polynomial boundedness, other solutions can lead to total cross sections growing no faster than $[\ln(\ln s)]^2$.

Froissart-bound saturation can occur as in Refs. 2 and 3 if (a) the Froissart bound is not satisfied off the mass shell, (b) the scattering amplitude is not strongly damped off the mass shell, or (c) the assumption of polynomial boundedness in Ref. 6 is wrong. We feel though, in the light of our equation, that it is difficult to argue against constant total cross sections on the grounds that this would be an accident of nature.

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Model for p-p diffraction

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Previous work on π-π scattering is generalized to include the p-p case. A fixed-pole solution is found for the p-p scattering amplitude. After approximating the residue of this pole, we compare with high-energy data from the CERN ISR. The one-parameter model explains several aspects of these data.

I. INTRODUCTION

In a previous paper[1] we presented a model for π-π scattering at infinite energy. The basic assumption of this model is that the exact scattering amplitude is given by the eikonal iteration of some set of Feynman graphs which are two-pion reducible in the crossed channel. The eikonal approximation is a natural one for high-energy scattering since it is based on the intuitively appealing idea that two highly relativistic particles at some impact parameter b simply traverse classical trajectories through the region of interaction and are phase-shifted by an amount proportional to a potential. In a relativistic field theory this potential is assumed to be some set of Feynman graphs. One would expect the longest-range forces to be the most important ones which contribute to this potential, since these long-range forces determine the extent of the scattering region in b space, which in turn roughly determines the cross section. These ideas have been partially borne out in perturbation theory[2]; however, the final situation is not clear since in some models there is no unique classical path for the scattering particles,[3] and in others the inclusion of vertex corrections destroys the simple eikonal result.[4] Despite these complications it seems reasonable to assume that there do exist field theories which, when solved completely at asymptotic energies, have the eikonal form we consider here.

It is an experimental fact that charge exchange of any kind is unimportant for elastic scattering at infinite energies. The longest-range force with vacuum quantum numbers in the strong interactions is two-pion exchange. In perturbation theory, the potential for two-pion exchange is the sum of all two-pion reducible Feynman graphs. Thus we are led naturally to the assumption that the Born term of the eikonal series is just this set of Feynman graphs.

Because only the zero-charge exchange force is nonvanishing at infinite energies, we are justified in ignoring the isospin of the exchanged pions in the following. Also spin is conserved by scattering particles at high energies and we need only consider spin-nonflip amplitudes.

We combine these ideas with the crossed-channel Bethe-Salpeter equation to obtain coupled self-consistent integral equations for the elastic amplitudes. Using previous results, we expect that these equations satisfy a bound stronger than the Froissart bound, and we find that they have a solution which behaves like a fixed pole at $J=1$.

A form suitable for analytic continuation in momentum transfer is presented for a certain class of eikonal amplitudes. We then make a reasonable approximation for the residue of the fixed-pole Born term, and compare this to the CERN data.

II. THE BETHE-SALPETER EQUATION FOR COUPLED CHANNELS

We define the normalization of spin-nonflip amplitudes by

$$\operatorname{Im} T_{ab}(s,t,m_a{}^2,m_a{}^2,m_b{}^2,m_b{}^2)=2|\vec{K}|\sqrt{s}\,\sigma_{\text{tot}}\,, \quad (2.1)$$

where $T_{ab}(s,t,u_{a1},u_{a2},u_{b1},u_{b2})=T_{ab}(Q,P,P')$ is the scattering amplitude for two stable particles a and b, in general off-shell, and

$$\begin{aligned} &s=(P+P')^2, \quad t=Q^2,\\ &u_{a1}=(P+\tfrac{1}{2}Q)^2, \quad u_{a2}=(P-\tfrac{1}{2}Q)^2, \qquad (2.2)\\ &u_{b1}=(P'-\tfrac{1}{2}Q)^2, \quad u_{b2}=(P'+\tfrac{1}{2}Q)^2,\\ &\vec{K}=\text{c.m. momentum of one particle.}\end{aligned}$$

