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Double-parton scattering contribution to production of jet pairs with large rapidity separation at the LHC

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For the first time in the literature, we discuss the production of a four-jet final state in proton-proton collisions at the LHC through the mechanism of double-parton scattering (DPS), in the context of jets with large rapidity separation. This is the region where searches for a Balitsky-Fadin-Kuraev-Lipatov (BFKL) signal are planned and/or being performed. The DPS contributions are calculated within the so-called factorized ansatz, and each step of DPS is calculated in the leading order (LO) collinear approximation. The LO pQCD calculations are shown to give a reasonably good description of recent CMS and ATLAS data on inclusive jet production; therefore, this formalism can be used to estimate the DPS effects. We demonstrate that the relative contribution (with respect to single parton scattering dijets and to the BFKL Mueller-Navelet jets) of DPS is growing at large rapidity distance between jets. This is consistent with our experience from previous studies of DPS effects in the case of open and hidden charm production. The calculated differential cross sections, as a function of rapidity distance between the jets that are the most remote in rapidity, are compared with recent results of leading logarithm and next-to-leading logarithm BFKL calculations for the Mueller-Navelet jet production at $\sqrt{s} = 7$ TeV. The DPS contribution to widely rapidity separated jet production is carefully studied for the present energy $\sqrt{s} = 7$ TeV, and also at the nominal LHC energy $\sqrt{s} = 14$ TeV and in different ranges of jet transverse momenta. The differential cross section as a function of dijet transverse momenta as well as two-dimensional $(p_T(y_{\rm min}) \times p_T(y_{\rm max}))$-plane correlations for DPS mechanism are also presented. Some ideas as to how the DPS effects could be studied in the case of four-jet production are suggested.

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I. INTRODUCTION

It is reasonable to expect that large-rapidity-distance jets are more decorrelated in azimuth than jets placed close in rapidity. About 25 years ago Mueller and Navelet predicted strong decorrelation in relative azimuthal angle [1] of such jets, due to the exchange of the BFKL ladder between quarks (partons). The generic picture is presented in diagram (a) of Fig. 1. In a picture that is a bit simplified, quarks, antiquarks, and gluons are emitted forward and backward, whereas gluons emitted along the ladder populate rapidity regions in between the most forward and backward jets. Because of diffusion along the exchange ladder, the correlation between the most forward and the most backward jets is small. This was a simple picture obtained within leading-logarithmic BFKL formalism [1–6]. In Ref. [7], so-called consistency constrain was also imposed. Recent higher-order BFKL calculations slightly modified this simple picture [8–17], leading to smaller azimuthal decorrelation in rapidity. Recently the next-to-leading logarithm (NLL) corrections were calculated both to the Green's function and to the jet vertices. The effect of the NLL correction is large and leads to significant lowering of the cross section. So far only averaged values of $\langle \cos(n\phi_{jj}) \rangle$ over available phase space, or even their ratios, have been studied experimentally [18]. More detailed studies are necessary to verify this type of calculation. In particular, the approach should reproduce dependence on the rapidity distance between the jets emitted in opposite hemispheres, and more detailed two-dimensional dependences on transverse momenta of the both jets. Large-rapidity-distance jets can be produced only at high energies, where the rapidity span is large due to kinematics. A first experimental trial of a search for the Mueller-Navelet (MN) jets was made by the D0 Collaboration [19]. In their study rapidity distance between jets was limited to 5.5 units only. Nonetheless, they have observed a broadening of the ϕ_{jj} distribution with growing rapidity distance between jets. However, theoretical interpretation of the broadening is not clear. The dijet azimuthal correlations were also studied in collinear next-to-leading-order approximation [20]. The LHC opens a new possibility to study the decorrelation effect quantitatively, at distances in rapidity

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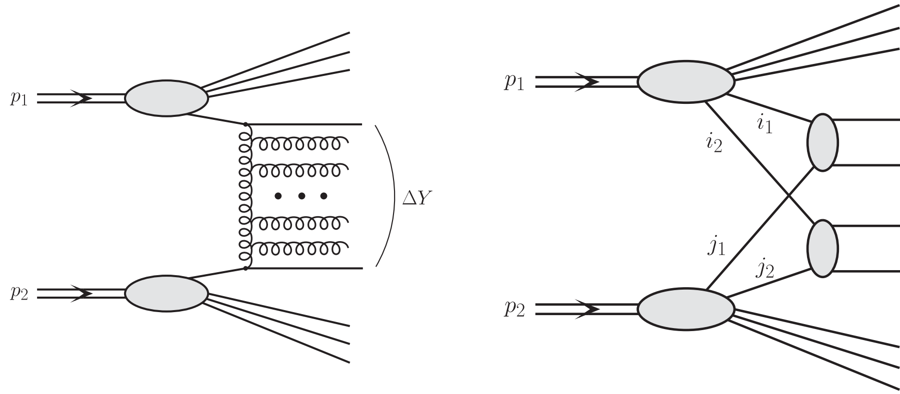


FIG. 1. A diagrammatic representation of the Mueller-Navelet jet production (left diagram) and of the double-parton scattering mechanism (right diagram).

as yet unavailable experimentally. The first experimental data measured at $\sqrt{s} = 7$ TeV are expected soon [21].

