Majorana neutrinos in $e\gamma$ colliders from an effective Lagrangian approach

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We study the production of heavy Majorana neutrinos (N) in $e^-\gamma$ colliders. We consider the tree-level process $e^-\gamma \to W^- N$, which allows for lepton number violation via the decay $N \to l^+ 2j$. We follow the approach of an effective theory, where we consider all possible gauge invariant and nonrenormalizable operators of lowest dimension, which determine the interactions of the Majorana neutrino with the standard particles. We study the total cross section for different masses of the Majorana neutrino and different center-of-mass energies. We give an estimate for the range of the Majorana neutrino masses which may be detected in a $e^-\gamma$ collider.

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I. INTRODUCTION

Remarkable progress has been made in understanding the nature of the neutrino sector in particle physics. The phenomenon of neutrino oscillations is particularly important since it gives experimental evidence for the nonvanishing mass of the standard neutrinos. Unlike the rest of the massive particles in the standard model, interactions of the Yukawa type are not well suited for the generation of masses of the order of 10^{-2} eV, which correspond to the neutrino masses. One possible scenario is the seesaw mechanism [1], which requires the existence of at least one type of heavy right-handed Majorana neutrinos N, with mass m_N, giving a small mass $m_\nu \sim 1/m_N$ to the neutrinos, which would explain neutrino oscillations. As indicated in [2], the parameters $U_{lN}(l = e, \mu, \tau)$ which determine the couplings of the interactions of the Majorana neutrino with the standard model particles, as in

$$\mathcal{L}^W_{V-A} = -\frac{g}{\sqrt{2}} U_{lN} \overline{N^c} \gamma^\mu P_L l W^+ + \text{H.c.} \tag{1}$$

where $P_L = (I - \gamma^5)/2$ is the chirality projection operator, turns out to be very small, even for Majorana masses as small as 100 GeV, in which case $U_{lN} \sim 10^{-7}$. This justifies an approach beyond the minimal seesaw mechanism in order to study new physics involving Majorana neutrinos.

In this paper we will study the process $e^-\gamma \to W^- N$ according to an effective model-independent Lagrangian approach. This process is interesting in this framework since it allows for nonconservation of lepton number. This is possible since the decay modes $N \to l^+ 2j$ are permitted for the final Majorana neutrino. The complete process $e^-\gamma \to W^- N \to W^- l^+ 2j$, where l^+ stands for an antilepton of any flavor and j for jets, is a clear signal of lepton number violation. A similar process has already been considered [3,4] although not in the context of an effective Lagrangian.

II. THE EFFECTIVE LAGRANGIAN

We parametrize the contributions of new physics effects by a set of five gauge-invariant nonrenormalizable operators of dimension six, $\mathcal{O}_i$. There exist operators of dimension five but they do not contribute to this particular scattering process. We also neglect higher dimension operators since their contributions are suppressed by the new physics scale Λ. The extended theory is thus represented by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{i=1}^{5} \frac{\alpha_i}{\Lambda^2} \mathcal{O}_i + \text{H.c.} \tag{2}$$

The first term in (2) contains the interactions in the standard model which are relevant to this process,

$$\begin{aligned}\mathcal{L}_{\text{SM}} = {} & -q_e \bar{e} \gamma^\mu e A_\mu + \frac{g}{\sqrt{2}} \bar{e}_L \gamma^\mu \nu_L W^-_\mu + \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W^+_\mu \\ & + i q_e (\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) W^{\mu -} A^\nu \\ & - i q_e (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) W^{\mu +} A^\nu \\ & + \frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu)(W^{\mu +} W^{\nu -} - W^{\mu -} W^{\nu +}). \end{aligned} \tag{3}$$

We have also introduced the dimensionless coupling constants α_i, one for each operator, with the list of operators being

$$\begin{aligned} &\mathcal{O}_1 = i(\phi^t \varepsilon D_\mu \phi)(\bar{N} \gamma^\mu l_R), \qquad \mathcal{O}_2 = (\bar{L} \sigma^{\mu\nu} N) \tilde{\phi} B_{\mu\nu}, \\ &\mathcal{O}_3 = (\bar{L} \sigma^{\mu\nu} \tau^i N) \tilde{\phi} W^i_{\mu\nu}, \qquad \mathcal{O}_4 = (\bar{L} D_\mu N) D^\mu \tilde{\phi}, \\ &\mathcal{O}_5 = (\overline{D_\mu L} N) D^\mu \tilde{\phi}. \end{aligned} \tag{4}$$