We define $I_{ab}(s,t,u_{a1},u_{a2},u_{b1},u_{b2})=I_{ab}(Q,P,P')$ to be the sum of all Feynman diagrams which are two-pion irreducible in the t channel with particles a and b as external legs. The Bethe-Salpeter equation is

$$T_{ab}(Q,P,P')=I_{ab}(Q,P,P')-\tfrac{1}{2}i\int\frac{d^4P''}{(2\pi)^4}I_{a\pi}(Q,P,P'')T_{\pi b}(Q,-P'',P')\Delta(P''-\tfrac{1}{2}Q)\Delta(P''+\tfrac{1}{2}Q)\,, \quad (2.3)$$

where $\Delta(P)$ is the full renormalized pion propagator. Taking the s-channel absorptive part, with all external masses below the first inelastic threshold, we find

$$\begin{aligned}\operatorname{Abs}T_{ab}(Q,P,P')=\operatorname{Abs}I_{ab}(Q,P,P')+\int\frac{d^4P''}{(2\pi)^4}&\operatorname{Abs}I_{a\pi}(Q,P,P'')\operatorname{Abs}I_{\pi b}(Q,-P'',P')\\ &\times\theta((P+P'')^2)\,\theta((P'-P'')^2)\Delta(P''-\tfrac{1}{2}Q)\Delta(P''+\tfrac{1}{2}Q)\,. \quad (2.4)\end{aligned}$$

Let us define the following symbolic notation:

$$\operatorname{Abs}A\times_t\operatorname{Abs}B=\int\frac{d^4P''}{(2\pi)^4}\operatorname{Abs}A(Q,P,P'')\operatorname{Abs}B(Q,-P'',P')\,\theta((P+P'')^2)\theta((P'-P'')^2)\Delta(P''+\tfrac{1}{2}Q)\Delta(P''-\tfrac{1}{2}Q)\,, \quad (2.5)$$

$$\operatorname{Abs}A\times_t\operatorname{Abs}A=(\operatorname{Abs}A)^2\,, \quad (2.6)$$

$$\operatorname{Abs}A\times_t(\operatorname{Abs}A)^{n-1}=(\operatorname{Abs}A)^{n-1}\times_t(\operatorname{Abs}A)=(\operatorname{Abs}A)^n. \quad (2.7)$$

Then (2.4) becomes

$$\operatorname{Abs}T_{ab}=\operatorname{Abs}I_{ab}+\operatorname{Abs}I_{a\pi}\times_t\operatorname{Abs}T_{\pi b}\,. \quad (2.8)$$

We now introduce the following functions:

$$\operatorname{Abs}T_{ab}(\lambda)=\lambda\operatorname{Abs}I_{ab}+\sum_{n=1}^{\infty}\lambda^{n+1}\operatorname{Abs}I_{a\pi}\times_t(\operatorname{Abs}I_{\pi\pi})^{n-1}\times_t\operatorname{Abs}I_{\pi b}\,, \quad (2.9)$$

$$\operatorname{Abs}T_{2ab}(\lambda)=\operatorname{Abs}T_{ab}(\lambda)-\lambda I_{ab}\,, \quad (2.10)$$

which satisfy the coupled equations

$$\mathrm{Abs}\, T_{ab}(\lambda) = \lambda\, \mathrm{Abs} I_{ab} + \lambda\, \mathrm{Abs} I_{a\pi} \times_t \mathrm{Abs}\, T_{\pi b}(\lambda). \tag{2.11}$$

These functions are useful because of the following relations:

$$\mathrm{Abs}\, T_{ab}(\lambda) = \mathrm{Abs}\, T_{ab} + (\lambda-1)[\mathrm{Abs}\, T_{ab} + \mathrm{Abs}\, T_{a\pi} \times_t \mathrm{Abs}\, T_{\pi b}] + \sum_{n=2}^{\infty} (\lambda-1)^n [\mathrm{Abs}\, T_{a\pi} \times_t (\mathrm{Abs}\, T_{\pi\pi})^{n-2} \times_t \mathrm{Abs}\, T_{\pi b} + \mathrm{Abs}\, T_{a\pi} \times_t (\mathrm{Abs}\, T_{\pi\pi})^{n-1} \times_t \mathrm{Abs}\, T_{\pi b}], \tag{2.13}$$

$$\left(\lambda \frac{d}{d\lambda} - 1\right) \mathrm{Abs}\, T_{ab}(\lambda) = \left(\lambda \frac{d}{d\lambda} - 1\right) \mathrm{Abs}\, T_{2ab}(\lambda) = \mathrm{Abs}\, T_{a\pi}(\lambda) \times_t \mathrm{Abs}\, T_{\pi b}(\lambda), \tag{2.12}$$

which can be checked by substitution. The two-pion, t-channel, reducible amplitude is given by $T_{2ab}(\lambda = 1) = T_{2ab}$.

