graphs which reverse the sign of the triple-Pomeron term. On the other hand, T. N. Ng and V. P. Sukhatme [University of Washington report, 1973 (unpublished)] use Gribov's Regge calculus with decoupling and find a negative contribution to the total cross section and also an excellent fit to the data.

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[30]These interactions can be easily pictured in impact-parameter space. If the simple multiperipheral graph is represented as a chain in this space which stretches between the target and projectile, then the absorptive graphs arise from interactions which occur between the ends of the chain, between one end and somewhere along the chain, and between two points along the chain. Absorption therefore clearly suppresses those configurations in which the chain is coiled up on itself—it prefers to be as straight as possible. One important effect of this phenomenon is to modify the distribution in the variables conjugate to the impact parameters namely, the relative transverse momenta.

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Decay $L^0 \to \nu_l \gamma$ in gauge theories of weak and electromagnetic interactions*

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Results are presented of a calculation of the rate for the decay $L^0 \to \nu_l \gamma$, where $L^0 = E^0$, M^0 is a neutral heavy lepton occurring in certain gauge theories of weak and electromagnetic interactions. This rate is compared with that for the semileptonic decay mode $L^0 \to l^- \pi^+$, and it is found that, although the former is of order α^3, whereas the latter is of order α^2, the $L^0 \to \nu_l \gamma$ rate can actually exceed the $L^0 \to l^- \pi^+$ rate in models which also have a charged heavy lepton L^+.

Non-Abelian gauge theories of weak and electromagnetic interactions employing the Higgs-Kibble mechanism of spontaneous symmetry breaking have evoked considerable interest since they provide a natural unification of these two classes of interactions and make possible a renormalizable theory of weak interactions.[1] Heavy leptons play an important role in such theories, being required in models which either do not have the neutral massive gauge boson Z, or have it but do not couple it to a $\bar{\nu}\gamma_\alpha \frac{1}{2}(1-\gamma_5)\nu$ neutral current.[2] Several models feature a neutral heavy lepton $L^0 = E^0, M^0$; these include the Georgi-Glashow (GG) model,[3] the second Prentki-Zumino (PZ) model,[4] and the 2-2, 3-2, and 2-3 models of Bjorken and Llewellyn Smith (B-LS).[5]

In this paper we calculate the rate for the decay $L^0 \to \nu_l \gamma$ in these five models. For definiteness we consider the heavy muon lepton M^0; our results apply equally well to the E^0. This decay is of interest, first, because, being completely leptonic, it is exactly calculable and independent of assumptions about how hadrons are included in the various models. Second, experimentally, in view of the difficulty of observing the final particles in the $L^0 \to \nu_l \gamma$ decay, one would like to determine how large the rate is, especially in comparison with the more easily observed charged modes. Since the GG model embodies the essential characteristics of a theory with heavy leptons, we concentrate on it and give only final results for the other models considered.

A brief review of the particle content of these theories may be helpful. The GG model is based on the group O(3) and has as gauge particles the charged intermediate vector bosons $W^\pm$ and the photon γ, but not the neutral gauge boson Z. The scalar fields form a self-conjugate triplet representation of O(3),

$$\begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix}.$$

After the spontaneous symmetry breakdown and shift in the ϕ field, $\phi^0_{\rm ph}=\phi^0-\langle\phi^0\rangle_0$ remains as the physical Higgs scalar, while the unphysical, charged scalars $\phi^\pm$ essentially become the longitudinal degrees of freedom of the (previously massless, now massive) $W^\pm$ fields, respectively, and can be transformed away by a gauge transformation to the unitary or U gauge. The muon leptonic sector of the model includes both a charged heavy lepton M^+ and a neutral heavy lepton M^0, the M^0 being mixed with the neutrino, with mixing angle β. The leptons are arranged in left-handed and right-handed triplets and a left-handed singlet as follows:

$$^{(3)}\psi_L=\begin{pmatrix} M^+ \\ M^0\cos\beta+\nu_\mu\sin\beta \\ \mu^-\end{pmatrix}_L,\quad ^{(3)}\psi_R=\begin{pmatrix} M^+ \\ M^0 \\ \mu^-\end{pmatrix}_R, \tag{1}$$

$$^{(1)}\psi_L=(M^0\sin\beta-\nu_\mu\cos\beta)_L.$$

Because both the multiplet containing $\nu_\mu\sin\beta$ and μ_L and the multiplet containing μ_R are triplets, the GG model is a 3-3 model in the B-LS classification. The strength of the weak interaction is determined by the parameter β; $g_W=\frac{1}{2}e\sin\beta$, so that

$$\frac{G_F}{\sqrt{2}}\equiv\frac{g_W{}^2}{m_W{}^2}=\frac{e^2\sin^2\beta}{4m_W{}^2}. \tag{2}$$

This formula also gives the mass of the W as

$$m_W=53|\sin\beta|\ \text{GeV}. \tag{3}$$

Finally, in the GG theory the lepton masses satisfy the relation

$$m_{M^+}+m_\mu=2m_{M^0}\cos\beta. \tag{4}$$

The other four models are based on the gauge group SU(2)$\times$U(1) and accordingly have as gauge bosons $W^\pm$, Z, and γ. The Z couples to a diagonal neutrino neutral current in the B-LS 2-2 and 2-3 theories, but not in the B-LS 3-2 theory or the second PZ theory. In each of these models the scalar fields form a complex doublet

$$\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}.$$

The field $\phi^0_{\rm ph}=(\frac{1}{2})^{1/2}(\phi^0+\phi^{0\dagger}-2\langle\phi^0\rangle_0)$ remains as the physical Higgs scalar while

$$\chi=\frac{(\phi^0-\phi^{0\dagger})}{i\sqrt{2}}$$

and $\phi^\pm$ become the longitudinal degrees of freedom of the Z and $W^\pm$, respectively. The analogs of Eqs. (2) and (3) for the Fermi constant and the mass of the W in these models are listed below for reference[6]:

$$\text{PZ:}\quad \frac{G_F}{\sqrt{2}}=\frac{e^2}{16m_W{}^2\sin^2\theta},$$

$$\Rightarrow m_W=26|\csc\theta|\ \text{GeV};$$

$$\text{B-LS 2-2 and B-LS 2-3:}\quad \frac{G_F}{\sqrt{2}}=\frac{e^2}{8m_W{}^2\sin^2\theta},$$

$$\Rightarrow m_W=37|\csc\theta|\ \text{GeV}; \tag{6}$$

$$\text{B-LS 3-2:}\quad \frac{G_F}{\sqrt{2}}=\frac{e^2\cos^2\eta}{4m_W{}^2\sin^2\theta},$$

$$\Rightarrow m_W=53|\cos\eta\csc\theta|\ \text{GeV}. \tag{7}$$

As regards the muon leptonic sector, each of these models, with the exception of the B-LS 2-2 model, has, in addition to the M^0, an M^+. For a detailed description of the arrangement of the leptons into multiplets one should consult Refs. 4 and 5; we only mention here that the second PZ model is of the 2-2 type (but differs from the B-LS 2-2 model in its particle content and couplings).

The S-matrix element for the decay $M^0\to\nu_\mu\gamma$ is

$$\begin{aligned}\langle\gamma(q),\nu_\mu(p')|S|M^0(p)\rangle &\\ &=-i(2\pi)^4\delta^4(p-p'-q)\\ &\quad\times\left[\frac{1}{(2\pi)^{9/2}}\frac{1}{(2q_0)^{1/2}}\frac{1}{(2p_0')^{1/2}}\left(\frac{m_{M^0}}{p_0}\right)^{1/2}\right]\mathfrak{M},\end{aligned} \tag{8}$$

