Why do we believe Newtonian gravitation at laboratory dimensions?*

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Recent proposals suggest types of inverse-square—law failures which can fit known astronomical data but which exhibit readily observable effects at laboratory dimensions. It is pointed out that there is no plausible way to infer from astronomical data that Newtonian gravitation applies in the laboratory. The experimental data are examined and it is found that past G measurements in the laboratory set only very loose limits on a possible variation in G and that present technology would allow considerable improvement.

INTRODUCTION

The investigation of Newtonian gravitation, in particular the inverse-square law, has received little attention over the years.[1] The reason for this is not difficult to find. Presuming that an inverse-square-law failure must take the form

$$R^{-2+\delta}\ , \tag{1}$$

then the observational data on the advance of the perihelion of Mercury gives[2] a very small value for δ. This value is so small that it is hopeless to look for an inverse-square-law failure in the laboratory. This result hinges upon the assumption that any failure in the inverse-square law must take the form given in Eq. (1).

A MORE GENERAL APPROACH

Recently, inverse-square-law failures of a more complicated nature have been suggested. Before giving these results we present a more general approach to discussing the problem. The Newtonian gravitational force law is written

$$F=\frac{GMm}{R^2}\ , \tag{2}$$

for F the attractive force between the masses M and m, R their separation, and G the gravitational constant. In the event that there is an R dependence beyond that in the R^{-2} factor we can conveniently represent it as a dependence of G on R. Hence we take, for the sake of argument,

$$F=\frac{G(R)Mm}{R^2}\ . \tag{3}$$

It should be pointed out that this procedure is more than merely convenient, since any effort to measure G at a given R will in fact give us $G(R)$ if there is an inverse-square-law failure.

This author has proposed a theory of gravitation related to the weak interaction[3] which yields an inverse-square-law failure[4] which can be represented in the form

$$G(R)=G_0[1+\alpha\ln R+\beta(\ln R)^2]^2\ , \tag{4}$$

where G_0 is a normalization constant, and α and β are constants. More recently, Fujii[5] and O'Hanlon[6] have proposed a failure of the general form

$$G(R)=G_0[1+\alpha(1+\beta R)e^{-\beta R}]\ , \tag{5}$$

where again G_0, α, and β are constants. Fujii derives his result using a field-theoretic approach, and O'Hanlon finds the same form using a metric theory.

Both of these functions have the property that for appropriate values for the constants they can fit astronomical data, including the perihelion of Mercury, while exhibiting a measurable failure at laboratory dimensions. It would be interesting to discuss the limits that experimental data put on these constants, but it is a major goal of this paper to point out that such considerations are largely meaningless. It is clear that Eqs. (4) and (5) are but special examples of an infinite class of rather simple[7] functions, $G(R)$, which are essentially flat at distances of about 10^{13} cm but develop appreciable curvature at distances of about 10 cm. The skeptic may verify this for himself by drawing curves on semilog paper which are essentially flat at 10^{13} cm but develop a slope at about 10 cm. It is clear that such functions need have no simple analytic form.

Further, it does not appear that there are any firmly established theoretical considerations which place limits on the type of functional form which $G(R)$ might take. As is well known, it has been impossible to relate gravitation to the other interactions, and hence gravitation has remained somewhat isolated theoretically and most other knowledge in physics cannot be brought to bear to make inferences about gravitation. The most prevelant theoretical approach to gravitation is, of course, the metric-theory approach of general relativity.

TABLE I. Measured values of G for various mass separations R.

Fig. 1 symbol	Author and date	$G(R)$ (10^{-8} dyn cm^2/g^2)	R (cm)	References and remarks
B	Boys, 1894	6.6576 ± 0.002	6.3	Refs. 9,10. Author's stated error.
Br	Braun, 1896	6.655 ± 0.002	8.6	Refs. 9,11. Author's stated error.
P	Poynting, 1891	6.6984 ± 0.029	32	Refs. 9,12. Error calculated from original paper.
RK-M	Richarz and Krigar-Menzel, 1898	6.685 ± 0.011	80	Refs. 9,13. Author's stated error.
H	Heyl, 1930	6.670 ± 0.005	13	Ref. 14. Error is average deviation from the mean.
HC	Heyl and Chrzanowski, 1942	6.673 ± 0.003	13	Ref. 15. Error is average deviation from the mean.
R	Rose *et al.*, 1969	6.674 ± 0.004	12	Ref. 16. Error is one standard deviation.

