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Transformation between current and constituent quarks and transitions between hadrons*

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The transformation from current- to constituent-quark basis states is discussed. Certain algebraic properties of the transformed vector and axial-vector currents are abstracted from the free-quark model and assumed to hold in nature. Supplemented by the partially conserved axial-vector current hypothesis and assumptions about the identification of the observed hadrons with simple constituent-quark states, the algebraic properties of the transformed currents are used to compute the pion and photon transitions between any two hadron states. General selection rules are stated. Many specific matrix elements for both meson and baryon decays are tabulated, and both their magnitudes and signs are compared with experiment.

I. INTRODUCTION

For almost a decade, the constituent-quark model[1,2] has given us a very successful classification scheme for most hadrons. In such a model nonexotic baryons are treated as composed of qqq, while mesons are $q\bar{q}$, with internal orbital angular momentum between the spin-$\frac{1}{2}$ quarks. As a result, particles fall in simple multiplets[3] of SU(6) $\times$O(3).

At the same time there has been a considerable amount of work on the classification of hadron states under the algebra of chiral SU(2)$\times$SU(2), or more generally, chiral SU(3)$\times$SU(3).[4-6] It is clear that hadron states, like the nucleon, share single irreducible representations of the chiral algebra with many other states, for the nucleon and many higher-mass N^*'s are connected by a generator of the algebra, Q_5, in the form of the pion field. Conversely, the nucleon state is a complicated mixture of many irreducible representations. Although important progress was made in both the purely theoretical and phenomenological classification of hadron states under chiral SU(3)$\times$SU(3) in the past,[7] previous work in this direction has suffered from being done on a case-by-case, somewhat *ad hoc* basis, with each hadron (or small set of hadrons) treated separately. Little systematic connection between the classification of different hadrons was found.

Recently, by relating these two classification schemes for hadrons, there has been what we consider major progress, principally due to the work of Melosh.[8] He approaches the problem by trying to relate two sets of operators: those of an SU(6)$_W$ of currents [including the SU(3)$\times$SU(3) formed by the vector and axial-vector charges] and an SU(6)$_W$ of strong interactions. However, we may equivalently consider the relation between their basis states: the irreducible representations of the algebra of currents (built up out of "current quarks") and the irreducible representations of the SU(6)$_W$ algebra of strong interactions (built up out of "constituent quarks").[9] Therefore, postulating a relation between the two algebras will give us the decomposition of physical hadron states, assumed to be simply identifiable with constituent quark states, into irreducible representations of the algebra of currents. A complete knowledge of the relation between the two algebras would then allow us to solve the problem of the classification of hadrons under the algebra of currents.

We shall assume that a transformation between constituent- and current-quark basis states exists. Without a detailed dynamics we cannot completely specify the transformation. However, we shall assume certain algebraic properties of the transformed axial-vector charge and first moment of the vector-current density. These algebraic properties are abstracted from the free-quark model, following the work of Melosh.[8] As a result, we have a theory of the algebraic structure of weak or electromagnetic transitions between hadrons. This theory is (1) simple in its algebraic properties, (2) systematic in treating all mesons and baryons in a unified way, and (3) definite in that it has a clear origin and structure, with the amplitudes related by Clebsch-Gordan coefficients and the decay widths related to the amplitudes involved in the theory in a nonarbitrary, known way.

Such a theory can be regarded as one more step in a program of abstracting algebraic properties from the free-quark model, without necessitating the reality either of free quarks themselves or of any picture of quarks bound in a "potential" from which they cannot escape. Abstraction from the free-quark model assures us that the assumed algebraic properties could be exact, and are at least consistent with relativity, invariance principles, etc. They are presumably the least complicated properties that one might expect to hold in the real world. The present theory greatly unifies the treatment of weak and electromagnetic transitions with the systematics of hadron spectroscopy, and produces well-defined quantitative predictions.

While vector-current-induced transitions are immediately testable through comparison with photon amplitudes, present weak-interaction data are generally insufficient to provide a test of transitions involving the axial-vector current. To provide tests of this part of the theory at the present time we must assume the partially conserved axial-vector current hypothesis (PCAC),[10] which relates matrix elements of the axial-vector charge taken between states at infinite momentum to those of the pion field taken between the same hadron states. With the assumption of PCAC, the theory becomes one of the algebraic structure of pion and photon transitions between hadrons. Expanding upon our previous work,[11] in this paper we investigate the general structure of such a theory and show what it predicts in detail for specific pion and photon transitions between both meson and baryon states.

As a theory of pion transitions, the present paper has much in common as far as general algebraic structure is concerned with both previous relativistic quark-model calculations[12] and certain broken-$SU(6)_W$ schemes.[13] We in fact regard this theory of current-induced transitions, supplemented by PCAC and/or vector-meson dominance, as providing a method of constructing a phenomenology of purely hadronic vertices and providing justification for some aspects of these other theoretical schemes.

In the next section we discuss the transformation from basis states of the $SU(6)_W$ of currents to that of strong interactions, and what we abstract of the algebraic properties found in the free-quark model by Melosh.[8] The applications of this algebraic structure to pion and photon decays, when supplemented by PCAC and assumptions on the identification of the observed hadrons with constituent quark states, is described in Sec. III. After stating general selection rules and comparing with other theoretical schemes, we tabulate specific matrix elements for pionic decays of mesons, pionic decays of baryons, and photonic decays of baryons in Secs. IV, V, and VI, respectively. Where possible a preliminary comparison with experiment of both their magnitudes and signs is made. We conclude with a discussion of the present theoretical and experimental situation and possible directions for further extension of the theory.

II. THE TRANSFORMATION FROM CURRENT TO CONSTITUENT QUARKS

Consider the algebra formed by the 16 vector and axial-vector charges, $Q^\alpha(t)$ and $Q_5^\alpha(t)$, which are simply integrals over all space of the time components of the corresponding currents measurable in weak and electromagnetic interactions:

$$Q^\alpha(t)=\int d^3x\, V_0^\alpha(\vec{x},t)\,, \tag{2.1a}$$

$$Q_5^\alpha(t)=\int d^3x\, A_0^\alpha(\vec{x},t)\,. \tag{2.1b}$$

Here α is an SU(3) index which runs from 1 to 8. At equal times these charges commute to form the algebra proposed by Gell-Mann,[14]

$$[Q^\alpha(t),Q^\beta(t)]=if^{\alpha\beta\gamma}Q^\gamma(t)\,, \tag{2.2a}$$

$$[Q^\alpha(t),Q_5^\beta(t)]=if^{\alpha\beta\gamma}Q_5^\gamma(t)\,, \tag{2.2b}$$

$$[Q_5^\alpha(t),Q_5^\beta(t)]=if^{\alpha\beta\gamma}Q^\gamma(t)\,. \tag{2.2c}$$

This is the algebra of chiral SU(3)×SU(3), for it can be easily shown that Eq. (2.2) is equivalent to the statement that the right-handed charges, $Q^\alpha+Q_5^\alpha$, and the left-handed charges, $Q^\alpha-Q_5^\alpha$, each form an SU(3), and that they commute with each other—hence, chiral SU(3)×SU(3). For $\alpha=1,2,3$ the Q^α's are the generators of isospin rotations; for $\alpha=1,\ldots,8$ they are the generators of SU(3). The last of Eqs. (2.2), sandwiched between nucleon states moving at infinite momentum in the z direction, yields the Adler-Weisberger sum rule.[15]

Taken between states at infinite momentum,[16] the Q^α's and Q_5^α's are "good" operators, i.e., they have finite (generally nonvanishing) values as $p_z\to\infty$. These values are the same as those of space integrals over the z components of the respective currents. If we adjoin to the space integrals of the time component of the vector currents and the z component of the axial-vector currents integrals over certain "good" tensor current densities, the SU(3)×SU(3) algebra between states at infinite momentum can be enlarged still further to form an $SU(6)_W$ algebra of 35 generators whose elements commute like the products of SU(3) and Dirac matrices, $\frac{1}{2}\lambda^\alpha$, $\frac{1}{2}\lambda^\alpha\beta\sigma_x$, $\frac{1}{2}\lambda^\alpha\beta\sigma_y$, $\frac{1}{2}\lambda^\alpha\sigma_z$. We refer to this algebra, introduced by Dashen and Gell-Mann[17] in 1965, as the $SU(6)_W$ of currents. We denote these generators collectively by F^i.

In what follows it will be convenient to label states or operators by their transformation properties under this algebra of currents. For this purpose we shall use just the SU(3)×SU(3) subalgebra of the whole $SU(6)_W$ algebra of currents to write

$$\{(A,B)_{S_z}, L_z\}\ ,$$

where A is the SU(3) representation under $Q^\alpha + Q_5^\alpha$, B the representation under $Q^\alpha - Q_5^\alpha$, and S_z is the eigenvalue of Q_5^0, the singlet axial-vector charge.[18] The quantity L_z is then defined in terms of the z component of the total angular momentum J as $L_z = J_z - S_z$. The "ordinary" (Q^α) SU(3) content of such a representation is just that of the direct product $A\times B$.

With such labeling it is clear that, for example,

$$Q_5^\alpha = \tfrac{1}{2}(Q^\alpha + Q_5^\alpha) - \tfrac{1}{2}(Q^\alpha - Q_5^\alpha)$$

transforms as $\{(8,1)_0, 0\} - \{(1,8)_0, 0\}$, while Q^α transforms as $\{(8,1)_0, 0\} + \{(1,8)_0, 0\}$. Representations of SU(3)×SU(3) can be built up from $(3,1)_{1/2}$, $(1,3)_{-1/2}$, $(1,\bar{3})_{1/2}$, and $(\bar{3},1)_{-1/2}$, which we define to be the *current quark* and *current antiquark* states with z spin projection $\pm\frac{1}{2}$. Therefore, if a nucleon at infinite momentum with $J_z = \frac{1}{2}$ acted under the algebra of currents as if it were simply composed of two current quarks with $S_z = \frac{1}{2}$ and one quark with $S_z = -\frac{1}{2}$ in a symmetrical wave function, we would have

$$|N\rangle = |\{(6,3)_{1/2}, 0\}\rangle\ . \tag{2.3}$$

However, the SU(3) content of $(6,3)_{1/2}$ is just that of an octet (including the nucleon) and a decuplet [including the Δ(1236)]. Since Q_5^α is a generator of SU(3)×SU(3), it can only connect this representation to itself, i.e., the nucleon to the nucleon or to the Δ(1236). Furthermore, such a classification of the nucleon gives $g_A = \frac{5}{3}$. The nucleon *cannot* be in such a simple representation. This is already apparent from the Adler-Weisberger sum rule[15] itself, for it shows that the nucleon is connected by a generator of the algebra, the axial-vector charge Q_5^α (in the form of the pion field through the use of PCAC), to many higher-mass N^*'s. Thus the nucleon and these N^*'s must be in the same representation of SU(3)×SU(3). Conversely, the nucleon state must span many different representations of the SU(3)×SU(3) of currents.

An attempt[19] to describe approximately the nucleon state in terms of a sum of irreducible representations of SU(3)×SU(3) yields

$$|N\rangle = \cos\theta\,|\{(6,3)_{1/2},0\}\rangle + \sin\theta(\sin\phi\,|\{(\bar{3},3)_{1/2},0\}\rangle + \cos\phi[\cos\psi\,|\{(8,1)_{3/2},-1\}\rangle + \sin\psi\,|\{(3,\bar{3})_{-1/2},1\}\rangle])\ , \tag{2.4}$$

where θ, ϕ, and ψ are parameters to be fitted phenomenologically. It is clear that parametrizing states in a manner resembling the complicated nucleon wave function in Eq. (2.4) is not the way to proceed in order to understand systematically the pionic decays of higher resonances. The number of phenomenological parameters would increase so as to render the approach essentially useless.

Instead, one may assume[8,9,20] that there exists a unitary operator V which transforms an irreducible representation (I.R.) of the algebra of currents into the physical state:

$$|\text{hadron}\rangle = V\,|\text{I.R., currents}\rangle\ . \tag{2.5}$$

The state $|\text{I.R., currents}\rangle$ is chosen as that irreducible representation of the algebra of currents which corresponds to baryons being built from just three current quarks, and mesons from quark-antiquark. Thus, for example, the complicated nucleon state in Eq. (2.4) is rewritten as

$$|N\rangle = V\,|\{(6,3)_{1/2},0\}\rangle\ . \tag{2.6}$$

All the complicated mixing of the real hadron states has been subsumed in the operator V.

In the following we will be interested in evaluating the hadronic matrix elements of a current, say Q_5^α. Using Eq. (2.5) we have

$$\langle\text{hadron}'|Q_5^\alpha|\text{hadron}\rangle = \langle\text{I.R.}',\ \text{currents}|V^{-1}Q_5^\alpha V|\text{I.R., currents}\rangle. \tag{2.7}$$

The complications of hadronic states under the algebra of currents have now been transferred to the effective operator $V^{-1}Q_5^\alpha V$, which may be studied as an independent object. Moreover, if the operator $V^{-}Q_5^\alpha V$ has simple transformation properties under the algebra of currents, the way is now open to evaluate systematically the matrix elements of Q_5^α between any two hadronic states.

Such is indeed the case in the free-quark model, as shown by Melosh.[8]

The operator V serves another useful purpose. It is easy to see that if we define a new set of generators

$$W^i = VF^iV^{-1}, \tag{2.8}$$

then the W^i *also form an* $SU(6)_W$ *algebra* and furthermore, from the definition of V in Eq. (2.5), *hadron states transform as irreducible representations* corresponding to the naive constituent-quark model of hadrons. We therefore call the basis states of this new $SU(6)_W$ "constituent quarks" and identify the algebra with that of the $SU(6)_W$ of strong interactions.[21] Equation (2.5) can therefore be be rewritten as

$$|\text{hadron}\rangle = |\text{I.R., constituents}\rangle = V|\text{I.R., currents}\rangle, \tag{2.9}$$

while Eq. (2.7) becomes

$$\langle\text{hadron}'|Q_5^\alpha|\text{hadron}\rangle = \langle\text{I.R.}', \text{constituents}|Q_5^\alpha|\text{I.R., constituents}\rangle = \langle\text{I.R.}', \text{currents}|V^{-1}Q_5^\alpha V|\text{I.R., currents}\rangle. \tag{2.10}$$

In the free-quark model, the $SU(6)_W$ of strong interactions would be identical with the $SU(6)_W$ of currents if the quarks were restricted to have momentum purely in the z direction ($p_\perp = 0$). It is the transverse momentum of quarks which is the reason for breaking the identity of the two symmetries. This is intuitive if we keep in mind that the $SU(6)_W$ of strong interactions was conceived as a collinear symmetry.

