores the stability of the vacuum [16,17] that is unfavored by the sixth order $SU(3)$ ’t Hooft interaction [21]. Whereas the ’t Hooft interaction was found a long time ago from QCD grounds [22], the eighth order term has already been considered in few approaches which do not necessarily exclude each other [23–26]. Since one can think about including progressively higher order effective quark interactions,it is interesting to note that the longstanding problem of the convergence of the QCD action in powers of quark currents [27–29] might also receive some insight from the microscopic calculations of multiquark interactions. Of course, eventually one might have to avoid double counting effects, which might be extremely difficult to assess. Although the emergence of such higher order interactions has already been addressed in the last decades, their contribution to the structure and reactions of the new (heavier) hadrons that were proposed to have multiquark structure (see, for example, Ref. [30]) is not really understood. In this work we derive low energy effective quark couplings due to the vacuum polarization with the chiral condensate.

A general quark effective action obtained by integrating out gluons from the QCD action can given by [19,26,31]

$$S_{\rm eff}[\bar{\psi},\psi] = \int_x \left[\bar{\psi}(i\partial - M)\psi - \frac{1}{2}\int_y j_\mu^b(x)(R_{bc}^{\mu\nu})^{-1}(x-y)j_\nu^c(y)\right] + S_A, \quad (1)$$

where the color quark current is $j_a^\mu = \bar{\psi}\lambda_a\gamma^\mu\psi$, the sums in color, flavor, and Dirac indices are implicit, the kernel $R_{bc}^{\mu\nu}$ is the gluon propagator that might depend on auxiliary variables, and the last term, S_A, corresponds to terms due the gluon integration, including the gluon determinant and ghost integration if needed, and eventually with dependence on auxiliary variables [26,32–34]. To investigate the flavor structure of the model, one might perform a Fierz transformation in the current-current interaction from which an NJL emerges. Several QCD condensates have been proposed besides the quark antiquark, and two gluon condensates have gained further attention: the order 2 condensate ($\langle A^2\rangle$) and the order 4 condensate ($\langle F^{\mu\nu}F_{\mu\nu}\rangle$). The interplay of the second gluon condensate with quark effective interactions was already considered (for example, in Ref. [35]). We wish to consider the former since it has been related to a possible effective gluon mass [36–38] that has seemingly been found in several other analytical calculations [39–46] and in numerical and lattice calculations [37,38,47–51]. Our departure point, therefore, is the NJL of Ref. [26] with a gluon condensate of order 2, although the NJL model can be obtained from QCD with different considerations [52,53]. A different approach was considered by Simonov within the instanton gas model to derive effective quark interactions [23–25] and we will not investigate if and to what extent these two approaches provide double counting effects.

The work is organized as follows. In Sec. II the Nambu–Jona-Lasinio model induced by the gluon condensate of order 2 ($\langle A_\mu A^\mu\rangle \sim \phi_0$) is considered, such that the quark content is separated into two components: the quasiparticle sector corresponding to the interacting quarks and the one corresponding to the condensed quarks, such that $\bar\psi\psi \to (\bar\psi\psi)_c + \bar\psi\psi$, preserving explicitly chiral symmetry. The variables $(\bar\psi\psi)_c$ are integrated out by introducing a set of usual auxiliary variables S_i, P_i that yields the scalar quark-antiquark condensate and a pseudoscalar variable. The (coupled) gap equations of the auxiliary variables $(S_{i,0}, P_{i,0}, \phi_0)$ are derived and solved in terms of a unique Euclidean covariant cutoff yielding results in perfect agreement with well-known effective masses from lattice and phenomenology. In Sec. III the quark determinant is expanded in powers of the quark field, or quark flavor currents, yielding polynomial effective quark interactions whose couplings depend on the two condensates $\langle\bar\psi\psi\rangle$ and $\langle A^2\rangle$. The values of the effective coupling constants are estimated using the same cutoff, or conversely the same gluon effective mass, as the one considered for the gap equations, yielding values comparable to those used in phenomenological fits in the literature. In the last section there is a summary.

