

## $B \rightarrow \pi\pi$ decays and effects of the next-to-leading order contributions

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In this paper we perform a systematic study for the three  $B \rightarrow (\pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0)$  decays in the perturbative QCD (pQCD) factorization approach with the inclusion of all currently known next-to-leading order (NLO) contributions from various sources. We found that (a) for the CP-averaged decay rates  $Br(B^0 \rightarrow \pi^+\pi^-)$  and  $Br(B^+ \rightarrow \pi^+\pi^0)$ , the NLO pQCD predictions agree with the data within one standard deviation; (b) for  $Br(B^0 \rightarrow \pi^0\pi^0)$ , however, although the NLO contributions can provide a  $\sim 100\%$  enhancement to the leading-order (LO) result, it is still not large enough to interpret the data; and (c) for the CP-violating asymmetries of  $B^0 \rightarrow \pi^+\pi^-$  decay, the NLO pQCD predictions for the central values of  $\mathcal{A}_{\pi\pi}$  and  $\mathcal{S}_{\pi\pi}$  have the same sign as the data but are still smaller in magnitude than the measured values. We also examined the relative strength of the LO and NLO contributions from different sources.

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As is well known, the standard model (SM) prediction for  $Br(B^0 \rightarrow \pi^0\pi^0)$  [1–3] is much smaller than the measured one, which has been known as the “ $\pi\pi$ ” puzzle in  $B \rightarrow \pi\pi$  decays [4,5]. In Ref. [3], the authors studied this puzzle by employing the pQCD approach [6–8], by including partial next-to-leading order (NLO) contributions known at that time, and found that  $Br(B^0 \rightarrow \pi^0\pi^0)$  can be increased from the leading-order (LO) prediction  $0.12 \times 10^{-6}$  to  $0.29 \times 10^{-6}$ .

In Refs. [9–11], very recently, the authors calculated the NLO twist-two and twist-three contributions to the form factors of  $B \rightarrow \pi$  transition in the pQCD approach. Here we will study the  $B \rightarrow \pi\pi$  decays again with the inclusion of these newly known NLO contributions to form factors to check their effects.

In the  $B$  rest frame, we assume that the light final state pion mesons are moving along the direction of  $n = (1, 0, \mathbf{0}_T)$  and  $v = (0, 1, \mathbf{0}_T)$ , respectively. We use  $x_i$  to denote the momentum fraction of the antiquark in each meson, and  $k_T$  for the corresponding transverse momentum. Using the light-cone coordinates, the  $B$  meson momentum  $P_B$  and the two final state pion mesons' momenta  $P_2$  and  $P_3$  can be written as

$$\begin{aligned} P_B &= \frac{M_B}{\sqrt{2}}(1, 1, \mathbf{0}_T), & P_2 &= \frac{M_B}{\sqrt{2}}(1 - r_3^2, r_2^2, \mathbf{0}_T), \\ P_3 &= \frac{M_B}{\sqrt{2}}(r_3^2, 1 - r_2^2, \mathbf{0}_T), \end{aligned} \quad (1)$$

where  $r_i = m_\pi/M_B$ . After the integration over the small components  $k_1^-, k_2^-,$  and  $k_3^+$  we find the decay amplitudes conceptually

$$\begin{aligned} \mathcal{A}(B \rightarrow M_2 M_3) &\sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \\ &\cdot \text{Tr}[C(t)\Phi_B(x_1, b_1)\Phi_\pi(x_2, b_2)\Phi_\pi(x_3, b_3) \\ &\times H(x_i, b_i, t)S_i(x_i)e^{-S(t)}], \end{aligned} \quad (2)$$

where  $b_i$  is the conjugate space coordinate of  $k_{iT}$ ,  $C(t)$  is the Wilson coefficient, the functions  $\Phi_B(x_1, b_1)$ ,  $\Phi_\pi(x_j, b_j)$  with  $j = (2, 3)$  are the wave functions of the initial  $B$  meson and the two final state pion mesons, respectively. The function  $H(k_1, k_2, k_3, t)$  is the hard kernel, while the jet function  $S_i(x_i)$  and the function  $e^{-S(t)}$  are the two Sudakov factors relevant for the considered  $B$  decays [8].

For the considered  $B \rightarrow \pi\pi$  decays, the corresponding weak effective Hamiltonian can be written as [12]

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* [C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)] \right. \\ &\quad \left. - V_{tb}V_{td}^* \left[ \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right] \right\} + \text{H.c.}, \end{aligned} \quad (3)$$

where  $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi constant,  $V_{ij}$  are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, the  $O_i$  ( $i = 1, \dots, 10$ ) are the local four-quark operators, and  $C_i(\mu)$  are the Wilson coefficients evaluated at scale  $\mu$  [12].