We need to mention that the operators $\mathcal{O}_i (i = 2, \ldots, 5)$ are naturally suppressed with respect to the operator $\mathcal{O}_1$ by a factor of $1/16\pi^2$, the reason being that these four operators are generated to one loop. This suppression factor must be reflected in the values of the coupling constants. The lepton l in $\mathcal{O}_1$ correspond to the right-handed SU(2) singlets e, μ and τ. In Eqs. (4) we recognize the various fields, $\phi = \binom{\varphi^+}{\varphi^0}$ is the

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electroweak isoscalar, $L = \binom{\nu_l}{l}_L$ is the left-handed doublet, e_R the right-handed singlet and $\tilde{\phi} = \varepsilon\phi^*$ with $\varepsilon = i\sigma_2$. We also have for the gauge fields and covariant derivative

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$
$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon_{ijk} W^j_\mu W^k_\nu,$$
$$D_\mu = \partial_\mu - igT^i W^i_\mu - ig' \frac{Y}{2} B_\mu,$$

where W^i_μ and B_μ are the gauge fields of the $SU(2)_L$ and $U(1)_Y$ groups, respectively, and g and g' the standard coupling constants of the respective groups.

After symmetry breaking around the standard vacuum $\phi_0 = \binom{0}{v}$, we can rewrite the Lagrangian (2) to obtain

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{i=1}^{7} \mathcal{L}_i + \text{noncontributing terms}, \tag{5}$$

where

$$\mathcal{L}_1 = \frac{\alpha_1 g v^2}{\sqrt{2}\Lambda^2}(\bar{l}_R \gamma^\mu N W^-_\mu - \bar{N}\gamma^\mu l_R W^+_\mu),$$
$$\mathcal{L}_2 = \frac{(2\alpha_2 v c_w + \alpha_3 v s_w)}{\Lambda^2}(\bar{\nu}\sigma^{\mu\nu} N + \bar{N}\sigma^{\mu\nu}\nu)\partial_\mu A_\nu,$$
$$\mathcal{L}_3 = \frac{\alpha_3 \sqrt{2} v}{\Lambda^2}(\bar{l}_L \sigma^{\mu\nu} N \partial_\mu W^-_\nu + \bar{N}\sigma^{\mu\nu} l_L \partial_\mu W^+_\nu),$$
$$\mathcal{L}_4 = \frac{\alpha_3 \sqrt{2} i g v s_w}{\Lambda^2}(\bar{l}_L \sigma^{\mu\nu} N W^-_\nu - \bar{N}\sigma^{\mu\nu} l_L W^+_\nu) A_\mu,$$
$$\mathcal{L}_5 = \frac{-\alpha_4 i g v}{\sqrt{2}\Lambda^2}(\bar{l}_L \partial^\mu N W^-_\mu - \partial^\mu \bar{N} l_L W^+_\mu),$$
$$\mathcal{L}_6 = \frac{-\alpha_5 i g v}{\sqrt{2}\Lambda^2}(\partial^\mu \bar{l}_L N W^-_\mu - \bar{N}\partial^\mu l_L W^+_\mu),$$
$$\mathcal{L}_7 = \frac{-\alpha_5 e g v}{\sqrt{2}\Lambda^2}(\bar{l}_L N W^-_\mu + \bar{N} l_L W^+_\mu) A^\mu. \tag{6}$$

There are many terms which do not contribute to this process, although the seven operators which are relevant contain the five coupling constants $\alpha_i, i = 1, \ldots, 5$.

III. THE $e^-\gamma$ COLLIDER

The basic idea behind an $e^-\gamma$ collider is the use of laser backscattering, which consists of pointing a laser to the parent e^+ beam of an e^-e^+ collider. The design and principal characteristics of the collider are discussed in the literature [5]. The energy spectrum and helicity distribution for the back-scattered photons depend on the polarization of the electron and laser photon beams. The spectrum is picked in the high energy region for the case of opposite polarizations of the electron and laser beams. Even in the case of unpolarized electrons and laser photons, the scattered photons possess a favorable distribution at high energies. In this work we will only consider the unpolarized case.