where $\mathfrak{M}$ is the invariant matrix element, $\mathfrak{M}=\epsilon^\mu\mathfrak{M}_\mu$, with ϵ^μ = photon polarization vector. For the general case of an off-shell photon (which would be relevant, for example, for the decay $M^0\to\nu_\mu l^+l^-$), $\mathfrak{M}_\mu$ is of the form

$$\begin{aligned}\mathfrak{M}_\mu=\bar{u}_\nu(p')\left(\frac{1+\gamma_5}{2}\right)\bigg[&C_1(q^2)\gamma_\mu+C_2(q^2)i\sigma_{\mu\nu}\frac{q^\nu}{m_{M^0}}\\ &+C_3(q^2)q_\mu\bigg]u_{M^0}(p).\end{aligned} \tag{9}$$

Electromagnetic current conservation or gauge invariance requires that

$$q^\mu\mathfrak{M}_\mu=0$$

or (with $q=p-p'$) (10)

$$C_1(q^2)m_{M^0}+C_3(q^2)q^2=0.$$

Hence, for the actual $M^0\to\nu_\mu\gamma$ decay, $C_1(0)=0$. It should be noted that individual diagrams give nonzero, and in fact divergent, contributions to $C_1(0)$, but these sum to zero in the total amplitude. Furthermore, because $\epsilon\cdot q=0$ for a real photon,

$C_3(0)$ gives no contribution to $\mathfrak{M}$. One therefore only needs to calculate $C_2(0)$. The amplitude must, of course, be non-Abelian gauge-invariant, that is, independent of the gauge parameter ξ which appears in the W and $\phi^\pm$ propagators. In order to examine the nature of the cancellation of ξ-dependent terms in the amplitude, we have performed the calculation in the general renormalizable or R_ξ gauge.[7] To facilitate the evaluation of the integrals we have used the approximation $m_L^2/m_W^2 \ll 1$, where $m_L = \max\{m_{M^+}, m_{M^0}\}$, and have thereby obtained a result accurate to lowest order in this presumably small quantity. This is a reasonable approximation, since the requirement that the weak correction to the anomalous magnetic moment of the muon not spoil the agreement between experiment and the theoretical quantum electrodynamics result (including the hadronic vacuum polarization contribution) constrains the mass of the heavy charged lepton in the GG model to be $m_{M^+} < 5$ GeV.[7,8] For moderate values of β, m_{M^0} is of comparable size. In contrast, from the above requirement on the muon $g-2$, coupled with the lower bound $m_{M^+} > 2$ GeV from a recent Caltech-NAL experiment[9] and the assumption that the diagram involving a virtual Higgs scalar is negligible, one finds that $m_W > 28$ GeV in the GG model.[10] In the other models considered the calculation of the muon anomalous magnetic moment does not constrain the masses of the neutral or charged heavy leptons.[11] It seems plausible, however, that in these theories, as in the GG theory, m_{M^0} and m_{M^+} should be of the order of a few GeV. Furthermore, in three of these theories m_W is guaranteed to be large; specifically, in the B-LS 2-2 and 2-3 models, $m_W > 37$ GeV, while in the second PZ model, $m_W > 26$ GeV. Finally, in the B-LS 3-2 model, for moderate values of the parameter η, m_W is again several tens of GeV. For this approximation to be useful, however, given the fact that terms in the integrals actually appear in the form $m_{M^+}^2/(m_W^2/\xi)$ or $m_{M^0}^2/(m_W^2/\xi)$, it is necessary to limit the gauge parameter ξ to the range $0 \le \xi \ll m_W^2/m_L^2$. This range includes as special cases $\xi = 0$ (U gauge) and $\xi = 1$ ('t Hooft-Feynman gauge).