In the present paper we will be primarily concerned with applications of the algebraic properties of the transformed axial-vector charge, $V^{-1}Q_5^\alpha V$, where Q_5 is defined in Eq. (2.1b), and the transformed first moment of the vector current, $V^{-1}D_\pm^\alpha V$, where $D_\pm^\alpha$ is defined as

$$D_\pm^\alpha = \int d^3x \left[\frac{\mp(x \pm iy)}{\sqrt{2}}\right] V_0(\vec{x}, t). \tag{2.11}$$

Taken between states at infinite momentum, commutators of Q_5^α lead to Adler-Weisberger sum rules,[15] while commutators of $D_\pm^\alpha$ lead to Cabibbo-Radicati sum rules.[22] Their properties under the algebra of currents are that

$$Q_5^\alpha \text{ transforms as } \{(8,1)_0 - (1,8)_0, 0\} \tag{2.12a}$$

and

$$D_\pm^\alpha \text{ transforms as } \{(8,1)_0 + (1,8)_0, \pm 1\}. \tag{2.12b}$$

In a free-quark model at $p_z = \infty$, either $V^{-1}Q_5^\alpha V$ or $V^{-1}D_\pm^\alpha V$ must connect only single-quark states to single-quark states; they thus have the general form

$$V^{-1}Q_5^\alpha V \text{ or } V^{-1}D_\pm^\alpha V = \int d^3x\, q^+(x)\Theta(\partial_\perp, \gamma_i)\tfrac{1}{2}\lambda^\alpha q(x), \tag{2.13}$$

where Θ is some function of the transverse derivatives ($\partial_\perp$) and the gamma matrices (γ_i). An explicit form of Θ was determined by Melosh,[8] while Eichten *et al.*[23] argue that a large class of such functions exist. Without having a detailed dynamical formalism we are unable to make use of an explicit form, even if it were given. What is important for our purpose is that the operator is a "single quark" operator; i.e., it depends only on the coordinates of a single quark and it does not create connected $q\bar{q}$ pairs.

It is this property that we abstract from the free-quark model and assume to hold in nature. In general, we assume that *the operators* $V^{-1}Q_5^\alpha V$ *and* $V^{-1}D_\pm^\alpha V$ *have the transformation properties of the most general linear combination of single quark operators consistent with SU(3) and Lorentz invariance.*

Exactly this is verified in the explicit free-quark-model calculations.[8,23] As SU(3) is assumed conserved there, we have $V^{-1}Q^\alpha V = Q^\alpha$. The operator $V^{-1}Q_5^\alpha V$, with $J_z = 0$, contains two terms which transform under SU(3)×SU(3) as $\{(8,1)_0 - (1,8)_0, 0\}$ and $\{(3,\bar{3})_1, -1\} - \{(\bar{3},3)_{-1}, 1\}$ and behave as components of $\underline{35}$'s of the full $SU(6)_W$ of currents. The operator $V^{-1}D_+^\alpha V$, with $J_z = 1$, is slightly more complicated, with three terms[24] which transform as $\{(8,1)_0 + (1,8)_0, 1\}$, $\{(3,\bar{3})_1, 0\}$, and $\{(\bar{3},3)_{-1}, 2\}$, again in $\underline{35}$'s of the $SU(6)_W$ of currents.[25] Thus, in spite of the enormous complication of V itself, we abstract these remarkably simple algebraic properties of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_+^\alpha V$ from the free-quark model and postulate them to hold in the real world. We proceed to apply this hypothesis to transitions between hadrons.

III. THE APPLICATION OF THE ALGEBRAIC STRUCTURE OF TRANSFORMED CURRENTS TO PHOTON AND PION DECAYS

To carry through this application of the algebraic structure of transformed currents to photon and pion decays of real hadrons, we need several additional physical assumptions. First, to relate matrix elements of Q_5 between states at infinite momentum to matrix elements of the pion field we need the PCAC hypothesis.[10] Explicitly, for $\alpha = 1, 2, 3$ we assume

$$\partial_\mu A_\mu^\alpha(x) = \frac{m_\pi^{\,2}}{\sqrt{2}} f_\pi \phi_\pi^\alpha(x), \tag{3.1}$$

where $A_\mu^\alpha(x)$ is the axial-vector current and f_π

$\simeq 135$ MeV is a constant related to the charged pion decay rate. The decay rate for hadron′ → hadron + π^- can then be computed in the narrow-resonance approximation in terms of matrix elements of $(1/\sqrt{2})(Q_5^1 - iQ_5^2)$ between states at infinite momentum as

$$\Gamma(\text{hadron}' \to \text{hadron} + \pi^-) = \frac{1}{(4\pi f_\pi^2)} \frac{p_\pi}{2J'+1} \frac{(M'^2 - M^2)^2}{M'^2} \sum_\lambda \left| \left\langle \text{hadron}', \lambda \left| \frac{1}{\sqrt{2}} (Q_5^1 - iQ_5^2) \right| \text{hadron}, \lambda \right\rangle \right|^2 , \qquad (3.2)$$

where p_π is the pion momentum and the sum extends over all the possible common helicities λ of the hadrons. The total width, Γ(hadron′ → hadron + π), may be obtained from Eq. (3.2) by adding the π^+ and π^0 widths, which are related by isospin Clebsch-Gordan coefficients. Equation (3.2) may also be obtained in a more clearly covariant way by considering the narrow-resonance approximation to the hadron′ intermediate-state contribution to the Adler-Weisberger sum rule obtained by taking Eq. (2.2c) between hadron states at infinite momentum. In either case, we see that the width for hadron′ → hadron + π is directly fixed by matrix elements of Q_5, up to the validity[26] of PCAC. As a result, there are *no arbitrary phase-space factors* in the calculation.

For photon decays we need no additional assumption to relate the width to the matrix element of the D_+^α operator of Eq. (2.11) taken between states at infinite momentum. We have directly that in the narrow-resonance approximation

$$\Gamma(\text{hadron}' \to \text{hadron} + \gamma) = \frac{e^2}{\pi} \frac{p_\gamma{}^3}{2J'+1} \sum_\lambda \left| \left\langle \text{hadron}', \lambda \left| D_+^3 + \frac{1}{\sqrt{3}} D_+^8 \right| \text{hadron}, \lambda - 1 \right\rangle \right|^2 , \qquad (3.3)$$

where e is the proton charge and p_γ the photon momentum, and the sum extends over all possible helicities λ. Note that although the definition of D_+^α in Eq. (2.11) involves only a first moment of the current, between states at infinite momentum all multipole amplitudes consistent with the spin and parity of the states enter matrix elements of D_+^α. Equation (3.3) may also be obtained from consideration of the narrow-resonance approximation to the hadron′ contribution to the Cabibbo-Radicati sum rule[22] on hadron states. Again we have no aribtrary phase-space factors.

In this paper we shall use the narrow-resonance-approximation expressions, Eqs. (3.2) and (3.3), for pion and photon decay widths in order to make a comparison of the theory with experiment. For broad resonances in the initial and/or final state, or for decays of resonances where the physically available phase space is small, such an approximation introduces non-negligible errors.[27] However, we view the present comparison as being sufficiently accurate as a first test of the theory, particularly in view of the experimental errors on values for pion or photon decay widths. When the situation warrants it, the values of

$$|\langle \text{hadron}' | Q_5^\alpha | \text{hadron} \rangle|^2$$

and

$$|\langle \text{hadron}' | D_+^\alpha | \text{hadron} \rangle|^2$$

should be determined irrespective of any approximation in terms of contributions to Adler-Weisberger and Cabibbo-Radicati sum rules, respectively.

Second, we need to identify the observed (nonexotic) hadrons with constituent quark states.[3] In other words, we assume that there is a portion of the physical Hilbert space which is well approximated by the single-particle states of the constituent quark model. For baryons, composed of qqq, we have the familiar SU(6) representations $\underline{56}$ $L=0^+$, $\underline{70}$ $L=1^-$, $\underline{56}$ $L=2^+$, etc., where L is the internal quark angular momentum. For mesons we have correspondingly the $q\bar{q}$ states $\underline{35}$ $L=0^-$, $\underline{1}$ $L=0^-$, $\underline{35}$ $L=1^+$, etc.

Third, we assume that states with different values of the quark spin as well as L_z and S_z are related as in the constituent-quark model, i.e., by the $SU(6)_W$ of strong interactions. This will allow us to relate different helicity states of a given hadron to each other.

Our calculation then proceeds as follows. The matrix elements (between states at infinite momentum) of Q_5^α or D_+^α which we wish to determine are those which enter the expressions for widths in Eqs. (3.2) and (3.3). In either case, we transform to an $SU(6)_W$-of-currents basis as in Eq. (2.10), so that $V^{-1}Q_5^\alpha V$ or $V^{-1}D_\pm^\alpha V$ are taken between irreducible representations of the algebra of currents. Now recall that we assume that results for the algebraic structure of $V^{-1}Q_5^\alpha V$ and $V^{-1}D_\pm^\alpha V$ which are found in the free-quark model are also to be found in nature. More specifically, we assume that these transformation properties are, respectively, the following:

$$V^{-1}Q_5^\alpha V \text{ transforms as } \{(8,1)_0-(1,8)_0, 0\} \text{ and } \{(3,\bar{3})_1,-1\}-\{(\bar{3},3)_{-1},1\}, \tag{3.4}$$

and

$$V^{-1}D_+^\alpha V \text{ transforms as } \{(8,1)_0+(1,8)_0, 1\} \text{ and } \{(3,\bar{3})_1,0\} \text{ and } \{(\bar{3},3)_{-1},2\}. \tag{3.5}$$

All these operators transform like components of $\underline{35}$'s under the $SU(6)_W$ of currents. We then know the algebraic properties (under the algebra of currents) of all terms of a transformed matrix element. Therefore we may use the Wigner-Eckart theorem and tables of Clebsch-Gordan coefficients to carry out the calculation from this point onward. Note that $SU(6)_W$ invariance of the transition operator under either the algebra of currents or that of strong interactions is not assumed—only the transformation properties of the various terms are needed in the calculation. We make no additional assumption in this paper that the $\{(8,1)_0-(1,8)_0,0\}$ term in $V^{-1}Q_5^\alpha V$ is related to Q_5^α, as in Ref. 28.

More explicitly, for a given matrix element of Q_5^α we write the initial and final hadron states with $J_z=\lambda$ in terms of states with definite quark L_z and S_z. This involves coupling internal L and S to form total J for each hadron. After transforming to an $SU(6)_W$-of-currents basis, the matrix element of the[29] $(8,1)_0-(1,8)_0$ or the $(3,\bar{3})_1-(\bar{3},3)_{-1}$ term in $V^{-1}Q_5^\alpha V$ can then be written by the Wigner-Eckart theorem applied to representations of the $SU(6)_W$ of currents as a reduced matrix element times the product of quark angular momentum, $SU(6)_W$, SU(3), and W-spin Clebsch-Gordan coefficients.[30-32] For example, suppose we were calculating the matrix element of the $(8,1)_0-(1,8)_0$ piece of $V^{-1}Q_5^\alpha V$ between initial and final states with common helicity λ, total angular momenta J and J', internal quark angular momenta L and L', quark spins S and S', SU(6) representations R and R', and SU(3) representations A and A', respectively. Then we have that

$$\langle R',A',L',S',J',\lambda,\text{ currents}|\{(8,1)_0-(1,8)_0,0\}|R,A,L,S,J,\lambda,\text{ currents}\rangle$$
$$=\sum_{S_z S_z'} \underbrace{(L'L_z'S'S_z'|J'\lambda)(LL_zSS_z|J\lambda)}_{\text{quark angular momentum Clebsch-Gordan coefficient}} \underbrace{(R'|\underline{35}|R)}_{SU(6)_W \text{ Clebsch-Gordan coefficient}} \underbrace{(A'|\underline{8}|A)}_{SU(3)\text{ Clebsch-Gordan coefficient}} \underbrace{(1\,0\,WW_z|W'W_z')}_{W\text{-spin Clebsch-Gordan coefficient}}$$
$$\times \underbrace{\langle R',L',L_z'\|(8,1)_0-(1,8)_0\|R,L,L_z\rangle}_{\text{reduced matrix element}}. \tag{3.6}$$

The W-spin Clebsch-Gordan coefficient follows since the $(8,1)_0-(1,8)_0$ operator has $W=1$ and $W_z=0$. For any state, $W_z=S_z$. For baryons, $\vec{W}=\vec{S}$, while for mesons we have the conventional correspondence[21]

$$\begin{aligned}
|W=1,\ W_z=1\rangle &= |S=1,\ S_z=1\rangle,\\
|W=1,\ W_z=0\rangle &= -|S=0,\ S_z=0\rangle,\\
|W=1,\ W_z=-1\rangle &= -|S=1,\ S_z=-1\rangle,\\
|W=0,\ W_z=0\rangle &= -|S=1,\ S_z=0\rangle.
\end{aligned} \tag{3.7}$$

The signs which result from using Eq. (3.7) to convert from quark spin to W spin are understood to be included in Eq. (3.6) in the $SU(6)_W$ Clebsch-Gordan coefficient.

The reduction of the $(3,\bar{3})_1-(\bar{3},3)_{-1}$ piece of $V^{-1}Q_5^\alpha V$ proceeds just as above, except that from Eq. (3.7) it transforms under W spin as $\{W=1,\ W_z=1\}+\{W=1,\ W_z=-1\}$. As a result, the sum in Eq. (3.6) is replaced by two sums involving the W-spin Clebsch-Gordan coefficients $(1\,1\,WW_z|W'W_z')$ and $(1\,-1\,WW_z|W'W_z')$. For photon decays we need only recall that $(8,1)_0+(1,8)_0$ is a $W=0$, $W_z=0$ object, while $(3,\bar{3})_1$ and $(\bar{3},3)_{-1}$ transform as $\{W=1,\ W_z=1\}$ and $-\{W=1,\ W_z=-1\}$. Since the net J_z initially and finally must be the same for either hadron' → hadron + π or hadron' → hadron + γ decays, and since the net value of $W_z=S_z$ must also be the same by the W-spin Clebsch-Gordan coefficient in Eq. (3.6) and its analogs, it follows that $L_z=J_z-S_z$ must also be additively conserved between the initial and final states (including the pion or photon operator).