The energy spectrum of the scattered high energy photons is defined by the Compton cross section,

$$f(x) = \frac{1}{\sigma_c}\frac{d\sigma_c}{dx} = N(\xi)\left[\frac{1}{1-x} + 1 - x - 4r(1-r)\right]. \tag{7}$$

The various symbols in (7) are defined by

$$x = \frac{\omega}{E_0} \le \frac{\xi}{\xi+1}, \qquad r = \frac{x}{\xi(1-x)},$$
$$N(\xi)^{-1} = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2}\right)\ln(\xi+1) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(\xi+1)^2}. \tag{8}$$

The variable x corresponds to the energy fraction carried by the scattered photon from the electron. E_0 and ω are the energies of the parent beam particles and the scattered photons respectively. The laser energy ω is chosen such that $\xi = 4\omega E_0/m_e^2$ has the optimum value of 4.8 in order to avoid the creation of e^+e^- pairs in the collision of the back-scattered photons with laser photons. The above equations may be found in [5], where a detailed treatment of the subject is presented.

IV. THE $e^-\gamma \to W^-N$ PROCESS AND THE RESULTS

So far we have presented the theoretical background in terms of an effective theory. We will make use of the Lagrangian (5) to find the scattering amplitude for the process. There are four Feynman diagrams, which are shown in Fig. 1. The scattering amplitude for this process is

$$\mathcal{M} = \sum_{ij} \mathcal{M}_i(\mathcal{L}_j), \tag{9}$$

where $\mathcal{M}_i(\mathcal{L}_j)$ is the contribution to the diagram i from $\mathcal{L}_j$. The various terms in (9) are

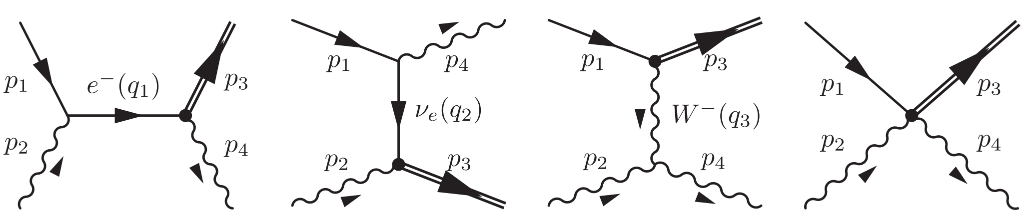

FIG. 1. Feynman diagrams for the process $e^-\gamma \to NW^-$.

$$\mathcal{M}_1(\mathcal{L}_1) = c_{11}\bar{u}_N\gamma^\mu\Delta_e(q_1)\gamma^\nu u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{11} = i\frac{\alpha_1 egv^2}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_1(\mathcal{L}_3) = c_{13}(ip_{4\lambda})\bar{u}_N\sigma^{\lambda\mu}\Delta_e(q_1)\gamma^\nu u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{13} = -i\frac{\sqrt{2}\alpha_3 ev}{\Lambda^2}$$

$$\mathcal{M}_1(\mathcal{L}_5) = c_{15}(ip_3^\mu)\bar{u}_N\Delta_e(q_1)\gamma^\nu u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{15} = \frac{\alpha_4 egv}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_1(\mathcal{L}_6) = c_{16}(-ip_1^\mu)\bar{u}_N\Delta_e(q_1)\gamma^\nu u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{16} = \frac{\alpha_5 egv}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_2(\mathcal{L}_2) = c_{22}(-ip_{2\lambda})\bar{u}_N\sigma^{\lambda\nu}\Delta_{\nu_e}(q_2)\gamma^\mu u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{22} = i\frac{2\alpha_2 gvc_w + \alpha_3 gvs_w}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_3(\mathcal{L}_1) = c_{31}a^{\nu\mu\rho}\bar{u}_N\gamma^\lambda u_e\Delta_W(q_3)_{\lambda\rho}\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{31} = -i\frac{\alpha_1 egv^2}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_3(\mathcal{L}_3) = c_{33}(iq_{3\eta})a^{\nu\mu\rho}\bar{u}_N\sigma^{\eta\lambda}u_e\Delta_W(q_3)_{\lambda\rho}\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{33} = i\frac{\sqrt{2}\alpha_3 ev}{\Lambda^2}$$

