PHYSICAL REVIEW D VOLUME 9, NUMBER 3 1 FEBRUARY 1974

Inclusive contributions to the rising inelastic proton-proton cross section*

Dennis Sivers and Frank von Hippel
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
(Received 24 May 1973)

Attention is directed to the nonscaling behavior of inclusive cross sections which may accompany a rising inelastic proton-proton cross section. In particular it is pointed out that the observed "threshold rise" of the invariant inclusive cross section for antibaryon production toward its scaling limit in the CERN ISR energy region sets a mass scale similar to that observed in the rising total cross section.

INTRODUCTION

Recent results from two CERN Intersecting-Storage-Rings (ISR) experiments[1,2] indicate that the inelastic p-p cross section (along with the elastic cross section) increases significantly above a c.m. energy $W = 20$ GeV. Since this energy is an order of magnitude larger than the masses of the known particles and resonances, it is natural to speculate that the rising cross section may provide indirect evidence for the onset of production of a new class of massive hadrons.[3] In this paper we point out that, while the search for new massive hadrons is of interest in its own right, such particles may not be required to set the scale for the energy dependence of the inelastic p-p cross section. Such an energy scale has already been observed in the "threshold rise" of the invariant inclusive cross section for antibaryon production in the ISR energy region.[4] Although the currently popular view is that the rising total cross section may presage an asymptotic $\ln^2 s$ behavior for the p-p total cross section,[5] our suggestion raises the possibility that the actual energy dependence in the ISR energy region may be more complex. Whatever the dynamics of the rising inelastic cross section, it is obviously an important challenge to experimentalists to analyze it into its inclusive components.

I. THE INELASTIC CROSS SECTION AND ITS INCLUSIVE COMPONENTS

The inelastic cross section is directly related to the invariant cross sections for the inclusive reactions

$$p + p \to c + \text{anything}. \tag{1}$$

If we define η_c as the average inelastic final-state energy fraction associated with particle type c,

$$\eta_c \sigma_{\text{in}} \equiv \tfrac{1}{2} \int dx_c\, d^2 p_{cT} \left(E_c \frac{d^3\sigma_c}{d^3 p_c} \right) , \tag{2}$$

energy conservation gives us a sum rule for the inelastic cross section in terms of the invariant inclusive cross sections[6]:

$$\sigma_{\text{in}}(W) = \sum_c \eta_c \sigma_{\text{in}} = \tfrac{1}{2} \sum_c \int dx_c\, d^2 p_{cT} \left[E_c \frac{d^3\sigma_c}{d^3 p_c} (W, x_c, p_{cT}) \right] . \tag{3}$$

Here

$$x_c \equiv 2 p_{cL}/W$$

is the usual Feynman scaling parameter. The cutoff in transverse momentum p_{cT} will make the integrals independent of the kinematic limit on p_{cT} at high energies, with the result that the only energy dependence of the integrals on the right-hand side of (3) will be through the explicit W dependence (i.e., nonscaling behavior) of the invariant inclusive cross sections—or through a singular dependence on x near the energy-dependent kinematic limit:

$$|x|_{\max} = 1 - \frac{(M_N + m_\pi)^2 + M_N{}^2 + 2p_T{}^2}{W^2} . \tag{4}$$

Unless such energy-dependent effects in the different terms in (3) cancel, there will be a corresponding energy dependence of the total cross section.

In fact, substantial cancellations in (3) occur between $W = 5$ and 20 GeV. The contributions associated with pions, kaons, and hyperons rise by about 5 mb, while the contribution of protons drops, producing an inelastic cross section which is remarkably constant.[7,8] From another point of view, the cancellation is even more dramatic: The probability of finding strange particles in the final state of p-p collision has grown to over 50% at $W = 20$ GeV.[9] The cross section for producing final states not containing strange particles has therefore dropped correspondingly. The dynamics of this cancellation remains mysterious. Explanations involving t-channel exchanges have trouble explaining the details of this kind of compensation and are incomplete in any case in that they do not include the different energy scales for the production of final states with strange particles and without.

Whatever the cause of the cancellations in the sum rule (3) which bring about an approximately constant inelastic cross section between $W=5$ and 20 GeV, there is no evidence for significant energy dependence of the invariant inclusive cross sections for pions, kaons, and leading protons in the ISR energy range.[10] Unfortunately, however, the experimental errors on these quantities are so large (10–25%) that they still may conceal significant contributions to the rise in the inelastic cross section. This situation poses an important challenge to experimentalists.