The general algebraic structure of the results is now apparent.[33,34] All the Q_5^α matrix elements taken between hadron states in two given SU(6) multiplets with given L_z and L_z' are related to at most one nonzero independent $SU(6)_W$ reduced matrix element, corresponding to the $\{(8,1)_0+(1,8)_0,0\}$, $\{(3,\bar{3})_1,-1\}$, or $-\{(\bar{3},3)_{-1},1\}$ piece of $V^{-1}Q_5^\alpha V$. Similarly, there is at most one independent $SU(6)_W$ reduced matrix element for photon decays between states in two given SU(6) multiplets with given values of L_z and L_z'. If L is zero, as is the case in essentially all cases of physical interest at the present, then of course $L_z=0$ and the L_z' dependence of the $SU(6)_W$ reduced matrix element becomes trivial [in particular, the $\{(3,\overline{3})_1, -1\}$ and $-\{(\overline{3},3)_{-1}, 1\}$ pieces of $V^{-1}Q_5^\alpha V$, with $L_z'=-1$ and $+1$, respectively, have the same reduced matrix element]. In such a case ($L=0$) there are at most two independent reduced matrix elements of Q_5^α and three independent reduced matrix elements of D_+^α taken between two given SU(6) multiplets.

Instead of describing pion decays in terms of the matrix elements taken between states with given helicity λ, one could describe them in terms of the orbital angular momentum l between the final hadron and pion. To carry through an identification of the l waves present, note that all the λ dependence of the decay amplitudes is given explicitly by the $SU(6)_W$ reduced matrix element and the product of three Clebsch-Gordan coefficients in Eq. (3.6) and its analog for the $(3,\overline{3})_1-(\overline{3},3)_{-1}$ term. From this we can deduce the l amplitudes involved in the decay. In particular, the W-spin Clebsch-Gordan coefficient implies that in vector form

$$\vec{W}'=\vec{W}+\vec{1}, \tag{3.8}$$

which is the same as

$$\vec{S}'=\vec{S}+\vec{1} \tag{3.9}$$

for baryons. Angular momentum conservation for the total decay and for the internal (quark) angular momentum and spin of each hadron are respectively

$$\vec{J}'=\vec{J}+\vec{1}, \tag{3.10}$$

$$\vec{J}'=\vec{L}'+\vec{S}', \tag{3.11}$$

and

$$\vec{J}=\vec{L}+\vec{S}. \tag{3.12}$$

Simple substitution of Eqs. (3.9), (3.11), and (3.12) into (3.10), together with the laws of addition of angular momentum, gives the result

$$|L-L'|-1\le l\le L+L'+1.$$

If $L'\ne L$, then $|L-L'|-1=||L-L'|-1|$. Since in the case $L=L'$ parity forces $l\ge 1=||L-L'|-1|$, we can write in either case[35]

$$||L-L'|-1|\le l\le |L+L'+1| \tag{3.13}$$

for pion decays of baryons. The same result in fact holds for mesons.[36]

In the particular case $L=0$ we have

$$|L'-1|\le l\le |L'+1| \tag{3.14}$$

and parity then forces the nontrivial result that the decays proceed only in the two partial waves

$$l=L'-1 \text{ and } L'+1, \tag{3.15}$$

where in principle other values could be present. Thus, in the particular case $L=0$ there are the same number of l values and reduced matrix elements. In general this is *not* true.

Direct manipulation of Eqs. (3.6) may be used to show that if $L'=L$, *and* the reduced matrix element in Eq. (3.6) is assumed[37] independent of $L_z=L_z'$, then the decays through $(8,1)_0-(1,8)_0$ proceed entirely in p wave ($l=1$), although all waves from $l=1$ to $l=2L+1$ are expected from Eq. (3.13). No such simplification holds for the $(3,\overline{3})_1-(\overline{3},3)_{-1}$ piece of $V^{-1}Q_5^\alpha V$ in general.

Similar results may be derived for photon decays if we simply replace $\vec{1}$ by the photon's angular momentum $\vec{j}_\gamma$, which is formed from the combination of its spin and orbital angular momentum. Thus we have in general that[38]

$$||L-L'|-1|\le j_\gamma\le |L+L'+1| \tag{3.16}$$

for photon decays between multiplets with internal quark angular momentum L and L', respectively.

The general algebraic structure of the theory presented here has much in common with relativistic quark-model calculations, such as those of Ref. 12. In fact, the results of Ref. 12 may be cast into a form which permits the complete identification of certain parameters there with the reduced matrix elements discussed here. However, the assumption of a "potential" in the quark-model calculations yields definite predictions of the reduced matrix elements themselves as they depend on masses and other parameters of the model, which is something we do not obtain using purely the algebraic structure discussed in Sec. II. Also very similar in algebraic structure, at least for decays to $L=0$ hadrons, are some broken-$SU(6)_W$ schemes.[13] The relation of such schemes, in particular l-broken $SU(6)_W$, to the present theory is discussed in detail in Ref. 33. The results of unbroken $SU(6)_W$ for pion transitions correspond to retaining only the $\{(8,1)_0-(1,8)_0, 0\}$ term in $V^{-1}Q_5^\alpha V$.

IV. THE PIONIC TRANSITIONS OF MESONS

With the basic features and assumptions of the theory described in the previous sections, we are in a position to apply it. We begin with the pionic decays of mesons. Only nonstrange meson decays will be discussed in detail, as all the corresponding strange-meson decay rates are related to those we calculate by SU(3). At the present time they add little to the experimental tests of the theory.

In the case of pion transitions among the lowest-

TABLE I. Decays of nonstrange $\underline{35}$ $L'=1$ mesons by pion emission to $\underline{35}$ $L=0$ mesons. The ω, σ, and f are assumed to be ideal mixtures of SU(3) singlets and octets so as to be composed on nonstrange quarks. Zweig's rule (Ref. 2) is used to relate decay amplitudes involving the $\underline{35}$ and $\underline{1}$ parts of the $\lambda=0$, ω, σ, and f. The η and H are assumed to be pure octet. The reduced matrix elements are defined in the text (see Ref. 41).

Decay		a	b	c	d
$A_2(I=1, J^{PC}=2^{++}) \to \pi^-\rho^+$,	$\lambda=1$	$-\frac{1}{8}\sqrt{3}$	$\frac{1}{12}\sqrt{6}$	$-\frac{1}{8}\sqrt{3}\,D$	$\frac{3}{80}D^2$
$B(I=1, J^{PC}=1^{+-}) \to \pi^-\omega^0$,	$\lambda=0$	$-\frac{1}{8}\sqrt{6}$	0	$-\frac{1}{24}\sqrt{6}\,(S+2D)$	$\frac{1}{96}(S^2+2D^2)$
	$\lambda=1$	0	$-\frac{1}{6}\sqrt{3}$	$-\frac{1}{24}\sqrt{6}\,(S-D)$	
$A_1(I=1, J^{PC}=1^{++}) \to \pi^-\rho^+$,	$\lambda=0$	0	$\frac{1}{6}\sqrt{6}$	$\frac{1}{12}\sqrt{3}\,(S-D)$	$\frac{1}{48}(2S^2+D^2)$
	$\lambda=1$	$\frac{1}{8}\sqrt{3}$	$\frac{1}{12}\sqrt{6}$	$\frac{1}{24}\sqrt{3}\,(2S+D)$	
$A_2(I=1, J^{PC}=2^{++}) \to \pi^-\eta^0$,	$\lambda=0$	$-\frac{1}{12}\sqrt{3}$	$\frac{1}{18}\sqrt{6}$	$-\frac{1}{12}\sqrt{3}\,D$	$\frac{1}{240}D^2$
$\delta(I=1, J^{PC}=0^{++}) \to \pi^-\eta$,	$\lambda=0$	$\frac{1}{24}\sqrt{6}$	$\frac{1}{9}\sqrt{3}$	$\frac{1}{24}\sqrt{6}\,S$	$\frac{1}{96}S^2$
$f(I=1, J^{PC}=2^{++}) \to \pi^-\pi^+$,	$\lambda=0$	$\frac{1}{4}$	$-\frac{1}{6}\sqrt{2}$	$\frac{1}{4}D$	$\frac{3}{160}D^2$
$\sigma(I=0, J^{PC}=0^{++}) \to \pi^-\pi^+$,	$\lambda=0$	$-\frac{1}{8}\sqrt{2}$	$-\frac{1}{3}$	$-\frac{1}{8}\sqrt{2}\,S$	$\frac{3}{64}S^2$
$H(I=0, J^{PC}=1^{+-}) \to \pi^-\rho^+$,	$\lambda=0$	$\frac{1}{8}\sqrt{2}$	0	$\frac{1}{24}\sqrt{2}\,(S+2D)$	$\frac{1}{96}(S^2+2D^2)$
	$\lambda=1$	0	$\frac{1}{6}$	$\frac{1}{24}\sqrt{2}\,(S-D)$	

[a]Coefficient of $\langle L'=1\|(8,1)_0-(1,8)_0\|L=0\rangle$.
[b]Coefficient of $\langle L'=1\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|L=0\rangle$.
[c]l-amplitude representation.
[d]g^2.

lying mesons, those in a $\underline{35}$ of SU(6) with quark angular momentum $L=0$, the situation is particularly simple. For $L'=0 \to L=0$ transitions only the $\{(8,1)_0-(1,8)_0, 0\}$ term in $V^{-1}Q_5^\alpha V$ can contribute and is purely $l=1$ in character. This single independent reduced matrix element then forces a relation between the Q_5 matrix elements for the two nonstrange transitions $\rho\to\pi$ and $\omega\to\rho$. Extracting these matrix elements from[39] $\Gamma(\rho\to\pi\pi)$ and from $\Gamma(\omega\to\pi\gamma)$ plus vector dominance, respectively, we find good agreement between theory and experiment.[40]

The pionic transitions from $\underline{35}$ $L'=1$ to $\underline{35}$ $L=0$ are more complicated. From Sec. III there are in general two independent reduced matrix elements,[41] which we write as

$$\langle L'=1\,\|\,(8,1)_0-(1,8)_0\,\|\,L=0\rangle$$

and

$$\langle L'=1\,\|\,(3,\bar{3})_1-(\bar{3},3)_{-1}\,\|\,L=0\rangle\,.$$

The coefficients of these two reduced matrix elements, calculated according to Eq. (3.6), for each possible nonstrange decay are presented in Table I, together with the possible hadron states which correspond to the constituent-quark-model states.

In Table I we list the amplitudes for the specific decay: hadron' $(L'=1)\to\pi^-+$hadron $(L=0)$. We have assumed that the η and H are purely octet in character, but have taken the ω, σ, and f to be ideally mixed combinations of singlets and octets, which results in their being composed of only nonstrange quarks. For decays involving the $\lambda=0$, ω, σ, and f mesons, Zweig's rule[2] has been invoked to relate the decay amplitudes which originate from the SU(6)_W $\underline{1}$ and $\underline{35}$ parts of their constituent quark states.[42] This gives a factor of $\sqrt{3}$ in the decay amplitudes over what is calculated under the assumptions that the ω, σ, and f are purely octet in character [and purely in a $\underline{35}$ of SU(6)_W].

For experimental reasons it is also useful to represent these helicity amplitudes in terms of amplitudes which correspond to definite angular momentum properties between the final hadron and pion. Recalling from the last section [Eq. (3.15)] that only $l=0$ and 2 are allowed here, there turns out to be a linear relation between the two reduced matrix elements and the two amplitudes with definite l properties, which we call S and D.[43] We choose the normalization such that

$$\begin{aligned}\langle L'=1\,\|\,(8,1)_0-(1,8)_0\,\|\,L=0\rangle &= \tfrac{1}{3}(S+2D),\\ \langle L'=1\,\|\,(3,\bar{3})_1-(\bar{3},3)_{-1}\,\|\,L=0\rangle &= +\tfrac{1}{4}\sqrt{2}\,(S-D),\end{aligned} \tag{4.1}$$

and therefore

$$S=D=\langle L'=1\,\|\,(8,1)_0-(1,8)_0\,\|\,L=0\rangle$$

if

$$\langle L'=1\,\|\,(3,\bar{3})_1-(\bar{3},3)_{-1}\,\|\,L=0\rangle$$

were to vanish. The representation of the helicity amplitudes in terms of S and D is also given in Table I. Finally, for completeness, we list for each decay the quantity

TABLE II. Comparison of predictions for $\underline{35}L'=1 \to \underline{35}\ L=0$ pionic decays from Table I with experiment. The predictions in the table correspond to $S=-2D$, i.e., the vanishing of $\langle L'=1\|(8,1)_0-(1,8)_0\|L=0\rangle$. Values of Γ(experimental) taken from Ref. 39.

Decay	Γ (predicted) (MeV)	Γ (experimental) (MeV)
$A_2(1310)\to\pi\rho$	77 (input)	77 ± 20
$B(1235)\to\pi\omega,\ \lambda=0$	0 (input)	dominantly $\lambda=1$
$B(1235)\to\pi\omega,\ \lambda=1$	76	100 ± 20 total width
$A_1(1070)\to\pi\rho,\ \lambda=0$	52	?
$A_1(1070)\to\pi\rho,\ \lambda=1$	26	
$A_2(1310)\to\pi\eta$	17	16 ± 4
$\delta(975)\to\pi\eta$	37	~60 total width
$f(1260)\to\pi\pi$	118	125 ± 25
$\sigma(760?)\to\pi\pi$	234	broad?

$$g^2=\frac{1}{2J'+1}\sum_{\alpha,\lambda}|\langle \text{hadron}',\lambda|Q_5^\alpha|\text{hadron},\lambda\rangle|^2, \qquad (4.2)$$

where the charge state of $|\text{hadron}'\rangle$ is fixed, but that of $|\text{hadron}\rangle$ is summed over along with the index α corresponding to the pion charge. Equation (3.2) shows that g^2 is the total pion decay width except for a factor $p_\pi(M'^2-M^2)^2/(4\pi f_\pi{}^2M'^2)$ which depends only on masses and the PCAC constant. As parity conservation establishes that the helicity $\pm\lambda$ matrix elements have the same magnitude, we need only calculate the $\lambda\geq0$ matrix elements, as in Table I, to carry out the sum over λ in Eq. (4.2).

A comparison of the results of Table I with the present experimental situation is contained in Table II. For this comparison we have used[39] $\Gamma(A_2\to\pi\rho)=77$ MeV and[44] $\Gamma_{\lambda=0}(B\to\pi\omega)=0$ as input. This latter condition is in agreement with experiments which see a dominantly transverse decay, and corresponds to setting[45]

$$\langle L'=1\|(8,1)_0-(1,8)_0\|L=0\rangle=0.$$

All amplitudes are then multiples of

$$\langle L'=1\|(3,\bar 3)_1-(\bar 3,3)_{-1}\|L=0\rangle\,.$$

The following are predictions of particular interest.

(1) $\Gamma(B\to\pi\omega)$ agrees within errors with $\pi\omega$ being the dominant (and so far, the only observed) mode out of a total B width[39] of 100 ± 20 MeV.