On the other hand, recent studies of multiparton interactions have shown that they may easily produce particles (objects) which are emitted at distances remote in rapidity. Good examples are the production of $c\bar{c}c\bar{c}$ [22–24] and the inclusive production of two J/ψ mesons [25,26]. Here we wish to concentrate on four-jet double-parton scattering (DPS) production with large distances between jets [see diagram (b) in Fig. 1]. Several suggestions about how to separate the four-jet DPS contribution from the single-parton scattering (SPS) contribution at midrapidities were discussed in Ref. [27].

In the present first exploratory study we shall report the first estimate of the DPS effects for jets with large rapidity separation within leading-order collinear approximation. This approximation will allow us to nicely illustrate the generic situation. We shall focus on the distribution in rapidity distance of the jets that are the most distant in rapidity. The DPS result will be compared to the distribution in rapidity distance for standard $2 \to 2$ SPS pQCD dijet as well as for the BFKL Mueller-Navelet dijet calculations. We shall identify the dominant partonic subprocesses important to understanding the situation in the small, but interesting, corner of the phase space of large positive and negative rapidities of jets. The calculation of distributions in rapidity distance will be supplemented by the analysis of correlations in the two-dimensional space of the transverse momenta of the two widely separated jets, or by calculation of distributions in transverse momentum imbalance of the jets or correlations in azimuthal angle between them.

II. BASIC FORMALISM

In the present calculation all partonic cross sections ($ij \to kl$) are calculated in leading-order only. The cross section for dijet production can be then written as

$$\frac{d\sigma(ij \to kl)}{dy_1 dy_2 d^2 p_t} = \frac{1}{16\pi^2 \hat{s}^2} \sum_{i,j} x_1 f_i(x_1, \mu^2) x_2 f_j(x_2, \mu^2) \overline{|\mathcal{M}_{ij\to kl}|^2}, \tag{2.1}$$

where y_1, y_2 are rapidities of the two jets (k and l) and p_t is the transverse momentum of one of them (they are identical). The parton distributions are evaluated at $x_1 = \frac{p_t}{\sqrt{s}}(\exp(y_1) + \exp(y_2))$, $x_2 = \frac{p_t}{\sqrt{s}}(\exp(-y_1) + \exp(-y_2))$, and $\mu^2 = p_t^2$ is used as the factorization and renormalization scale.

In our calculations we include all leading-order $ij \to kl$ partonic subprocesses (see, e.g., [28,29]). The K factor for dijet production is rather small, of the order of 1.1–1.3 (see, e.g., [30,31]), but can be easily incorporated in our calculations. Below we shall show that the leading-order approach gives results in sufficiently reasonable agreement with recent ATLAS [32] and CMS [33] data.

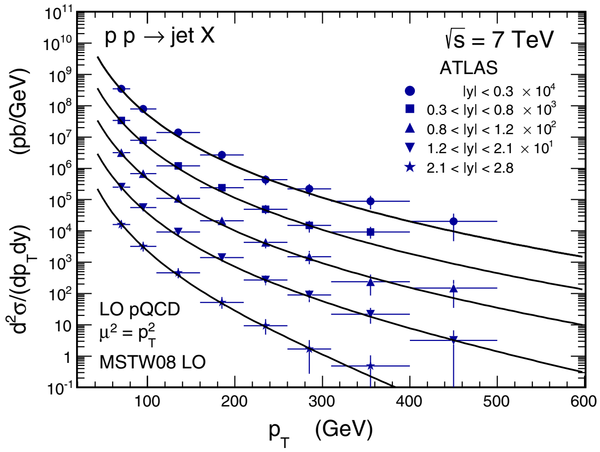
This simplified leading-order approach can, however, be used conveniently in our first estimate of DPS differential cross sections for jets widely separated in rapidity. In analogy to the production of $c\bar{c}c\bar{c}$ (see, e.g., [22] for our notation) one can write

$$\frac{d\sigma^{\rm DPS}(pp \to 4{\rm jets}X)}{dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2 p_{2t}} = \sum_{\substack{i_1, j_1, k_1, l_1 \\ i_2, j_2, k_2, l_2}} \frac{\mathcal{C}}{\sigma_{\rm eff}} \frac{d\sigma(i_1 j_1 \to k_1 l_1)}{dy_1 dy_2 d^2 p_{1t}} \frac{d\sigma(i_2 j_2 \to k_2 l_2)}{dy_3 dy_4 d^2 p_{2t}}, \tag{2.2}$$

where

$$\mathcal{C} = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } i_1 j_1 = i_2 j_2 \wedge k_1 l_1 = k_2 l_2 \\ 1 & \text{if } i_1 j_1 \neq i_2 j_2 \vee k_1 l_1 \neq k_2 l_2 \end{array} \right\}$$

and partons $i, j, k, l = g, u, d, s, \bar{u}, \bar{d}, \bar{s}$. The combinatorial factors include identity of the two subprocesses. Each step of the DPS is calculated in the leading-order approach [see Eq. (2.1)]. The quantity $\sigma_{\rm eff}$ has the dimension of the cross section and has a simple interpretation in the impact parameter representation [34]. Above y_1, y_2 and y_3, y_4