II. THE NJL AND THE SCALAR QUARK-ANTIQUARK CONDENSATE

The generating functional of the local $SU(3)$ (flavor) NJL limit of the action (1) is given by [1,26,52]

$$Z[\bar\eta,\eta] = \int \mathcal{D}[\bar\psi,\psi] exp\left\{ i\left[S_{NJL}[\bar\psi,\psi] + \int_x (\bar\eta\psi + \eta\bar\psi)\right]\right\}, \tag{2}$$

where

$$S_{NJL}[\bar\psi,\psi] = \int_x \left\{ \bar\psi(i\partial - M)\psi + \frac{g_4}{2}[(\bar\psi\lambda_i\psi)^2 + (\bar\psi\gamma_5\lambda_i\psi)^2]\right\} + S_A \tag{3}$$

for $i = 0, \ldots, N_f^2 - 1$. For the gluon determinant, $S_A = -\frac{i}{2}\mathrm{Tr}\log(R^{\mu\nu}_{ab})$, where Tr stands for the sum over all indices and spatial integration. The following truncated gluon propagator will be considered:

$$(R^{\mu\nu}_{bc})^{-1}(x-y) = \delta_{bc}\left[(\partial^2 + c\phi(x))\left(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right) + \frac{1}{\lambda}\partial_\mu\partial_\nu\right]^{-1}\delta^4(x-y), \tag{4}$$

where λ is the (covariant) gauge fixing parameter. The transverse effective gluon mass is therefore $M_G^2 = c\phi_0$, with ϕ_0 being an auxiliary variable for the gluon condensate $\langle A^2\rangle$; g_4 has dimension $(\text{mass})^{-2}$ and is of the order of N_c^{-1}, at least in the leading order. It is given by

$$g_4 = \frac{\beta}{M_G^2}, \tag{5}$$

where β is a numerical factor accounting for the color and flavor structure of the model for the $SU(3)$ $\beta = \frac{g^2}{9}$ and where g is the zero momentum QCD running coupling constant [48].

Let the quark field bilinears be separated into two components: the one corresponding to the quarks that condense ($(\bar\psi\psi)_c$)and the other to the interacting quasiparticle quarks, analogously to other formalisms; see, for example, Refs. [54,55]. This is done by considering that each quark bilinear, as well as the functional measure of the generating functional, will be written as

$$\bar\psi\psi \to (\bar\psi\psi)_c + \bar\psi\psi. \tag{6}$$

This way chiral symmetry will not be explicitly broken. A further analysis within a renormalization group approach is outside the scope of this work. Since $\mathrm{tr}[\gamma_5] = 0$ and then $\langle\bar\psi\gamma_5\lambda_i\psi\rangle = 0$, only even powers of the pseudoscalar bilinear will contribute. The resulting interaction term can be written as $\mathcal{L}_I = \mathcal{L}_q + \mathcal{L}_c + \mathcal{I}_{\text{int}}$, where

$$\begin{aligned}
\mathcal{L}_q &= g_4[(\bar\psi\lambda_i\psi)^2 + (\bar\psi i\gamma_5\lambda_i\psi)^2],\\
\mathcal{L}_c &= g_4[(\bar\psi\lambda_i\psi)_c^2 + (\bar\psi i\gamma_5\lambda_i\psi)_c^2],\\
\mathcal{I}_{\text{int}} &= g_4[\bar\psi\lambda_i\psi\cdot(\bar\psi\lambda_i\psi)_c + \bar\psi i\gamma_5\lambda_i\psi\cdot(\bar\psi i\gamma_5\lambda_i\psi)_c\\
&\quad + (\bar\psi\lambda_i\psi)_c\cdot\bar\psi\lambda_i\psi + (\bar\psi i\gamma_5\lambda_i\psi)_c\cdot\bar\psi i\gamma_5\lambda_i\psi].
\end{aligned} \tag{7}$$

The quark component $(\bar{\psi}\psi)_c$ can be integrated out by introducing the usual $SU(3)$ auxiliary variables S_a, P_a. For that, the above generating functional is multiplied by the following unity integrals:

$$1 = N' \int D[S_i, P_i] \exp\left[-\frac{i}{2c_s}\int_x [S_i^2 + P_i^2]\right], \quad (8)$$

where c_s is a constant to be fixed and N' a normalization constant. The fourth order quark interaction $\mathcal{L}_c$ cancels out if the following variable shifts with corresponding unit Jacobian are done: $S^i \to S^i + 2g_4(\bar{\psi}\lambda^i\psi)_c$ and $P^i \to P^i + 2g_4(\bar{\psi}i\gamma_5\lambda^i\psi)_c$, where we consider $c_s = 2g_4$. One is left with the following linearized action for the component $(\bar{\psi}\psi)_c$ in terms of the auxiliary variables:

$$S_{NJL} \to \int_x \left[\int_y \bar{\psi}_c(S^{-1}(x-y))\psi_c - \frac{1}{4g_4}(S_i^2 + P_i^2) + \bar{\psi}(i\partial - M)\psi + g_4[(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2]\right] + S_A. \quad (9)$$