$$\mathcal{M}_3(\mathcal{L}_5) = c_{35}(ip_4^\lambda)a^{\nu\mu\rho}\bar{u}_N u_e\Delta_W(q_3)_{\lambda\rho}\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{35} = -\frac{\alpha_4 egv}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_3(\mathcal{L}_6) = c_{36}(-ip_1^\lambda)a^{\nu\mu\rho}\bar{u}_N u_e\Delta_W(q_3)_{\lambda\rho}\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{36} = -\frac{\alpha_5 egv}{\sqrt{2}\Lambda^2}$$

$$\mathcal{M}_4(\mathcal{L}_4) = c_{44}\bar{u}_N\sigma^{\nu\mu}u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\nu}, \qquad c_{44} = -i\frac{\sqrt{2}\alpha_3 gvs_w}{\Lambda^2}$$

$$\mathcal{M}_4(\mathcal{L}_7) = c_{47}\bar{u}_N u_e\mathcal{E}^*_{W\mu}\mathcal{E}_{\gamma\mu}, \qquad c_{47} = -\frac{\alpha_5 egv}{\sqrt{2}\Lambda^2}$$

with

$$a^{\nu\mu\rho} = (q_3 - p_2)^\mu g^{\nu\rho} + (p_2 + p_4)^\rho g^{\mu\nu} - (p_4 + q_3)^\nu g^{\mu\rho}.$$

We make use of the helicity formalism [6,7] to evaluate the scattering amplitude $\mathcal{M}$. This is a convenient method since it produces relatively short expressions and will permit us study polarizations of initial and final states in a very direct way in more detailed future treatments. The initial and final particles are represented in the following expressions:

$$u_e = \frac{s(p_{11}, p_{12})}{m_e}u(p_{11}, +1) + u(p_{12}, -1), \qquad p_{11} + p_{12} = p_1 \tag{10}$$

$$\bar{u}_N = \frac{t(p_{32}, p_{31})}{m_N}\bar{u}(p_{31}, +1), \qquad p_{31} + p_{32} = p_3 \tag{11}$$

$$\mathcal{E}^\mu_\gamma(\lambda_2) = \frac{1}{\sqrt{4p_2.k_2}}\bar{u}(k_2, \lambda_2)\gamma^\mu u(p_2, \lambda_2), \qquad k_2 = (1, 0, 1, 0) \tag{12}$$

$$\mathcal{E}^\mu_W = \sqrt{\frac{3}{8\pi m_w}}\bar{u}(p_{41}, -1)\gamma^\mu u(p_{42}, -1), \qquad p_{41} + p_{42} = p_4. \tag{13}$$

Note that Eq. (11) makes explicit the fact that the Majorana neutrino is right handed. The requirement on k_2 in Eq. (12) is not to be collinear with the momentum of the photon, thus $k_2 = (1, 0, 1, 0)$ is a good auxiliary vector. We should mention that only the state of the photon is explicitly dependent on the polarization. Equations (10), (11), and (13) correspond to only one polarization state along some four vector (one for each case) which is not necessary to be specified, the reason being that the sum over polarizations is equivalently replaced by three integrations over the solid angles Ω_1, Ω_3 and Ω_4, corresponding to the auxiliary pair of vectors (p_{i1}, p_{i2}), $i = 1, 3, 4$.

We plug this expressions into (9) in order to numerically evaluate the amplitude. The total cross section is numerically calculated from the expression

$$\sigma = \int_{x_{\min}}^{x_{\max}} \mathrm{d}x f(x)\int_{\mathrm{ps}} \mathrm{d}\sigma(x), \tag{14}$$

where "ps" stands for phase space. The limits in the integral over the energy distribution of the photons are

$$x_{\min} = \frac{(m_N + m_W)^2 - m_e^2}{2E^2 + 2E\sqrt{E^2 - m_e^2}} \quad \text{and} \quad x_{\max} = 0.8277. \tag{15}$$