In general R_ξ gauge, in lowest order, there are twelve possible diagrams which contribute to the decay $M^0 \to \nu_\mu \gamma$ (see Fig. 1). These can be classified into six types, one set of six having μ^- as the internal virtual lepton, and the other set having M^+ as the virtual lepton, with appropriate changes in the charges of the $W^\pm$ and $\phi^\pm$ involved [graphs 1(a), ..., 1(f)]. In U gauge only graphs 1(a) and 1(b) are present, since the others include unphysical, charged scalars $\phi^\pm$ on internal lines. These twelve diagrams are all present in the GG model. However, for models with no M^+, such as the B-LS 2-2 model, or models with an M^+ but no $WM^+\nu_\mu$ coupling (and consequently no $\phi M^+ \nu_\mu$ coupling), such as the B-LS 2-3 model, only the six diagrams having virtual μ^- contribute In addition to these twelve graphs involving only virtual μ^-, M^+, $W^\pm$, and/or $\phi^\pm$ on the internal lines, there are also, in theories with a $ZM^0\nu_\mu$ coupling, graphs involving a virtual Z.[12] But each such graph contributes only to $C_1(0)$, which is irrelevant to the decay amplitude for the reason stated above.

Table I shows the contributions to $C_2(0)$ of each of the six types of graphs, summed for both μ^- and M^+ terms, in the GG model. Graphs 1(e) and 1(f) are, for our range of ξ, smaller by a factor which is $\lesssim \xi m_L^2/m_W^2 \ll 1$ (in which the mass ratio arises from the mass-dependent $\phi^\pm$-lepton couplings) and are therefore negligible. It is interesting to analyze how the non-Abelian gauge-dependent terms cancel among the diagrams. The W propagator in general R_ξ gauge,

$$i\,\Delta_{\alpha\beta}(k) = -i\left(g_{\alpha\beta} - \frac{k_\alpha k_\beta (1-\xi^{-1})}{k^2 - m_W^2/\xi}\right)\Big/ (k^2 - m_W^2)$$
$$= \frac{-i\,(g_{\alpha\beta} - k_\alpha k_\beta/m_W^2)}{k^2 - m_W^2} - \frac{i k_\alpha k_\beta/m_W^2}{k^2 - m_W^2/\xi}\,, \tag{11}$$

has, as its only gauge-dependent part, an unphysical term with a pole at $k^2 = m_W^2/\xi$. ξ-dependent terms originating from this part of the W propa-

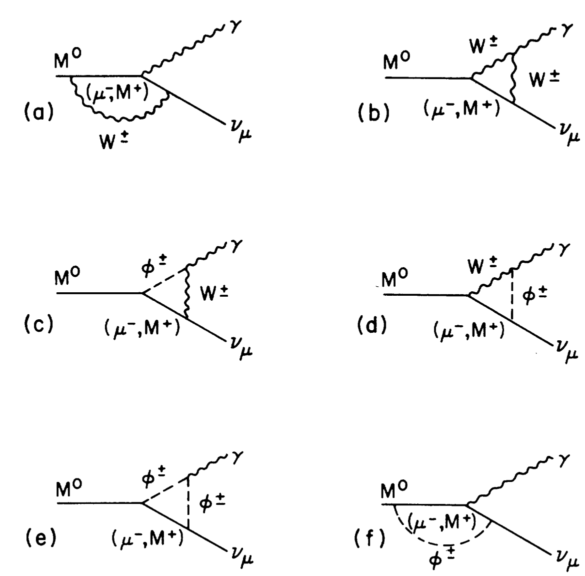

FIG. 1. Diagrams contributing to the decay $M^0 \to \nu_\mu \gamma$ in general R_ξ gauge.

gator in diagrams with internal W lines must cancel against similar terms from diagrams involving virtual $\phi^{\pm}$ with propagators

$$i\Delta(k)=\frac{i}{k^2-m_W{}^2/\xi}\ . \tag{12}$$

From the structure of the graphs, it is evident that the ξ-dependent part of graph 1(a) should cancel with graph 1(f), while the ξ-dependent part of graph 1(b) should cancel with the sum of graphs 1(c), 1(d), and 1(e). Since graphs 1(e) and 1(f) are negligible, at least for our range of ξ, it follows that graph 1(a) must be individually non-Abelian gauge-invariant, and graph 1(b) must have its gauge-dependent part cancel with graphs 1(c) and 1(d). As can be seen from Table I, this is exactly what happens. The final invariant amplitude in the GG model is