To make use of these equations we assume that the amplitudes $\mathrm{Abs}\, T_{ab}$ are strongly damped off the mass shell, and in particular that they satisfy the Froissart bound off the mass shell when the external masses are below the first inelastic threshold. We have shown[5] that these assumptions lead to a high-energy bound for $T_{2\pi\pi}$. This result generalizes to the present case. With the same assumption about polynomial boundedness as in Ref. 5 we find

$$|T_{2ab}(s,t)| < s^{1+\epsilon}, \quad s \to \infty \tag{2.14}$$

with ϵ an arbitrarily small positive number. Reasonableness then demands

$$|T_{2ab}(s,t)| < c(t)\, s (\ln s)^m. \tag{2.15}$$

Using the exact Froissart bound in (2.12) and (2.13) suggests [recall that only a weak form of the Froissart bound was needed to establish (2.14) (see Ref. 5)]

$$|T_{2ab}(s,t)| < c(t)\, s (\ln s)^5. \tag{2.16}$$

Stronger assumptions are needed to establish this than are needed for (2.14). We therefore take it as a reasonable possibility.

III. THE EIKONAL ASSUMPTION

The eikonal form for the high-energy amplitudes we take to be

$$T_{ab}(s,b) = 2is\left[1 - \exp\left(\frac{i}{2s} T_{2ab}(s,b)\right)\right], \tag{3.1}$$

where

$$T_{ab}(s,b) = \int \frac{d^2q}{(2\pi)^2} e^{i\vec{q}\cdot\vec{b}}\, T_{ab}(s, -\vec{q}^2) \tag{3.2}$$

and

$$T_{2ab}(s,b) = \int \frac{d^2q}{(2\pi)^2} e^{i\vec{q}\cdot\vec{b}}\, T_{2ab}(s, -\vec{q}^2). \tag{3.3}$$

The large-b behavior of T_{2ab} is forced by analytic properties of Feynman graphs to be

$$T_{2ab}(s,b) \leq a(s) \frac{e^{-2\mu b}}{(2\mu b)^{1/2}}, \quad b \to \infty, \quad \mu = \text{pion mass}. \tag{3.4}$$

Combining (3.1), (3.4), and the bound (2.15) leads, without any additional assumptions, to a bound on the total cross section. The argument for the π-π case was presented in Ref. 1. The same argument applies when the external particles are not pions and we have

$$\sigma_{ab} \leq c_{ab} [\ln(\ln s)]^2. \tag{3.5}$$

If we use the probable result (2.16), we find that the coefficient c_{ab} is determined to be $c_{ab} = 25(\pi/\mu^2)$, and we have

$$\sigma_{ab} \leq 25 \frac{\pi}{\mu^2} [\ln(\ln s)]^2. \tag{3.6}$$

These bounds are certainly not rigorous results. They depend on strong assumptions. These are in short the following:

(a) The eikonal form is correct for the exact scattering amplitude.

(b) Only two-pion reducible graphs are important in the eikonal Born term.

(c) The Froissart bound is satisfied off shell by the full scattering amplitudes, and these amplitudes are also strongly damped off the mass shell.

(d) The functions $T_{ab}(\lambda)$ are moderately well-behaved functions of λ.

Having established limits on the growth of cross sections in our model, we now exhibit a solution to our equations. To do this we make the following assumption which we call the multiperipheral assumption:

$$\mathrm{Abs}\, T_{2ab}(s,t;\lambda) = \beta_{2ab}(s,t;\lambda) s^{\alpha_{ab}(t;\lambda)} + \text{nonleading terms}. \tag{3.7}$$

Equation (2.12) then becomes

$$\text{Abs}\, T_{2ab}(s,t;\lambda)=\frac{1}{\ln s}\left[\frac{d\alpha_{ab}(t;\lambda)}{d\lambda}\right]^{-1}\times \text{Abs}\, T_{a\pi}(\lambda)\times_t \text{Abs}\, T_{\pi b}(\lambda)\,. \tag{3.8}$$