The lower limit above is calculated by requiring that the minimum initial energy is the energy necessary to produce a Majorana neutrino and a W boson at rest, and the upper limit $x_{\max} = 4.8/(4.8+1)$ is an optimal value, as mentioned in Sec. III. The differential cross section is written as

$$d\sigma(x) = \frac{\overline{|\mathcal{M}(x)|^2}}{2xE_{\mathrm{cm}}^2} d\Pi_2, \qquad d\Pi_2 = \frac{|\vec{p}_3|_{\mathrm{cm}}}{8\pi E_{\mathrm{cm}}} dz, \qquad (16)$$

where $z = \cos(\theta)$ and θ is the angle of the scattered Majorana neutrino with respect to the beam axis in the center of mass frame. The expression for the squared amplitude is then

$$\overline{|\mathcal{M}(x)|^2} = \frac{1}{4}\sum_{\lambda_2} \frac{1}{2\pi}\int d\Omega_1 \frac{1}{2\pi}\int d\Omega_3 \times \int d\Omega_4 |\mathcal{M}(x,\lambda_2,\Omega_1,\Omega_3,\Omega_4)|^2. \qquad (17)$$

Since we are interested in violation of lepton number, we consider the process $e^-\gamma \to W^- l^+ 2j$, where j stands for jets and l for any lepton flavor. This is possible since the decay $N \to l^+ 2j$ is allowed. This Majorana neutrino decay was studied in [8], where all possible effective operators of dimension six involving quarks were considered. The branching was found to depend on the Majorana neutrino mass, varying between 0.2 and 0.3 for the masses considered here. Thus we calculate the cross section as

$$\sigma(e^-\gamma \to W^- l^+ 2j) = \sigma(e^-\gamma \to W^- N) \times \mathrm{Br}(N \to l^+ 2j). \qquad (18)$$

In order to have a quantitative estimate for the possible detection of Majorana neutrinos, we assume a representative one-year integrated luminosity of 100 fb^{-1}. We set the new physics scale $\Lambda = 1$ TeV. As for the coupling constants α_i, we assume them to be of order one unless the operators are generated to one loop. Thus we initially set $\alpha_1 = 1$ and $\alpha_i = 1/16\pi^2$ for $i = 2,3,4,5$. We also take into account the constraints due to double-β decay. We argue that the operator $\mathcal{O}_1$ gives the dominant contribution to the double β-decay process, coming from $\mathcal{L}_1$ in (6). Apart from $\mathcal{L}_2$, the other operators in (6) give negligible contributions to the double β-decay process for the following two reasons: (1) $\mathcal{L}_3$, $\mathcal{L}_5$, and $\mathcal{L}_6$ contribute with low momentum factors due to the derivatives of the fields, and (2) $\mathcal{L}_4$ and $\mathcal{L}_7$ generate a final photon with small phase space of order $(E_\gamma^{\max}/v)^2$, with $v = 246$ GeV and $E_\gamma^{\max}$ the maximum available energy for the final photon, which is of order ~MeV. Thus, the double β-decay process puts limits only on α_1.

In [8] an upper limit was found for the coupling α_1,

$$\alpha_1 \leq 7.8 \times 10^{-2} \sqrt{\frac{m_N}{100\ \mathrm{GeV}}}\ \mathrm{GeV}^{-2}, \qquad (19)$$

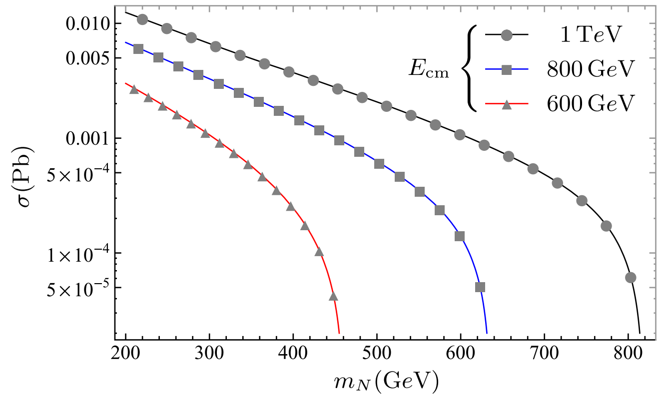

FIG. 2 (color online). Cross section for $e^-\gamma \to NW^-$.

which we take for the value of α_1 This corresponds to a scale $\Lambda = 1$ TeV. See also [9–12].