$$\mathfrak{M}=\frac{e^3\sin\beta}{4\pi^2}\left[\frac{m_{M^0}(m_{M^+}-m_\mu)}{m_W{}^2}\right]\times\left[\bar{u}_\nu(p')\left(\frac{1+\gamma_5}{2}\right)i\epsilon^\mu\sigma_{\mu\nu}\frac{q^\nu}{m_{M^0}}\,u_{M^0}(p)\right]. \tag{13}$$

One should note that this amplitude arises only from the right-handed part of the $WM^0\mu^-$ and WM^0M^+ vertices, and from the corresponding parts of the $\phi M^0\mu^-$ and ϕM^0M^+ vertices.[13] Similarly, the left-handed part of the $W\nu\mu^-$ and $W\nu M^+$ vertices give zero contribution to the quantity which, for the neutrino, plays a role analogous to that of $C_2(0)$ in the decay $M^0\to\nu_\mu\gamma$, namely, the neutrino's anomalous magnetic moment. It is for this reason that the neutrino, which is entirely left-handed (this of course requires it to be massless) and hence has only left-handed couplings, has no anomalous magnetic moment, as is evident from the fact that such a moment would be given by the expression[14]

$$\bar{u}_\nu(p')\tfrac{1}{2}(1+\gamma_5)\bar{F}_2(0)i\sigma_{\mu\nu}q^\nu\tfrac{1}{2}(1-\gamma_5)u_\nu(p)=0\ . \tag{14}$$

From the amplitude (13) one calculates the rate in the GG model:

$$\Gamma(M^0\to\nu_\mu\gamma)=\frac{\alpha^3}{4\pi^2}\left[\frac{m_{M^0}(m_{M^+}-m_\mu)}{m_W{}^2}\right]^2 m_{M^0}\sin^2\beta. \tag{15}$$

For the B-LS 3-2 model, which, like the GG model, has an M^+, one finds a similar rate[6]:

$$\Gamma(M^0\to\nu_\mu\gamma)_{3-2}=\frac{\alpha^3}{8\pi^2}\left(\frac{m_{M^0}m_{M^+}}{m_W{}^2}\right)^2 m_{M^0}\cos^2\eta\csc^4\theta\,, \tag{16}$$

where η and θ are parametric angles defined in Eq. (7). The $m_{M^0}m_\mu/m_W{}^2$ term is missing because the $WM^0\mu$ vertex has no right-handed part. For the B-LS 2-2 and 2-3 models, as previously noted, only the six graphs with internal muons are present. Accordingly, the $m_{M^0}m_{M^+}/m_W{}^2$ term is absent from the amplitude, and the rates are smaller by the factor $(m_\mu/m_{M^+})^2\ll 1$:

$$\Gamma(M^0\to\nu_\mu\gamma)_{2-2}=\frac{\alpha^3}{16\pi^2}\left(\frac{m_{M^0}m_\mu}{m_W{}^2}\right)^2 m_{M^0}\csc^4\theta\,, \tag{17}$$

$$\Gamma(M^0\to\nu_\mu\gamma)_{2-3}=\frac{\alpha^3}{8\pi^2}\left(\frac{m_{M^0}m_\mu}{m_W{}^2}\right)^2 m_{M^0}\csc^4\theta\,. \tag{18}$$

Finally, the second PZ model has only left-handed couplings for the WM^0M^+ and $WM^0\mu$ vertices and consequently gives, to within our approximation, a zero decay rate.