In this equation,

$$S^{-1}(x-y) = [i\partial - M^* + 2g_4(\lambda_i(\bar{\psi}\lambda_i\psi) + \lambda_i i\gamma_5(\bar{\psi}i\gamma_5\lambda_i\psi))]\delta^4(x-y), \quad (10)$$

being that the effective mass (matrix) already receives the contribution from the auxiliary variables S_i that will not vanish in the vacuum, i.e.,

$$M^* = M + S_i\lambda_i. \quad (11)$$

By integrating out the component $(\bar{\psi}\psi)_c$, the resulting effective action for the quasiparticle quarks with the auxiliary variables reads

$$S_{\text{eff}} = i\text{Tr}\ \ln\ [-iS^{-1}(x-y)] + \int_x \left[-\frac{1}{4g_4}(S_i^2 + P_i^2) + \bar{\psi}(i\gamma_\mu\partial^\mu - M^*)\psi + g_4[(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2]\right] + S_A. \quad (12)$$

Before expanding this expression in terms of the quark bilinears, it is desirable to derive a set of gap equations to determine the ground state by extremizing this effective action with respect to the auxiliary variables, ϕ_0, S_i, P_i.

A. Ground state: Gluon condensate of order 2 and the chiral scalar quark-antiquark condensate

The gluon sector of the effective action (3) will be replaced by

$$S_A = -\int_x \left[\frac{c}{4}\phi^2\right] - \frac{i}{2}\text{Tr}\ \ln\ [R_{bc}^{\mu\nu}(x-y)], \quad (13)$$

so that the effective action (12) can be written as

$$S_{\text{eff}} = i\text{Tr}\ \ln\ [S^{-1}(x-y)] - \frac{i}{2}\text{Tr}\ \ln\ [R_{bc}^{\mu\nu}(x-y)] + \int_x \left[-\frac{1}{4g_4}(S_i^2 + P_i^2) - \frac{c}{2}\phi^2 + \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi + g_4[(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2]\right]. \quad (14)$$

In the ground state, quasiparticle fields are zero $\bar{\psi}, \psi \to 0$ and the effective potential for the gluon and quark-antiquark condensates (ϕ_0, S_i, besides the pseudoscalar variable P_i) can be extremized. These equations are the following:

$$\left.\frac{\partial S_{\text{eff}}}{\partial \phi}\right|_{\phi=\phi_0, S^i=S_0^i} = 0, \qquad \left.\frac{\partial S_{\text{eff}}}{\partial S_i}\right|_{S^i=S_0^i, \phi=\phi_0} = 0, \qquad \left.\frac{\partial S_{\text{eff}}}{\partial P_i}\right|_{S^i=S_0^i, \phi=\phi_0} = 0, \quad (15)$$

being that in all these equations $P_0^i = 0$ is a trivial and necessary solution. These gap equations, in the Euclidean momentum space, will be regularized by a covariant cutoff yielding

$$\phi_0 = 3(N_c^2 - 1)\int \frac{d^4k_E}{(2\pi)^4}\left(\frac{1}{k_E^2 + M_G^2}\right) + \frac{9}{4g^2}(S_u^2 + S_d^2 + S_s^2),$$
$$S_f^0 = \frac{16g^2N_c}{9c\phi_0}\int \frac{d^4k_E}{(2\pi)^4}\left(\frac{M_f^*}{k_E^2 + M_f^{*2}}\right),$$
$$P_f^0 = 0, \quad (16)$$

where $f = u, d, s$ stands for up, down, strange quarks and the effective mass can be written for each quark flavor as $M_f^* = M_f + S_f^0$.