The  $B$  meson is treated as a very good heavy-light system with the wave function in the form of

$$\Phi_B = \frac{i}{\sqrt{2N_c}} (\not{P}_B + m_B) \gamma_5 \phi_B(\mathbf{k}_1). \quad (4)$$

Here we adopted the  $B$ -meson distribution amplitude  $\phi_B(x, b)$  widely used for example in Refs. [1,13],

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$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp \left[ -\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2 \right], \quad (5)$$

where the  $b$  dependence was included through the second term in the exponential function, the shape parameter  $\omega_b = 0.40 \pm 0.04$  has been fixed [8] from the fit to the  $B \rightarrow \pi$  form factors derived from lattice QCD and from light-cone sum rule [14], and finally the normalization factor  $N_B$  depends on the value of  $\omega_b$  and  $f_B$  defined through the normalization relation  $\int_0^1 dx \phi_B(x, b=0) = f_B / (2\sqrt{6})$ . The wave functions of the final state pion mesons and the relevant distribution amplitudes  $\phi_\pi^{A,P,T}$  are of the same

form as those adopted in Refs. [3,15–17]. The Gegenbauer moments  $a_i^\pi$  and other parameters are adopted from Refs. [3,18],

$$\begin{aligned} a_2^\pi &= 0.25, & a_4^\pi &= -0.015, & \rho_\pi &= m_\pi / m_{0\pi}, \\ \eta_3 &= 0.015, & \omega_3 &= -3.0, \end{aligned} \quad (6)$$

with  $m_{0\pi}$  the chiral mass of the pion.

The  $B \rightarrow \pi\pi$  decays have been studied by employing the pQCD factorization approach at the LO [1] or partial NLO level [3]. The total decay amplitude at the leading order for the three  $B \rightarrow \pi\pi$  decays are the following,

$$\begin{aligned} \mathcal{M}_{\text{LO}}(B^0 \rightarrow \pi^+ \pi^-) &= \frac{G_F}{\sqrt{2}} \left\{ \lambda_u [a_1 f_\pi F_{e\pi}^{V-A} + c_1 M_{e\pi}^{V-A} + a_2 f_B F_{a\pi}^{V-A} + c_2 M_{a\pi}^{V-A}] \right. \\ &\quad - \lambda_t \left[ (a_4 + a_{10}) f_\pi F_{e\pi}^{V-A} + (a_6 + a_8) f_\pi F_{e\pi}^{SP} + (c_3 + c_9) M_{e\pi}^{V-A} + (c_5 + c_7) M_{e\pi}^{V+A} \right. \\ &\quad + \left( 2a_3 + a_4 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) f_B F_{a\pi}^{V-A} + \left( 2a_5 + \frac{1}{2}a_7 \right) f_B F_{a\pi}^{V+A} + \left( a_6 - \frac{1}{2}a_8 \right) f_B F_{a\pi}^{SP} \\ &\quad \left. \left. + \left( c_3 + 2c_4 - \frac{1}{2}c_9 + \frac{1}{2}c_{10} \right) M_{a\pi}^{V-A} + \left( c_5 - \frac{1}{2}c_7 \right) M_{a\pi}^{V+A} + \left( 2c_6 + \frac{1}{2}c_8 \right) M_{a\pi}^{SP} \right] \right\}, \quad (7) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\text{LO}}(B^0 \rightarrow \pi^0 \pi^0) &= \frac{1}{\sqrt{2}} \frac{G_F}{\sqrt{2}} \left\{ \lambda_u \left[ -a_2 f_\pi F_{e\pi}^{V-A} - c_2 M_{e\pi}^{V-A} + a_2 f_B F_{a\pi}^{V-A} + c_2 M_{a\pi}^{V-A} \right] \right. \\ &\quad - \lambda_t \left[ \left( -\frac{3}{2}a_7 \right) f_\pi F_{e\pi}^{V+A} + \left( a_4 - \frac{3}{2}a_9 - \frac{1}{2}a_{10} \right) f_\pi F_{e\pi}^{V-A} + \left( a_6 - \frac{1}{2}a_8 \right) f_\pi F_{e\pi}^{SP} \right. \\ &\quad + \left( c_3 - \frac{1}{2}c_9 - \frac{3}{2}c_{10} \right) M_{e\pi}^{V-A} + \left( c_5 - \frac{1}{2}c_7 \right) M_{e\pi}^{V+A} + \left( -\frac{3}{2}c_8 \right) M_{e\pi}^{SP} \\ &\quad + \left( 2a_3 + a_4 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) f_B F_{a\pi}^{V-A} + \left( 2a_5 + \frac{1}{2}a_7 \right) f_B F_{a\pi}^{V+A} + \left( a_6 - \frac{1}{2}a_8 \right) f_B F_{a\pi}^{SP} \\ &\quad \left. \left. + \left( c_3 + 2c_4 - \frac{1}{2}c_9 + \frac{1}{2}c_{10} \right) M_{a\pi}^{V-A} + \left( c_5 - \frac{1}{2}c_7 \right) M_{a\pi}^{V+A} + \left( 2c_6 + \frac{1}{2}c_8 \right) M_{a\pi}^{SP} \right] \right\}, \quad (8) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\text{LO}}(B^+ \rightarrow \pi^+ \pi^0) &= \frac{1}{\sqrt{2}} \frac{G_F}{\sqrt{2}} \left\{ \lambda_u [(a_1 + a_2) f_\pi F_{e\pi}^{V-A} + (c_1 + c_2) M_{e\pi}^{V-A}] \right. \\ &\quad - \lambda_t \left[ \left( \frac{3}{2}a_9 + \frac{3}{2}a_{10} \right) f_\pi F_{e\pi}^{V-A} + \left( \frac{3}{2}a_7 \right) f_\pi F_{e\pi}^{V+A} + \left( \frac{3}{2}a_8 \right) f_\pi F_{e\pi}^{SP} \right. \\ &\quad \left. \left. + \left( \frac{3}{2}c_9 + \frac{3}{2}c_{10} \right) M_{e\pi}^{V-A} + \left( \frac{3}{2}c_7 \right) M_{e\pi}^{V+A} + \left( \frac{3}{2}c_8 \right) M_{e\pi}^{SP} \right] \right\}, \quad (9) \end{aligned}$$