(2) $\Gamma(f\to\pi\pi)$ is in excellent agreement with experiment. Use of a d-wave phase-space factor (and relating the coupling constant to that of $A_2\to\pi\rho$ as in Table I) instead of the PCAC-dictated factor changes the prediction by more than a factor of 2, destroying the agreement.

(3) $\Gamma(A_2\to\pi\eta)$ is in excellent agreement with experiment.

(4) We predict a relatively narrow $A_1\to\pi\rho$ with a dominantly longitudinal character. This is obviously not the nonresonance observed[46] in $\pi^\pm p\to(3\pi)^\pm p$, and there is no established state with which to compare our prediction.

(5) $\Gamma(\delta\to\pi\eta)$ agrees with the roughly known[39] total width.

(6) We have somewhat arbitrarily assigned the σ a mass of 760 MeV. While it is gratifying that the resulting $\Gamma(\sigma\to\pi\pi)$ is broad, the uncertainties in identifying the nonstrange quark state with an observed $J^{PC}=0^{++}$ hadron are very large.

Overall, we find that experiment and theory compare very favorably for $L'=1\to L=0$ pionic decays of mesons. Encouraged by this, we consider $L'=1\to L=1$ transitions. A list of the possible transition amplitudes appears in Table III. We note that, including the dependence on L_z' and L_z,

TABLE III. Transitions of nonstrange $\underline{35}$ $L'=1$ mesons to other $\underline{35}$ $L=1$ mesons by pion emission. The notation for labeling the states is as in Table I, with the σ, D, and f assumed to be ideal mixtures of SU(3) singlets and octets so as to be composed of nonstrange quarks. Zweig's rule (Ref. 2) is used to relate decay amplitudes involving the $SU(6)_W$ $\underline{35}$ and $\underline{1}$ parts of the $\lambda=0$ f, D, and σ, and the $\lambda=1$ f and D. The reduced matrix elements are defined in the text (see Ref. 47).

Decay	a	b	c
$B\to\pi^-\delta^+,\ \lambda=0$	$\frac{1}{6}$	0	$\frac{1}{4}\sqrt2$
$B\to\pi^-A_1^+,\ \lambda=1$	0	$-\frac{1}{12}\sqrt6$	$\frac{1}{8}\sqrt3$
$A_2\to\pi^-B^+,\ \lambda=0$	$-\frac{1}{6}\sqrt2$	0	$\frac14$
$\lambda=1$	0	$-\frac{1}{12}\sqrt6$	$-\frac18\sqrt3$
$D\to\pi^-\delta^+,\ \lambda=0$	0	$-\frac16\sqrt2$	$\frac14$
$D\to\pi^-A_1^+,\ \lambda=1$	$\frac{1}{12}\sqrt3$	0	$\frac18\sqrt6$
$D\to\pi^-A_2^+,\ \lambda=0$	0	$-\frac16$	$-\frac14\sqrt2$
$\lambda=1$	$-\frac{1}{12}\sqrt3$	0	0
$A_1\to\pi^-\sigma,\ \lambda=0$	0	$\frac16\sqrt2$	$-\frac14$
$f\to\pi^-A_1^+,\ \lambda=0$	0	$-\frac16$	$-\frac14\sqrt2$
$\lambda=1$	$-\frac{1}{12}\sqrt3$	0	0
$f\to\pi^-A_2^+,\ \lambda=1$	$\frac{1}{12}\sqrt3$	0	$-\frac18\sqrt6$
$\lambda=2$	$\frac16\sqrt3$	0	0
$H\to\pi^-B^+,\ \lambda=1$	0	0	0

[a]Coefficient of

$\langle L'=1,\ L_z'=0\|(8,1)_0-(1,8)_0\|L=1,\ L_z=0\rangle$.

[b]Coefficient of

$\langle L'=1,\ L_z'=1\|(8,1)_0-(1,8)_0\|L=1,\ L_z=1\rangle$.

[c]Coefficient of

$\langle L'=1,\ L_z'=1\|(3,\bar3)_1-(\bar3,3)_{-1}\|L=1,\ L_z=0\rangle$.

TABLE IV. Decays of nonstrange $\underline{35}\,L'=2$ mesons by pion emission to $\underline{35}\ L=0$ mesons. The ω, ω_3, D_2, and ω' are assumed to be ideal mixtures of SU(3) singlets and octets so as to be composed of nonstrange quarks. Zweig's rule is used to relate decay amplitudes involving the $\underline{35}$ and $\underline{1}$ parts of these states. The reduced matrix elements are defined in the text (see Ref. 49).

Decay	a	b	c	d
$g(I=1,\ J^{PC}=3^{--})\to\pi^-\pi^+,\ \lambda=0$	$-\frac{1}{10}\sqrt{5}$	$+\frac{1}{20}\sqrt{30}$	$-\frac{1}{10}\sqrt{5}F$	$\frac{1}{140}F^2$
$g(I=1,\ J^{PC}=3^{--})\to\pi^-\omega^0,\ \lambda=1$	$-\frac{1}{30}\sqrt{30}$	$+\frac{1}{10}\sqrt{5}$	$-\frac{1}{30}\sqrt{30}F$	$\frac{1}{105}F^2$
$F_1(I=1,\ J^{PC}=2^{--})\to\pi^-\omega^0,\ \lambda=0$	0	$\frac{1}{4}\sqrt{3}$	$\frac{1}{10}\sqrt{2}(P-F)$	$\frac{1}{300}(3P^2+2F^2)$
$\lambda=1$	$\frac{1}{12}\sqrt{6}$	$\frac{1}{8}$	$\frac{1}{60}\sqrt{6}(3P+2F)$	
$\rho'(I=1,\ J^{PC}=1^{--})\to\pi^-\pi^+,\ \lambda=0$	$\frac{1}{30}\sqrt{30}$	$\frac{3}{20}\sqrt{5}$	$\frac{1}{30}\sqrt{30}P$	$\frac{1}{90}P^2$
$\rho'(I=1,\ J^{PC}=1^{--})\to\pi^-\omega^0,\ \lambda=1$	$-\frac{1}{60}\sqrt{30}$	$-\frac{3}{40}\sqrt{5}$	$-\frac{1}{60}\sqrt{30}P$	$\frac{1}{180}P^2$
$A_3(I=1,\ J^{PC}=2^{-+})\to\pi^-\rho^-,\ \lambda=0$	$-\frac{1}{6}\sqrt{3}$	0	$-\frac{1}{30}\sqrt{3}(2P+3F)$	$\frac{1}{150}(2P^2+3F^2)$
$\lambda=1$	0	$-\frac{1}{8}\sqrt{6}$	$-\frac{1}{10}(P-F)$	
$\omega_3(I=0,\ J^{PC}=3^{--})\to\pi^-\rho^+,\ \lambda=1$	$\frac{1}{30}\sqrt{30}$	$-\frac{1}{10}\sqrt{5}$	$\frac{1}{30}\sqrt{30}F$	$\frac{1}{35}F^2$
$D_2(I=0,\ J^{PC}=2^{--})\to\pi^-\rho^+,\ \lambda=0$	0	$-\frac{1}{4}\sqrt{3}$	$-\frac{1}{10}\sqrt{2}(P-F)$	$\frac{1}{100}(3P^2+2F^2)$
$\lambda=1$	$-\frac{1}{12}\sqrt{6}$	$-\frac{1}{8}$	$-\frac{1}{60}\sqrt{6}(3P+2F)$	
$\omega'(I=0,\ J^{PC}=1^{--})\to\pi^-\rho^+,\ \lambda=1$	$\frac{1}{60}\sqrt{30}$	$\frac{3}{40}\sqrt{5}$	$\frac{1}{60}\sqrt{30}P$	$\frac{1}{60}P^2$

[a] Coefficient of $\langle L'=2\,\|\,(8,1)_0-(1,8)_0\,\|\,L=0\rangle$.
[b] Coefficient of $\langle L'=2\,\|\,(3,\bar{3})_1-(\bar{3},3)_{-1}\,\|\,L=0\rangle$.
[c] l-amplitude representation.
[d] g^2.

there are three possible independent reduced matrix elements,[47,48] two from the $(8,1)_0-(1,8)_0$ term and one from the $(3,\bar{3})_1-(\bar{3},3)_{-1}$ term in $V^{-1}Q_5^\alpha V$, respectively. Of all these transitions only $D\to\pi\delta$ is both kinematically allowed and presently observed.[39] Both future partial-width measurements and the extraction of coupling constants from exchanges in two-body scattering amplitudes may permit experimental checks of these relations in the future.

Of more relevance to present experiments are the pionic decays $\underline{35}\ L'=2\to\underline{35}\ L=0$. The decay matrix elements are listed in Table IV, both in terms of the reduced matrix elements[49]

$$\langle L'=2\,\|\,(8,1)_0-(1,8)_0\,\|\,L=0\rangle$$

and

$$\langle L'=2\,\|\,(3,\bar{3})_1-(\bar{3},3)_{-1}\,\|\,L=0\rangle$$

and in terms of amplitudes P and F corresponding to values of $l=1$ and 3. Their relation is

$$\begin{aligned}&\langle L'=2\,\|(8,1)_0-(1,8)_0\,\|L=0\rangle=\tfrac{1}{5}(2P+3F)\,,\\&\langle L'=2\,\|(3,3)_1-(3,3)_{-1}\,\|L=0\rangle=\tfrac{2}{15}\sqrt{6}(P-F)\,.\end{aligned}\qquad(4.3)$$

Again, the quantity g^2 is given in the last column of Table IV for the various possible decays.

There is a paucity of detailed information with which to check these predictions, but some preliminary indications are available. For example, we predict $\Gamma(g\to\pi\pi)/\Gamma(g\to\pi\omega)\simeq 1.5$, while one analysis[50] gives 1.4 ± 0.7. Similarly, experiment[39] for $\Gamma(g\to\pi\pi)$ and Table IV give $\Gamma(\omega_3\to\pi\rho)\sim 120$ MeV, while it is known that $\pi\rho$ is the dominant decay mode of the $\omega(1675)$ with a total width of 141 ± 17 MeV.

A problem of current interest is the classification and decay modes of a second $I=1$, $J^{PC}=1^{--}$ vector meson, the $\rho'(1600)$. It may be either a member of a "radially excited" $\underline{35}\ L=0$, part of a $\underline{35}\ L=2$ (see Table IV), or some mixture of the two. The most surprising experimental observation concerning its decay modes is the lack of a strong $\rho'\to\pi\pi$ amplitude ($\rho'\to\rho\pi\pi$ is dominant).[51] As no other p-wave decay of the type $\underline{35}\ L'=2\to\underline{35}\ L=0$ has been observed, we cannot predict $\Gamma(\rho'\to\pi\pi)$ from Table IV on the assumption it lies in the $\underline{35}\ L=2$ multiplet. However, whether the ρ' and its ω' partner are in a $\underline{35}\ L'=2$ or in a $\underline{35}\ L'=0$ "radial excitation," the smallness of $\Gamma(\rho'\to\pi\pi)$ and the theory forces $\Gamma(\omega'\to\pi\rho)$ and $\Gamma(\rho'\to\pi\omega)$ to be small also.[52] Observation of any two of these decay modes could provide an interesting test of the theory and of the classification of the corresponding states.

While we have calculated other meson decays, e.g. $\underline{35}\ L'=2\to\underline{35}\ L=1$, nothing particularly simple or presently testable emerges from the straightforward calculations. The over-all situation, however, is quite encouraging. There is not only general success for $\underline{35}\ L'=1\to\underline{35}\ L=0$ decays, but consistency with[45]

TABLE V. Decays of nonstrange $\underline{70}$ $L'=1$ baryons into $\underline{56}$ $L=0$ baryons by pion emission. States in the $\underline{70}$ $L'=1$ are labeled by J^P and [SU(3) multiplet]$^{2S+1}$, where S is the quark spin. The reduced matrix elements are defined in the text (see Ref. 54).

Decay			a	b	c	d
D_{15}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$\frac{1}{30}\sqrt{5}$	$-\frac{1}{30}\sqrt{5}$	$\frac{1}{30}\sqrt{5}D$	$\frac{1}{360}D^2$
$\frac{5}{2}^-[8]^4$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{60}\sqrt{10}$	$\frac{1}{60}\sqrt{10}$	$-\frac{1}{60}\sqrt{10}D$	$\frac{1}{180}\,7D^2$
		$\lambda=\frac{3}{2}$	$-\frac{1}{30}\sqrt{15}$	$\frac{1}{30}\sqrt{15}$	$-\frac{1}{30}\sqrt{15}D$	
D_{13}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{80}\sqrt{5}$	$\frac{1}{80}\sqrt{5}$	$-\frac{1}{80}\sqrt{5}D$	$\frac{1}{2160}D^2$
$\frac{3}{2}^-[8]^4$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$+\frac{1}{180}\sqrt{10}$	$\frac{7}{90}\sqrt{10}$	$+\frac{1}{180}\sqrt{10}(5S-4D)$	$\frac{1}{540}(25S^2+16D^2)$
		$\lambda=\frac{3}{2}$	$\frac{1}{20}\sqrt{10}$	$\frac{1}{30}\sqrt{10}$	$\frac{1}{180}\sqrt{10}(5S+4D)$	
S_{11}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{18}$	$-\frac{1}{9}$	$-\frac{1}{18}S$	$\frac{1}{216}S^2$
$\frac{1}{2}^-[8]^4$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$\frac{1}{36}\sqrt{2}$	$-\frac{1}{36}\sqrt{2}$	$\frac{1}{36}\sqrt{2}D$	$\frac{1}{108}D^2$
D_{13}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$\frac{1}{9}\sqrt{2}$	$-\frac{1}{9}\sqrt{2}$	$\frac{1}{9}\sqrt{2}D$	$\frac{1}{54}D^2$
$\frac{3}{2}^-[8]^2$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{9}$	$-\frac{1}{18}$	$-\frac{1}{18}(S+D)$	$\frac{1}{54}(S^2+D^2)$
		$\lambda=\frac{3}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{18}(S-D)$	
S_{11}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{9}$	$-\frac{2}{9}$	$-\frac{1}{9}S$	$\frac{1}{54}S^2$
$\frac{1}{2}^-[8]^2$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$\frac{1}{18}\sqrt{2}$	$-\frac{1}{18}\sqrt{2}$	$\frac{1}{18}\sqrt{2}D$	$\frac{1}{27}D^2$
D_{33}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$-\frac{1}{36}\sqrt{2}$	$\frac{1}{36}\sqrt{2}$	$-\frac{1}{36}\sqrt{2}D$	$\frac{1}{432}D^2$
$\frac{3}{2}^-[10]^2$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$-\frac{2}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}(S+D)$	$\frac{5}{216}(S^2+D^2)$
		$\lambda=\frac{3}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{9}(S-D)$	
S_{31}	$\to \pi^- N^+$,	$\lambda=\frac{1}{2}$	$\frac{1}{36}$	$\frac{\ }{18}$	$\frac{1}{36}S$	$\frac{1}{432}S^2$
$\frac{1}{2}^-[10]^2$	$\to \pi^- \Delta^+$,	$\lambda=\frac{1}{2}$	$\frac{1}{9}\sqrt{2}$	$-\frac{1}{9}\sqrt{2}$	$\frac{1}{9}\sqrt{2}D$	$\frac{5}{108}D^2$

[a] Coefficient of $\langle L'=1\|(8,1)_0-(1,8)_0\|L=0\rangle$.
[b] Coefficient of $\langle L'=1\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|L=0\rangle$.
[c] l-amplitude representation.
[d] g^2.

$$\langle L'=1\|(8,1)_0-(1,8)_0\|L=0\rangle=0,$$

so that only one reduced matrix element describes adequately all such decays. For $\underline{35}$ $L'=2$ $\to\underline{35}$ $L=0$ there is consistency with the meager available data, all on $l=3$ decays, but no check on the $l=1$ decays, which would allow us to fix the ratio of the two reduced matrix elements.

