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Scalar-tensor theory of gravitation and spontaneous breakdown of scale invariance

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A version of Brans-Dicke theory of massive scalar and massless tensor fields is given. A connection to a spontaneously broken scale invariance is shown.

Recently O'Hanlon,[1] Acharya and Hogan[2] have shown that a generally covariant theory of gravitation can accommodate a *massive* scalar field in addition to the massless tensor field.[3] This seems to shed a new light on the scalar-tensor theory of gravitation[4] by showing a close connection with another intriguing hypothesis in the theory of quantized fields: the spontaneous breaking of scale invariance.[5]

We may first consider the simplest model in which the Brans-Dicke-type scalar field[4] plays, at the same time, the role of the field of a dilaton—a Nambu-Goldstone boson of scale invariance.[6] We find that this model is satisfactory in the sense that the weak-field approximation to the scalar field in the manner of Brans and Dicke corresponds to introducing a "shifted field" in the manner of Goldstone.[7] The vacuum expectation value of the scalar field gives the gravitational constant, on one hand, and finite masses of matter particles, on the other hand. Perhaps the only drawback of this economical model is that, as will be shown later, one can hardly conceive any experiments which test the theory.

In order to have testable predictions we then consider the second simplest model in which in addition to the Brans-Dicke field another scalar field is introduced. We find that the two fields get mixed with each other and result in a massless field and a massive field.[8] It turns out that the former field is decoupled from the static matter, while the latter manifests itself in the non-Newtonian part of the static gravitational potential, as suggested previously in a different approach.[9] The present model provides us with a better understanding of why the force range of this unusual part is expected to be most likely a macroscopic distance roughly of the order of $(Gm_N{}^4)^{-1/2} \sim 10^5$ cm, where G is the Newtonian gravitational constant while m_N is the nucleon mass ($c = \hbar = 1$ throughout). We maintain general covariance. The tests of general relativity remain unaffected.

We assume the Lagrangian density

$$\mathcal{L} = (-g)^{1/2}\left(\tfrac{1}{2}f^{-2}\phi^2 R - \tfrac{1}{2}\phi_{,\mu}\phi^{,\mu} - \tfrac{1}{2}\psi_{,\mu}\psi^{,\mu} - \tfrac{1}{2}\Phi_{,\mu}\Phi^{,\mu} + L_1 + L_2\right). \qquad (1a)$$

Here the scalar field ϕ has been chosen to have a "normal" dimension, i.e., the dimension of mass. Obviously the dimensionless constant f^2 plays the role of the Brans-Dicke constant ω. Another scalar field ψ has been introduced. In addition to these we consider for simplicity only the scalar and isoscalar "nucleon" field Φ.[10] Including other particles interacting with each other does not affect the results. The interaction Lagrangian consists of two parts:

$$-L_1 = \sum_{r=0}^{4} c_r\,\phi^{4-r}\psi^r\,, \qquad (1b)$$

$$-L_2 = \tfrac{1}{2}(g_1{}^2\phi^2 + g_2{}^2\psi^2)\Phi^2\,. \qquad (1c)$$

So far we have no dimensional constants, so that complete scale invariance holds and all the particles are still massless.

One derives the equations

$$G_{\mu\nu}=f^2\phi^{-2}T_{\mu\nu}-\phi^{-2}(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)\phi^2, \quad (2a)$$

$$f^{-2}\phi R+\Box\phi+\partial L'/\partial\phi=0, \quad (2b)$$

$$\Box\psi+\partial L'/\partial\psi=0, \quad (2c)$$

$$\Box\Phi+\partial L'/\partial\Phi=0, \quad (2d)$$

where $L'=L_1+L_2$ and the Belinfante tensor $T_{\mu\nu}$ includes the contribution from the matter as well as from ϕ and ψ; ∇_μ is a covariant derivative and $\Box=\nabla^\lambda\nabla_\lambda$. Contracting (2a) yields $-R=f^2\phi^{-2}T-3\phi^{-2}\Box\phi^2$ which is to be substituted into (2b). Further transforming the trace T by using the explicit form of $T_{\mu\nu}$ and (2c), (2d), we cast (2b) into the form

$$\Box\phi=-\phi^{-1}(\phi_{,\mu}\phi^{,\mu})-\tfrac{1}{2}Z\phi^{-1}\Box(\psi^2+\Phi^2), \quad (3)$$

where $Z=(1+6f^{-2})^{-1}$.