where  $\lambda_u = V_{ub}^* V_{ud}$ ,  $\lambda_t = V_{tb}^* V_{td}$  and the Wilson coefficients  $a_i$  are the same as those defined in Ref. [3]. The 11 decay amplitudes  $F_{e\pi, a\pi}^{V\pm A}$ ,  $F_{e\pi, a\pi}^{SP}$ ,  $M_{e\pi, a\pi}^{V\pm A}$ , and  $M_{a\pi}^{SP}$  in Eqs. (7)–(9) are obtained by evaluating analytically the Feynman diagrams as shown in Fig. 1 and have been given for example in Refs. [1,3].

In the framework of the pQCD factorization approach, the NLO contributions should include the following pieces from rather different sources:

- (1) The Wilson coefficients  $C_i(m_W)$  at NLO level [12], the renormalization group (RG) evolution matrix  $U(\mu, m_W, \alpha)$  at NLO level [12], and the strong coupling constant  $\alpha_s(\mu)$  at two-loop level [5].
- (2) The NLO contributions from the vertex corrections (VC), the quark-loops (QL), and the chromomagnetic penguin operator  $O_{8g}$  (MP) as given in Refs. [3,9,11,19,20].

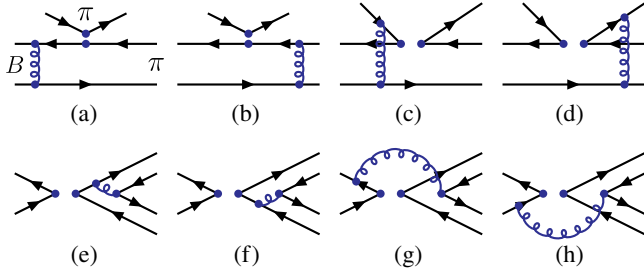


FIG. 1 (color online). Feynman diagrams that may contribute to the  $B \rightarrow \pi\pi$  decays in the pQCD approach at leading order.

- (3) The NLO twist-two and twist-three contributions to the form factors (FF) of the  $B \rightarrow \pi$  transition as calculated in Refs. [9,11].

The still missing NLO parts in the pQCD approach are the  $O(\alpha_s^2)$  contributions from hard spectator diagrams and

annihilation diagrams, as illustrated by Fig. 5 of Ref. [16]. According to the general arguments as presented in Ref. [3] and explicit numerical comparisons of the contributions from different sources for  $B \rightarrow K\eta^{(\prime)}$  decays [16], one generally believes that these still missing NLO parts are high-order corrections to small quantities, and therefore could be neglected safely.

For details of the calculations about those NLO contributions from the vertex corrections, the quark loops and the chromomagnetic penguins  $O_{8g}$  and the explicit expressions of these NLO contributions, one can see Refs. [3,19]. The NLO vertex corrections can be taken into account by the proper replacements of the Wilson coefficients  $a_i(\mu)$ , as presented explicitly for example in Eqs. (50),(51) of Ref. [16]. For the NLO contributions from the quark loops, for example, the corresponding decay amplitudes are of the form

$$\begin{aligned} \mathcal{M}^{(\text{QL})}(B^0 \rightarrow \pi^0\pi^0) &= \frac{G_F}{\sqrt{2}} \frac{8\pi}{\sqrt{6}} C_f^2 M_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \\ &\times \{ [(1+x_3)\phi_\pi^A(x_2)\phi_\pi^A(x_3) + r_\pi(1-2x_3)(\phi_\pi^P(x_3)\phi_\pi^A(x_2) + \phi_\pi^T(x_3)\phi_\pi^A(x_2)) \\ &+ 2r_\pi\phi_\pi^A(x_3)\phi_\pi^P(x_2)] \cdot E^{q_l}(t_q, l^2) \cdot h_e(x_1, x_3, b_1, b_3) \\ &+ [2r_\pi x_1\phi_\pi^A(x_3)\phi_\pi^P(x_2) + 2r_\pi\phi_\pi^P(x_3)\phi_\pi^A(x_2)] \cdot E^{q_l}(t'_q, l^2) \cdot h_e(x_3, x_1, b_3, b_1) \}, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{M}^{(\text{QL})}(B^0 \rightarrow \pi^+\pi^-) &= \frac{G_F}{\sqrt{2}} \frac{8\pi}{\sqrt{6}} C_f^2 M_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \\ &\times \{ [(1+x_3)\phi_\pi^A(x_2)\phi_\pi^A(x_3) + r_\pi(1-2x_3)(\phi_\pi^P(x_3)\phi_\pi^A(x_2) + \phi_\pi^T(x_3)\phi_\pi^A(x_2)) \\ &+ 2r_\pi\phi_\pi^A(x_3)\phi_\pi^P(x_2)] \cdot E^{q_l}(t_q, l^2) \cdot h_e(x_1, x_3, b_1, b_3) \\ &+ [2r_\pi x_1\phi_\pi^A(x_3)\phi_\pi^P(x_2) + 2r_\pi\phi_\pi^P(x_3)\phi_\pi^A(x_2)] \cdot E^{q_l}(t'_q, l_2) h_e(x_3, x_1, b_3, b_1) \}, \end{aligned} \quad (11)$$

$$\mathcal{M}^{(\text{QL})}(B^+ \rightarrow \pi^+\pi^0) = 0, \quad (12)$$

where  $r_\pi = m_0^\pi/m_B$ , and the terms proportional to  $r_\pi^2$  are not shown in above equations. The function  $E^{q_l}(t_q, l^2)$ ,  $h_e(x_i, b_i)$  and other relevant parameters can be found for example in Appendix B of Ref. [16]. It is straightforward to

find the NLO contributions  $\mathcal{M}^{(\text{MP})}(B \rightarrow \pi\pi)$  from the  $O_{8g}$  insertion correction [3,16,19].