In these equations, the only free parameters are the QCD coupling constant, which will be given by the zero momentum running coupling $\alpha_s \equiv \frac{g^2}{4\pi} = \frac{8.92}{N_c}$ [48], and the current quark masses, $m_u = 3$ MeV, $m_d = 6$ MeV, and $m_s = 91$ MeV [56]. With these four Lagrangian parameters there is one covariant cutoff that solves the four gap equations, fixing $N_c = 3$. Solutions for typical values of the effective gluon mass, found in lattice calculations, are shown in Table I with the resulting effective quark masses. For example, with $M_G \simeq 650$ MeV [37,39,48,50] it yields

TABLE I. Values of the mass correction calculated with the parameters fitted from the gap equations (16)—M_G, Λ, and quark current masses $m_u = 3$ MeV, $m_d = 6.6$ MeV, $m_s = 90.6$ MeV—and gauge coupling g from Ref. [48], considering the NJL coupling constant (g_4) obtained from the effective model.

$M_G[\Lambda]$ (MeV)	$\Delta m_u[M_u^*]$ (MeV)	$\Delta m_d[M_d^*]$ (MeV)	$\Delta m_s[M_s^*]$ (MeV)	$g_4(\mathrm{GeV}^{-2})$	$\tilde{g}_4(\mathrm{GeV}^{-2})$	$\tilde{g}_6(\mathrm{GeV}^{-5})$	$g_1^{(8)}(\mathrm{GeV}^{-8})$	$g_2^{(8)}(\mathrm{GeV}^{-8})$
600 [651]	271 [274]	273 [281]	303 [395]	12	2.8	−1100	6526	1957
650 [706]	294 [297]	296 [303]	328 [419]	10	2.5	−748	3469	1041
700 [760]	312 [319]	319 [326]	352 [352]	8	2.2	−520	1930	579
800 [870]	359 [365]	366 [372]	401 [492]	6	1.7	−278	670	201

$M_u^* = 297$ MeV, $M_d^* = 303$ MeV, and $M_s^* = 419$ MeV for $\Lambda = 706$ MeV. As expected, this cutoff is the usual NJL cutoff [2–4] and certainly higher than the Λ_{QCD} [47–49]. In Ref. [35] a similar model was developed by considering the order 4 gluon condensate ($\langle F_{\mu\nu}^2 \rangle$) with similar reasonably good results.

III. EFFECTIVE QUARK INTERACTIONS AND COUPLING CONSTANTS

In this section the effective quark model (12) will be expanded in the lowest order derivative expansion [57] in powers of bilinears $\bar{\psi}\Gamma\psi$ (where Γ is any combination of flavor λ_i and γ_5) and the gluon sector S_A of the model plays no role from here on. While on one hand an expansion of this kind might impose certain limitations on the resulting values of the effective couplings since it is a perturbative treatment, on the other hand it might require instead a weak strength of the quark fields to assure its validity. Nevertheless, it might be a safe starting point for investigating higher order quark effective interactions. The expansion has the following shape:

$$\begin{aligned} S_{\mathrm{eff}} \simeq\; & S_{\mathrm{eff},(0)}[\phi_0, S_i, P_i] \\ & + \frac{1}{1!1!}\int_{x_1,x_2} \frac{\delta^2 S_{\mathrm{eff}}}{\delta\bar{\psi}(x_1)\delta\psi(x_2)}\bigg|_{\psi=\bar{\psi}=0} \bar{\psi}(x_1)\psi(x_2) \\ & + \frac{1}{2!2!}\int_{x_1,x_2,x_3,x_4} \frac{\delta^4 S_{\mathrm{eff}}}{\delta\bar{\psi}(x_1)\delta\psi(x_2)\delta\bar{\psi}(x_3)\delta\psi(x_4)}\bigg|_{\psi=\bar{\psi}=0} \\ & \times \bar{\psi}(x_1)\psi(x_2)\bar{\psi}(x_3)\psi(x_4) + \mathrm{h.o.}, \end{aligned} \tag{17}$$

where $\int_{x_1,x_2} = \int dx_1 \int dx_2$, h.o. stands for (even) higher order derivatives, and the odd powers must disappear since they are calculated for $\psi, \bar{\psi} \to 0$ at the end.