Two possibilities may be distinguished: (i) The rising inelastic cross section is due to a proportionate increase in each contribution to (3), as would be the case if each contribution had reached its relative asymptotic value, or (ii) some contributions to (3) are still varying rapidly at ISR energies. In fact, even at the current level of experimental accuracy, there is evidence that two contributions to the inelastic cross section are rising rapidly: (i) the apparently singular behavior at $|x| = 1$ of the invariant cross section for inclusive proton production, and (ii) the rising antiproton contribution to (3) in the ISR energy region. Contribution (i) has already been discussed by Capella[11] so we will only review it briefly here.

Near $|x|=1$ the limiting proton distribution from inelastic proton scattering is sharply peaked[11] in a manner similar to the $(1-|x|)^{-1}$ behavior predicted by a triple Pomeranchuk-Regge expression. This singular behavior results in a $\ln(1-|x|_{\max})$ term in the proton contribution on the right-hand side of (3) after integration is over. From (4) it will be seen that this results in a $\ln W^2$ behavior. Based on Capella's parameterization we would estimate that this effect contributes about 1 mb to the total 3–4-mb rise of the inelastic cross section measured between $W=20$ and 53 GeV at the ISR.

II. "THRESHOLD RISE" IN BARYON PAIR PRODUCTION

A large mass scale has already been observed in the energy dependence of inclusive antiproton production. The invariant cross section for this reaction is known to rise by an order of magnitude between $W=7$ and $W=50$ GeV.[4] In Sec. IV we will discuss briefly why multiparticle phase-space effects result in this "threshold rise" for antiproton production appearing at c.m. energies an order of magnitude higher than those which would be expected on the basis of pure energy considerations. Here we consider the effect of this rise on the inelastic p-p cross section.

Since baryons are produced in pairs, the rise in the production of antibaryons will be accompanied by a corresponding rise in the number of baryons. In the absence of experimental measurements adequate to separate the resulting rising proton component from the dominant "leading proton" component in the inclusive proton spectrum at ISR energies, we will make the plausible assumption that the inclusive spectrum of the "pair produced" baryons and antibaryons is the same. As a consequence, the contributions of antibaryons in (3) should be doubled to take into account the full effect of the pair production.

The only antibaryons which are observed experimentally at the ISR are antiprotons. We will assume that antineutrons are produced with approximately equal cross sections. To take into account the production of the strange members of the antibaryon octet, we note that they are typically produced with low momenta and will therefore decay into final states containing antinucleons within a distance of a few centimeters. In a single-armed spectrometer experiment we expect therefore that all antibaryons decaying into antiprotons will be counted as antiprotons. The branching ratios for decays of the strange members of the antibaryon octet are such that, if they were all produced with equal probability, there would be almost equal numbers of antiprotons and antineutrons in their decay products. We will therefore assume that the inclusive production of antineutrons—through intermediate strange antibaryons as well as directly—is equal to that of antiprotons. If we neglect the small contribution due to the energy carried by other decay products of the strange antibaryons, our estimate of the contribution of $B\bar{B}$ pairs in (3) will then be four times the contribution of the antiprotons (as measured in a spectrometer experiment). At the lowest ISR energy we arrive at a figure of about 0.6 mb, and this appears to approximately double at the highest ISR energy.[4] Our estimate is therefore that the growth of $B\bar{B}$ production in the ISR energy range can account directly for an increase of approximately 0.6 mb in the total cross section through Eq. (3).

III. CLUSTER REPRESENTATION

Indirectly the rising inclusive cross section for pair production might result in a much larger increase in the total cross section. This can easily be seen in a cluster representation of the inelastic cross section[12]:

$$\sigma_{in}(s, Z) = \sum_{\{n_i\}} (z_1)^{n_1}(z_2)^{n_2}\cdots\sigma_{n_1,n_2,\ldots}(s)$$
$$= \exp\left\{\sum_{\{n_i\}} \frac{(z_1)^{n_1}}{n_1!}\frac{(z_2)^{n_2}}{n_2!}\cdots\Phi_{n_1,n_2,\ldots}(s)\right\}. \quad (5)$$

Here σ_{n_1,n_2} is the partial cross section for the production of n_1 particles of type 1, n_2 of type 2, etc., and (5) defines the Φ_{n_1,n_2} as the exclusive cluster functions for the same set of particles. The physical inelastic cross section is obtained when all Z_i are set equal to unity.