Very recently, the NLO twist-two and twist-three contributions to the form factors  $f^{+,0}(q^2)$  of  $B \rightarrow \pi$  transition have been calculated in Refs. [9,11]. When these NLO contributions taken into account, the form factor  $f^+(q^2)$ , for example, can be written in the form

$$\begin{aligned} f^+(q^2)|_{\text{NLO}} &= 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\ &\times \left\{ r_\pi [\phi_\pi^P(x_2) - \phi_\pi^T(x_2)] \cdot \alpha_s(t_1) \cdot e^{-S_{B\pi}(t_1)} \cdot S_t(x_2) \cdot h(x_1, x_2, b_1, b_2) \right. \\ &+ \left[ (1+x_2\eta)(1+F_{T2}^{(1)}(x_i, \mu, \mu_f, q^2))\phi_\pi^A(x_2) + 2r_\pi \left( \frac{1}{\eta} - x_2 \right) \phi_\pi^T(x_2) - 2x_2 r_\pi \phi_\pi^P(x_2) \right] \\ &\cdot \alpha_s(t_1) \cdot e^{-S_{B\pi}(t_1)} \cdot S_t(x_2) \cdot h(x_1, x_2, b_1, b_2) \\ &\left. + 2r_\pi \phi_\pi^P(x_2)(1+F_{T3}^{(1)}(x_i, \mu, \mu_f, q^2)) \cdot \alpha_s(t_2) \cdot e^{-S_{B\pi}(t_2)} \cdot S_t(x_2) \cdot h(x_2, x_1, b_2, b_1) \right\}, \end{aligned} \quad (13)$$

with the NLO twist-two and twist-three correction factors

$$F_{T2}^{(1)}(x_i, \mu, \mu_f, q^2) = \frac{\alpha_s(\mu_f)C_F}{4\pi} \left[ \frac{21}{4} \ln \frac{\mu^2}{m_B^2} - \left( \frac{13}{2} + \ln r_1 \right) \ln \frac{\mu_f^2}{m_B^2} + \frac{7}{16} \ln^2(x_1 x_2) + \frac{1}{8} \ln^2 x_1 \right. \\ \left. + \frac{1}{4} \ln x_1 \ln x_2 + \left( -\frac{1}{4} + 2 \ln r_1 + \frac{7}{8} \ln \eta \right) \ln x_1 + \left( -\frac{3}{2} + \frac{7}{8} \ln \eta \right) \ln x_2 \right. \\ \left. + \frac{15}{4} \ln \eta - \frac{7}{16} \ln^2 \eta + \frac{3}{2} \ln^2 r_1 - \ln r_1 + \frac{101\pi^2}{48} + \frac{219}{16} \right], \quad (14)$$

$$F_{T3}^{(1)}(x_i, \mu, \mu_f, q^2) = \frac{\alpha_s(\mu_f)C_F}{4\pi} \left[ \frac{21}{4} \ln \frac{\mu^2}{m_B^2} - \frac{1}{2} (6 + \ln r_1) \ln \frac{\mu_f^2}{m_B^2} + \frac{7}{16} \ln^2 x_1 - \frac{3}{8} \ln^2 x_2 \right. \\ \left. + \frac{9}{8} \ln x_1 \ln x_2 + \left( -\frac{29}{8} + \ln r_1 + \frac{15}{8} \ln \eta \right) \ln x_1 + \left( -\frac{25}{16} + \ln r_2 + \frac{9}{8} \ln \eta \right) \ln x_2 \right. \\ \left. + \frac{1}{2} \ln r_1 - \frac{1}{4} \ln^2 r_1 + \ln r_2 - \frac{9}{8} \ln \eta - \frac{1}{8} \ln^2 \eta + \frac{37\pi^2}{32} + \frac{91}{32} \right], \quad (15)$$

where  $r_i = m_B^2/\xi_i^2$  with the choice of  $\xi_1 = 25m_B$  and  $\xi_2 = m_B$  [9],  $\eta = 1 - q^2/m_B^2$  with  $q^2 = (P_1 - P_3)^2$  is the energy fraction carried by the meson which picks up the spectator quark of the  $B$  meson,  $\mu$  ( $\mu_f$ ) is the renormalization (factorization) scale, the hard scale  $t_{1,2}$  are chosen as the largest scale of the propagators in the hard  $b$ -quark decay diagrams [9,11], and the function  $S_f(x_2)$  and hard function  $h(x_i, b_j)$  can be found in Refs. [9,11]. For  $B \rightarrow \pi\pi$  decays, the large recoil region corresponds to the energy fraction  $\eta \sim O(1)$ . We here also set  $\mu = \mu_f = t$  in order to minimize the NLO contribution to the form factors [11,21].