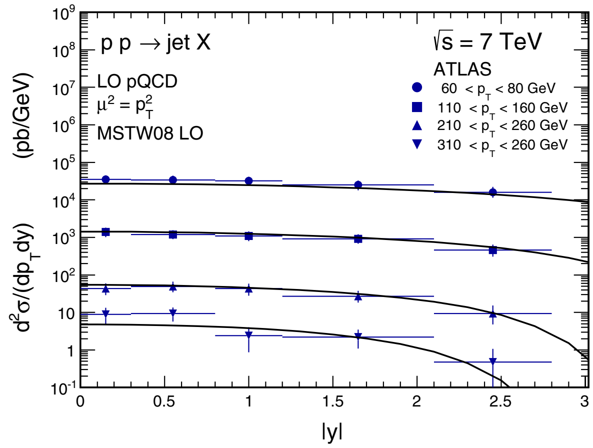


FIG. 2 (color online). Transverse momentum distribution of jets for different regions of the jet rapidity (left panel) and corresponding rapidity distribution of jets with different cuts in p_t, as specified in the right panel. The theoretical calculations were performed with the MSTW08 set of parton distributions [53]. The data points were obtained by the ATLAS Collaboration [32].

are rapidities of partons (jets) in “first” and “second” partonic subprocess, respectively. The p_{1t} and p_{2t} are respective transverse momenta.

Experimental data from the Tevatron [35] and the LHC [36–38] provide an estimate of σ_{eff} in the denominator of formula (2.2). As in our recent paper [24] we take $\sigma_{\mathrm{eff}} = 15$ mb, which is the world average value for different processes in a similar range of energies. A detailed analysis of the σ_{eff} parameter based on various experimental data can be found, e.g., in Refs. [39,40].

The physics of multiparton interactions is rather intricate. The theory of DPS is being developed quickly (see, e.g., [41,42]). In a more general case[1] the four-jet DPS cross section can be written somewhat schematically as

$$\begin{aligned}\sigma^{\mathrm{DPS}}_{pp\to 4\mathrm{jets}} \approx{}& \frac{\mathcal{C}}{\sigma_{\mathrm{eff},2v2}}\Sigma_{ijkl}\int dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t} D_{ik}(x_1,x_2,\mu_1^2,\mu_2^2) D_{jl}(x_1{}',x_2{}',\mu_1^2,\mu_2^2)\frac{d\sigma_{ij\to\mathrm{jet1jet2}}}{dy_1dy_2d^2p_{1t}}\frac{d\sigma_{kl\to\mathrm{jet3jet4}}}{dy_3dy_4d^2p_{2t}}\\ &+\frac{\mathcal{C}}{\sigma_{\mathrm{eff},2v1}}\Sigma_{ijkl}\int dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t}(\hat{D}_{ik}(x_1,x_2,\mu_1^2,\mu_2^2) D_{lj}(x_1{}',x_2{}',\mu_1^2,\mu_2^2)\\ &+D_{ik}(x_1,x_2,\mu_1^2,\mu_2^2)\hat{D}_{lj}(x_1{}',x_2{}',\mu_1^2,\mu_2^2))\frac{d\sigma_{ij\to\mathrm{jet1jet2}}}{dy_1dy_2d^2p_{1t}}\frac{d\sigma_{kl\to\mathrm{jet3jet4}}}{dy_3dy_3d^2p_{2t}} + 1v1\ \mathrm{term}. \end{aligned} \tag{2.3}$$

The C factor is combinatorial and was defined in the context of the factorized ansatz [see Eq. (2.2)]. The D_{ik}, D_{jl} functions represent the so-called double-parton distribution functions (DPDF). The conventional DPDFs describe nonperturbative correlations between two partons in the proton, and they undergo special QCD evolution (see, e.g., [45–47]). The $\hat{D}_{ik}$, $\hat{D}_{il}$ functions are more effective and correspond to perturbative parton splitting in the ladder (see, e.g., [48,49]). Although the leading logarithm formalism was given [48], until now they have not been calculated numerically. The effective parameters $\sigma_{\mathrm{eff},2v2}$ and $\sigma_{\mathrm{eff},2v1}$ do not need to be the same and are not well known. The last $1v1$ double-splitting (parton splitting on both sides) term corresponds rather to loop contributions of the $2\to 4$ category, and is rather not of the DPS character. The issue is not fully resolved at present.