V. PIONIC TRANSITIONS OF BARYONS

In our discussion we will concentrate on the transitions between the $\underline{70}$ $L=1$ and $\underline{56}$ $L=2$ to the ground state $\underline{56}$ $L=0$, although we will also mention briefly the transitions inside the lowest multiplet. As in the meson case, we discuss mainly the nonstrange baryon decays. Our choice of multiplets discussed is motivated by the fact that they are the only ones for which a fairly complete experimental comparison can be made. In particular, we will examine three points: (1) Decay widths into πN and $\pi\Delta$, (2) phases of amplitudes in $\pi N \to N^* \to \pi\Delta$, and (3) F/D values.

For transitions of the type $\underline{56}$ $L'=0\to\underline{56}$ $L=0$, only the $\{(8,1)_0-(1,8)_0, 0\}$ term in $V^{-1}Q_5^\alpha V$ contributes. The two predictions made by the theory are the following:

(1) $F/D=\frac{2}{3}$ for the baryon decays.

This is tested directly by the axial-vector contribution to the weak leptonic decays of the baryon octet, without need for PCAC. This prediction agrees with experiment.[39]

$$(2)\ \left\langle \Delta^0, \lambda=\tfrac{1}{2}\left|\frac{1}{\sqrt{2}}(Q_5^1-iQ_5^2)\right|p, \lambda=\tfrac{1}{2}\right\rangle=(\tfrac{2}{5}\sqrt{2})\left\langle n, \lambda=\tfrac{1}{2}\left|\frac{1}{\sqrt{2}}(Q_5^1-iQ_5^2)\right|p, \lambda=\tfrac{1}{2}\right\rangle.$$

This prediction also agrees with experiment.[53]

For $\underline{70}$ $L'=1\to\underline{56}$ $L=0$ transitions there are two independent amplitudes. Table V gives the results for a neutral resonance in the $\underline{70}$ $L'=1$ decaying into $\pi^-\Delta^+$ and π^-p in terms of reduced matrix elements of the $(8,1)_0-(1,8)_0$ and $(3,\bar{3})_1-(\bar{3},3)_{-1}$ terms in $V^{-1}Q_5^\alpha V$. Since experimental phase-shift-analysis results are usually presented in terms of

TABLE VI. Decays of nonstrange $\underline{56}$ $L'=2$ baryons into $\underline{56}$ $L=0$ baryons by pion emission. States in the $\underline{56}$ $L'=2$ are labeled by J^P and $[\mathrm{SU(3)\ multiplet}]^{2S+1}$, where S is the quark spin. The reduced matrix elements are defined in the text (see Ref. 54).

Decay		a	b	c	d
F_{15} $\frac{5}{2}^+[8]^2$	$\to \pi^- N^+,\ \lambda=\frac{1}{2}$	$-\frac{1}{9}\sqrt{3}$	$\frac{2}{9}$	$-\frac{1}{9}\sqrt{3}F$	$\frac{1}{54}F^2$
	$\to \pi^- \Delta^+,\ \lambda=\frac{1}{2}$	$-\frac{2}{45}\sqrt{6}$	$-\frac{2}{45}\sqrt{2}$	$-\frac{2}{225}\sqrt{6}(3P+2F)$	$\frac{16}{3375}(3P^2+2F^2)$
	$\lambda=\frac{3}{2}$	0	$-\frac{4}{45}\sqrt{3}$	$-\frac{4}{75}(P-F)$	
P_{13} $\frac{3}{2}^+[8]^2$	$\to \pi^- N^+,\ \lambda=\frac{1}{2}$	$\frac{1}{9}\sqrt{2}$	$\frac{1}{9}\sqrt{6}$	$\frac{1}{9}\sqrt{2}P$	$\frac{1}{54}P^2$
	$\to \pi^- \Delta^+,\ \lambda=\frac{1}{2}$	$\frac{4}{45}$	$-\frac{2}{45}\sqrt{3}$	$\frac{2}{225}(P+9F)$	$\frac{8}{3375}(P^2+9F^2)$
	$\lambda=\frac{3}{2}$	0	$\frac{2}{45}\sqrt{3}$	$\frac{2}{75}(P-F)$	
F_{37} $\frac{7}{2}^+[10]^4$	$\to \pi^- N^+,\ \lambda=\frac{1}{2}$	$\frac{4}{105}\sqrt{7}$	$-\frac{8}{315}\sqrt{21}$	$\frac{4}{105}\sqrt{7}F$	$\frac{4}{525}F^2$
	$\to \pi^- \Delta^+,\ \lambda=\frac{1}{2}$	$-\frac{2}{105}\sqrt{14}$	$\frac{4}{315}\sqrt{42}$	$-\frac{2}{105}\sqrt{14}F$	$\frac{1}{70}F^2$
	$\lambda=\frac{3}{2}$	$-\frac{2}{105}\sqrt{70}$	$\frac{4}{315}\sqrt{210}$	$-\frac{2}{105}\sqrt{70}F$	
F_{35} $\frac{5}{2}^+[10]^4$	$\to \pi^- N^+,\ \lambda=\frac{1}{2}$	$-\frac{2}{315}\sqrt{42}$	$\frac{4}{315}\sqrt{14}$	$-\frac{2}{315}\sqrt{42}F$	$\frac{8}{4725}F^2$
	$\to \pi^- \Delta^+,\ \lambda=\frac{1}{2}$	$\frac{2}{315}\sqrt{21}$	$\frac{38}{315}\sqrt{7}$	$\frac{2}{1575}\sqrt{21}(21P-16F)$	$\frac{1}{9450}(147P^2+128F^2)$
	$\lambda=\frac{3}{2}$	$\frac{2}{35}\sqrt{14}$	$\frac{2}{315}\sqrt{42}$	$\frac{2}{525}\sqrt{14}(7P+8F)$	
P_{33} $\frac{3}{2}^+[10]^4$	$\to \pi^- N^+,\ \lambda=\frac{1}{2}$	$-\frac{2}{45}\sqrt{2}$	$-\frac{2}{45}\sqrt{6}$	$-\frac{2}{45}\sqrt{2}P$	$\frac{4}{675}P^2$
	$\to \pi^- \Delta^+,\ \lambda=\frac{1}{2}$	$\frac{2}{45}$	$-\frac{4}{45}\sqrt{3}$	$-\frac{2}{225}(4P-9F)$	$\frac{1}{1350}(16P^2+9F^2)$
	$\lambda=\frac{3}{2}$	$-\frac{2}{15}$	$-\frac{4}{45}\sqrt{3}$	$-\frac{2}{75}(4P+F)$	
P_{31} $\frac{1}{2}^+[10]^4$	$\to \pi^- N^+,\ \lambda=\frac{1}{2}$	$\frac{2}{45}\sqrt{2}$	$\frac{2}{45}\sqrt{6}$	$\frac{2}{45}\sqrt{2}P$	$\frac{8}{675}P^2$
	$\to \pi^- \Delta^+,\ \lambda=\frac{1}{2}$	$-\frac{2}{45}$	$-\frac{2}{45}\sqrt{3}$	$-\frac{2}{45}P$	$\frac{1}{270}P^2$

[a] Coefficient of $\langle L'=2\|(8,1)_0-(1,8)_0\|L=0\rangle$.
[b] Coefficient of $\langle L'=2\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|L=0\rangle$.
[c] l-amplitude representation.
[d] g^2.

amplitudes of definite l, we also use these. The relation between the two sets of reduced matrix elements is given by[54]

$$\langle \underline{70}\ L'=1\|(8,1)_0-(1,8)_0\|\underline{56}\ L=0\rangle=\tfrac{1}{3}(S+2D), \tag{5.1}$$

$$\langle \underline{70}\ L'=1\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|\underline{56}\ L=0\rangle=\tfrac{1}{3}(S-D). \tag{5.2}$$

Equations (5.1) and (5.2) define the normalization of the reduced matrix elements S and D. These matrix elements are not to be confused with those appearing in Sec. IV in meson decays. Note that in principle a g wave could also be present here, but we predict its absence by Eq. (3.14). For convenience we have also listed in each case the number g^2 defined previously as

$$g^2=\frac{1}{(2J'+1)}\sum_{\lambda,\alpha}|\langle \mathrm{hadron}',\lambda|Q_5^\alpha|\mathrm{hadron},\lambda\rangle|^2,$$

which is related by only momentum- and mass-dependent factors to the partial width for hadron′ $\to$ hadron + π [see Eq. (3.2)]. We have expressed g^2 in terms of S and D in table V.

Table VI gives the above quantities for the $\underline{56}$ $L'=2\to\underline{56}$ $L=0$ transitions. In this case,[54]

$$\langle \underline{56}\ L'=2\|(8,1)_0-(1,8)_0\|\underline{56}\ L=0\rangle=\tfrac{1}{5}(2P+3F), \tag{5.3}$$

$$\langle \underline{56}\ L'=2\|(3,3)_1-(3,3)_{-1}\|\underline{56}\ L=0\rangle=\tfrac{1}{5}\sqrt{3}(P-F) \tag{5.4}$$

The h-wave amplitude which could also appear is predicted to vanish by Eq. (3.14).

Before comparing the experimental partial widths with theory, we must note that mixing is possible *within* the $\underline{70}$ $L=1$ multiplet.[55] In this multiplet there are two D_{13} resonances, with $S=\frac{1}{2}$ and $S=\frac{3}{2}$, mixtures of which may form the physically observed states. Similarly the two observed S_{11} resonances may be mixtures of $S=\frac{1}{2}$ and $S=\frac{3}{2}$ states in the $\underline{70}$. To eliminate the complications posed by the mixing, in this paper we will compare with experiment only the sum of squares of reduced matrix elements (g^2) for two resonances which may be mixtures of the quark-model $S=\frac{1}{2}$ and $\frac{3}{2}$ states. This quantity is independent of the mixing angle. By using this we do, however, pay the price of losing some information, and a later, more complete fit will have to deal with mixing.[55]

The manner of comparison of the theory with experiment is made unambiguous by the use of

TABLE VII. Decays of 70 $L'=1$ and 56 $L'=2$ baryons into 56 $L=0$ baryons by pion emission. All rates are fixed by the D_{13} and S_{11} decays to πN for the 70 $L'=1$ decays, and by the F_{15} and P_{31} decays to πN for the 56 $L'=2$ decays.[63] For two states which may be mixed, a combination of widths which is independent of mixing is used and listed under Γ(predicted).

Decay	Γ(predicted) (MeV)	Γ(experimental)[39,57] (MeV)
$D_{13}(1520)\to(\pi N)_d$ $D_{13}(1700)\to(\pi N)_d$	Γ(1520) +0.50 Γ(1700) = 79 MeV (input)	79 ± 20
$D_{13}(1520)\to(\pi\Delta)_d$ $D_{13}(1700)\to(\pi\Delta)_d$	Γ(1520) +0.243 Γ(1700) = 30 MeV	10 ± 6
$S_{11}(1535)\to(\pi\Delta)_d$ $S_{11}(1715)\to(\pi\Delta)_d$	Γ(1535) +0.264 Γ(1715) = 35 MeV	not seen
$D_{15}(1670)\to(\pi N)_d$	21 MeV	56 ± 14
$D_{15}(1670)\to(\pi\Delta)_d$	82 MeV	84 ± 21
$S_{31}(1640)\to(\pi\Delta)_d$	81	52 ± 20
$D_{33}(1690)\to(\pi N)_d$	19	32 ± 9
$D_{33}(1690)\to(\pi\Delta)_d$	55	not seen
$S_{11}(1535)\to(\pi N)_s$ $S_{11}(1715)\to(\pi N)_s$	Γ(1535) +0.505 Γ(1715) = 116 (input)	116 ± 55
$D_{13}(1520)\to(\pi\Delta)_s$ $D_{13}(1700)\to(\pi\Delta)_s$	Γ(1520) +0.243 Γ(1700) = 46	19 ± 10
$S_{31}(1640)\to(\pi N)_s$	18	48 ± 9
$D_{33}(1690)\to(\pi\Delta)_s$	61	172 ± 60
$F_{15}(1688)\to(\pi N)_f$	84 (input)	84 ± 25
$F_{37}(1950)\to(\pi N)_f$	74	92 ± 20
$F_{37}(1950)\to(\pi\Delta)_f$	65	37 ± 18
$F_{35}(1880)\to(\pi N)_f$	14	36 ± 18
$F_{35}(1880)\to(\pi\Delta)_f$	77	16 ± 16
$P_{33}(\quad)\to(\pi\Delta)_f$		?
$F_{15}(1688)\to(\pi\Delta)_f$	12	not seen
$P_{13}(1860)\to(\pi\Delta)_f$	57	not seen
$P_{31}(1860)\to(\pi N)_p$	75 (input)	75 ± 25
$P_{31}(1860)\to(\pi\Delta)_p$	8	not seen
$P_{33}(\quad)\to(\pi N)_p$		?
$P_{33}(\quad)\to(\pi\Delta)_p$		?
$F_{35}(1880)\to(\pi\Delta)_p$	44	not seen
$P_{13}(1860)\to(\pi N)_p$	118	75 ± 25
$P_{13}(1860)\to(\pi\Delta)_p$	5	not seen
$F_{15}(1688)\to(\pi\Delta)_p$	15	22 ± 7