In the numerical calculations, we use the following input parameters [4,5] (all masses and decay constants in units of GeV)

$$\begin{aligned} f_B &= 0.21, & f_\pi &= 0.13, & m_\pi &= 0.14, \\ m_{0\pi} &= 1.4, & M_B &= 5.28, & m_b &= 4.8, \\ m_c &= 1.5, & M_W &= 80.41, & \tau_{B^0} &= 1.53 \text{ ps}, \\ \tau_{B^+} &= 1.641 \text{ ps}. \end{aligned} \quad (16)$$

For the CKM matrix elements, we adopt the Wolfenstein parametrization with the CKM parameters as given in Ref. [5]:  $A = 0.832 \pm 0.017$ ,  $\lambda = 0.2246 \pm 0.0011$ ,  $\bar{\rho} = 0.130 \pm 0.018$ , and  $\bar{\eta} = 0.350 \pm 0.013$ .

We first calculate the pQCD predictions for the form factor  $F_0^{B \rightarrow \pi}(0)$  for the  $B \rightarrow \pi$  transition at the LO and NLO level, respectively, and find numerically that

$$F_0^{B \rightarrow \pi}(0) = \begin{cases} 0.27 \pm 0.05, & \text{LO,} \\ 0.28_{-0.06}^{+0.05}, & \text{NLO.} \end{cases} \quad (17)$$

We find that the NLO twist-two and twist-three contributions are similar in magnitude but have opposite sign; the  $\sim 15\%$  enhancement to the central value of the LO pQCD

prediction is therefore largely canceled by the inclusion of the NLO twist-three contribution. The pQCD predictions as given in Eq. (17) agree very well with those obtained from the QCD sum rule or other methods.

Using the input parameters and the wave functions as given in previous sections, it is easy to calculate the CP-averaged branching ratios for the considered three  $B \rightarrow \pi\pi$  decays. When all currently known NLO contributions are taken into account, we find the following NLO pQCD predictions for the CP-averaged branching ratios:

$$\begin{aligned} Br(B^0 \rightarrow \pi^+ \pi^-) &= [7.67_{-1.85}^{+2.63} (\omega_b)_{-1.39}^{+1.53} (f_B)_{-1.28}^{+1.44} (a_\pi^2)] \times 10^{-6}, \\ Br(B^+ \rightarrow \pi^+ \pi^0) &= [4.27_{-1.01}^{+1.42} (\omega_b)_{-0.77}^{+0.85} (f_B)_{-0.75}^{+0.82} (a_\pi^2)] \times 10^{-6}, \\ Br(B^0 \rightarrow \pi^0 \pi^0) &= [0.23_{-0.05}^{+0.08} (\omega_b)_{-0.04}^{+0.05} (f_B)_{-0.03}^{+0.04} (a_\pi^2)] \times 10^{-6}, \end{aligned} \quad (18)$$

where the major theoretical errors are induced by the uncertainties of  $\omega_b = 0.4 \pm 0.04$  GeV,  $f_B = 0.21 \pm 0.02$  GeV, and the Gegenbauer moment  $a_\pi^2 = 0.25 \pm 0.15$ , respectively.

In Table I, we show the pQCD predictions for the CP-averaged branching ratios of the three  $B \rightarrow \pi\pi$  decays when the NLO contributions from different sources are included step by step. The label ‘‘LOWC’’ means the pQCD predictions from the LO Feynman diagrams as illustrated in Fig. 1 but calculated numerically by using the Wilson coefficients  $C_i(m_W)$  and the RG evolution matrix  $U(t, m, \alpha)$  at the NLO level. The labels ‘‘+VC,’’ ‘‘+QL,’’ and ‘‘+MP’’ mean the LOWC results plus the NLO contribution from the vertex corrections (VC), the quark loops (QL), and the chromomagnetic penguin (MP), respectively. The label ‘‘NLO’’ means all currently known NLO contributions, including the very recently known NLO twist-two and twist-three contributions to the  $B \rightarrow \pi$  transition form factor [9,11], are all taken into account

TABLE I. The pQCD predictions for the CP-averaged branching ratios (in unit of  $10^{-6}$ ). The meaning of the labels has been explained in the text.

Channel	LO	NLOWC	+VC	+QL	+MP	NLO	QCDF[20]	Data [4]
$B^0 \rightarrow \pi^+\pi^-$	7.46	6.65	6.91	7.02	6.87	$7.67^{+3.47}_{-2.64}$	8.9	$5.10 \pm 0.19$
$B^+ \rightarrow \pi^+\pi^0$	3.54	4.23	3.54	–	–	$4.27^{+1.85}_{-1.47}$	6.0	$5.48^{+0.35}_{-0.34}$
$B^0 \rightarrow \pi^0\pi^0$	0.12	0.24	0.27	0.29	0.21	$0.23^{+0.19}_{-0.15}$	0.3	$1.91^{+0.22}_{-0.23}$

and all theoretical errors from different sources are added in quadrature. In the last two columns of Table I, for the sake of comparison, we also list the measured values as given by HFAG [4] and those QCDF predictions as given in Ref. [20].