The cluster functions containing single antibaryons are related directly to the antibaryon multiplicity:

$$\langle n_{\bar{B}}\rangle = \sum_{\{n_j\}} n_{\bar{B}}\Phi\cdots n_{\bar{B}}\cdots(s). \quad (6)$$

Consequently, if we neglect multiple antibaryon production, an approximation which should be good in the ISR energy region, we may use (5) to obtain the approximation

$$\sigma_{in}(s) \cong [1+\langle n_{\bar{B}}(s)\rangle]\tilde{\sigma}_{in}(s). \quad (7)$$

Here $\tilde{\sigma}_{in}$ is the same as σ_{in} except that the clusters containing antibaryons in (5) have been suppressed.

The antibaryon multiplicity rises from 0.13 to 0.32 in the ISR energy region.[9] If we were to neglect the effects through unitarity of antibaryon production on cluster functions not containing antiprotons, Eq. (7) would imply that antibaryon production results in an increase of σ_{in} by a factor of 1.17 or about 6 mb in this energy region. Of course this observation can only be suggestive since we do not know how $\tilde{\sigma}_{in}(s)$ should vary in this region. Our discussion does raise the possibility, however, that the "threshold rise" of antibaryon production may "feed through" into nonscaling contributions to the inclusive cross sections for the production of other final particles.

IV. THE ENERGY SCALE FOR HEAVY-PARTICLE PRODUCTION

It has been suggested[3] that the rise of the inelastic cross section in the ISR energy region may be a threshold effect due to the onset of the production of particles with masses of the order of 10 GeV. There are two bits of evidence which would indicate, however, that, if threshold effects contribute to the rise, the masses of the particles involved may well be substantially smaller.

The first piece of evidence comes from the elastic cross section. The energy dependence in the elastic cross section in the ISR energy region occurs at small t.[13] If this reflects a change in the transparency of the proton associated with the increasing inelastic cross section, then the change must be occurring at the "edge" of the proton. It seems to us implausible that particles with a mass comparable to the total c.m. energy could be produced in such "peripheral" collisions.

The second more direct indication which we have that the production of less massive particles might be involved is the fact that the "threshold rise" of antiproton production is observed to still be present at ISR energies.

The energy scale for the rise of the inclusive antiproton production cross section in the ISR energy region may be understood as a kinematic reflection of the fact that, even at these energies, the average energies of the produced pions is rather low. At $W=20$ GeV there are an average of 10 pions produced per inelastic collision.[9] The leading nucleons take off typically half the total energy, leaving the pions with an average c.m. energy of a GeV or so. In this situation the production of a $B\bar{B}$ pair at a cost of 2–4 GeV energy obviously results in a significant reduction in the multiparticle phase space available to the accompanying pions. These kinematic effects have been calculated quantitatively in simple phase-space models.[7,14]

CONCLUSION

In conclusion, we have discussed in this paper the relationship between the total inelastic cross section for p-p scattering in the ISR energy range and the properties of inclusive cross sections. We have concentrated particularly on the contribution of the nonscaling "threshold rise" of baryon pair production in this energy region on the rising inelastic cross section. It is obvious using Eq. (3) that, if such threshold effects are still important at ISR energies, it may be premature to speculate that the asymptotic behavior of the total cross section is finally emerging.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]U. Amaldi *et al.*, Phys. Lett. 44B, 112 (1973).

[2]S. R. Amendolia *et al.*, Phys. Lett. 44B, 119 (1973).

[3]A recent speculation along these lines has been offered by M. S. Chanowitz and S. D. Drell, Phys. Rev. Lett. 30, 807 (1973).

[4]A. Bertin *et al.*, Phys. Lett. 42B, 493 (1972).

[5]See, e.g., E. Leader and U. Maor, Phys. Lett. 43B, 505 (1973).

[6]T. T. Chou and C. N. Yang, Phys. Rev. Lett. 25, 1072 (1970).

[7]D. Sivers, Phys. Rev. D 8, 4004 (1973).

[8]The p-p inelastic cross section is about as constant as the total cross section between P_{lab} of 20 and 60 GeV/c. In this interval, the total cross section falls by 0.62 ± 0.17 mb [S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971)]. The elastic cross section drops even faster by 1.15 ± 0.20 mb in the same energy interval [G. G. Beznogikh *et al.*, Phys. Lett. 43B, 85 (1973)], resulting in a net rise of about 0.5 ± 0.3 mb in the inelastic cross section.