Returning to the GG model and comparing the $M^0\to\nu_\mu\gamma$ decay with a typical semileptonic decay mode, $M^0\to\mu^-\pi^+$, for which the rate is[5]

$$\Gamma(M^0\to\mu^-\pi^+)_{GG}=\frac{G_F{}^2m_{M^0}{}^3f_\pi{}^2}{16\pi}\,\frac{1+\cos^2\beta}{\sin^2\beta}=\frac{\pi}{8}\alpha^2\frac{m_{M^0}{}^3f_\pi{}^2}{m_W{}^4}\sin^2\beta(1+\cos^2\beta) \tag{19}$$

(with $f_\pi=0.93m_\pi$ the pion decay constant), we obtain the branching ratio

$$\frac{\Gamma(M^0\to\nu_\mu\gamma)_{GG}}{\Gamma(M^0\to\mu^-\pi^+)_{GG}}=\frac{2\alpha}{\pi^3}\left(\frac{m_{M^+}-m_\mu}{f_\pi}\right)^2(1+\cos^2\beta)^{-1}. \tag{20}$$

A similar result holds in the B-LS 3-2 model. Numerically, the branching ratio is

TABLE I. Contributions of diagrams in general R_ξ gauge to $C_2(0)$ in the Georgi-Glashow model. The entries for each of the six types of diagrams represent the sum of the μ^- and the M^+ contributions. Entries should be multiplied by $(e^3/16\pi^2)\sin\beta[m_{M^0}(m_{M^+}-m_\mu)/m_W{}^2]$ to obtain the contribution to $C_2(0)$.

Diagrams	Contribution
1(a)	2
1(b)	$2-\frac{\xi}{\xi-1}\left(1-\frac{\ln\xi}{\xi-1}\right)$
[1(c)+1(d)]	$\frac{\xi}{\xi-1}\left(1-\frac{\ln\xi}{\xi-1}\right)$
1(e)	0
1(f)	0
Total	4

$$\frac{\Gamma(M^0 \to \nu_\mu \gamma)_{GG}}{\Gamma(M^0 \to \mu^- \pi^+)_{GG}} \simeq 0.28 \frac{m_{M^+}{}^2}{\mathrm{GeV}^2} \frac{(1-m_\mu/m_{M^+})^2}{1+\cos^2\beta} . \tag{21}$$

For $m_{M^+} \sim$ a few GeV this is a large ratio; indeed, for $m_{M^+} > 2$ GeV it is larger than unity. For reference, as mentioned earlier, $m_{M^+} > 2$ GeV and, in the GG model, $m_{M^+} < 5$ GeV.[7-9]

This is thus a case where, rather surprisingly, a leptonic decay formally of order α^3 can compete with and dominate over a semileptonic decay of order α^2, in theories with an M^+ and a right-handed part to the WM^0M^+ vertex, because of the occurrence of a large factor $(m_{M^+}/f_\pi)^2$ in the branching ratio. This is similar to what happens in the anomalous magnetic-moment calculation. There, for theories without an M^0, such as the Weinberg[15] and Lee-Prentki-Zumino[4,16] models, $a_\mu^W \sim G_F m_\mu^2$. In contrast, for models with an M^0 and both a right-handed and left-handed part to the $WM^0\mu$ vertex, such as the GG theory, diagrams involving a virtual M^0 occur and give an anomalous magnetic moment larger by the factor m_{M^+}/m_μ; specifically,[7,8]

$$(a_\mu^W)_{GG} \sim -\frac{\alpha}{\pi} \frac{m_\mu m_{M^0}}{m_W{}^2} \cos\beta$$
$$\sim -\frac{G_F}{2\sqrt{2}\,\pi^2} m_\mu m_{M^+} \csc^2\beta .$$