In Fig. 2, we show the cross section for the process $e^-\gamma \to W^- N$ as a function of the Majorana neutrino mass. Three center-of-mass energies for the e^+e^- collider are considered: 600 GeV, 800 GeV, and 1 TeV.

As for the background, we consider the process $e^-\gamma \to W^- W^- W^+ \nu_e$. This process gives the same detected final state as our lepton-violating process, since one of the W^- may decay into two jets and the W^+ may decay into an anti lepton l^+ and a neutrino ν_l. The final standard neutrinos avoid detectors. Thus the cross section may be written as

$$\sigma(e^-\gamma \to W^- l^+ 2j \nu_e \nu_l) = \sigma(e^-\gamma \to W^- W^- W^+ \nu_e) \times \mathrm{Br}(W^- \to 2j) \times \mathrm{Br}(W^+ \to l^+ \nu_l). \qquad (20)$$

We have used the CompHEP package [13] to evaluate the background cross section, obtaining 1.2×10^{-3} pb for $E_{\mathrm{cm}} = 600$ GeV, 4.8×10^{-3} pb for $E_{\mathrm{cm}} = 800$ GeV, and 1.1×10^{-2} pb for $E_{\mathrm{cm}} = 1$ TeV. According to (20) and the representative luminosity, and also taking $\mathrm{Br}(W^- \to 2j) \simeq 0.68$ and $\mathrm{Br}(W^+ \to l^+ \nu_l) \simeq 0.1$ from [14], we obtain the

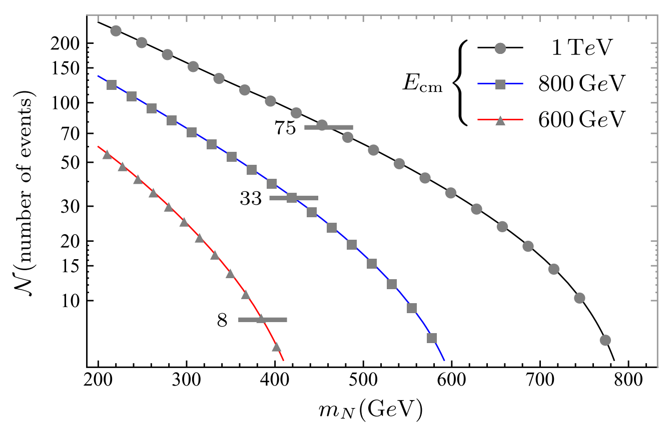

FIG. 3 (color online). Number of events $\mathcal{N}$ for $e^-\gamma \to W^- l^+ 2j$.

number of events $\mathcal{N}_{\rm SM} = 8$, $\mathcal{N}_{\rm SM} = 33$, and $\mathcal{N}_{\rm SM} = 75$ for the three values of the center-of-mass energies, respectively.

The main result of this study is shown in Fig. 3. Here we compare the number of events $\mathcal{N}$ generated by the process $e^-\gamma \to W^- l^+ 2j$, to the number of events $\mathcal{N}_{\rm SM}$ predicted by the standard model, due to the process $e^-\gamma \to W^- l^+ 2j\nu_e\nu_l$. We note that the number of standard events coincides with the number of the nonstandard events at the Majorana neutrino masses of 390, 420, and 460 GeV, corresponding to the center-of-mass energies of 600 GeV, 800 GeV, and 1 TeV, respectively. This indicates that neutrino masses up to approximately these values may be detected, and for masses beyond these values there would not be sufficient events to clearly identify new physics. It may be argued that, since we have set the new physics scale Λ equal to 1 TeV, we could not rely on the $E_{\rm cm} = 1$ TeV calculated events, and also probably the $E_{\rm cm} = 800$ GeV plot, which is perhaps too close to the scale. These near-the-scale values are justified by the arbitrariness of the chosen values for the coupling constants $\alpha_i (i = 1, \ldots, 5)$. We have taken α_1 equal to the upper limit imposed by double-β decay limits, and $\alpha_i = 1/16\pi^2 (i = 2, \ldots, 5)$ for the other four operators, which are generated to one loop. In this sense we are just setting an order of magnitude for the coupling constants as well as the scale. We should also mention that, because of the same reason, the values for the limit detectable masses of the Majorana neutrino are of course not exact, but give us an estimate for these detectable masses.

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