The phase of T_{2ab} is determined by the even-signature constraint at $\lambda=1$,

$$T_{2ab}(s,t)=\frac{e^{i\pi\alpha_{ab}(t)/2}}{2\sin[\pi\alpha_{ab}(t)/2]}\,\text{Abs}\, T_{2ab}(s,t). \tag{3.9}$$

Let us remark here that the assumptions we have made so far are only for $t\leqslant 0$. Several of our assumptions become suspect in the region $t>(2\mu)^2$. In particular, there is no reason for the long-range forces to dominate the Born term in this region. This is clear from the results of Sec. IV. Thus we restrict our model only to physical values of t.

Equations (3.1), (3.8), and (3.9) yield an integral equation for $T_{ab}(s,t)$. They have solutions

$$T_{ab}(s,t,u_{1a},u_{2a},u_{1b},u_{2b})=i\beta_{ab}(t,u_{1a},u_{2a},u_{1b},u_{2b})s \tag{3.10}$$

and

$$T_{2ab}(s,t,u_{1a},u_{2a},u_{1b},u_{2b})=i\beta_{2ab}(t,u_{1a},u_{2a},u_{1b},u_{2b})s\,. \tag{3.11}$$

The verification that these are in fact solutions is the same as the π-π case of Ref. 1.

Since T_{2ab} is the sum of two-pion reducible Feynman graphs it is reasonable to assume that it satisfies a fixed-s dispersion relation. Taking the Fourier transform of this, ignoring subtractions and bound-state poles, we find

$$T_{2ab}(s,b)=\frac{1}{4\pi^2 i}\int_{(2\mu)^2}^{\infty} dM^2[\text{Disc}\, T_{2ab}(S,M^2)]\times K_0(Mb). \tag{3.12}$$

Comparing (3.11) with (3.12) we can expect to have

$$\beta_{2ab}(b)=\frac{1}{4\pi^2 i}\int_{(2\mu)^2}^{\infty} dM^2[\text{Disc}\,\beta_{2ab}(M^2)]K_0(Mb). \tag{3.13}$$

$\beta_{2ab}(b)$ must be real and positive to satisfy unitarity in our model. Putting these into (3.1) we find

$$T_{ab}(s,b)=2is\left[1-\exp\left(-\int_{(2\mu)^2}^{\infty} dM^2\rho_{ab}(M^2)K_0(Mb)\right)\right], \tag{3.14}$$

$$\rho_{ab}(M^2)=\frac{-i}{8\pi^2}\,\text{Disc}\,\beta_{2ab}(M^2)\,. \tag{3.15}$$

We have not been able to solve our integral equations for the residue functions because of off-shell problems. In order to test (3.14) we propose the following approximate form:

$$T_{ab}(s,b)\approx 2is\{1-\exp[-c_{ab}K_0(2\mu b)]\}, \tag{3.16}$$

which should approximate (3.14) well for large values of b. For p-p scattering we have then

$$T_{pp}(s,t)=4\pi is\int_0^\infty db\, b\, J_0(qb)\times\{1-\exp[-c_{pp}K_0(2\mu b)]\}, \tag{3.17}$$

$$\sigma_{\text{tot}}=4\pi\int_0^\infty db\, b\{1-\exp[-c_{pp}K_0(2\mu b)]\}, \tag{3.18}$$

$$\sigma_{\text{el}}=2\pi\int_0^\infty db\, b\{1-\exp[-c_{pp}K_0(2\mu b)]\}^2, \tag{3.19}$$

$$\sigma_{\text{in}}=2\pi\int_0^\infty db\, b\{1-\exp[-2c_{pp}K_0(2\mu b)]\}, \tag{3.20}$$

$$\frac{d\sigma}{dt}=|T_{pp}|^2/(16\pi s^2). \tag{3.21}$$

This one-parameter description is compared to the new CERN data[6] in Sec. V. Certain properties of Eq. (3.17) at large q may be extracted analytically; we discuss these in Sec. IV.