PCAC,[26] which connects the partial width for hadron′ → hadron + π to ⟨hadron′|Q_5^α|hadron⟩, as in Eq. (3.2). Usual comparisons of symmetry predictions with experiment introduce *ad hoc* barrier factors taken from nonrelativistic potential theory. Typically these factors are proportional to p^{2l+1}. Particularly if phase space for a given decay is rather small, the difference between the use of PCAC and a barrier-factor prescription can be significant. As a fairly extreme example, consider the Δ(1950) decays into πN and $\pi\Delta$. Since the Δ(1950) has $J^P=\frac{7}{2}^+$ we are treating f-wave decays. If we were to use our results for g^2 (Table VI) as the coefficient of a barrier factor (p^7) or in the PCAC expression for the width we would find that

$$\left[\frac{\Gamma(\Delta(1950)\to\pi N)}{\Gamma(\Delta(1950)\to\pi\Delta)}\right]_{p^{2l+1}} \simeq 2.8\left[\frac{\Gamma(\Delta(1950)\to\pi N)}{\Gamma(\Delta(1950)\to\pi\Delta)}\right]_{\text{PCAC}}. \tag{5.5}$$

Similar differences can appear in evaluating the relative contribution of two partial waves in the same decay. While at present uncertainties in the data are in many cases even larger than the differences discussed above, in principle we are forced to use the PCAC expressions, and future experiments should permit a discrimination between the different results for widths.[56]

Table VII compares the experimental partial widths[39,57,58] of the 70 $L=1$ and 56 $L=2$ baryons with theory. We have chosen to fit the S, P, D, and F parameters to certain decays rather than doing an over-all least-squares fit. We observe that the agreement of experiment with theory is only qualitative, and that large experimental errors in the matrix elements are involved. One of the strongest disagreements is in the decays of the $D_{15}(1670)$, which cannot be mixed within the $\underline{70}$ $L=1$ multiplet. The disagreement is in fact sharper than is apparent in Table VII, since the errors quoted on the πN and $\pi\Delta$ widths are correlated by the reasonably well-known inelasticity. While the theory predicts that less than 20% of the width is due to the πN decay, experiment indicates a 40% branching ratio.[39]

We must emphasize at this point that a large ambiguity exists in evaluating the partial widths of resonances even when phase-shift-analysis results are known. In the case of strongly inelastic resonances, different ways of extracting resonance couplings may be used, such as extrapolations to the pole, K-matrix fits, Breit-Wigner fits, etc. These give widely varying estimates of partial widths. For example, the width of the $D_{13}(1520)$ decay to $\pi\Delta$ changes from 24 to 53 MeV, depending on whether one used coupling estimates from the Argand diagram or a T-matrix pole fit.[58]

In addition to predicting relative magnitudes, the theory also predicts relative signs of amplitudes for inelastic scattering. For the reaction $\pi N \to N^* \to \pi\Delta$ we can compare our predictions to recent isobar-model phase-shift analyses.[57-59] Table VIII lists the theoretically predicted phases coming from the $(8,1)_0-(1,8)_0$ and $(3,\bar{3})_1-(\bar{3},3)_{-1}$ pieces of $V^{-1}Q_5^\alpha V$ and the experimental results.[57,58] The theoretical predictions are of two kinds. First are those involving amplitudes with the same (l) partial wave in both the incoming and outgoing channels and which are therefore proportional to squares of matrix elements. These have well-defined signs regardless of the relative magnitudes of the reduced matrix elements of the $(8,1)_0 - 1,8)_0$ and $(3,3)_1-(3,3)_{-1}$ terms. The second kind of of sign prediction depends on this relative magnitude,[60] and may help us in deducing which term is is dominant for pion decays from one SU(6) multiplet to another.[61]

At present the experimental situation disagrees with the theory even for predictions of the first kind, as seen in Table VIII. We note, however, that the only disagreement is between the $D_{13}(1520)$ couplings and all other couplings. This sign cannot be changed by mixing the two D_{13} states. If the sign of this resonance could be reversed, one would have complete agreement between theory and experiment. We note that the analysis on which we base our comparison suffers from the lack of data between 1540 MeV and 1650 MeV, i.e., between the $D_{13}(1520)$ and the other resonances in the $\underline{70}$ $L=1$ and $\underline{56}$ $L=2$. The relative phases of

TABLE VIII. Signs of resonant amplitudes in $\pi N \to N^* \to \pi\Delta$ for N^*'s in the $\underline{70}$ $L=1$ and $\underline{56}$ $L=2$. The $S_{11}(1550)$ and $D_{13}(1520)$ are taken as dominantly the quark spin $S=\frac{1}{2}$ states, while the $S_{11}(1715)$ and $D_{13}(1700)$ are assumed to be dominantly $S=\frac{3}{2}$ within the $\underline{70}$ $L=1$. The arbitrary over-all phase is chosen so that the $DD_{15}(1670)$ amplitude is negative.

	a	b	c	d
$\underline{70}$ $L=1$	$DS_{13}(1520)$	−	+	−
	$DD_{13}(1520)$	+	+	−
	$SD_{11}(1550)$	+	−	?
	$SD_{31}(1640)$	+	−	−
	$DS_{33}(1690)$	−	+	+
	$DD_{33}(1690)$	+	+	?
	$DD_{15}(1670)$	−	−	−
	$DS_{13}(1700)$	−	+	+
	$DD_{13}(1700)$	−	−	?
	$SD_{11}(1715)$	+	−	?
$\underline{56}$ $L=2$	$FP_{15}(1688)$	+	−	+
	$FF_{15}(1688)$	−	−	?
	$PP_{13}(1860)$	+	+	?
	$PF_{13}(1860)$	−	+	?
	$FF_{37}(1950)$	+	+	+
	$FP_{35}(1880)$	+	−	?
	$FF_{35}(1880)$	+	+	+
	$PP_{33}(\quad)$	−	−	?
	$PF_{33}(\quad)$	−	+	?
	$PP_{31}(1860)$	−	−	?

[a] Amplitude in $\pi N \to \pi\Delta$. The first letter refers to the πN partial wave, the second to the $\pi\Delta$ partial wave (see Refs. 57, 58).

[b] Theoretical sign from $(8,1)_0-(1,8)_0$ term in $V^{-1}Q_5^\alpha V$ (see Ref. 59).

[c] Theoretical sign from $(3,\bar{3})_1-(\bar{3},3)_{-1}$ term in $V^{-1}Q_5^\alpha V$ (see Ref. 59).

[d] Experiment (Refs. 57 and 58).

amplitudes above and below the gap are determined by continuity and K-matrix fits. It is not inconceivable that a different solution across the gap may still be found which will reverse the sign of the $D_{13}(1520)$ relative to other resonances. If the present solution persists then our results, along with other quark-model and broken-SU(6)$_W$ results,[61] are in very serious conflict with experiment.

If a new solution, with reversed sign of the $D_{13}(1520)$, were to exist it would have a $(3,\bar{3})_1 - (\bar{3},3)_{-1}$-dominated transition for the $\underline{70}$ $L=1$, and an $(8,1)_0-(1,8)_0$-dominated transition for the $\underline{56}$ $L=2$. This would agree with the solution to be discussed in the next section resulting from an analysis of signs in pion photoproduction. Such a solution would be consistent with the approach taken in this paper, since we may have different terms dominate different transitions.[62] The presence of just the $(3,\overline{3})_1-(\overline{3},3)_{-1}$ reduced matrix element could fit the data on decay widths of baryons in the $\underline{70}$ $L=1$, just as it did for mesons in the $\underline{35}$ $L=1$ (see Sec. IV). In that case we would have $S^2=4D^2$. Holding the d-wave widths as they are in Table VII, this would bring the S_{31} and D_{33} predictions into agreement with experiment while worsening the S_{11} and D_{13} agreement.[63] In the case of $\underline{56}$ $L=2$ baryon decays, the presence of only the $(8,1)_0-(1,8)_0$ reduced matrix element results in $P=F$, which is consistent with the results presented in Table VII as it stands.[63]

Finally, we mention F/D ratios. Unlike meson decays, where F/D follows from charge conjugation, the baryon F/D values are predictions of the theory. Given the nonstrange baryon decay amplitudes and F/D ratios, all strange baryon decays are predicted up to questions of mixing and SU(3) breaking. Both the $(8,1)_0-(1,8)_0$ and the $(3,\overline{3})_1-(\overline{3},3)_{-1}$ terms have identical F/D ratios, since both belong to the $W=1$ part of a $\underline{35}$ representation. The predicted values for F/D are $\frac{2}{3}$, $\frac{5}{3}$, and $-\frac{1}{3}$ for the $\underline{56}$ $W=\frac{1}{2}$, $\underline{70}$ $W=\frac{3}{2}$, and $\underline{70}$ $W=\frac{1}{2}$ states, respectively. The $\underline{56}$ $L=2$ prediction agrees with the experimental value,[64] while the $\underline{70}$ $L=1$ situation is complicated by the mixing discussed above.[55,64]

In all our discussions we have neglected possible mixing between different SU(6) multiplets.[65] While such mixing may modify some of our predictions which disagree with experiment, it does so only at the expense of considerably complicating the simple quark-model picture.

VI. PHOTON TRANSITIONS

In Secs. II and III we have discussed the kinematic and algebraic properties of the first moment of the vector current with $J_z=\pm1$,

$$D_\pm^\alpha=\int d^3x\left[\mp\frac{(x\pm iy)}{\sqrt{2}}\right]V_0^\alpha(\vec{x},t)\,, \tag{6.1}$$

taken between states at infinite momentum. In particular, as shown by Eq. (3.5), all helicity amplitudes for real photon transitions are proportional to matrix elements of $D_+^3+(1/\sqrt{3})D_+^8$ between states at infinite momentum. Such matrix elements of D_+^α are equal to those of the three terms in $V^{-1}D_+^\alpha V$ found in Eq. (3.2) taken between those irreducible representations of the $SU(6)_W$ of currents which correspond to forming baryons out of qqq and mesons out of $q\overline{q}$. In this section we discuss some of the results obtainable in this way for $\underline{56}$ $L'=0\to\underline{56}$ $L=0$ and $\underline{70}$ $L'=1\to\underline{56}$ $L=0$ baryon transitions, presenting many of the general features of photon transitions in the process. We leave a complete discussion of both baryon and meson photon decays to another paper.[38,66]

TABLE IX. Photon transition amplitudes for $\underline{56}$ $L'=0\to\underline{56}$ $L=0$. Matrix elements of $D_+^3+(1/\sqrt{3})D_+^8$ are considered ($J_z=+1$ photons), and λ denotes J_z of the decaying N or Δ. See text.

Decay	Coefficient of $\langle L'=0\Vert(3,\overline{3})_1\Vert L=0\rangle$
$N^+\to\gamma N^+$, $\lambda=\frac{1}{2}$	$-\frac{2}{15}\sqrt{5}$
$N^0\to\gamma N^0$, $\lambda=\frac{1}{2}$	$+\frac{4}{45}\sqrt{5}$
$\Delta^+\to\gamma N^+$, $\lambda=\frac{1}{2}$	$-\frac{2}{45}\sqrt{10}$
$\lambda=\frac{3}{2}$	$-\frac{2}{45}\sqrt{30}$

In Table IX we present the results for matrix elements of the rotated dipole operator for $\underline{56}$ $L'=0\to\underline{56}$ $L=0$ transitions.[67] Only the term in $V^{-1}D_+^\alpha V$ [see Eq. (3.2)] which transforms as $\{(3,\overline{3})_1,0\}$ can make a nonzero contribution, for $L_z=0$ in both the initial and final states. All matrix elements are therefore proportional to the single reduced matrix element

$$\langle\underline{56}\ L'=0\Vert(3,\overline{3})_1\Vert\underline{56}\ L=0\rangle.$$

For transitions between two octet members of the $\underline{56}$, this term is characterized by an F/D value of $\frac{2}{3}$.

In physically interpreting the matrix element in Table IX, there is a slight subtlety. A direct evaluation of the matrix element of D_+^α between one-nucleon states shows that the result is proportional to the anomalous magnetic moment (in fact $\langle N,\lambda=\frac{1}{2}|D_+|N,\lambda=-\frac{1}{2}\rangle=-\sqrt{2}\,\mu_A$ at infinite momentum). However, as shown by Melosh,[8] a careful calculation of $V^{-1}D_+^\alpha V$ between one-nucleon states at infinite momentum gives a result which has the transformation properties of Eq. (3.2) minus a term which is exactly equal to the Dirac moment. Therefore, adding the Dirac moment to the anomalous moments to form the total moment, we see that the terms in Eq. (3.2) are to be interpreted as being proportional to the total moment when taken between the same initial and final states.

With this in mind, we see immediately that Table IX gives

$$\frac{\mu_T(n)}{\mu_T(p)}=-\frac{2}{3}\,, \tag{6.2}$$

the old SU(6) result, which is rather close to experiment.[39] Furthermore, the ratio of $\sqrt{3}$ between the $\lambda=\frac{3}{2}$ and $\frac{1}{2}$ amplitudes for $\Delta\to\gamma N$ corresponds to a pure magnetic dipole transition with

$$\frac{\mu^*}{\mu_T(p)}=\tfrac{2}{3}\sqrt{2}\,, \tag{6.3}$$

if we use the relation

$$\langle\Delta,\lambda=\tfrac{1}{2}|D_+|N,\lambda=-\tfrac{1}{2}\rangle=\frac{\mu^*}{\sqrt{2}}$$

and the relation between

$$\langle N, \lambda = \tfrac{1}{2} | D_+ | N, \lambda = -\tfrac{1}{2} \rangle$$

and the magnetic moment of the nucleon given above. A phenomenological analysis[68] of the data for pion photoproduction gives a value for $\mu^*/\mu_T(p)$ which is 1.28 ± 0.03 times the right-hand side of Eq. (6.3). However, this is the result of finding the residue at the Δ pole in $\gamma N \to \pi N$. In our approach one should evaluate μ^* by taking the Δ contribution to the Cabibbo-Radicati sum rule (see Sec. III). This results in a value[69] of $\mu^*/\mu_T(p)$ which is 0.9 ± 0.1 times the right-hand side of Eq. (6.3), i.e., in quite satisfactory agreement with the theory. Equations (6.2) and (6.3) are standard SU(6) results, as is to be expected since the $(3, \overline{3})_1$ term in $V^{-1} D_+^\alpha V$ has the same transformation properties as the magnetic moment operator[70] used in SU(6).

