$K^+ \to \pi^+ \gamma\gamma$ decay in a current-current quark model and a unified approach to weak radiative kaon decays*

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$K^+ \to \pi^+\gamma\gamma$ decay as mediated by $\mathcal{H}_W^{(\text{parity-violating})}$ is studied in a modified fermion-loop model. The effect of $\mathcal{H}_W^{(\text{p.v.})}$ is simulated by a phenomenologically constructed parity-violating meson-baryon-baryon interaction without introducing any new parameters. We find that the predicted branching ratio $r = \Gamma(K^+ \to \pi^+\gamma\gamma)/\Gamma(K^+ \to \text{all}) = 0.64\times 10^{-6}$ is of the same order of magnitude as that of the tree-graph model of Moshe and Singer.

Recently it was shown in a series of papers[1-3] that the fermion-loop model,[4] suitably modified[1-3] for weak interactions, is successful in providing (i) a qualitative[1] explanation for $K_2^0 \to \gamma\gamma$ decay, (ii) a predicted branching ratio for the CP-conserving decay $K_2^0 \to \pi^+\pi^-\gamma^2$,

$$r_0 = R(K_2^0 \to \pi^+\pi^-\gamma)/R(K_2^0 \to \text{all modes}) = 3.0\times 10^{-4},$$

consistent with the tree-graph estimate $2.6\times 10^{-4} < r_0 < 4\times 10^{-4}$ of Moshe and Singer[5] and below the present[6] experimental upper limit ($r_0 < 4\times 10^{-4}$), and (iii) a predicted branching ratio

$$r_\pm = \frac{R(K^\pm \to \pi^\pm\pi^0\gamma;\ 55\text{ MeV} \leq T_{\pi^\pm} \leq 90\text{ MeV})}{R(K^\pm \to \text{all modes})} = 1.6\times 10^{-5}$$

in excellent agreement with the recent experiment of Abrams *et al.*[7] At the same time, one noted the remarkable parallelism and consistency of these predictions for processes mediated by $\mathcal{H}_W^{(\text{parity-conserving})}$ with those of the tree-graph model.[5]

Although one of us (R.R.) has discussed recently[8] the natural enlargement of the fermion-loop model to include also the effects of $\mathcal{H}_W^{(\text{parity-violating})}$, thus enabling consideration of the baryon-antibaryon contributions to the $K_2^0 - K_1^0$ mass difference, where *both* $\mathcal{H}_W^{(\text{p.c.})}$ and $\mathcal{H}_W^{(\text{p.v.})}$ play roles, as was noted there,[8] that calculation does not provide a very stringent test of this now complete yet *still a zero-parameter model for weak decays*.[9] On the other hand, the calculation of $K^+ \to \pi^+\gamma\gamma$ decay in this (now) unified fermion-loop model *does* furnish such a test and, moreover, will be seen to provide another example of the remarkable correlation between the zero-parameter fermion-loop model[1-3,8] and the tree approximation,[5] albeit for a radiative decay process mediated principally by $\mathcal{H}_W^{(\text{p.v.})}$.

In their tree-graph analysis of $K^+ \to \pi^+\gamma\gamma$, Moshe and Singer[5] divide the contributing Feynman graphs into six groups, of which the first five are found to give relatively small contribution.[10] The last group, which Moshe and Singer find to be the dominant contributor to the decay amplitude, arises from the use of a phenomenological Lagrangian with vector-gauge particles and are shown in Fig. 1. The PVV part of the Lagrangian of Ref. 5 entails also four-particle vertices by virtue of the self-interaction term appearing in the covariant curl $V_{\mu\nu}$.[5] These additional four-particle vertices generated by the $V\times V$ term in $V_{\mu\nu}$ have their strength determined by the three-particle ones, this being a direct result of the Yang-Mills form of the Lagrangian. The matrix element for the diagrams of Fig. 1 is[5]