IV. ANALYTIC CONTINUATION IN q

We consider here the integral

$$I=\int_0^\infty db\, b J_0(qb)\times\left[1-\exp\left(-\int_{(2\mu)^2}^{\infty} dM^2\rho(M^2)K_0(Mb)\right)\right],\quad t=-q^2 \tag{4.1}$$

for arbitrary $\rho(M^2)$ and $\text{Re}\,q>0$. The representation (4.1) diverges when $|\text{Im}\,q|>2\mu$. Therefore we note that

$$J_0(qb)=\tfrac{1}{2}[H_0^1(qb)+H_0^2(qb)]\,. \tag{4.2}$$

When (4.2) is placed into (4.1) we are left with two integrals. The integral with H_0^1 can be rotated to the positive imaginary b axis and the one with H_0^2 to the negative imaginary b axis. The contributions from the infinite quarter circles vanish. We find

$$I=\frac{1}{2}\int_0^{i\infty}db\,bH_0^1(qb)\left[1-\exp\left(-\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)K_0(Mb)\right)\right]$$
$$+\frac{1}{2}\int_0^{-i\infty}db\,bH_0^2(qb)\left[1-\exp\left(-\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)K_0(Mb)\right)\right]. \tag{4.3}$$

We have assumed that no singularities have been encountered in deforming the contour in arriving at (4.3). If such singularities are encountered, they can be included as pole or cut terms. Equation (4.3) can be written

$$I=\frac{1}{2}\int_0^{i\infty}db\,bH_0^1(qb)\times\left[\exp\left(-\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)K_0(e^{-i\pi}Mb)\right)-\exp\left(-\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)K_0(Mb)\right)\right]. \tag{4.4}$$

In this form, the integral is convergent for all $\mathrm{Re}\,q>0$. Equation (4.4) is the limit of the Sommerfeld-Watson transform when the direct-channel energy gets very large. The details of this will appear elsewhere.

We note that for q real

$$H_0^1(e^{i\pi/2}q|b|)=-\frac{2i}{\pi}K_0(q|b|), \tag{4.5}$$

so that when q gets large the integrand of (4.4) is damped exponentially by a factor $e^{-q|b|}$. This means that we can replace the expression in square brackets by its small b limit as was noted by Lévy and Sucher.[7] This yields

$$I\approx\frac{1}{2}\int_0^{i\infty}db\,bH_0^1(qb)\times\left[\exp\left(\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)\ln(e^{-i\pi}Mb)\right)-\exp\left(\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)\ln(Mb)\right)\right], \tag{4.6}$$

where we have assumed that the integrals

$$g=\int_{(2\mu)^2}^{\infty}dM^2\ln(M)\rho(M^2) \tag{4.7}$$

and

$$A=\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2) \tag{4.8}$$

exist. Doing the integration (4.6) we find

$$2\int d^2b\,J_0(qb)\left[1-\exp\left(-\int_{(2\mu)^2}^{\infty}dM^2\rho(M^2)K_0(Mb)\right)\right]\approx 8e^{g}\sin(\tfrac{1}{2}\pi A)\frac{1}{q^{2+A}}2^A\left[\Gamma\left(\frac{2+A}{2}\right)\right]^2,\quad q\text{ large}. \tag{4.9}$$

For the approximation (3.17) we have

$$2\int d^2b\,J_0(qb)\{1-\exp[-cK_0(2\mu b)]\}\approx 8(2\mu)^c\sin(\tfrac{1}{2}\pi c)\frac{1}{q^{2+c}}2^c\left[\Gamma\left(\frac{2+c}{2}\right)\right]^2. \tag{4.10}$$

We remark that when (4.1) is valid (i.e., $|\mathrm{Im}\,q|<2\mu$) one expects the lighter-mass exchanges in the eikonal function, the long-range forces, to be the most important. This is because of the asymptotic behavior

$$K_0(Mb)\approx\left(\frac{\pi}{2}\right)^{1/2}\frac{e^{-Mb}}{(Mb)^{1/2}},$$

which damps the large-mass exchanges for b real. When only (4.4) is valid ($|\mathrm{Im}\,q|>2\mu$, $\mathrm{Re}\,q>0$), b is imaginary in the argument of $K_0(Mb)$, and there is no reason for the lighter-mass exchanges to dominate the eikonal function. It is for this reason that we do not expect to be able to continue our model to $t>(2\mu)^2$ as it stands since one of our main assumptions is the domination of long-range forces.