Experimentally, it would be troublesome if the $M^0 \to \nu_\mu \gamma$ decay were a main mode because of the problem of detecting the photon (and the virtual impossibility of detecting the neutrino). Fortunately, the charged decay modes $M^0 \to \mu^-\mu^+\nu_\mu$ and $M^0 \to \mu^- e^+ \nu_e$ are dominant over the neutrino-photon mode. Our rate for $M^0 \to \nu_\mu \gamma$, combined with the result for $M^0 \to \mu^- l^+ \nu_l$ in the GG model,[5]

$$\Gamma(M^0 \to \mu^- l^+ \nu_l)_{GG} \simeq \frac{G_F{}^2 m_{M^0}{}^5}{192\pi^3}\left(\frac{1+\cos^2\beta}{\sin^2\beta} - \frac{4\cos\beta}{\sin^2\beta}\frac{m_\mu}{m_{M^0}}\right), \tag{22}$$

gives (here neglecting terms of order m_μ/m_{M^0}) the branching ratio

$$\frac{\Gamma(M^0 \to \nu_\mu\gamma)_{GG}}{\Gamma(M^0 \to \mu^-\mu^+\nu_\mu)_{GG} + \Gamma(M^0 \to \mu^- e^+ \nu_e)_{GG}} \simeq \frac{12\alpha}{\pi}\left(\frac{m_{M^+}}{m_{M^0}}\right)^2 (1+\cos^2\beta)^{-1}$$
$$\simeq \frac{48\alpha}{\pi} \frac{\cos^2\beta}{1+\cos^2\beta}$$
$$\lesssim 0.12 . \tag{23}$$

This branching ratio is of comparable size in the B-LS 3-2 model, while for the B-LS 2-2 and 2-3 models it is smaller by the factor $(m_\mu/m_{M^+})^2 \ll 1$.

These results also apply to the $E^0 \to \nu_e \gamma$ decay, with appropriate changes in the lepton masses.

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[5]J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973).

[6]We use θ as a generic symbol for the parametric angle in the second PZ and B-LS models; each model can, of course, have a different value for θ. This notation should not cause any confusion.

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$$(a_\mu^W)_{\phi^0 \text{graph}} \sim \frac{\alpha}{8\pi}\left(\frac{m_\mu m_{M^+}}{m_W{}^2}\right)\left(\frac{m_\mu m_{M^+}}{m_{\phi^0}{}^2}\right),$$

is negligible because $m_\mu m_{M^+}/m_{\phi^0}{}^2 \ll 1$. This is a plausible assumption since in all models known at the present time, m_{ϕ^0} is unbounded from above.

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even if $m_{M^+} > m_{M^0}$, the decay of the M^+ into M^0 plus other particles should not be an important decay mode, simply from considerations of phase space.

[10]The lower bound $M_W > 28$ GeV is derived from the results of Refs. 7 and 8, using the fact that $m_{M^+} > 2$ GeV and the assumption that $m_\mu m_{M^+}/m_{\phi^0} << 1$, so that the diagram with a virtual Higgs scalar gives a negligible contribution to a_μ^W. In Ref. 8 the authors, using the best lower bound on m_{M^+} then available, $m_{M^+} > 0.5$ GeV, were able to conclude that $m_W > 18$ GeV in the GG model. For graphs showing the restrictions on m_{M^+} as a function of m_W (and a_μ^W and m_{ϕ^0}), see Fig. 4 of Ref. 7 and Fig. 2 of Ref. 8.

[11]The reason for this is that the constraint on m_{M^+} arises from the large contribution to a_μ^W by the diagrams involving a virtual M^0 (only one in U gauge). These diagrams give a term proportional to

$$-\frac{\alpha}{2\pi}\frac{m_\mu m_{M^0}}{m_W^2}b_L b_R$$

and a smaller term proportional to

$$-\frac{\alpha}{2\pi}\frac{m_\mu^2}{m_W^2}\left(\frac{b_L^2+b_R^2}{4}\right),$$

where b_L and b_R are the coefficients of the left- and right-handed parts of the $WM^0\mu$ vertex,

$$\Gamma^\alpha_{WM^0\mu} = ie\overline{M}^0\gamma^\alpha\left[b_L\tfrac{1}{2}(1-\gamma_5)+b_R\tfrac{1}{2}(1+\gamma_5)\right]\mu .$$

In the GG model, $b_L = \cos\beta$ and $b_R = 1$, whence

$$(a_\mu^W)_{GG} \sim -\frac{\alpha}{\pi}\frac{m_\mu m_{M^0}}{m_W^2}\cos\beta \sim -\frac{\alpha}{2\pi}\frac{m_\mu m_{M^+}}{m_W^2}.$$

This gives the constraint on m_{M^+}, as a function of m_W. However, for the other four models considered, either $b_L = 0$ or $b_R = 0$, so that $\alpha_\mu^W \sim (\alpha/8\pi)(m_\mu^2/M_W^2)$ (as in models without an M^0) and there is no constraint on m_{M^+} or m_{M^0}.