This short summary of the present situation shows that no precise calculations are possible in the moment. The problem is being studied [49–52]. We think, therefore, that it is better at present to use the more phenomenological approach based on the simplified factorized ansatz [see Eq. (2.2)] with σ_{eff} extracted from empirical studies. We leave the very interesting problem of separation into different DPS categories for future separate studies. We also wish to keep the present analysis, the first in the literature in the context of the Mueller-Navelet jets, as simple as possible. We think that the simplified approach, which is reasonably well under phenomenological control, is better than the more theoretically advanced approach that is not fully understood in details, at least in the moment. Further studies are badly needed. One could, and should, return to the problem once there is a better quantitative

[1]The most general case should include parton spin correlations, color correlations, or even interference effects [42–44]. The inclusion of these effects seems to be rather complicated and goes beyond the scope of our present paper.

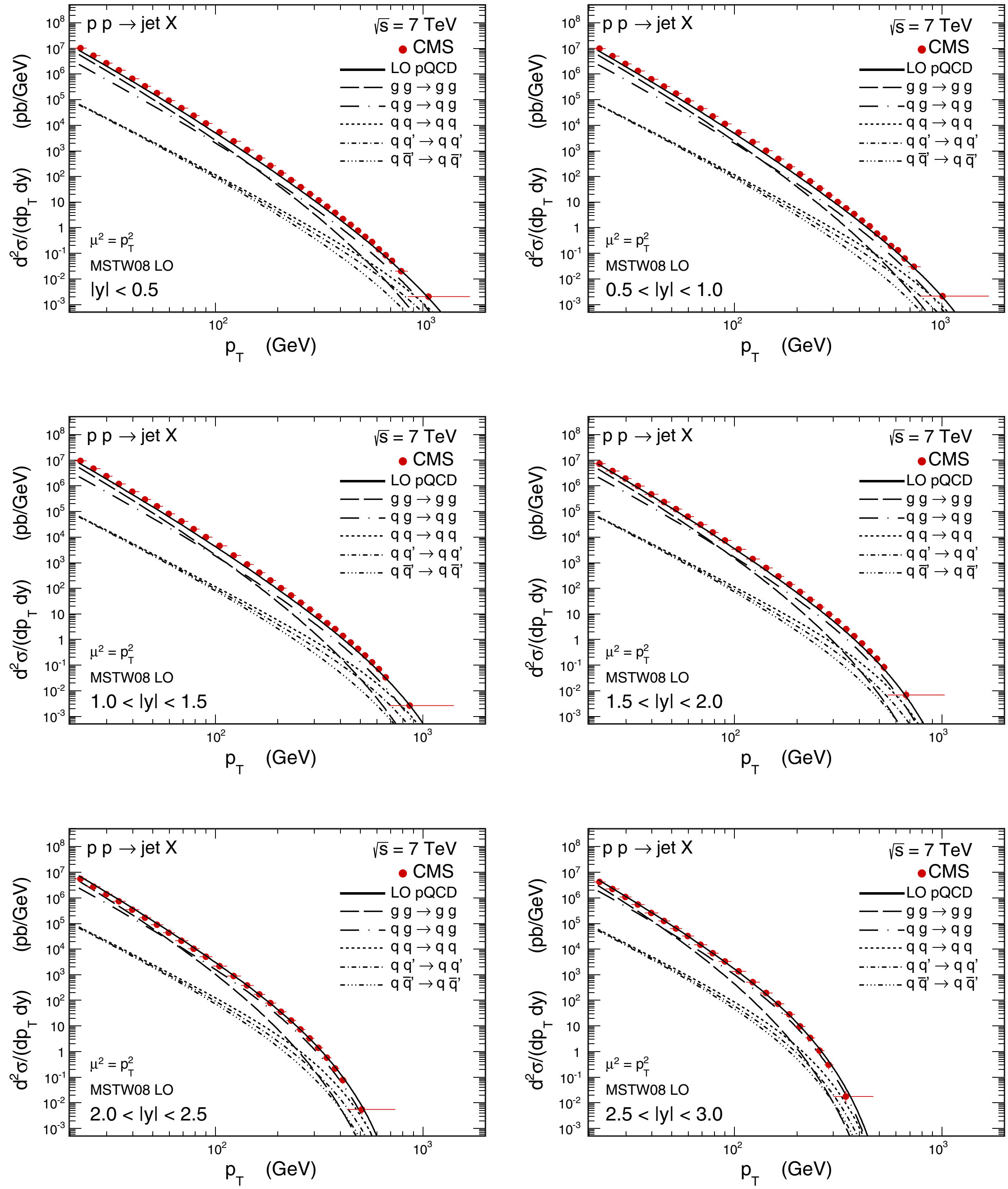


FIG. 3 (color online). The same as in the previous figure but now together with the CMS experimental data [33]. In addition, we show decomposition into different partonic components, as explained in the figure caption.

phenomenological understanding of the problem with a more advanced theoretical framework.