The second order term produces the following contribution:

$$\begin{aligned} S_{\mathrm{eff}}^{(2)} = g_4 \int_x & \mathrm{tr}[(S_0(x-y)\lambda_i)(\bar{\psi}\lambda_i\psi) \\ & + (S_0(x-y)i\gamma_5\lambda_i)(\bar{\psi}i\gamma_5\lambda_i\psi)], \end{aligned} \tag{18}$$

where $S_0(x-y) = S(x-y)|_{\bar{\psi},\psi\to 0}$ and tr stands for the traces of discrete indices. The operatorial coefficients of the quark bilinears will be resolved separately from the quark-antiquark bilinears. Those operators when resolved in momentum space will contribute to the effective coupling constants. This way, expression (18) can then be rewritten as

$$S_{\mathrm{eff}}^{(2)} = \int_x \mathrm{tr}\left\{-g_4\lambda_0\left(\int \frac{d^4k}{(2\pi)^4}\frac{1}{\gamma\cdot k - M^*}\right)\right\}\bar{\psi}(x)\lambda_0\psi(x), \tag{19}$$

where the local limit was considered and $\mathrm{tr}[\gamma_5] = 0$, $\mathrm{tr}[\lambda_i] = 0$ for $i \neq 0$. The traces were calculated yielding one mass term for each of the quark flavors. Therefore this second order term of the expansion produces a correction to the quark masses that can be written, in the Euclidean momentum space, as

$$\Delta m_f = 16 g_4 N_c \int \frac{d^4k_E}{(2\pi)^4}\frac{M_f^*}{k_E^2 + M_f^{*2}}. \tag{20}$$

It is interesting to emphasize that this expression is different from the effective mass given by expression (11), although it was calculated in terms of the same parameters and cutoff considered in the last section for the gap equations. It is worth remembering that we adopt the model in which the NJL coupling constant is inversely proportional to the gluon effective mass, expression (5), and the resulting effective quark masses m^* (and M^*) are shown in Table I for different values of M_G. Although the departing point was a $U(3)$ NJL model, from here on the calculations will be restricted to the $SU(3)$ model.

The fourth order term in expression (17) is calculated next for zero momentum exchange. The operators which are not contracted with the quark fields will be resolved, yielding the coupling constant. This guarantees the chiral invariance of the original interaction. In the limit of zero momentum transfer, this term can be written as

$$\begin{aligned} S_{\mathrm{eff}}^{(4)} &= -16 g_4^2 N_c \mathrm{tr}\int_x \left[\int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{\gamma\cdot k - M^*}\right)^2 \lambda_j^2\right] \\ &\quad \times \{(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2\} \\ &= \tilde{g}_4 \int_x \{(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2\}, \end{aligned} \tag{21}$$

where $\gamma_5^2 = I$ and $\mathrm{tr}(\lambda_i\lambda_j) = 2\delta_{ij}$. This is a one-loop correction to the NJL coupling constant which will be calculated by regularizing the integral for a Euclidean four-momenta cutoff. This coupling constant can be written as

$$\tilde{g}_4 = 4g_4^2 N_c \int \frac{d^4 k_E}{(2\pi)^4} \sum_f \frac{k_E^2 - M_f^{*2}}{(k_E^2 + M_f^{*2})^2}. \tag{22}$$

In Table I values for this coupling for different values of the effective gluon mass from the gap equation are shown.

The sixth order terms of the expansion will similarly be given by

$$S_{\rm eff}^{(6)} = \frac{1}{3!3!} \int_{x_{i=1,2,\ldots,6}} \frac{\delta^6 S_{\rm eff}}{\delta\bar{\psi}(x_6)\delta\psi(x_5)\delta\bar{\psi}(x_4)\delta\psi(x_3)\delta\bar{\psi}(x_2)\delta\psi(x_1)}\bigg|_{\psi=\bar{\psi}=0} \bar{\psi}(x_6)\psi(x_5)\bar{\psi}(x_4)\psi(x_3)\bar{\psi}(x_2)\psi(x_1). \tag{23}$$

After the factorization of the operators that yield the effective coupling constant, similar to the previous terms, the following identity is used:

$$\mathrm{tr}(\lambda_i \lambda_j \lambda_k) = D_{ijk}, \tag{24}$$

being that in the $SU(3)$ case it reduces to $D_{ijk} = 2(d_{ijk} + i f_{ijk})$, where d_{ijk} and f_{ijk} are the symmetric and antisymmetric $SU(3)$ tensors. Expression (23) can then be written as

$$\begin{aligned} S_{\rm eff}^{(6)} = {} & 32 g_4^3 N_c \int_x \int \frac{d^4 k}{(2\pi)^4} \sum_{f=u,d,s} \left(\frac{1}{(\gamma\cdot k - M_f^*)^3} \right) \\ & \times \left\{ \frac{d_{ijk}}{18} [(\bar{\psi}\lambda_i\psi)(\bar{\psi}\lambda_j\psi)(\bar{\psi}\lambda_k\psi) - 3(\bar{\psi}i\gamma_5\lambda_i\psi)(\bar{\psi}i\gamma_5\lambda_j\psi)(\bar{\psi}\lambda_k\psi)] \right\}, \end{aligned} \tag{25}$$