$$\mathfrak{M}^{(\text{MS})} = \frac{4}{3}\frac{G_{\text{NL}}e^2}{\sqrt{2}\,g_\rho^{\,2}}\left(\frac{m_\rho}{m_V}\right)^2 h\,\epsilon^{\alpha\beta\mu\nu}\Big\{C_K(1+\epsilon_1)\Big[K_\nu\epsilon_\beta\epsilon'_\mu(q-q')_\alpha + K_\nu q'_\alpha q_\beta\Big(\frac{K\cdot\epsilon}{K\cdot q}\,\epsilon'_\mu - \frac{K\cdot\epsilon'}{K\cdot q}\,\epsilon_\mu\Big)\Big]$$
$$- C_\pi(1-\tfrac{1}{2}\epsilon_1+\tfrac{3}{4}\epsilon_2)\Big[p^+_\nu\epsilon_\beta\epsilon'_\mu(q-q')_\alpha + p^+_\nu q'_\alpha q_\beta\Big(\frac{K\cdot\epsilon}{K\cdot q}\,\epsilon'_\mu - \frac{K\cdot\epsilon'}{K\cdot q}\,\epsilon_\mu\Big)\Big]\Big\}\,, \qquad (1)$$

where the ϵ_1 are the SU(3)-breaking parameters of Ref. 5 and $h^2/4\pi = 0.1/[m_\pi^{\,2}(1+\epsilon_1)^2]$, m_V is the SU(3)-symmetric vector-meson mass with $m_V = 847$ MeV, and $g_\rho^{\,2}/4\pi \simeq 2.6$. The presence of bremsstrahlung-like terms in Eq. (1) is dictated by the requirement of gauge invariance in the presence of SU(3) breaking.[11] On the other hand, it is instructive to note that the SU(3)-symmetric limit

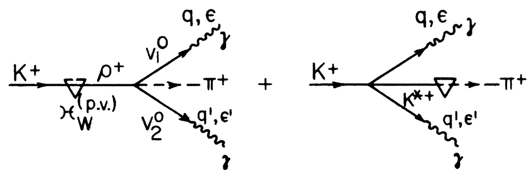

FIG. 1. The classes of diagrams comprising the dominant contribution to $K^+ \to \pi^+\gamma\gamma$ decay in the model of Moshe and Singer.

$$\mathfrak{M}^{(MS)}_{SU(3)} = \frac{4}{3}\frac{G_{NL}\, e^2}{\sqrt{2}\,(g_\rho^{(0)})^2}\, h^{(0)} C \epsilon^{\alpha\beta\mu\nu}(q+q')_\nu\, \epsilon_\beta \epsilon'_\mu (q-q')_\alpha , \tag{2}$$

with $\epsilon_i = 0$, $C_K = C_\pi = C$, is gauge-invariant by itself by virtue of four-momentum conservation $K = p^+ + q + q'$.

Following Ref. 8 we adjoin now to the earlier version[1-3] of the modified (for the weak interaction) fermion-loop model in which *only the parity-conserving part* of the weak Hamiltonian was replaced by the equivalent weak Hamiltonian in Gronau's[12] parametrization

$$\mathcal{H}_W^{(p.c.)} = -\sqrt{2}\, F\, \mathrm{Tr}([\bar{B}, B]\lambda_6) + \sqrt{2}\, D\, \mathrm{Tr}(\{\bar{B}, B\}\lambda_6) , \tag{3}$$

the effective (parity-violating) Hamiltonian,

$$\mathcal{H}_W^{(p.v.)} = -\bar{\psi}_j\, c\, d_{i6l}\left(-i f_{ljk} + \frac{\delta}{\phi}\, d_{ljk}\right)\gamma^\mu \psi_k \partial_\mu \phi_i , \tag{4}$$

which is also assumed valid for off-baryon-mass-shell calculations. [Note that in so doing we have introduced *no new parameters* into the model: Eq. (4) yields Gronau's[12] "K^* contribution to S waves," essential in his fit of S-wave hyperon-decay amplitudes.] δ/ϕ is the D/F ratio for the γ_μ coupling at the strong VBB vertex and the constant c is obtained from the measured $K_S^0 \to \pi^+\pi^-$ decay width, $c = 3.2 \times 10^{-9}$ MeV^{-1}.

Neglecting those contributions mediated by $\mathcal{H}_W^{(p.c.)}$ which have been shown to be small,[5] one finds the fermion-loop (FL) graphs mediated by $\mathcal{H}_W^{(p.v.)}$, which can contribute to $K^+ \to \pi^+\gamma\gamma$ shown in Fig. 2. It is straightforward to show that no contribution to $\mathfrak{M}^{(FL)}$ is made by the group of bremsstrahlung graphs [see Fig. 2(b)] as in the corresponding tree-graph analysis.[5] Indeed, we find that only the contribution from the graphs in Fig. 2(a) survives. We assert that *these graphs comprise the fermion-loop analog of the appropriate dominant contributor of Moshe and Singer*[5] (which class is likewise mediated by $\mathcal{H}_W^{(p.v.)}$). These graphs yield the gauge-invariant amplitude

$$\mathfrak{M}^{(\text{fermion loop})} = (16 K_0^+ p_0^+ q q')^{1/2} \times \langle \gamma(q)\gamma(q')\pi^+(p^+)\text{out}\,|\mathcal{H}_W^{(p.v.)}(0)|\,K^+(K)\rangle .$$