We now introduce the shifted fields σ_1 and σ_2 by $\phi=v_1+\sigma_1$ and $\psi=v_2+\sigma_2$, where v_1 and v_2 are the vacuum expectation values. Equations (2c) and (3) are then put into the form

$$\Box\sigma_i=\delta_i+\gamma_{ij}\sigma_j+J_i. \quad (4)$$

We find $\delta_1=-xZ\delta_2$, $\delta_2=v_1^3F(x)$, where

$$F(x)=c_1+2c_2x+3c_3x^2+4c_4x^3, \quad (5)$$

with $x\equiv v_2/v_1$. We require $\delta_i=0$. In addition to the normal solution $v_1=0$, one may have a self-consistent solution with $v_1\neq 0$, $F(x)=0$. We assume that this is indeed the case. Note that only the ratio x is determined in terms of c_r, while v_1 is left arbitrary. Other symbols in (4) are

$$\gamma_{11}=-xZ\gamma_{21}, \quad \gamma_{21}=v_1^2(3c_1+4c_2x+3c_3x^2), \quad (6a)$$

$$\gamma_{12}=-xZ\gamma_{22}, \quad \gamma_{22}=2v_1^2(c_2+3c_3x+6c_4x^2), \quad (6b)$$

$$J_1=-xZJ_2-\tfrac{1}{2}Zv_1^{-1}\Box\Phi^2, \quad J_2=g_2^2v_2\Phi^2=\zeta v_2^{-1}\rho, \quad (6c)$$

$$\rho=m_N^2\Phi^2, \quad (6d)$$

where

$$m_N^2=g_1^2v_1^2+g_2^2v_2^2. \quad (7)$$

Here m_N is the spontaneously generated nucleon mass,[11] and

$$\zeta=(g_2v_2)^2[(g_1v_1)^2+(g_2v_2)^2]^{-1}.$$

In (6c) and (6d) we have dropped terms higher order in σ_i.

In the "simplest" model in which there is no ψ we have $\zeta=0$. Equations (4), (6c), and (6d) show that σ_i are then decoupled from the static matter. Note that the last term of (6c) vanishes if the momentum transfer from the nucleon is neglected. There are no other terms which contain the nucleon field explicitly. Although there are other neglected terms which are higher order in σ_i, inclusion of their virtual couplings to the nucleon should be illegal in the spirit of the present Lagrangian model. This is the reason why the model without ψ is scarcely tested by present experiments. Brans and Dicke's original result in which the massless scalar field gives observable effects would follow if the nucleon mass were assumed to be a consequence of an explicit breakdown of scale invariance.

Now (4) can be diagonalized to give

$$(\Box-\lambda_i)\sigma_i'=J_i', \quad (8)$$

where $\sigma_i=\alpha_{ij}\sigma_j'$ and $(\alpha^{-1}\gamma\alpha)_{ij}=\delta_{ij}\lambda_i$. The eigenvalues can be calculated to be

$$\lambda_1=0, \quad \lambda_2=\gamma_{22}-xZ\gamma_{21}\equiv\mu^2. \quad (9)$$

We also find by ignoring $\Box\Phi^2$ that

$$J_1'=0, \quad J_2'=(\alpha_{22})^{-1}v_2^{-1}\zeta\rho. \quad (10)$$

The first part of Eqs. (10) shows that the massless field σ_1' is decoupled from the static matter. Consider a static mass point with the mass M composed of nucleons.[12] The solution of (8) thus gives

$$\sigma_1'=0, \quad \sigma_2'=-\zeta v_2^{-1}(M/\alpha_{22})(e^{-\mu r}/4\pi r).$$