where the antisymmetric component is zero. This term has exactly the flavor structure of the $SU(3)$ determinantal 't Hooft interaction [2–4] that can be written as

$$\begin{aligned} S_{\rm eff}^{(6)} = \tilde{g}_6 \int_x \frac{d_{ijk}}{18} & [(\bar{\psi}\lambda_i\psi)(\bar{\psi}\lambda_j\psi)(\bar{\psi}\lambda_k\psi) \\ & - 3(\bar{\psi}i\gamma_5\lambda_i\psi)(\bar{\psi}i\gamma_5\lambda_j\psi)(\bar{\psi}\lambda_k\psi)], \end{aligned} \tag{26}$$

where the effective coupling, with the same covariant Euclidean momentum cutoff, is given by

$$\tilde{g}_6 = -32 g_4^3 N_c \int \frac{d^4 k_E}{(2\pi)^4} \sum_{f=s,d,u} \frac{3k_E^2 M_f^* - M_f^{*3}}{(k_E^2 + M_f^{*2})^3}. \tag{27}$$

This coupling constant is related to the usual definition of the 't Hooft term (κ in Refs. [2,3,16,21]) by $\tilde{g}_6 = \frac{9}{16}\kappa$. Numerical values for this coupling constant, with the same parameters as before, are shown in Table I. They are all negative in agreement with phenomenological values.

The eighth order term is calculated from the following derivative:

$$\int_{x_{i=1\ldots8}} \frac{\delta^8 S_{\rm eff}}{\delta\bar{\psi}(x_8)\delta\psi(x_7)\delta\bar{\psi}(x_6)\delta\psi(x_5)\delta\bar{\psi}(x_4)\delta\psi(x_3)\delta\bar{\psi}(x_2)\delta\psi(x_1)}\bigg|_{\psi=\bar{\psi}=0} \bar{\psi}(x_8)\psi(x_7)\bar{\psi}(x_6)\psi(x_5)\bar{\psi}(x_4)\psi(x_3)\bar{\psi}(x_2)\psi(x_1). \tag{28}$$

This term will be also resolved in the limit of zero momentum exchange. With the same factorization of the traces, it can be written as

$$\begin{aligned} S_{\rm eff}^{(8)} = {} & -16 \frac{(2g_4)^4 N_c}{4!4!} \int d^4 x \int_{x_{i=1\ldots8}} \mathrm{tr}[\lambda_i\lambda_j\lambda_k\lambda_l] \left(\sum_f \frac{1}{(i\partial - M_f^*)^4} \right) \\ & \times \sum_{a\neq b\neq c\neq d=1,3,5,7; e\neq f\neq g\neq h=2,4,6,8} [(\bar{\psi}N_{ae}^i\psi)(\bar{\psi}N_{bf}^j\psi)(\bar{\psi}N_{cg}^k\psi)(\bar{\psi}N_{dh}^l\psi)], \end{aligned} \tag{29}$$

where different combinations of the operators $N_{ae}^i \equiv (\lambda_i + i\gamma_5\lambda_i)_{ae}$ were found; they are defined below. The traces over $(\gamma_5)^{2n}$ (n integer) were calculated for these coefficients. Only two types of terms in expression (29) are nonzero, namely those for which the flavor structure of the quark bilinears arranges in the following forms:

$$K_1 \to \mathrm{tr}(N_{14}N_{32})(N_{56}N_{78}), \tag{30}$$

$$K_2 \to \mathrm{tr}(N_{16}N_{74}N_{52}N_{38}). \tag{31}$$

All the other terms either disappear since $\mathrm{tr}[\gamma_5] = \mathrm{tr}[\lambda_i] = \mathrm{tr}[\gamma_5^3] = 0$ or reduce to one of these two terms.