However, as O'Hanlon's work amply demonstrates, even metric theories can produce an inverse-square-law failure if they are slightly modified. It is, therefore, an open question as to the kinds of $G(R)$ which might emerge from further modifications of the metric theories.

Since our present knowledge does not seem to restrict $G(R)$ in any way (except for satisfying astronomical data), it appears that there is no plausible way to extrapolate data on the inverse-square law from the orbit of Mercury to the laboratory. Indeed, extrapolations of this kind (over 12 orders of magnitude) are generally rejected out of hand in other areas of physics.[8]

Evidently, to assess the validity of the inverse-square law at distances of a few centimeters we must examine laboratory data of measurements of G itself. We shall see that these data are not especially reassuring, and in particular that present-day technology does allow a considerable improvement.

LABORATORY DATA ON $G(R)$

Laboratory measurements of G have not been numerous, and many of the values come from older data. In selecting the older data I have deferred to the judgment of a contemporary of that period, Poynting,[9] in assessing the merits of each work. Poynting flatly rejects all geological measurements on the basis that the mass distributions are too poorly known. In Table I,[9-16] I present all the older data which stated errors of less than 0.5% and which were not criticized by Poynting as "provisional" or containing systematic errors. I further rejected one datum where iron-attracting masses were used, because of paramagnetic effects.[17] It should be noted that Poynting regarded the work by Boys and that by Braun as the most authoritative data of that period (1910).

Also included in Table I are all of the more recent data for which the mass separations could be found.[18] For all data, the mass separations, R, have been determined to 5%.[19]

The $G(R)$ data in Table I have been plotted vs. $\ln R$ in Fig. 1. If $G(R)$ were constant, we should expect the data to lie on a flat, straight line. Clearly they do not, and there even appears to be an appreciable trend in the data. The fact that the

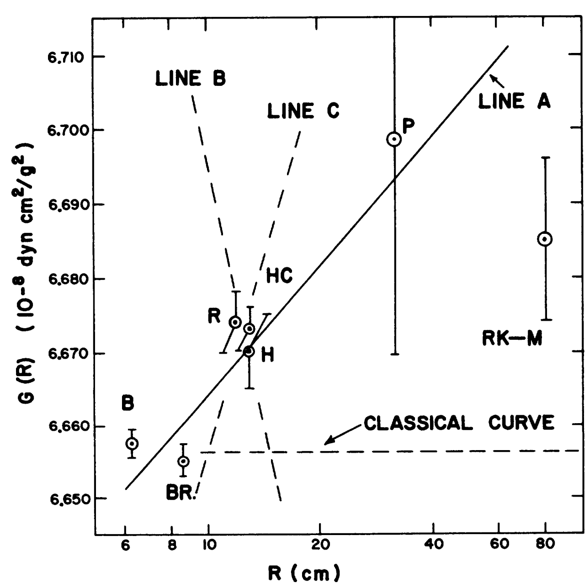

FIG. 1. Semilog plot of G at the different R.

Poynting datum lies along a line determined by the two other data clumps suggests that line A might be a plausible fit to the data; however, the error bars preclude taking line A very seriously. Line A does, however, suggest that more investigation of $G(R)$ in the laboratory is desirable.

It may be felt that we should confine our attention to only the most recent data. Line B and line C are constructed to pass through the error limits of these data, and hence represent the limits on any variation in $G(R)$ as determined by modern data. These limits are obviously not very stringent.