Now we turn to the evaluations of the CP-violating asymmetries of  $B \rightarrow \pi\pi$  decays in pQCD approach. For  $B^+ \rightarrow \pi^+\pi^0$  decays, the LO and NLO pQCD predictions for the direct CP-violating asymmetries  $\mathcal{A}_{CP}$  are the following:

$$\mathcal{A}_{CP}^{\text{dir}}(B^\pm \rightarrow \pi^\pm\pi^0) = \begin{cases} -4.7\%, & \text{LO,} \\ -5.6\%, & \text{NLO.} \end{cases} \quad (19)$$

For  $B^0 \rightarrow \pi^+\pi^-$  and  $\pi^0\pi^0$  decays, the time-dependent decay rate is defined as [22]

$$\begin{aligned} \mathcal{P}(\Delta t, q) &= \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \{1 + q[\mathcal{A}_{\pi\pi} \cos(\Delta m_d \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m_d \Delta t)]\}. \end{aligned} \quad (20)$$

where  $\Delta t = t_{\pi\pi} - t_{\text{tag}}$ ,  $\tau_{B^0}$  is the  $B^0$  lifetime,  $\Delta m_d$  is the mass difference between the two mass eigenstates of the neutral  $B^0$  meson, and  $q = +1(-1)$  when  $f_{\text{tag}} = B^0(\bar{B}^0)$ . The parameters  $\mathcal{A}_{\pi\pi}$  and  $\mathcal{S}_{\pi\pi}$  are the direct and mixing-induced CP-violating parameters, respectively, and have been defined as the form of

$$\mathcal{A}_{\pi\pi} = \frac{|\lambda_{\pi\pi}|^2 - 1}{1 + |\lambda_{\pi\pi}|^2}, \quad \mathcal{S}_{\pi\pi} = \frac{2\text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad (21)$$

where  $\lambda_f = \frac{q\bar{A}_f}{pA_f}$  depends on the parameters related to the  $B^0 - \bar{B}^0$  mixing and to the decay amplitudes of  $B^0/\bar{B}^0 \rightarrow f$  with the CP eigenstate  $f$ .

Using the input parameters and the wave functions as given in previous sections, we calculate the CP-violating asymmetries for  $B^0 \rightarrow (\pi^+\pi^-, \pi^0\pi^0)$  decays and list the numerical results in Table II. The labels ‘‘NLOWC,’’ ‘‘+VC,’’ ‘‘+QL,’’ ‘‘+MP,’’ and ‘‘NLO’’ in Table II have the same meaning as those in Table I. The major theoretical errors as given in Table II are induced by the uncertainties of input parameters of  $\omega_b$ , and  $a_2^\pi$ . As a comparison, we also list currently available measured values for  $\mathcal{A}_{\pi\pi}$  and  $\mathcal{S}_{\pi\pi}$  for  $B^0 \rightarrow \pi^+\pi^-$  decay in last column.

From the numerical values as listed in Table I and II, one can see the following points:

- (i) For the decay rates  $Br(B^0 \rightarrow \pi^+\pi^-)$  and  $Br(B^0 \rightarrow \pi^+\pi^-)$ , the NLO pQCD predictions agree with the data within  $1\sigma$  error since the theoretical errors are still large.
- (ii) For  $B^0 \rightarrow \pi^0\pi^0$  decay, although the NLO contributions provide about  $\sim 100\%$  enhancement to the LO result, it is still much smaller than the measured one. The so-called ‘‘ $\pi\pi$ ’’ puzzle is still an open problem. The contribution from the soft Glauber gluon [23], or the inclusion of the charm content effect through the tetramixing of  $\pi - \eta - \eta' - \eta_c$  as proposed in Ref. [24], may be the possible ways out of this crisis, but it needs more studies.
- (iii) For the CP-violating asymmetries of  $B^0 \rightarrow \pi^+\pi^-$  decay, the NLOpQCD predictions for the central values of  $\mathcal{A}_{\pi\pi}$  and  $\mathcal{S}_{\pi\pi}$  have the same sign with the measured values, close to them, but still smaller in magnitude than the data (no overlap for the error bands at  $2\sigma$  level). Evaluations for those still missing NLO contributions may play a key role to interpret the data of  $\mathcal{A}_{\pi\pi}$  and  $\mathcal{S}_{\pi\pi}$ .
- (iv) For  $B^+ \rightarrow \pi^+\pi^0$  decay, its direct CP violation is small in size. For  $B^0 \rightarrow \pi^0\pi^0$  decay, however, the pQCD predictions for their CP-violating asymmetries are

TABLE II. The LO and NLO pQCD predictions for the direct and mixing-induced CP asymmetries for  $B^0 \rightarrow \pi^+\pi^-$  and  $\pi^0\pi^0$  decays. The world averages as given in Ref. [4] are listed in the last column.

Mode	LO	NLOWC	+VC	+QL	+MP	NLO	Data [4]
$\mathcal{A}_{\pi\pi}$	0.27	0.25	0.26	0.13	0.12	$0.12^{+0.04}_{-0.06}$	$+0.31 \pm 0.05$
$\mathcal{S}_{\pi\pi}$	-0.28	-0.40	-0.39	-0.49	-0.41	$-0.40^{+0.05}_{-0.04}$	$-0.66 \pm 0.06$
$\mathcal{A}_{CP}^{\text{dir}}(\pi^0\pi^0)$	-0.10	-0.51	0.61	0.69	0.74	$0.78^{+0.05}_{-0.08}$	–
$\mathcal{A}_{CP}^{\text{mix}}(\pi^0\pi^0)$	-0.02	0.61	0.67	0.41	0.50	$0.47^{+0.02}_{-0.11}$	–

TABLE III. The LO pQCD predictions for the numerical values (in unit of  $10^{-4}$ ) of the individual and total decay amplitudes of  $B^0/\bar{B}^0 \rightarrow (\pi^+\pi^-, \pi^0\pi^0)$  and  $B^\pm \rightarrow \pi^\pm\pi^0$  decays, as well as the ratios  $R_{\text{LO}}$ .