V. COMPARISON WITH EXPERIMENTAL RESULTS

We wish to review here some of the results on p-p scattering obtained at CERN[6]:

(a) In the region $|t|>0.3$ GeV^2, $d\sigma/dt$ does not change very much over the range of CERN energies ($\sqrt{s}=31$ to 53 GeV). In particular, the position of the minimum at $t\approx 1.4$ GeV^2 moves, if at all, by less than 0.1 GeV^2 over this range.

(b) For smaller momentum transfers, $|t|<0.3$ GeV^2, the diffractive pattern is changing quite noticeably with s. The general trend is for upward arching in the small-t region.

(c) Total cross sections are observed to rise at CERN consistent with a $\ln s$ or $(\ln s)^2$ growth fit by $\sigma_{tot}\approx\{38.4+0.5[\ln(s/s_0)]^2\}$ mb, $s_0=137$ GeV^2.

(d) The ratio of the elastic to the total cross sections is $\sigma_{el}/\sigma_{tot} \approx 0.17$, and is roughly constant over the CERN range.

Result (a) supports a fixed-pole model. Any model which saturates the Froissart bound must have the first diffractive zero move to $t=0$ as $1/\ln s^2$. In most models for growing cross sections being considered at present one must have $\sigma_{el}/\sigma_{tot} = 0.5$ asymptotically, in poor agreement with the present value. Thus most proponents of growing cross sections must ultimately argue that CERN energies are not yet in the asymptotic region.

Because of results (b) and (c), we also believe that the asymptotic limit has not been reached, since in our model we have the bound $\sigma_{tot} < c[\ln(\ln s)]^2$ and also the fixed-pole solution leading to constant cross sections. We take the antithetical point of view from proponents of rising cross sections and we believe that these effects will eventually turn out to be transient.

We have tried to fit the CERN data using (3.17)–(3.21). Since the position of the minimum at $|t| = 1.4$ GeV^2 is very stable with increasing s, we chose c_{pp}, our one parameter, to produce a zero in $T_{pp}(s,t)$ at this point. We find the value $c_{pp} = 2.64$. The results for $d\sigma/dt$ are shown in Fig. 1. Note that the over-all normalization is determined by c; we do not have an over-all normalization constant as in the droplet model of Chou, Wu, and Yang.[8]

The model curve fits the data points quite well in the region 0.2 GeV$^2 < |t|$. It also shows the deviation from exponential behavior as observed in this region. For smaller t the model curve arcs upward and deviates from the experimental points. This is consistent with the trend of the CERN data; however, as the experimental slope is increasing in this region, it is completely believable that it will approach the curve shown.

Our model curve deviates from the experimental $d\sigma/dt$ for large q. This deviation is what one would expect from our approximation. We have approximated the integral $\int_{(2\mu)^2}^{\infty} dM^2 \rho(M^2) \times K_0(Mb)$ by $c_{pp} K_0(2\mu b)$. One expects that $c_{pp} < \int_{(2\mu)^2}^{\infty} dM^2 \rho(M^2)$. Comparing (4.9) with (4.10) we see that our model curve drops too slowly for large q since c_{pp} will be smaller than $\int_{(2\mu)^2}^{\infty} dM^2 \rho(M^2)$, which determines the large-q behavior.

We find for the ratio of σ_{el} to σ_{tot}

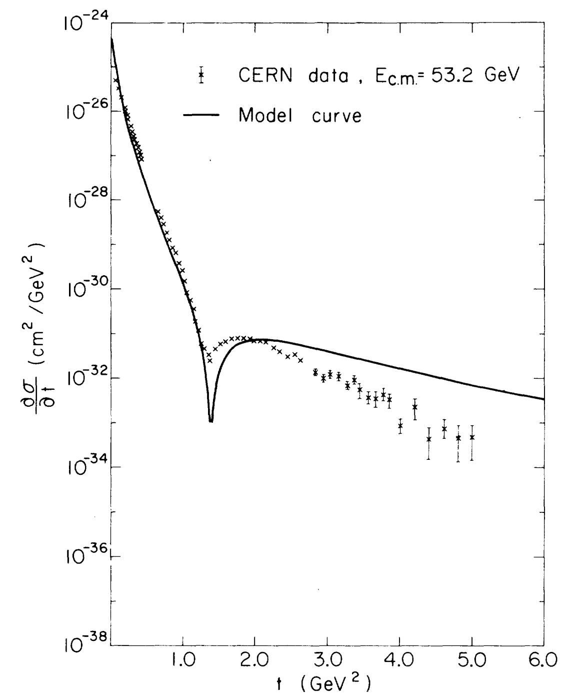

FIG. 1. p-p differential cross section.

$$\frac{\sigma_{el}}{\sigma_{tot}} = 0.2 \,. \tag{5.1}$$

This is to be compared with the experimental result of 0.17.