A convenient, dimensionless way in which to parametrize a possible R dependency of G is by way of the logarithmic derivative. Using lines A, B, and C, we might conclude that

$$\left.\frac{d[\ln G(R)]}{d(\ln R)}\right|_{R\simeq 10\text{ cm}} = 3.4\times 10^{-3}\begin{pmatrix}+7.5\times 10^{-3}\\ -20.\times 10^{-3}\end{pmatrix}$$

gives about as good a summary of our knowledge of $G(R)$ at distances of a few centimeters as any.

The rather broad limits that present data place on $G(R)$ are especially interesting because present-day technology allows a substantial improvement in these limits. In particular we believe that our laboratory will be able to determine

$$\left.\frac{d[\ln G(R)]}{d(\ln R)}\right|_{R\simeq 10\text{ cm}}$$

to within $\pm 0.5\times 10^{-3}$ in the near future.

CONCLUSION

We have argued that the extrapolation of astronomical data to the laboratory system provides no plausible information on $G(R)$ at distances of a few centimeters, and that laboratory data on $G(R)$ are far more meager than they need be. Hence we conclude that further laboratory research on $G(R)$ is significant and necessary.

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[1]Most books concerning gravitation scarcely mention the inverse-square law in a critical context at all. See, for example, R. H. Dicke, *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York, 1965); D. Brouwer and G. M. Clemence, *Methods of Celestial Mechanics* (Academic, New York, 1961); R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965).

[2]Simon Newcomb [in "Gravitation," *Encyclopaedia Britannica*, 11th edition (1910) Vol. 4, p. 384] gives $\delta\simeq 1.6\times 10^{-7}$. For this value, a laboratory experiment could only exhibit at most a part-per-million discrepancy. If the general-relativity correction is included the situation becomes worse by one order of magnitude.

[3]D. R. Long, Bull. Am. Phys. Soc. 12, 1057 (1967).

[4]D. R. Long, Bull. Am. Phys. Soc. 15, 1640 (1970); D. R. Long, E. W. S. C. Cheney Report No. 1, 1971 (unpublished).

[5]Y. Fujii, Nature Phy. Sci. 234, 5 (1971).

[6]J. O'Hanlon, Phys. Rev. Lett. 29, 137 (1972).

[7]A simple function in the sense, say, of having a first or second derivative which is monotonically increasing or decreasing.

[8]It is sometimes claimed that the "consistency" of the gravitational attraction of the various bodies of the solar system supports the constancy of G. A moment's reflection shows this is not true, because the mass values for these bodies are calculated from the laboratory value of G and there is no accurate way to check them independently.

[9]J. H. Poynting, in "Gravitation," *Encyclopaedia Britannica*, 11th edition (1910), Vol. 4, p. 384.

[10]C. V. Boys, Philos. Trans. R. Soc. Lond. A, part i, 1 (1895).

[11]K. Braun, S. J. Deukschr. Akad. Wiss. Wien, Math-naturw. Cl. 64, 187 (1896).

[12]J. H. Poynting, Philos. Trans. R. Soc. Lond. A 182, 565 (1891).

[13]F. Richarz and O. Krigar-Menzel, Nature 55, 296 (1898).

[14]P. Heyl, J. Res. Natl. Bur. Stand. (U.S.) 5, 1234 (1930).

[15]P. Heyl and P. Chrzanowski, J. Res. Natl. Bur. Stand. (U.S.) 29, 1 (1942).

[16]R. D. Rose, H. M. Parker, R. A. Lowry, A. R. Kuhlthau, and J. W. Beams, Phys. Rev. Lett. 23, 655 (1969).

[17]The Wilsing datum of Ref. 9.

[18]For one recent measurement see L. Facy and C. Pontikis, C. R. Acad. Sci. (Paris) 270, 15 (1970); and C. R. Acad. Sci. (Davis) 272, 1397 (1971); 274, 437 (1972). The data given do not determine the separation, and the authors have failed to provide that information. I have also come across a reference to a measurement by J. Renner of Budapest in 1970. I have not located the details of this measurement.

[19]The separation in the Richarz and Krigar-Menzel work has been approximately corrected for the fact that one of the masses was grossly aspherical.