[12]Of the four models possessing a Z, only the second PZ model has a nonzero $ZM^0\nu_\mu$ coupling.

[13]This statement is most clearly evident in U gauge, where one has only to consider the WM^0M^+ and $WM^0\mu^-$ vertices, which are, respectively,

$$\mp ie\overline{X}\gamma^\alpha\{\cos\beta[\tfrac{1}{2}(1-\gamma_5)]+\tfrac{1}{2}(1+\gamma_5)\}M^0,$$

where $\overline{X} = \overline{M}^+, \overline{\mu}^-$. Then the fact that the final amplitude arises only from the right-handed part of this vertex is made manifestly obvious by the absence of $\cos\beta$ terms in the amplitude. Although $\cos\beta$-dependent terms do contribute to each of the individual graphs of type 1(a) and 1(b), they cancel between the μ^- and M^+ graphs for each type. The reason for this cancellation is that the left-handed part of the WM^0M^+ ($WM^0\mu^-$) vertex contains a chiral projection operator $\frac{1}{2}(1-\gamma_5)$ which, when commuted through the terms in the numerator of the integral for a given graph until it multiplies the $\frac{1}{2}(1+\gamma_5)$ for the outgoing neutrino, leaves only terms which depend only on m_{M^0}, not m_{M^+} or m_μ. These terms are thus the same for a μ^- graph and an M^+ graph of the same type. Because the amplitudes for the μ^- graphs have opposite signs from those for the M^+ graphs, and because the denominators of the integrals for the μ^- graphs of a given type differ from those of the integrals for the M^+ graphs only by the interchange of m_μ^2 and $m_{M^+}^2$, both small compared to m_W^2, the above-mentioned terms cancel between these μ^- and M^+ graphs for each type of graph, to leading order in the parameter $m_L^2/m_W^2 << 1$. In general R_ξ gauge there are also the ϕM^0M^+ and $\phi M^0\mu^-$ vertices to consider. The complication is that corresponding to the right-handed part of the WM^0M^+ ($WM^0\mu^-$) vertex there is both a right-handed and a left-handed part to the ϕM^0M^+ ($\phi M^0\mu^-$) vertex, and similarly for the left-handed part of the WM^0M^+ ($WM^0\mu^-$) vertex. However, for graph 1(c), these terms originating from the part of the ϕM^0M^+ ($\phi M^0\mu^-$) vertex with corresponds to the left-handed part of the WM^0M^+ ($WM^0\mu^-$) vertex (that is, the $\cos\beta$-dependent terms) cancel in the manner described above. Finally, for graph 1(d), there are no $\cos\beta$-dependent terms in the amplitude.

[14]As defined here, $\tilde{F}_2(0)$ has dimensions of inverse mass. The situation with regard to $C_2(0)$ in M^0 decay differs from that with regard to $\tilde{F}_2(0)$ for the neutrino in that for $C_2(0)$ the left-handed contributions do not vanish for any individual graph [except for graph 1(d)], but rather cancel to leading order in m_L^2/m_W^2, between μ^- and M^+ graphs. For the neutrino $\tilde{F}_2(0)$, the (necessarily left-handed) contributions of each μ^- and M^+ graph are individually and exactly zero. Note that, since the left-handed terms in $C_2(0)$ are proportional to m_{M^0}, they would, as in the neutrino case, vanish for each graph separately as $m_{M^0} \to 0$.

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[16]B. W. Lee, Phys. Rev. D 6, 1188 (1972).