The decays from $\underline{70}$ $L' = 1$ to $\underline{56}$ $L = 0$ (we consider only the nucleon in the $\underline{56}$ here)[38] are somewhat more complicated. We first of all note that although the $\{(\overline{3}, 3)_{-1}, 2\}$ term in $V^- D_+ V$ cannot contribute because $|\Delta L_z| \leq 1$ for $L' = 1 \to L = 0$ decays, both the $\{(8,1)_0 + (1,8)_0, 1\}$ and $\{(3, \overline{3})_1, 0\}$ terms do contribute. Hence, everything depends on two reduced matrix elements, whose coefficients for the various decays are presented in Table X. Second, the $\{(8,1)_0 + (1,8)_0, 1\}$ term is purely electric dipole in character if analyzed in terms of multipoles, as can be proven directly.[38] The $\{(3, \overline{3})_1, 0\}$ term is not simple in this way. Third, we note that the Moorhouse quark model selection rule[71] forbidding $\gamma p \to N^*$, where the N^* has quark spin $S = \frac{3}{2}$, is correctly reflected in Table X.

In fact, there is a one-to-one correspondence between Tables IX and X and the results of quark-model calculations[72]: The $(8,1)_0 + (1,8)_0$ term in $V^{-1} D_+^\alpha V$ corresponds to the photon interacting with the convection current, while the $(3, \overline{3})_1$ term corresponds to the interaction with the quark magnetic moments. Of course, explicit quark-model calculations with, say, harmonic potentials give the reduced matrix elements as well, something we do not obtain at all with the theory under discussion.

For $\underline{56}$ $L' = 2 \to \underline{56}$ $L = 0$ decays all three terms in Eq. (3.2) can contribute and the situation in general becomes more complicated than the quark-model calculations referred to above. We defer a detailed discussion of this and the comparison of decay widths to another publication.[38]

Just as for pionic decays, the relative signs of the amplitudes for photon transitions are an important test of the theory. The signs of amplitudes in $\gamma N \to \pi N$ have already been compared with certain quark-model calculations[73] and found to be in agreement. The correspondence in general algebraic structure of these models with the present theory leads us to immediately conclude that the signs are consistent with this theory. In fact, since $\gamma N \to N^* \to \pi N$ involves the product of the γN and πN couplings, information on both kinds of transitions is obtainable. A detailed analysis[74] shows that all the observed signs of amplitudes for $\gamma N \to \pi N$ involving intermediate N^*'s in the $\underline{70}$ $L = 1$ multiplet are consistent with the theory if the signs of the S and D amplitudes of Sec. V are opposite, i.e., if the amplitudes have the signs given by the $(3, \overline{3})_1 - (\overline{3}, 3)_{-1}$ term in $V^{-1} Q_5^\alpha V$. This lends further support to the existence of a solution to the $\pi N \to \pi \Delta$ phase shifts with this property, such such as that discussed in Sec. V. The signs of amplitudes for $\underline{56}$ $L = 2$ intermediate states in

TABLE X. Decays of nonstrange $\underline{70}$ $L' = 1$ baryons into neutrons and protons in the $\underline{56}$ $L = 0$ by photon emission. States in the $\underline{70}$ $L' = 1$ are labeled by J^P and [SU(3) multiplet]$^{2S+1}$, where S is the quark spin. λ denotes the helicity (J_z) of the state in the $\underline{70}$. The photon has $J_z = +1$, corresponding to the operators $D_+^3 + (1/\sqrt{3}) D_+^8$.

a			b	c
D_{15} $\frac{5}{2}^-[8]^4$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	0	0
		$\lambda = \frac{3}{2}$	0	0
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	0	$\frac{1}{30}\sqrt{5}$
		$\lambda = \frac{3}{2}$	0	$\frac{1}{30}\sqrt{10}$
D_{13} $\frac{3}{2}^-[8]^4$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	0	0
		$\lambda = \frac{3}{2}$	0	0
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	0	$-\frac{1}{90}\sqrt{5}$
		$\lambda = \frac{3}{2}$	0	$-\frac{1}{30}\sqrt{15}$
S_{11} $\frac{1}{2}^-[8]^4$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	0	0
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	0	$-\frac{1}{18}$
D_{13} $\frac{3}{2}^-[8]^2$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	$-\frac{1}{12}\sqrt{2}$	$\frac{1}{6}\sqrt{2}$
		$\lambda = \frac{3}{2}$	$-\frac{1}{12}\sqrt{6}$	0
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	$+\frac{1}{12}\sqrt{2}$	$-\frac{1}{18}\sqrt{2}$
		$\lambda = \frac{3}{2}$	$+\frac{1}{12}\sqrt{6}$	0
S_{11} $\frac{1}{2}^-[8]^2$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{6}$
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	$+\frac{1}{6}$	$+\frac{1}{18}$
D_{33} $\frac{3}{2}^-[10]^2$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	$-\frac{1}{12}\sqrt{2}$	$-\frac{1}{18}\sqrt{2}$
		$\lambda = \frac{3}{2}$	$-\frac{1}{12}\sqrt{6}$	0
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	$-\frac{1}{12}\sqrt{2}$	$-\frac{1}{18}\sqrt{2}$
		$\lambda = \frac{3}{2}$	$-\frac{1}{12}\sqrt{6}$	0
S_{31} $\frac{1}{2}^-[10]^2$	$\to \gamma N^+$,	$\lambda = \frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{18}$
	$\to \gamma N^0$,	$\lambda = \frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{18}$

[a] Decay.
[b] Coefficient of $\langle L' = 1 \| (8,1)_0 + (1,8)_0 \| L = 0 \rangle$ (see text).
[c] Coefficient of $\langle L' = 1 \| (3, \overline{3})_1 \| L = 0 \rangle$ (see text).

$\gamma N \to \pi N$ is consistent with the present theory, but as only f-wave πN resonances have established γN couplings, we are unable[74] to conclude anything yet on the relative signs of P and F, as defined in Sec. V.

In general, photon amplitudes provide a particularly clean test of the theory with no use of PCAC being necessary. As such, the good agreement found is expecially significant. The agreement of signs in $\gamma N \to \pi N$ therefore lends strong support to the theory in general, and to the πN (and $\pi\Delta$) decays of the $\underline{70}$ $L=1$ baryons being dominated by the $(3,\overline{3})_1 - \overline{(\overline{3},3)_{-1}}$ term of $V^{-1}Q_5^\alpha V$ in particular.

VII. SUMMARY AND DISCUSSION

In the first few sections of this paper we have indicated how the introduction of a transformation V from current- to constituent-quark basis states helps unify the discussion of finding the decomposition of hadron states at infinite momentum into irreducible representations of the algebra of currents. While no one has yet been able to completely specify this transformation because of the lack of a detailed dynamics of hadrons, it is possible to guess at certain algebraic properties of the transformed currents.

In particular, following the work of Melosh,[8] we have abstracted from the free-quark model the algebraic properties [under the $\mathrm{SU}(6)_W$ of currents] of the transformed axial-vector charge, $V^{-1}Q_5^\alpha V$, and the transformed first moment of the vector current, $V^{-1}D_+^\alpha V$. These transformed operators, taken between known irreducible representations of the algebra of currents, are equal to the untransformed operators, Q_5^α and D_+^α, taken between hadron states built out of constituent quarks. With the use of PCAC, matrix elements of Q_5^α are related to those of the pion field. Matrix elements of D_+^α are proportional to real photon transition amplitudes. As a result we have an elegant and beautiful theory of the algebraic structure of pion or photon transitions between hadrons based on the one assumption of abstraction of certain algebraic properties of the free-quark model. We stress that at this stage it is worthy of being called a theory, and not a phenomenology, in that the algebraic properties assumed have a clear origin and could be exact, and in that our basic assumption is consistent with relativity and invariance principles. In the resulting theory, as applied to actual physical transitions, amplitudes are related in a straightforward way by Clebsch-Gordan coefficients, and decay widths are in turn related to these amplitudes in a nonarbitrary, known way.

When the theory is applied to the pionic decays of mesons, the results of a comparison with experiment are very encouraging. Both for $\underline{35}$ $L'=0 \to \underline{35}$ $L=0$ transitions, where only the $(8,1)_0 - \overline{(1,8)_0}$ term in $V^{-1}Q_5^\alpha V$ contributes, and for $\underline{35}$ $L'=1 \to \underline{35}$ $L=0$ transitions, where both the $\overline{(8,1)_0 - (1,8)_0}$ and $(3,\overline{3})_1 - (\overline{3},3)_{-1}$ terms in $V^{-1}Q_5^\alpha V$ can contribute, there is good agreement with experiment. In the case of $\underline{35}$ $L'=1 \to \underline{35}$ $L=0$, moreover, the experimental results suggest the dominance of

$$\langle L'=1 \| (3,\overline{3})_1 - (\overline{3},3)_{-1} \| L=0\rangle$$

over

$$\langle L'=1 \| (8,1)_0 - (1,8)_0 \| L=0\rangle;$$

in fact there is consistency with the vanishing of the latter reduced matrix element. For $\underline{35}$ $L'=2 \to \underline{35}$ $L=0$ decays, present data are rather sparse, but what data do exist are also quite consistent with the theory. Further experiments on $\underline{35}$ $L'=2$ decays (especially p waves) would be of considerable interest in this regard.

The situation for pionic decay widths of baryons is not quite so encouraging; there are failures by factors of 2 to 3 in our comparison of theory and experiment. However, given our assumption of simple identification of physical states with those of the constituent-quark model [no mixing of different SU(6) multiplets], the theoretical use of the narrow-resonance approximation in computing decay widths, and the experimental difficulties in assigning widths to broad, inelastic resonances, the present situation with regard to baryon pionic decay widths is reasonable.

Of more crucial importance is the situation with regard to the relative signs of resonant amplitudes in $\pi N \to \pi\Delta$. The present experimental analysis[57,58] of $\pi N \to \pi\pi N$ produces relative signs which disagree with those predicted for baryons in the $\underline{70}$ $L=1$. If this situation persists, then we will have to face at least one of the following alternatives: (1) There is large mixing of different SU(6) multiplets, thereby invalidating our identification of the observed hadrons with simple quark model states. (2) The use of the full $\mathrm{SU)6)}_W \times \mathrm{O}(3)$ to relate different quark spin states is invalid, and only a weaker set of relations holds, such as those following from chiral SU(3)×SU(3). (3) The algebraic properties abstracted for $V^{-1}Q_5^\alpha V$ from the free-quark model do not hold in the real world. None of these possibilities is particularly appealing, nor does any of them explain the success with meson decays or the agreement of theory and experiment for the signs found[74] in $\gamma N \to \pi N$.

On the other hand, suppose that another solution to the $\pi N \to \pi\Delta$ phase shifts is found which gives relative signs in agreement with the theory. Presumably this must come from reversal of the

$D_{13}(1520)$ signs relative to those of other $\underline{70}$ $L=1$ resonances above the "gap" in the data. The resulting situation would then indicate dominance of the

$$\langle L'=1 \| (3,\bar{3})_1 - (\bar{3},3)_{-1} \| L=0\rangle$$

reduced matrix element, in agreement with the photoproduction results.[74] With more than 20 signs of resonant amplitudes in $\gamma N \to \pi N$ and $\pi N \to \pi\Delta$ in agreement with the theory, there would then be strong support for the theory as a valid description of *both* photon and pion transitions, as well as for the identification of the observed hadron states with those of the constituent-quark model.

We would then possess a viable theory of weak- and electromagnetic-current-induced transitions between hadrons at $q^2=0$. With the identification of the observed hadrons to good approximation with constituent-quark states and the use of PCAC, a powerful approximate theory of all pion and photon decays results. This approximate theory can then be extended to include vector-meson transitions if we make an additional assumption, that of vector-meson dominance. Demanding consistency between the two ways of treating $A_2 \to \pi\rho$, for example, then leads to connections between the reduced matrix elements involved in π transitions and those in ρ transitions. However, inasmuch as the longitudinal and transverse electromagnetic currents have independent reduced matrix elements in this theory, the $\lambda=0$ and $\lambda=\pm 1$ helicities of the vector meson are not necessarily tied together for us in the way that they are in a model which starts with the strong interactions being symmetrical under $SU(6)_W$ (and then breaks the symmetry in some way) and for which the $\lambda=0$ pion and $\lambda=0,\pm 1$ ρ are related.[75] Therefore, although we can duplicate such models by making additional assumptions relating various reduced matrix elements, we are not forced to do so.

This brings us to one of the important extensions of the present theory. Namely, one might construct a phenomenology of purely hadronic vertices by "tieing on" the phenomenology at points of overlap with the present theory of current transitions plus the assumptions of PCAC and/or vector-meson dominance.[76] One could provide justification for some of the broken-$SU(6)_W$ schemes that have been devised, and in the process see clearly the level of approximation and the additional assumptions necessary to obtain their results. In this way it might be possible to construct a full phenomenology of purely hadronic vertices, including, but not restricted to, those involving pseudoscalar or vector mesons.

Other directions for extension of the theory include an investigation of the predictions for strangeness-changing pseudoscalar-meson decays and the use of kaon PCAC. At the same time, the phenomenological analysis should be extended to cover the pionic decays of strange particles.

Of more fundamental interest is an investigation of mass formulas within this theory. Such an investigation has already proved very interesting and profitable in the framework of the previous work on finding the representations of current algebra exhibited by hadrons at infinite momentum.[77,78] A preliminary look at this problem in the present framework indicates that it may be rather complicated.[79]

Finally, of major interest is the extension of the theory to values of $q^2 \neq 0$. For single currents, this permits the interrelating of resonance weak and electromagnetic excitation form factors. For products of currents, the bilocal operators are candidates for investigation,[80] their matrix elements being measurable in deep-inelastic scattering. Clearly, a large class of problems of great interest involving the structure of hadrons as probed by weak and electromagnetic currents is investigatable from this new point of view.

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[25]We need only consider D_+^α with $J_z=+1$, since all matrix elements of D_-^α are related to those of D_+^α by parity.

[26]Intrinsic to the use of PCAC is a ~10% error for matrix elements. There is no contradiction between Eq. (3.2) and the powers of momentum (p^{2l+1}) expected in an expression for a decay width from arguments based on threshold factors—the additional powers of momentum are implicitly contained in the matrix element of Q_5^α. Massive or massless final pions have momenta which typically differ by less than 10% for any given decay which we treat in this paper.

[27]See in particular the case of the $\Delta(1236)$ discussed by S. L. Adler and F. J. Gilman, Phys. Rev. 156, 1598 (1967).

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[29]When the context makes the meaning clear, we will often drop the L_z label for the transformation properties of operators.

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[32]W-spin Clebsch-Gordan coefficients are just those of SU(2). We use the Condon-Shortley phase conventions with the W-spin state of the relevant piece of $V^{-1}Q_5^\alpha V$ or the $V^{-1}D_+^\alpha V$ taken first.