To rewrite expression (29), the following $SU(3)$ relations were used:

$$\begin{aligned} &\mathrm{tr}(\lambda_i\lambda_j\lambda_k\lambda_l) = 16\left(\frac{1}{12}\delta^{ij}\delta^{kl} + \frac{1}{8}h^{ija}h^{akl}\right) \\ &\quad \text{for } h_{ija} = d_{ija} + if_{ija}, \\ &\quad h_{ija}h_{akl} = d_{ija}d_{akl} - f_{ija}f_{akl} + i(d_{ija}f_{akl} + f_{ija}d_{akl}), \\ &\quad \mathrm{tr}\lambda_i\lambda_j = 2\delta_{ij}, \end{aligned} \tag{32}$$

as well as the $SU(3)$ Jacobi identity and the (anti)symmetry of the tensor d_{ija} (f_{ija}). By considering a simplified notation with $s_i = \bar{\psi}\lambda_i\psi$ and $p_i = \bar{\psi}\lambda_i\gamma_5\psi$, the first structure, expression (30), can be written as

$$K_1 \to \frac{16}{12}[s_i^2 + p_i^2]^2, \tag{33}$$

where $h_{iia} = 0$ and $\mathrm{tr}[\gamma_5] = \mathrm{tr}[\gamma_5^3] = 0$.

For the second term, it follows that

$$\begin{aligned} K_2 \to 4\frac{16}{12}(s_i^2 + p_i^2)^2 + \frac{16}{8}[&d_{ija}d_{akl}(s_is_js_ks_l + p_ip_jp_kp_l \\ &+ 2s_is_jp_kp_l) - 4f_{ija}f_{akl}s_ip_js_kp_l] \end{aligned} \tag{34}$$

Now the chiral projectors, $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$, can be used to rewrite the terms above. By resolving all the traces (Dirac, flavor and color) of the corresponding coefficients, back to expression (29), it yields

$$\begin{aligned} S_{\mathrm{eff}}^{(8)} = \tilde{g}_8 \int_x \Bigg\{ &\left(\frac{16}{12} + 4\frac{16}{12}\right)(\bar{\psi}P_R\psi\bar{\psi}P_L\psi)^2 \\ &+ \frac{16}{8}(\bar{\psi}P_R\psi\bar{\psi}P_L\psi\bar{\psi}P_R\psi\bar{\psi}P_L\psi)\Bigg\}, \end{aligned} \tag{35}$$

where the following effective coupling constant was defined, using Euclidean momenta with the same cutoff as before:

$$\begin{aligned} \tilde{g}_8 = 16 \times 16 \frac{(2g_4)^4 N_c}{4!4!} \int \frac{d^4k_E}{(2\pi)^4} \\ \times \left(\sum_f \frac{k_E^4 - 6k_E^2 M_f^{*2} + M_f^{*4}}{(k_E^2 + M_f^{*2})^2 (k_E^2 + M_f^{*2})^2}\right). \end{aligned} \tag{36}$$

The two terms in expression (35) are precisely the most general $SU(3)$ chiral invariant Lagrangian interactions considered by Osipov *et al.* [16,17], which can be rewritten as

$$\begin{aligned} \mathcal{L}_{\mathrm{eff},8} = g_2^{(8)}&(\bar{\psi}P_R\psi\bar{\psi}P_L\psi\bar{\psi}P_R\psi\bar{\psi}P_L\psi) \\ &+ g_1^{(8)}(\bar{\psi}P_R\psi\bar{\psi}P_L\psi)^2. \end{aligned} \tag{37}$$

The couplings $g_1^{(8)} = \tilde{g}_8\frac{80}{12}$ and $g_2^{(8)} = \tilde{g}_8\frac{16}{8}$ have different relative weights in expression (35) which behave in the way suggested in Refs. [16,17] for the stability of the ground state, i.e., $g_1^{(8)} > g_2^{(8)}$. In Table I, some values for the effective coupling constants $g_1^{(8)}$ and $g_2^{(8)}$ are given as functions of the same values for free parameters and gluon effective mass (or cutoff) from the gap equations.