This model's predictions for asymptotic cross sections are

$$\sigma_{tot} = 114 \text{ mb}, \tag{5.2}$$

$$\sigma_{el} = 23.7 \text{ mb}, \tag{5.3}$$

$$\sigma_{in} = 90.3 \text{ mb}. \tag{5.4}$$

Equation (5.2) is much higher than the present experimental value of

$$\sigma_{tot} \approx 43 \text{ mb}, \quad \sqrt{s} = 53 \text{ GeV}, \tag{5.5}$$

and so is consistent with rising cross sections in the range of energies presently available. We are disappointed that the numbers are so high. We hope that a more accurate approximation for the eikonal function will give lower numbers for (5.2)–(5.4).

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Deep-inelastic contribution for the neutral current

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The effective coupling constants are calculated for the processes $\nu p \to \nu p$ and $K^+ \to \pi^+ \nu\bar{\nu}$ by taking account of the deep-inelastic contribution. The formal light-cone analysis is applied to get the momentum dependence of the structure functions. It is shown that the lowest-order perturbation is small and that the deep-inelastic contribution for the coupling constant is arranged in the form of sum rules which are derived from U(6) × U(6) quark algebra for the process $\nu p \to \nu p$. Sum rules are also obtained for the decay process $K^+ \to \pi^+ \nu\bar{\nu}$ by using SU(3) symmetry where the K^+ and π^+ meson mass difference is neglected. By comparing with the experimental upper bound for the neutral current the following results are obtained: The weak boson mass should be less than 11 GeV/c^2 and the local theory of the weak interaction is valid up to 50 GeV.

I. INTRODUCTION

Recently a renormalizable theory of the weak interaction was proposed.[1] In this connection the problem of the neutral current has been investigated by various authors.[2,3] The absence of the neutral current puts restrictions on the model of the weak interaction; if we assume a charged vector boson as a medium of the weak interaction, we may get the validity of the local theory of the weak interaction from the experimental upper bounds for the neutral current.[4] As knowledge about the structure of the weak interaction is still scarce, the conditions obtained from the neutral current may be quite significant.

In this article we assume that the weak interaction is mediated by a vector boson; the weak boson is supposed to be a local field and couples to the current locally. Hence the neutral current is produced as an effect of fourth or higher order in the coupling constant. Instead of computing the S matrix we calculate the effective coupling constant for the neutral current by taking into account the deep-inelastic contribution as well as the lowest-order perturbation, which is shown to be small. By comparing the coupling constant with the experimental upper bound, one might expect to get the validity of the weak interaction because the expression is cutoff-dependent.

To evaluate the deep-inelastic contribution we apply the formal light-cone analysis to the structure functions to get the scaling laws. As the result, the effective coupling constants for the process $\nu p \to \nu p$ are shown to be simply expressed in terms of the sum rules which are derived by assuming U(6)×U(6) quark algebra. For the $K^+ \to \pi^+\nu\bar{\nu}$ decay process the sum rules are also obtained by assuming SU(3) symmetry where the $K\pi$ mass difference is neglected. Owing to this expression we are able to get not only the upper bound for the cutoff but also the restriction for the weak-boson mass.

This paper is organized as follows: In Sec. II we give a perturbation calculation. In Sec. III we calculate the deep-inelastic contribution to the effective coupling constants, and show that they are expressed in the form of sum rules. In Sec. IV we derive the upper bound for the weak-boson mass and the cutoff. Section V is devoted to discussion.

II. THE PERTURBATION CALCULATION

We investigate the semileptonic process νp νp and $K^+ \to \pi^+\nu\bar{\nu}$ with recourse to the perturbation theory under the assumption that the weak interaction is mediated by a charged vector boson. For these processes we calculate the effective coupling constants by taking a low-energy limit in the S