III. NUMERICAL RESULTS

Before we shall show our results for the DPS rapidity-distant-jet correlations, we wish to verify the quality of the description of observables for inclusive jet production. In Fig. 2 we show distributions in the jet transverse momentum for different intervals of jet (pseudo)rapidity (left panel) and distribution in jet (pseudo)rapidity for different intervals of jet transverse momentum (right panel). In this calculation, we have used the MSTW08 parton distribution functions [53]. The agreement with recent ATLAS data

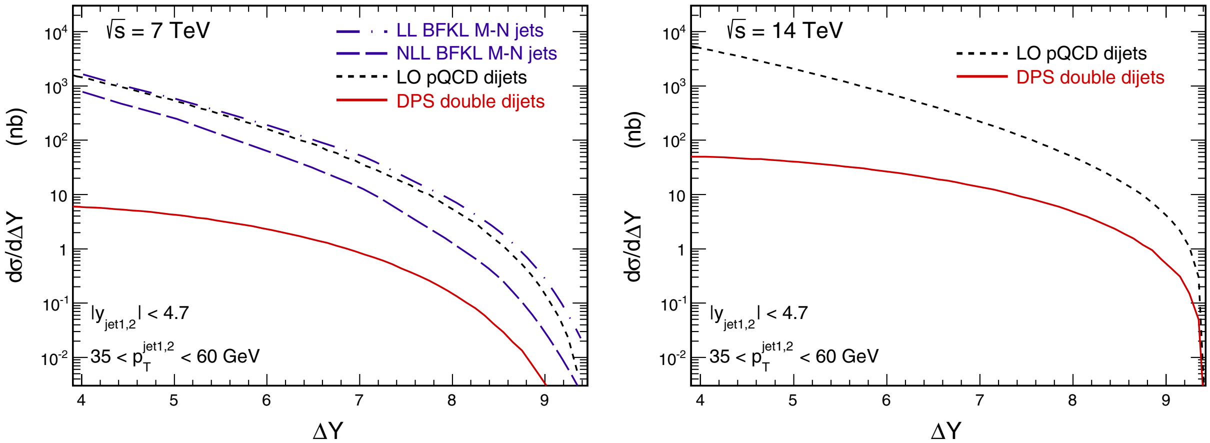


FIG. 4 (color online). Distribution in rapidity distance between jets (35 GeV $< p_t <$ 60 GeV) with maximal (the most positive) and minimal (the most negative) rapidities. The collinear pQCD result is shown by the short-dashed line and the DPS result by the solid line. The calculation has been performed for $\sqrt{s} = 7$ TeV (left panel) and $\sqrt{s} = 14$ TeV (right panel). For comparison we show also results for the classical BFKL Mueller-Navelet jets in leading-logarithm and next-to-leading-order-logarithm approaches from Ref. [15].

[32] is fairly reasonable, which allows us to use the same distributions for our first evaluation of the DPS effects for large rapidity distances between jets.

In Fig. 3 we compare our calculation with the CMS Collaboration data [33]. In addition, we show contributions of different partonic mechanisms. In all rapidity intervals the gluon-gluon and quark-gluon (gluon-quark) contributions clearly dominate over the other contributions, and in practice it is sufficient to include only these subprocesses in further analysis.

Now, having shown rather good description of the LHC experimental jet distributions, we shall proceed to the jets with large rapidity separation. In Fig. 4 we show the distribution in the rapidity distance between two jets in leading-order collinear calculation and between the jets that are the most distant in rapidity in the case of four DPS jets. In this calculation we have included cuts characteristic for the CMS experiment [21]: y_1, $y_2 \in (-4.7, 4.7)$, p_{1t}, $p_{2t} \in (35\text{ GeV}, 60\text{ GeV})$. For comparison we also show results for the LL and NLL BFKL calculation for MN jets from Ref. [15]. For this kinematics the DPS jets give a sizeable relative contribution to the two-jet case only at large rapidity distance. However, the four-jet (DPS) and dijet (LO SPS) final state can be easily distinguished; in principle, one can concentrate on the DPS contribution which is interesting by itself.[2] The NLL BFKL cross section (long-dashed line) is smaller than that for the LL BFKL cross section (long dashed-dotted line) as well as than the LO collinear approach (short-dashed line). At $\sqrt{s} = 7$ TeV our DPS contribution constitutes only about ~20% of the NLL BFKL at the rapidity distance $\Delta Y \approx 8$–9. However, at the higher energy $\sqrt{s} = 14$ TeV, we expect the DPS contribution to be almost identical to the NLL BFKL cross section (not shown in the figure due to lack of relevant, and rather involved, calculation) in the similar range of rapidity distance and, therefore, potentially important. In this context a measurement of absolutely normalized cross sections as a function of rapidity distance seems very important. Here (and in Fig. 5) we show the simple collinear SPS dijet contribution only for reference. This may be not the best estimate of the SPS contribution. The BFKL tries to do it better.