With the aid of the trace identity[13]

$$\mathrm{Tr}(d_i f_j f_k f_l) = \tfrac{3}{4} i \sum_n (d_{ijn} f_{nkl} + d_{lin} f_{njk} + d_{ikn} f_{njl}) , \tag{5}$$

one finds in the zeroth approximation in an expansion in external invariants

$$\mathfrak{M}^{(\text{fermion loop})} = \frac{e^2 g c}{4\phi}\,\frac{(f\delta + d\phi)}{2\pi^2 m} \times \epsilon^{\alpha\beta\mu\nu}(q+q')_\nu \epsilon_\beta \epsilon'_\mu (q-q')_\alpha , \tag{6}$$

where, as in Refs. 1–3, 8, and 12, we take $g^2/4\pi = 14.6$, $\delta/\phi = -0.5$, $d/f = 1.8$, and a "mean" baryonic mass m of 1 GeV. It is natural to compare our result, Eq. (5), with the SU(3)-symmetric limit of Moshe and Singer[5] since we have taken SU(3) to be conserved at vertices and have ignored the breaking in baryonic masses as well. We find that the fermion-loop model prediction, with *no adjustable parameters*,

$$A^{(\text{fermion loop})} = \frac{e^2 g c}{4\phi}\,\frac{(f\delta + d\phi)}{2\pi^2 m} = 2.3 \times 10^{-14}\ \text{MeV}^{-2} , \tag{7a}$$

with

$$\frac{\Gamma^{(\text{fermion loop})}(K^+ \to \pi^+\gamma\gamma)}{\Gamma(K^+ \to \text{all})} = 0.64 \times 10^{-6} , \tag{7b}$$

compares remarkably with the SU(3)-symmetric limit of Moshe and Singer[5,14]:

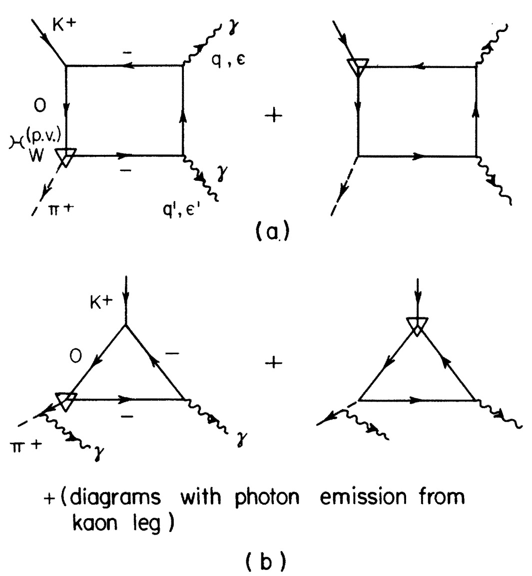

FIG. 2. Classes of diagrams contributing to $K^+ \to \pi^+\gamma\gamma$ decay in the fermion-loop model. (a) Fermion-loop analog of the tree-graph model contributing to $K^+ \to \pi^+\gamma\gamma$ decay. (b) Bremsstrahlung graphs which make no contribution to $K^+ \to \pi^+\gamma\gamma$ decay.

$$A^{(\mathrm{MS})}_{(\mathrm{SU}(3))} = \frac{4}{3}\,\frac{G_{\mathrm{NL}}e^2}{\sqrt{2}\,(g_\rho^{(0)})^2}\,h^{(0)}C$$

$$= 2.0\times 10^{-14}\ \mathrm{MeV}^{-2}, \qquad (8a)$$

with

$$\frac{\Gamma^{(\mathrm{MS})}_{(\mathrm{SU}(3))}(K^+ \to \pi^+\gamma\gamma)}{(K^+ \to \mathrm{all})} = 0.56\times 10^{-6}. \qquad (8b)$$

For completeness, the pion kinetic energy spectrum for the fermion-loop model[5]

$$\frac{d\Gamma}{dt} = (A^{(\text{fermion loop})})^2\,\frac{m_K{}^5}{16\pi^3}\left[t - \tfrac{1}{2}\left(1 - \frac{m_\pi}{m_K}\right)\right]^2 \times\left[\left(t + \frac{m_\pi}{m_K}\right)^2 - \frac{m_\pi{}^2}{m_K{}^2}\right]^{1/2} \qquad (9)$$

is plotted in Fig. 3.