The values of the masses and effective coupling constants shown in Table I are in very good agreement with phenomenological fits in the investigation of the spontaneously broken chiral symmetry and light hadron structure [2,16,17]. The effective quark masses m_f^* are very close to the values for the effective masses M_f^* obtained from the scalar quark condensates S_f, but a little smaller for the strange quark mass. This seems to justify the use of M_f^* as equivalent to m_f^*. The NJL effective coupling constants g_4 and $\tilde{g}_4$ are of the order of the usual coupling considered in different versions of the model [2,3,16,17]. The sixth order coupling constant is slightly smaller than the phenomenological fits for the 't Hooft coupling ($\kappa \simeq -770 \to -1100\ \mathrm{GeV}^{-5}$) as it was considered in Refs. [16,17,21], since $\kappa = \frac{16}{9}\tilde{g}_6$. Finally, the eight order terms are also slightly higher than the values considered in the phenomenological fits by Osipov and collaborators [16,21], following nearly the systematics for the two different coupling constants, i.e., $g_1 > g_2$, which is required by the vacuum stability conditions analysis found in those references. Both $g_1^{(8)}$ and $g_2^{(8)}$ are positive, although they compare well with the phenomenological fits in the ranges of $g_1 \sim 1000 \to 6000\ \mathrm{GeV}^{-8}$ and $g_2 \sim -130 \to 320\ \mathrm{GeV}^{-8}$.

IV. SUMMARY AND CONCLUSIONS

In this work, low energy effective quark interactions were derived by considering vacuum polarization for a QCD-based Nambu–Jona-Lasinio model, where the NJL coupling is proportional to the zero momentum QCD coupling constant g^2 and inversely proportional to the effective gluon mass [26]. The quark field was separated into two components, one that condenses into scalar quark-antiquark condensate and another one corresponding to interacting quarks, the quasiparticles of the model. By integrating out the first component with the introduction of the usual scalar/pseudoscalar variables, an effective model for the interacting quarks in terms of the vacuum values of three auxiliary variables, ϕ^0, S_i^0, P_i^0, was obtained. Since these variables can be associated with typical condensates of the QCD vacuum, their gap equations were calculated and solved by extremizing the effective potential at zero quark field and by considering a unique covariant

Euclidean cutoff, as usually done. The resulting quark and gluon effective masses, M_f^* and M_G, are basically the same as those obtained in different approaches. The quark determinant was expanded in powers of bilinears $\bar{\psi}\Gamma\psi$, yielding corrections to the interacting quark (quasiparticles) masses and effective multiquark couplings. The resulting quark effective coupling constants were calculated by factorizing each term of the determinant expansion following nearly the lines of the lowest order derivative expansion, yielding chiral invariant multiquark interactions in terms of the same covariant cutoff fixed in the solution of the gap equations, with the model being nonrenormalizable. Besides, if on one hand this calculation might be seen as a first analysis to be supplemented by a renormalization group investigation, on another hand, the values for the masses and effective coupling constants shown in Table I are in very good agreement with phenomenological fits in the investigation of the spontaneously broken chiral symmetry and light hadron structure [2,16]. The vacuum polarization corrections for the effective interacting quark masses (m_f^*) are very close to the values for the effective masses M_f^* from the scalar quark condensates $S_{0,f}$, but a little smaller for the strange quark mass. This seems to allow the identification of both masses, $m_f^* \sim M_f^*$. The correction to the NJL coupling constant $\tilde{g}_4$ is smaller than the usual coupling considered in different versions of the model but of the same order of magnitude [2,16]. The sixth order coupling constant has a slightly smaller modulus than the phenomenological fits for the 't Hooft coupling as considered in Refs. [16,21], $\kappa \simeq -770 \to -1100\ \mathrm{GeV}^{-5}$, by identifying $\kappa = \frac{16}{9}\tilde{g}_6$. Finally the eighth order term is in good agreement with the values considered in the phenomenological fits by Osipov and collaborators [16,21], following nearly the systematics for the two different coupling constants, i.e., $g_1 > g_2$, which is required by the vacuum stability conditions analysis found in those references. They have values $g_1 \sim 1000 \to 6000\ \mathrm{GeV}^{-8}$ and $g_2 \sim -130 \to 320\ \mathrm{GeV}^{-8}$. It is interesting to note that some of the resulting effective quark interactions correspond to the ones obtained from other derivations based on instanton physics, in particular the sixth order interactions and seemingly the eighth order one [22,23]. Although these two different calculations might provide a sort of double counting of QCD effects, i.e., from instanton physics and QCD condensates, the discussion of this issue is outside the scope of the present work. The approach considered in this work is a systematic framework which allows for improvements and accounts for further effects, in particular from the QCD vacuum. Therefore it might be valuable for the analysis of the stability of the QCD expansion in quark currents. Moreover, it allows for computing corrections to different effective quark interactions other than those found here, such as those derivative couplings neglected above. This program would help to pin down the correct physical value for the effective couplings at the desired energy scale.

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