As for the BFKL Mueller-Navelet jets, the DPS contribution grows with decreasing jet transverse momenta. Therefore, let us now discuss results for even smaller transverse momenta. In Fig. 5 we show the rapidity-distance distribution for the even smaller lowest limit for transverse momentum of the jet. A measurement of such minijets may, however, be difficult. Now the DPS contribution may even exceed the standard SPS dijet contribution, especially at the nominal LHC energy. We have checked that lowering of the jet p_t upper limit does not improve the situation significantly. How to measure such (mini)jets is an open issue. In principle, one could measure, for instance, correlations of semihard ($p_t \sim 10$ GeV) neutral pions with the help of so-called zero-degree calorimeters, which are installed by all major LHC experiments. Other possibilities could be considered as well.

Now we wish to concentrate ourselves on correlations between transverse momenta of the rapidity-distant jets. In our case the large-rapidity-distance jets are coming from different partonic scatterings and are, therefore, quite uncorrelated. In Fig. 6 we present our results. The (p_{1t}, p_{2t}) distribution for the DPS mechanism is rather different than similar distributions for dijet SPS [54] and MN jets [4]. In our approximation the dijets from the SPS as well as the jets from the same partonic scattering in DPS are correlated along the $p_{1t} = p_{2t}$ diagonal [54] (see the straight diagonal

[2]The present Mueller-Navelet BFKL calculations concentrate on the jets that are the most distant in rapidity, and do not pay attention to what is going on in between them.

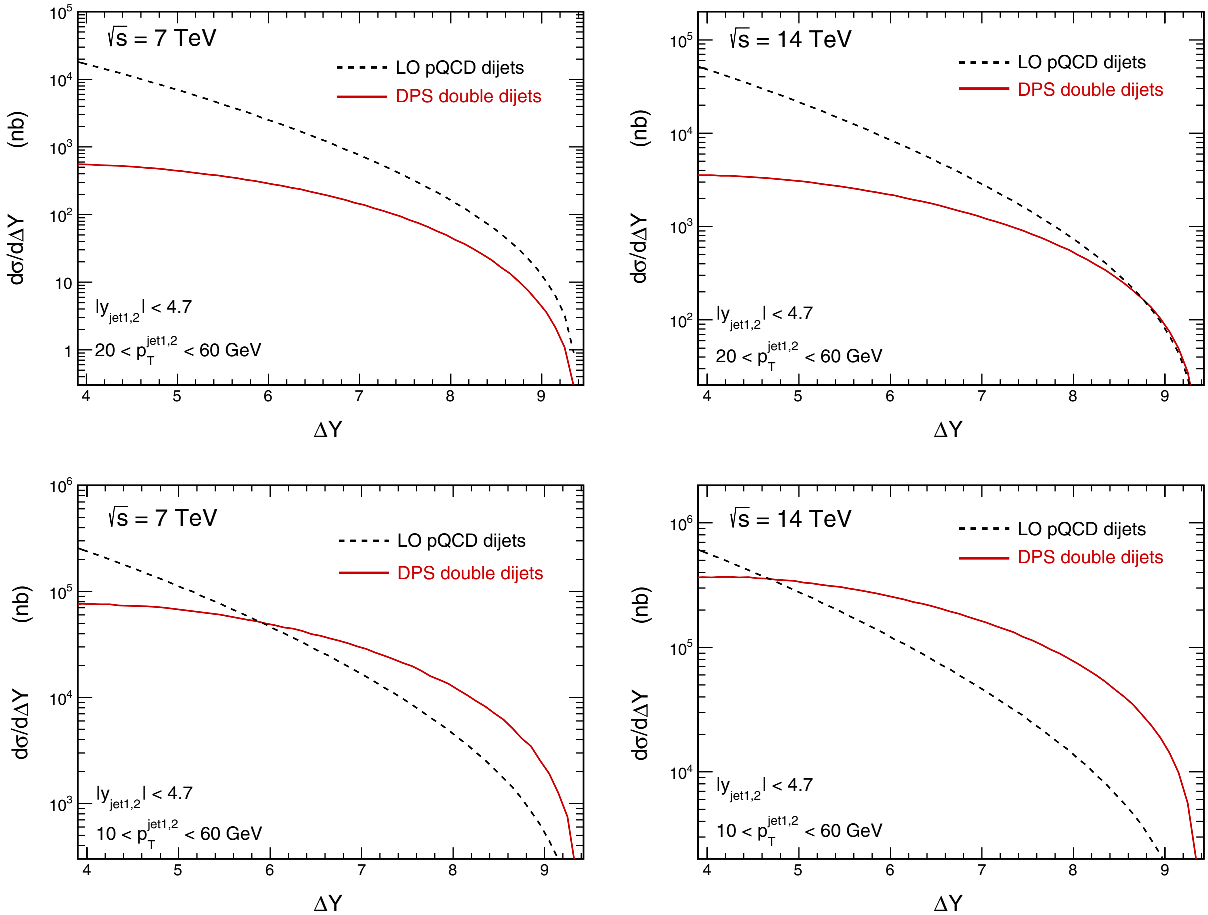


FIG. 5 (color online). The same as in the previous figure, but now for a somewhat smaller lower cut on minijet transverse momentum.