In closing we emphasize that this persistent agreement between predictions of the extended fermion-loop model[1-3] and the tree-graph analysis[5] warrants further investigation. It would certainly be desirable to improve the present experimental limit, which is unfortunately one order of magnitude[15] above our predicted branching ratio (7b).

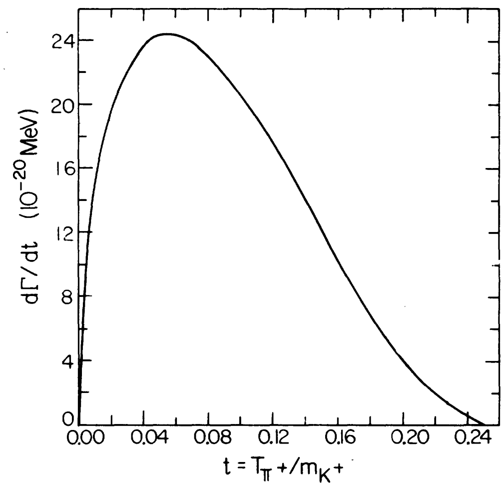

FIG. 3. Pion kinetic-energy spectrum for $K^+ \to \pi^+\gamma\gamma$ in the fermion-loop model [Eq. (8)].

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[2]R. Rockmore and T. F. Wong, Phys. Rev. D 7, 3425 (1973).

[3]R. Rockmore, J. Smith, and T. F. Wong, Phys. Rev. D 8, 3224 (1973).

[4]J. Steinberger, Phys. Rev. 46, 1180 (1949).

[5]M. Moshe and P. Singer, Phys. Rev. Lett. 27, 1685 (1971); Phys. Rev. D 6, 1379 (1972). In this discussion our attention is focused on this, the most successful of such (current-current) models, which also seems the "simplest," in having a minimum of neutral currents.

[6]R. C. Thatcher *et al.*, Phys. Rev. 174, 1674 (1968).

[7]R. J. Abrams *et al.*, Phys. Rev. Lett. 29, 1118 (1972).

[8]R. Rockmore, Phys. Rev. D 8, 3226 (1973).

[9]One can only say that the extended loop model is not inconsistent with present theoretical understanding of the $K_2^0 - K_1^0$ mass difference.

[10]The contribution of the fermion-loop analogs of these which are mediated by $\mathcal{H}_W^{(\mathrm{p.c.})}$ will be discussed elsewhere.

[11]Note that the partial width for $K^+ \to \pi^+\gamma\gamma$ is calculated in the radiation gauge in this case [P. Singer (private communication)].

[12]M. Gronau, Phys. Rev. Lett. 28, 188 (1972); Phys. Rev. D 5, 118 (1972).

[13]The discerning reader will note that since $\delta/\phi \neq 0$, the present calculation is *not* a mere transcription of the MS Lagrangian for $PVVV$ vertices into the language of the fermion-loop model as would otherwise be the case. Consequences of the identity (Ref. 5) and others for the fermion-loop model will be taken up elsewhere.

[14]On the other hand [M. Moshe and P. Singer, unpublished and private communication], they predict

$$\frac{\Gamma^{(\mathrm{MS})}(K^+ \to \pi^+\gamma\gamma)}{\Gamma(K^+ \to \mathrm{all})} = (2.4 \pm 0.7)\times 10^{-6}$$

in the case of SU(3) breaking (see Ref. 5). This is slightly below the lower limit given in Ref. 5 and results from a new over-all fit of their model to several radiative K^+ decays which includes $K^+ \to \pi^+\pi^0\gamma$ not considered up to now.

[15]M. Chen *et al.*, Phys. Rev. Lett. 20, 73 (1968); J. H. Klems *et al.*, *ibid.* 25, 473 (1970); Phys. Rev. D 4, 66 (1971); D. Ljung, Phys. Rev. Lett. 28, 523 (1972); D. Ljung and D. Cline, Phys. Rev. D 8, 1307 (1973); R. J. Abrams *et al.*, BNL Report No. 18017, 1973 (unpublished).