With the aid of the relation $\alpha_{12}=-xZ\alpha_{22}$, we obtain

$$\sigma_1=\zeta v_1^{-1}ZM(e^{-\mu r}/4\pi r), \quad \sigma_2=-(xZ)^{-1}\sigma_1. \quad (11)$$

We write $g_{\mu\nu}=\eta_{\mu\nu}+fv_1^{-1}h_{\mu\nu}$ and apply the weak-field approximation to (2a). Also introducing

$$\chi_{\mu\nu}=h_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}h-2f^{-1}\eta_{\mu\nu}\sigma_1,$$

and imposing

$$\chi_{\mu\nu}{}^{,\nu}=0,$$

we obtain

$$\Box\chi_{\mu\nu}=-2fv_1^{-1}T_{\mu\nu}, \quad (12)$$

where we have again dropped terms higher order in σ_i'. The static potential V which describes a force acting on a nucleon is obtained most easily by writing the nucleon part of the Hamiltonian in the form

$$H_N\approx m_N^2\Phi^2+m_N^2V\Phi^2.$$

We find that V consists of two parts; $V=V_1+V_2$, where

$$V_1=-\tfrac{1}{2}fv_1^{-1}h_{00}$$

is the usual "geometric part," while V_2 arises when one replaces ϕ^2 and ψ^2 in (1c) by $2v_1\sigma_1$ and $2v_2\sigma_2$, respectively. Combining (11) and the similar static solution of (12), we finally obtain

$$V(r) = -G_\infty(M/r)(1 + ae^{-\mu r}),\quad a = \xi^2[(3+\tfrac{1}{2}f^2)^{-1} + 2x^{-2}f^{-2}], \tag{13}$$

where $G_\infty = f^2/8\pi v_1^2$ is the gravitational constant which controls the long-range force. The potential (13) is essentially the same as that suggested previously,[9] and also derived by O'Hanlon,[1] and Acharya and Hogan.[2,13] The short-distance limit of (13) is given by $V(r) \to -G_0(M/r)$, where $G_0/G_\infty = 1 + a$.

The long-range force comes only from the geometric part. As was shown in Refs. 1 and 2, the tests of general relativity remain unaffected as long as $r \gg \mu^{-1}$. On the other hand, the finite-range force comes from both of the geometric and the nongeometric parts. It then follows that a particle falls off a geodesic as far as the (usually small) effect of the finite-range force is concerned. Nevertheless the equivalence principle still holds since V_2 comes from the very term, Eq. (1c), which creates the *inertial* mass.

Finally we try to evaluate μ. We notice that no scalar meson has ever been established among observed particles. We conclude that the coupling constants g_1 and g_2 must be much smaller than that of the weak interaction. As a tentative choice we assume that the interactions of the scalar fields, if any, are as weak as the gravitational interaction. Let g represent g_1 and g_2 which are assumed to share the same order of magnitude. The above assumption amounts to

$$g \sim (Gm_N^2)^{1/2} \sim 10^{-19}. \tag{14}$$

Also suppose v_1 and v_2 are of the same order of magnitude as each other and are represented by v. From (7) and (14) one finds $v \sim g^{-1}m_N \sim 10^{19}m_N$. We further assume $c_r \sim g^4$. This is obviously based on the plausibility argument that each ϕ or ψ field carries a constant g. The solution x of $F(x) = 0$ [see (5)] will then be of the order unity. From (6a), (6b), and (9) it now follows that $\mu^2 \sim v^2g^4$, as long as f is also of the order unity. Again using (7) we obtain $\mu \sim gm_N$ or

$$\mu^{-1} \sim g^{-1}m_N^{-1} \sim 10^{19}m_N^{-1} \sim 10^5 \text{ cm},$$

as expected. Consequences of the possible intermediate-range gravity with such a force range have been discussed in detail[9,14] to show that the available measurements are not accurate enough to give a final judgement. Improving the experiments is urged to test the theory and search for a possible link between particle physics and gravitation.

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[12] We can show that the results remain unchanged if the interaction among nucleons is included, as long as one averages over time the resultant fields with many periods of the motion inside the object.

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