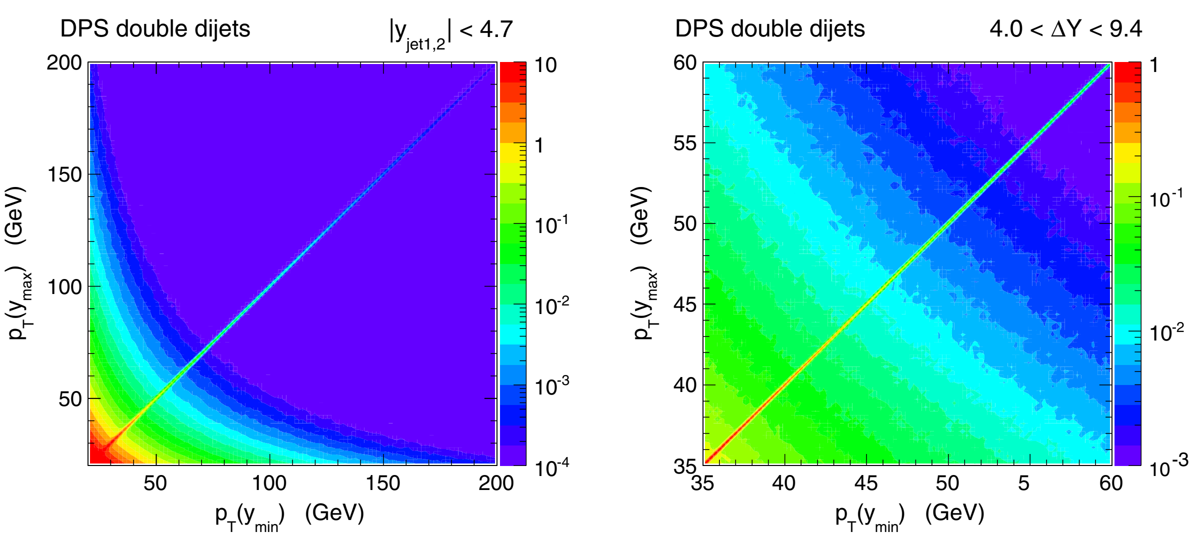


FIG. 6 (color online). The double-differential distribution in transverse momenta of jet with minimal $[p_T(y_{\rm min})]$ and maximal $[p_T(y_{\rm max})]$ rapidities. In the left panel we show the result for the full range of rapidities, and in the right panel we show the result only for large-rapidity separations of jets, as defined previously.

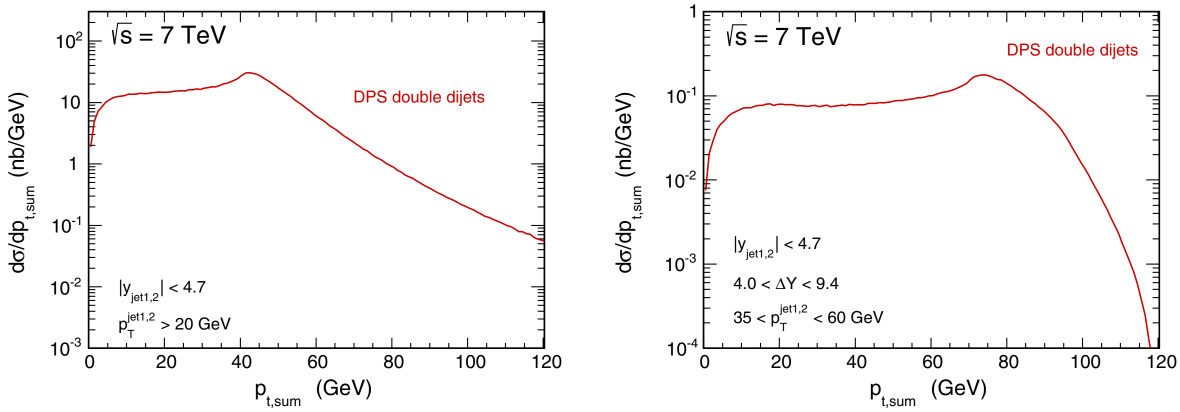


FIG. 7 (color online). Distribution in transverse momentum imbalance (vector sum of transverse momenta of jets) between the jets with "minimal" and "maximal" rapidities.