[33]The algebraic structure of the theory considered here and its relation to previous work, specifically broken-$SU(6)_W$ schemes, has also been considered by A. J. G. Hey, J. L. Rosner, and J. Weyers, Nucl. Phys. B61, 205 (1973). See also the work of A. J. G. Hey and J. Weyers, Phys. Lett. 44B, 263 (1973).

[34]We thank J. Rosner for a discussion of the results of Ref. 33 and for pointing out to us the dependence of the reduced matrix elements on L_z and L_z' in the general case unless additional assumptions are made.

[35]D. Horn and S. Meshkov (unpublished).

[36]This may be proven by brute force starting from Eq. (3.6) and using Eq. (3.7) to relate W-spin and quark spin states.

[37]The p-wave character of $(8,1)_0-(1,8)_0$-induced decays between multiplets with $L'=L$ was incorrectly stated in Ref. 11 to follow without any additional assumption. The additional assumption holds if $SU(6)_{W,\mathrm{strong}}$ is conserved and in the theory outlined in Ref. 28, but not in general.

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[40]We extract the amplitude for $\omega\to\pi\rho$ from $\Gamma(\omega\to\pi\gamma)$ using $\gamma_\rho^2/4\pi=0.6$ and the model of M. Gell-Mann *et al*. [Phys. Rev. Lett. 8, 261 (1962)]. Comparison of the prediction with the amplitude for $\rho\to\pi\pi$, as obtained from $\Gamma(\rho\to\pi\pi)$, shows agreement (to 10%). See the

discussion in Ref. 28, particularly footnote 13.

[41]Since the $\{(8,1)_0-(1,8)_0,0\}$ piece of $V^{-1}Q_5^{\alpha}V$ does not change L_z, we have $L_z=L_z'=0$ for the corresponding $SU(6)_W$ reduced matrix element

$$\langle L'=1,\,L_z'=0\|\{(8,1)_0-(1,8)_0,0\}\|L=0,\,L_z=0\rangle\,,$$

which we write as

$$\langle L'=1\|(8,1)_0-(1,8)_0\|L=0\rangle.$$

For the $\{(3,\bar{3})_1,-1\}-\{(\bar{3},3)_{-1},1\}$ piece of $V^{-1}Q_5^{\alpha}V$ we have similarly

$$\langle L'=1,\,L_z'=-1\|\{(3,\bar{3})_1,-1\}\|L=0,\,L_z=0\rangle$$
$$=\langle L'=1,\,L_z'=+1\|-\{(\bar{3},3)_{-1},+1\}\|L=0,\,L_z=0\rangle$$
$$\equiv\langle L'=1\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|L=0\rangle.$$

The amplitudes for these meson decays are also treated in Ref. 33.

[42]The longitudinal ω composed of nonstrange quarks is partly in $\{(8,1)_0+(1,8)_0,0\}$ and in $\{(1,1),0\}$, which respectively transform as a $\underline{35}$ and a $\underline{1}$ of $SU(6)_W$. Zweig's rule then forbids the ϕ, which is the orthogonal state to the ω and composed purely of strange quarks, to decay to two mesons composed of nonstrange quarks (such as π and B). This relates the $\underline{1}$ and $\underline{35}$ decay amplitudes. Similar results hold for parts of the $\lambda=0$ σ and f, the $\lambda=1$ D and f, etc.

[43]These are similar to amplitudes defined by Colglazier and Rosner (Ref. 13).

[44]D. Cohen *et al.*, Phys. Rev. D $\underline{8}$, 23 (1973); M. Afzal *et al.*, Nuovo Cimento $\underline{15A}$, 61 (1973); R. Ott, Ph.D. thesis, Univ. of California (unpublished); LBL Report No. LBL-1547, 1972 (unpublished) and references therein; G. Lynch (private communication); see also, Y. Eisenberg (private communication).

[45]This follows automatically in the theory presented in Ref. 28, where the $(8,1)_0-(1,8)$ term in $V^{-1}Q_5^{\alpha}V$ is proportional to a generator and cannot connect two different L representations.

[46]See, for example, R. Klanner, in *Experimental Meson Spectroscopy—1972,* proceedings of the Third International Conference, Philadelphia, 1972, edited by A. H. Rosenfeld and K.-W. Lai (A.I.P., New York, 1972), p. 164.

[47]The two independent reduced matrix elements involving the $\{(8,1)_0-(1,8)_0,0\}$ term are

$$\langle L'=1,\,L_z'=0\|(8,1)_0-(1,8)_0\|L=1,\,L_z=0\rangle$$

and

$$\langle L'=1,\,L_z'=1\|(8,1)_0-(1,8)_0\|L=1,\,L_z=1\rangle$$
$$=\langle L'=1,\,L_z'=-1\|(8,1)_0-(1,8)_0\|L=1,\,L_z=-1\rangle.$$

For the $\{(3,\bar{3})_1-1\}-\{(\bar{3},3)_{-1}\}$ term we have

$$\langle L'=1,\,L_z'=-1\|\{(3,\bar{3})_1,-1\}\|L=1,\,L_z=0\rangle$$
$$=\langle L'=1,\,L_z'=+1\|-\{(\bar{3},3)_{-1},1\}\|L=1,\,L_z=0\rangle$$
$$=-\langle L'=1,\,L_z'=0\|\{(3,\bar{3})_1,-1\}\|L=1,\,L_z=1\rangle$$
$$=-\langle L'=1,\,L_z'=0\|-\{(\bar{3},3)_{-1},1\}\|L=1,\,L_z=-1\rangle$$
$$\equiv\langle L'=1,\,L_z'=1\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|L=1,\,L_z=0\rangle.$$

Explicit computation shows that setting the two $(8,1)_0-(1,8)_0$ reduced matrix elements equal yields a pure p-wave decay amplitude arising from the single resulting $(8,1)_0-(1,8)_0$ amplitude, as discussed in general in Sec. III (see Ref. 37).

[48]Our previous claim that the $(3,\bar{3})_1-(\bar{3},3)_{-1}$ term for $L'=1\to L=1$ pion decays is also p-wave (see Ref. 11) is not correct because of the mistaken omission of minus signs in the reduced matrix element relations given in footnote 47. We thank Jon Rosner for discussions on this and the corresponding results of Ref. 33.

[49]The two independent reduced matrix elements we take to be

$$\langle L'=2\|(8,1)_0-(1,8)_0\|L=0\rangle$$
$$\equiv\langle L'=2,\,L_z'=0\|\{(8,1)_0-(1,8)_0,\,0\}\|L=0,\,L_z=0\rangle$$

and

$$\langle L'=2\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|L=0\rangle$$
$$\equiv\langle L'=2,\,L_z'=-1\|\{(3,\bar{3})_{-1},\,1\}\|L=0,\,L_z=0\rangle$$
$$=\langle L'=2,\,L_z'=1\|-\{(\bar{3},3)_{-1},\,1\}\|L=0,\,L_z=0\rangle.$$

[50]R. H. Graham and T. S. Yoon, Phys. Rev. $\underline{6}$, 336 (1972).

[51]See the review by M. Davier, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 104.

[52]See also the discussion by J. L. Rosner, CERN Report No. TH-1632, 1973 (unpublished).

[53]With the definitions

$$\langle n,\lambda=\tfrac{1}{2}|(1/\sqrt{2})(Q_5^1-iQ_5^2)|p,\lambda=\tfrac{1}{2}\rangle=g_A/\sqrt{2}$$

and

$$\langle\Delta^0,\lambda=\tfrac{1}{2}|(1/\sqrt{2})(Q_5^1-iQ_5^2)|p,\lambda=\tfrac{1}{2}\rangle=\tfrac{1}{2}g^*$$

the Adler-Weisberger sum rule reads

$$g_A^{\,2}-g^{*2}+\text{(higher-mass contributions)}=1.$$

The prediction then is that $g^*=\frac{4}{5}g_A\simeq1.0$, while the experimental value of g^* lies in the range 0.8 to 1.05 (see Ref. 5, particularly footnote 13).

[54]For $\underline{70}$ $L'=1\to\underline{56}$ $L=0$, the two independent reduced matrix elements are taken to be

$$\langle\underline{70}\;L'=1\|(8,1)_0-(1,8)_0\|\underline{56}\;L=0\rangle$$
$$\equiv\langle\underline{70}\;L'=1,\,L_z'=0\|\{(8,1)_0-(1,8)_0,\,0\}\|\underline{56}\;L=0,\,L_z=0\rangle$$

and

$$\langle\underline{70}\;L'=1\|(3,\bar{3})_1-(\bar{3},3)_{-1}\|\underline{56}\;L=0\rangle$$
$$\equiv\langle\underline{70}\;L'=1,\,L_z'=-1\|\{(3,\bar{3})_1,\,-1\}\|\underline{56}\;L=0,\,L_z=0\rangle$$
$$=\langle\underline{70}\;L'=1,\,L_z'=+1\|-\{(\bar{3},3)_{-1},\,+1\}\|\underline{56}\;L=0,\,L_z=0\rangle.$$

The situation for $\underline{56}$ $L'=2\to\underline{56}$ $L=0$ is analogous. Amplitudes equivalent to S and D or P and F are defined by Petersen and Rosner (Ref. 13).

[55]For a recent discussion, see D. Faiman and D. Plane, Nucl. Phys. $\underline{B50}$, 379 (1972).

[56]See, in this connection, the discussion of $\Gamma(f\to\pi\pi)$ in Sec. IV, where experiment favors the PCAC relation.

[57] D. Herndon *et al.*, LBL Report No. LBL-1065, 1972 (unpublished); R. Cashmore (private communication); U. Mehtani *et al.*, Phys. Rev. Lett. 29, 1634 (1973).

[58] R. Cashmore, *Baryon Resonances—73* (Purdue Univ. Press, West Lafayette, 1973), p. 53.

[59] The forward, s-channel helicity partial-wave amplitude for $\pi_1 N \to \pi_2 \Delta$, $T^J_{0\,1/2,\,0\,1/2}$, is related to the experimental partial-wave amplitude $T^J_{ll'}$ of Ref. 57 by

$$\begin{aligned} T^J_{0\,1/2,\,0\,1/2} = {} & (\text{positive number}) \\ & \times (l\ 0\ \tfrac{1}{2}\tfrac{1}{2} | J\ \tfrac{1}{2})(l'\ 0\ \tfrac{3}{2}\tfrac{1}{2} | J\ \tfrac{1}{2}) \\ & \times (1\ I_z^{(\pi_1)}\ \tfrac{1}{2} I_z^{(N)} | I^{(N^*)}\ I_z^{(N^*)}) \\ & \times (\tfrac{3}{2} I_z^{(\Delta)}\ 1\ I_z^{(\pi_2)} | I^{(N^*)}\ I_z^{(N^*)}) T^J_{ll'}, \end{aligned}$$

where l and l' are the orbital angular momentum in the initial and the final state, respectively. Note the ordering in the isospin Clebsch-Gordan coefficients.

[60] In other words, it depends on the sign of S/D and P/F, which are positive [negative] for the $(8,1)_0 - (1,8)_0$ $[(3,\overline{3})_1 - (\overline{3},3)_{-1}]$ term in $V^{-1} Q_5^\alpha V$.

[61] The relative signs of resonant amplitudes in theories with generally similar algebraic structure have been considered by R. G. Moorhouse and N. H. Parsons [Glasgow reports, 1973 (unpublished)] in the quark model, and by D. Faiman and J. L. Rosner, Phys. Lett. 45B, 357 (1973) in a broken $SU(6)_W$ scheme.

[62] The theory presented in Ref. 28 would demand that only the $(3,\overline{3})_1 - (\overline{3},3)_{-1}$ reduced matrix element is nonvanishing for both $\underline{70}\ L'=1 \to \underline{56}\ L=0$ and $\underline{56}\ L'=2 \to \underline{56}\ L=0$ transitions (see footnote 45). A scheme with domination of even [odd] L' multiplet decays by $(8,1)_0 - (1,8)_0$ $[(3,\overline{3})_1 - (\overline{3},3)_{-1}]$ is discussed by Buccella *et al.* (Ref. 20).

[63] Table VII is calculated with the S_{11}, D_{13}, P_{31}, and F_{15} decays to πN used as input to obtain S^2, D^2, P^2, and F^2 of Tables V and VI, respectively. The experimental widths used as input yield $S^2 = 2.65$, $D^2 = 2.37$, $P^2 = 1.11$, and $F^2 = 1.34$ if $f_\pi = 135$ MeV.

[64] See, for example, the discussion of S. Meshkov, in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics*, edited by E. Gotsman (Weizmann Science Press, Jerusalem, 1971), p. 252.

[65] The possible mixing of $\underline{70}\ L=2$ states with the $\underline{56}\ L=2$ has been recently discussed by D. Faiman, J. L. Rosner, and J. Weyers, Nucl. Phys. B57, 45 (1973).

[66] Some weaker results, using only the $SU(3) \times SU(3)$ subalgebra of $SU(6)_W$, have been obtained by A. Love and D. V. Nanopoulos, Phys. Lett. 45B, 507 (1973).

[67] Such transitions were also considered by Melosh (Ref. 8).

[68] R. H. Dalitz and D. G. Sutherland, Phys. Rev. 146, 1180 (1966).

[69] We use the results of F. J. Gilman and H. J. Schnitzer [Phys. Rev. 150, 1362 (1966)] and S. L. Adler and F. J. Gilman (Ref. 27), updated by recent photoproduction amplitude analysis, to calculate μ^*.

[70] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Lett. 13, 514 (1964).

[71] R. G. Moorhouse, Phys. Rev. Lett. 16, 772 (1966).

[72] For example, see D. Faiman and A. W. Hendry, Phys. Rev. 173, 1720 (1968); R. P. Feynman, M. Kislinger, and F. Ravndal (Ref. 12).

[73] R. G. Moorhouse and H. Oberlack, Phys. Lett. 43B, 44 (1973).

[74] F. J. Gilman and I. Karliner, Phys. Lett. 46B, 426 (1973).

[75] W. Petersen and J. L. Rosner, Phys. Rev. D 7, 747 (1973).

[76] We thank Professor M. Gell-Mann for several discussions suggesting this general framework for a phenomenology of strong-interaction vertices.

[77] F. J. Gilman and H. Harari, Phys. Rev. Lett. 19, 723 (1967); F. J. Gilman and H. Harari (Ref. 6).

[78] S. Weinberg, Phys. Rev. 177, 2604 (1969).

[79] F. J. Gilman and M. Kugler (unpublished); F. Buccella *et al.*, Nuovo Cimento Lett. 8, 244 (1973).

[80] S. P. de Alwis (Ref. 23); H. J. Melosh (Ref. 8 and unpublished work).