Decay	$\mathcal{M}^{a+b}$	$\mathcal{M}^{c+d}$	$\mathcal{M}^{\text{anni}}$	$\mathcal{M}_{\text{LO}}$	$R_{\text{LO}}$
$B^0 \rightarrow \pi^+\pi^-$	$-1.40 - i2.32$	$0.094 + i0.022$	$0.11 + i0.48$	$-1.19 - i1.81$	$7.33:0.009:0.25:4.72$
$B^+ \rightarrow \pi^+\pi^0$	$-0.61 - i1.50$	$-0.073 - i0.048$	–	$-0.69 - i1.54$	$2.62:0.008:0.00:2.85$
$B^0 \rightarrow \pi^0\pi^0$	$-0.31 - i0.05$	$0.13 + i0.08$	$0.01 + i0.26$	$-0.17 + i0.29$	$0.10:0.020:0.07:0.11$
$\bar{B}^0 \rightarrow \pi^+\pi^-$	$-1.40 + i2.32$	$-0.041 - i0.090$	$0.11 + i0.30$	$-1.33 + i2.53$	$7.33:0.010:0.10:8.16$
$B^- \rightarrow \pi^-\pi^0$	$-0.61 + i1.50$	$0.020 + i0.085$	–	$-0.59 + i1.58$	$2.62:0.008:0.00:2.85$
$\bar{B}^0 \rightarrow \pi^0\pi^0$	$-0.31 + i0.05$	$-0.03 - i0.15$	$0.05 + i0.19$	$-0.29 + i0.08$	$0.10:0.020:0.04:0.30$

large in size and may be measurable in the running LHCb and future super-B experiments.

From the numerical results as listed in Table I–II, one can see that the LO pQCD predictions could be changed significantly after the inclusion of the NLO contributions. We here will check the relative strength for those LO contributions from different kinds of Feynman diagrams, and then examine the effects of the NLO contributions from different sources.

In Table III we show the central values of the pQCD predictions for the numerical values (in unit of  $10^{-4}$ ) of the decay amplitude from different Feynman diagrams at the LO level. The label “ $\mathcal{M}^{a+b}$ ” (“ $\mathcal{M}^{c+d}$ ”) means the decay amplitude of the factorizable emission diagrams Figs. 1(a) and 1(b) (the spectator diagrams Figs. 1(c) and 1(d)). The label “ $\mathcal{M}^{\text{anni}}$ ” means the decay amplitude from the four annihilation diagrams Figs. 1(e)–1(h). The term  $\mathcal{M}_{\text{LO}}$  means the full LO decay amplitude. The ratio  $R_{\text{LO}}$  in Table III is defined as the form of

$$R_{\text{LO}} = |\mathcal{M}^{a+b}|^2 : |\mathcal{M}^{c+d}|^2 : |\mathcal{M}^{\text{anni}}|^2 : |\mathcal{M}_{\text{LO}}|^2. \quad (22)$$

From the numerical results as listed in Table III, one can find the following points:

- (i) At the leading order, the two factorizable emission diagrams do provide the dominant contribution. For  $B^0/\bar{B}^0 \rightarrow \pi^+\pi^-$  and  $B^\pm \rightarrow \pi^\pm\pi^0$  decays, we find numerically that

$$|\mathcal{M}^{a+b}|^2 \gg |\mathcal{M}^{c+d}|^2 \quad \text{or} \quad |\mathcal{M}^{\text{anni}}|^2. \quad (23)$$

For  $B^0/\bar{B}^0 \rightarrow \pi^0\pi^0$  decay, although  $|\mathcal{M}^{a+b}|^2$  is still larger than  $|\mathcal{M}^{\text{anni}}|^2$ , the annihilation diagrams for this

decay do have a small real part but a large imaginary part, which in turn result in an effective contribution to its branching ratio and also provide the large strong phase required to produce the large CP violation.

- (ii) By comparing  $\mathcal{M}^i$  for  $B^0/B^+$  decays and their CP conjugated  $\bar{B}^0/B^-$  decays, one can see that the amplitude  $\mathcal{M}^{a+b}$  does not has the strong phase,  $\mathcal{M}^{c+d}$  has a small strong phase, but the annihilation diagrams (i.e.,  $\mathcal{M}^{\text{anni}}$ ) do provide the dominant large strong phase. This feature confirmed the general expectation again [16] in the pQCD factorization approach: The strong phase needed to produce large CP violation for the two-body charmless hadronic  $B$  meson decays really comes from the annihilation diagrams.