[9]M. Antinucci *et al.*, Nuovo Cimento Lett. 6, 121 (1973). We assume that up to $W = 20$ kaons are mostly produced singly. We assume also that $\pi^0 = \frac{1}{2}(n_{\pi^+} + n_{\pi^-})$.

[10]E. Lillethun, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 211.

[11]A. Capella, Phys. Rev. D 8, 2047 (1973).

[12]A. H. Mueller, Phys. Rev. D 4, 150 (1971); B. R. Webber, Nucl. Phys. B43, 541 (1972).

[13]B. Barbiellini *et al.*, Phys. Lett. 39B, 663 (1972).

[14]R. Jengo, A. Krzywicki, and B. Petersson, Phys. Lett. 43B, 397 (1973).

PHYSICAL REVIEW D VOLUME 9, NUMBER 3 1 FEBRUARY 1974

Transverse-momentum distribution of inclusive pions*

Stephen S. Pinsky † and Paul R. Stevens

Department of Physics, University of California, Riverside, California 92502

(Received 1 June 1973)

We study the transverse-momentum distribution of pions that come from the decay of ρ's. We find that the inclusive transverse-momentum distribution of such pions will exhibit a turning over as $k_\perp^2 \to 0$. This effect is strictly kinematic and will appear in any model where π's come from the decay of unpolarized low-mass clusters which have a $p_\perp^2$ distribution that is peaked at $p_\perp^2 = 0$.

Recent studies of multiplicity distributions have considered in some detail the possibility that the observed distributions come from the decay of some primary-cluster distribution. For example,[1] if one were to assume that these primary clusters are produced independently and then decay uniformly, we would have a definite prediction for the prong cross-section distribution. Comparison of such a model with the data indicates that there are between one and two charged particles per cluster. Along this line, others[2] have taken these clusters to be resonances in attempts to understand various distributions of neutrals.

We would like to consider the implication of such models on the transverse-momentum distribution of inclusive pions. For simplicity we will consider a specific model, however our results are far more general. In the model we will consider here we will assume the following:

(i) A substantial number of produced π's come from ρ's.

(ii) The inclusive ρ distribution is peaked in $p_\perp^2$ at $p_\perp^2 = 0$.

(iii) The ρ's are produced unpolarized.

Let us now consider the π distribution that results from the decay of such a primary ρ distribution,

$$\omega \frac{d^3\sigma}{d^3k}\bigg| = \int \frac{d^3p}{E}\,\frac{d^3\sigma}{d^3p/E}\,\frac{1}{2\pi}\,\delta\left(\frac{(P-K)^2}{2} - \tfrac{1}{2}m^2\right),$$

following the calculational technique of Ref. 3. We break up p into components parallel ($p_\parallel'$) and perpendicular ($p_\perp'$) to k. Using the δ function we do the integration over $\cos\theta$, θ being the angle between k and p. Thus

$$\omega \frac{d^3\sigma}{d^3k}(\omega, k_\perp^2; s) = \frac{1}{2\pi k}\int dE\,d\varphi\,\frac{d^3\sigma}{d^3p/E}(E, p_\perp^2; s),$$

where φ is the azmuthal angle of p around k. At this point it is convenient to decompose p into components perpendicular and parallel to the beam direction, since it is in this form that $d^3\sigma/(d^3p/E)$ has simple properties, in particular scaling. In the following discussion we will assume that we are at high enough energies so that we need only consider the scaling piece of $d^3\sigma/(d^3p/E)$. Thus

$$\omega \frac{d^3\sigma}{d^3k} = \frac{1}{2\pi M^2}\int_0^{\overline{Q^2}} dQ^2 d\varphi \times \frac{d^3\sigma}{d^3p/E}\left(E = E_- + \frac{k}{M^2}Q^2, p_\perp^2, s\right),$$

where

$$p_\perp^2 = \left(\frac{k_\parallel}{k}Q_x\alpha + \frac{p_\parallel'}{k}k_\perp\right)^2 + Q_y^2\alpha^2,$$

and

$$Q^2 = \frac{M^2}{k}(E - E_-), \quad E_\mp = \frac{M^2}{2m^2}(\omega \mp k\eta),$$