line in Fig. 6). In principle, one could eliminate this region by dedicated cuts. How the situation looks in the BFKL calculation can be already seen from a simple LL calculation [4]. The CMS Collaboration could make such two-dimensional studies. Another alternative are studies of distributions in the transverse momentum imbalance $\vec{p}_{t,\mathrm{sum}} = \vec{p}_{1t} + \vec{p}_{2t}$ between the rapidity-distant jets. In Fig. 7 we show distributions for the full range of rapidity distances (left panel) as well as for large-rapidity-distance jets (right panel). The first choice is rather conventional for standard dijet analyses and the second is typical for the Mueller-Navelet jet studies. The DPS mechanism generates situations with large transverse momentum imbalance. This could be used, in addition, to enhance the content of the DPS effects by taking a lower cut on the dijet imbalance. The transverse momentum imbalance for jets remote in rapidity is bigger than that for jets close in rapidity. The corresponding distribution for Mueller-Navelet jets has its maximum at very small $p_{t,\mathrm{sum}}$. It would be interesting to calculate, in the future, the transverse momentum imbalance distribution for SPS dijets as well as for the BFKL Mueller-Navelet jets. This clearly goes beyond the scope of this short paper.

Finally we wish to discuss azimuthal correlations between the jets distant in rapidity. The azimuthal angle distributions for the Mueller-Navelet jets were calculated by many groups, and we will not repeat such calculations here. The DPS jets are fully uncorrelated, at least in our approach. This is expected to be different for the SPS dijets [delta function $\delta(\phi - \pi)$ in the leading-order collinear approach] as well as for the classical Mueller-Navelet jets. The SPS dijet azimuthal correlations, as well as the transverse momentum imbalance distribution, could be easily calculated in the k_t-factorization approach [54–58]. In this approach one avoids singularities present in the fixed-order collinear approximation.

A contamination of the large-rapidity-distance jets by the DPS effects may distort the information on genuine Mueller-Navelet jets and make the comparison with the BFKL calculation not fully conclusive. A detailed analysis of this contamination will be a subject of our future studies.

IV. CONCLUSIONS AND DISCUSSION

In the present paper we have discussed for the first time how the double-parton scattering effects may contribute to large-rapidity-distance dijet correlations, and may potentially shadow the BFKL signal from the Mueller-Navelet jets. The present exploratory calculation has been performed intentionally in leading-order approximation to understand and explore the general situation. This means, also, that each step of DPS was calculated in collinear pQCD leading order. We have shown that leading-order calculation provides a quite adequate description of inclusive jet distributions when confronted with recent results obtained by the ATLAS and CMS Collaborations. We have identified the dominant partonic pQCD subprocesses relevant for the production of jets with large rapidity distance.

We have concentrated ourselves on distributions in rapidity distance between the jets that are the most distant in rapidity. The results of the dijet SPS mechanism have been compared to the DPS mechanism. We have performed calculations relevant for planned CMS experimental analysis. We have shown that the contribution of the DPS mechanism increases with increasing distance in rapidity between jets. This is analogous to similar observations made already for the production of $c\bar{c}c\bar{c}$ [22–24] and $J/\psi J/\psi$ mesons [25,26]. For comparison we have also shown some recent predictions for the Mueller-Navelet jets in the LL and NLL BFKL framework taken from the literature. For the CMS configuration our DPS contribution is smaller than the SPS dijet contribution even at high-rapidity distances, but is only slightly smaller than that for the NLL BFKL calculation known from the literature. The DPS final-state topology is clearly different than that for the SPS dijets (four versus two jets), which may help to disentangle the two mechanisms experimentally. Of course, SPS three- and four-jet final states should be included in more refined analyses of distributions in rapidity distance.

We have shown also that the relative effect of DPS could be increased by lowering the transverse momenta of jets, but such measurements can be difficult if not impossible. Alternatively one could study correlations of semihard pions distant in rapidity. Correlations of two neutral pions could be done, at least in principle, with the help of zero-degree calorimeters present at each of the main detectors at the LHC. This type of study requires further analyses, taking hadronization effects also into account.

The DPS effects are interesting not only in the context of how they contribute to distribution in rapidity distance and how they can distort signal of genuine Mueller-Navelet jets, but also *per se*. For such studies one would need to measure four jets: two distant in rapidity (probably outside of the CMS central detector) and two in between (probably with the help of the CMS central detector). One could make use of correlations in jet transverse momenta, jet imbalance, and azimuthal correlations to enhance or lower the contribution of DPS. We expect that in the case of DPS, the most remote jets would be almost decorrelated. Further detailed Monte Carlo studies are required to settle the real experimental program of such studies.

At present we have used the factorized approach justified from phenomenology. It seems rather simple in the context of a recent progress in understanding DPS on the theoretical side. The general framework, except for traditional DPS processes ($4 \to 4$), should include also perturbative parton splitting processes (at least $3 \to 4$), as discussed recently. Quantitative details of the more advanced fromework have to be worked out. Clearly more work in this direction, and also in the context of jets with large rapidity separation, is needed.

ACKNOWLEDGMENTS

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