In Table IV the label “ $\Delta\mathcal{M}_{\text{FF}}$ ” describes the total modification due to the inclusion of both the NLO twist-two and twist-three contributions to the  $B \rightarrow \pi$  transition form factors [9,11], it is indeed very small in size due to the strong cancellation between the NLO twist-2 and twist-3 part. The label “ $\Delta\mathcal{M}_{\text{NLO}}$ ” denotes the changes with respect to “ $\mathcal{M}_{\text{LO}}$ ” induced by the inclusion of all currently known NLO contributions, and finally we define the total decay amplitude at the NLO level as  $\mathcal{M}_{\text{NLO}} = \mathcal{M}_{\text{LO}} + \Delta\mathcal{M}_{\text{NLO}}$  and the ratio  $R_{\text{NLO}}$  as  $R_{\text{NLO}} = |\mathcal{M}_{\text{NLO}}|^2/|\mathcal{M}_{\text{LO}}|^2$ , which measures the effects of the NLO contributions to the considered decays directly.

From the pQCD predictions for the numerical values of the decay amplitudes as listed in Table IV, we find the following points:

- (i) As illustrated by the numbers in third column, the contributions from the NLO contributions to the  $B \rightarrow \pi$  transition form factors are indeed very small.

TABLE IV. The same as in Table III but for  $\Delta\mathcal{M}_{\text{FF}}$ ,  $\Delta\mathcal{M}_{\text{NLO}}$ , and  $\mathcal{M}_{\text{NLO}}$  for  $B/\bar{B} \rightarrow \pi\pi$  decays. The ratios  $R_{\text{NLO}}$  are also listed in last column.

Decay	$\mathcal{M}_{\text{LO}}$	$\Delta\mathcal{M}_{\text{FF}}$	$\Delta\mathcal{M}_{\text{NLO}}$	$\mathcal{M}_{\text{NLO}}$	$R_{\text{NLO}}$
$B^0 \rightarrow \pi^+\pi^-$	$-1.20 - i1.82$	$-0.07 - i0.13$	$-0.17 - i0.17$	$-1.37 - i1.99$	1.23
$B^+ \rightarrow \pi^+\pi^0$	$-0.69 - i1.54$	$-0.05 - i0.08$	$-0.24 - i0.06$	$-0.93 - i1.60$	1.20
$B^0 \rightarrow \pi^0\pi^0$	$-0.17 + i0.29$	$0.00 - i0.01$	$0.12 - i0.09$	$-0.05 + i0.20$	0.38
$\bar{B}^0 \rightarrow \pi^+\pi^-$	$-1.33 + i2.53$	$-0.08 + i0.12$	$-0.14 - i0.23$	$-1.47 + i2.30$	0.92
$B^- \rightarrow \pi^-\pi^0$	$-0.59 + i1.58$	$-0.03 + i0.10$	$0.12 + i0.21$	$-0.47 + i1.79$	1.20
$\bar{B}^0 \rightarrow \pi^0\pi^0$	$-0.29 + i0.09$	$-0.02 + i0.01$	$-0.28 - i0.26$	$-0.57 - i0.17$	3.85

The reason is the large cancellation between the NLO twist-two and twist-three pieces.

- (ii) For  $B^\pm \rightarrow \pi^\pm \pi^0$  decays, the inclusion of all NLO contributions leads to a 20% enhancement to the LO one. For  $B^0/\bar{B}^0 \rightarrow \pi^+ \pi^-$  decay, the effects of NLO contribution to the decay amplitude of the  $B^0 \rightarrow \pi^+ \pi^-$  decay and its CP conjugated decay are rather different: about 20% enhancement to the former case, but 9% decrease to  $\bar{B}^0 \rightarrow \pi^+ \pi^-$  decay mode. And finally provide a 3% enhancement to its CP-averaged branching ratio.
- (iii) For  $B^0/\bar{B}^0 \rightarrow \pi^0 \pi^0$  decays, the NLO contributions themselves and their effects on the LO decay amplitudes are rather different for  $B^0 \rightarrow \pi^0 \pi^0$  decay and its CP-conjugated decay mode:

$$\Delta\mathcal{M}_{\text{NLO}} = \begin{cases} 0.12 - i0.09, & \text{for } B^0 \rightarrow \pi^0 \pi^0, \\ -0.28 - i0.26, & \text{for } \bar{B}^0 \rightarrow \pi^0 \pi^0, \end{cases} \quad (24)$$

$$R_{\text{NLO}} = \begin{cases} 0.38, & \text{for } B^0 \rightarrow \pi^0 \pi^0, \\ 3.85, & \text{for } \bar{B}^0 \rightarrow \pi^0 \pi^0, \end{cases} \quad (25)$$

due to the very different interference patterns between  $\mathcal{M}_{\text{LO}}$  and  $\Delta\mathcal{M}_{\text{NLO}}$  for these two decay modes. The total enhancement to the CP-averaged decay rate  $Br(B^0/\bar{B}^0 \rightarrow \pi^0 \pi^0)$  is around 100%.

In short, we made a systematic study for the  $B \rightarrow \pi\pi$  decays in the pQCD factorization approach with the inclusion of all currently known NLO contributions to the considered decays. We find the following points:

- (i) For  $B^0 \rightarrow \pi^+ \pi^-$  and  $\pi^+ \pi^0$  decays, the NLO pQCD predictions for their CP-averaged branching ratios agree well with the measured values within one standard deviation.
- (ii) For the CP-averaged branching ratio  $Br(B^0/\bar{B}^0 \rightarrow \pi^0 \pi^0)$ , however, although the NLO contributions can provide  $\sim 100\%$  enhancement to the LO result, it is still much smaller than the measured one. The so-called “ $\pi\pi$ ” puzzle is still an open problem.
- (iii) We examined the relative strength for those LO and NLO contributions from different sources.

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