

## $K_S^0$ regeneration on nuclei and the coherent production model\*

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We use the coherent production model to calculate the energy dependence of the forward  $K_S^0$  regeneration amplitude on nuclear targets. The agreement with experiment is satisfactory.

In this note we wish to apply the coherent production model<sup>1</sup> to the analysis of the  $K_S^0$  regeneration reaction on nuclei,

$$K_L^0 A \rightarrow K_S^0 A, \quad (1)$$

where  $A$  is the mass number. The data for this reaction<sup>2,3</sup> for hydrogen, copper, and lead targets show two distinctive features:

(a) The forward differential cross sections,  $d\sigma/dt$ , are proportional to  $P_L^{-n_A}$  for  $2.5 \leq P_L \leq 7.5$  GeV/c, where  $P_L$  is the laboratory momentum of incoming  $K_L$ , and  $n_A$  is a constant for a fixed  $A$ . There is no apparent Pomeron exchange, as expected, since charge conjugation,  $c = -1$ , must be exchanged in the  $t$  channel.

(b) For each nuclear target, the regeneration

phases,  $\phi_A$ , are near  $-45^\circ$ ,  $|\phi_A + 45^\circ| \leq 15^\circ$ . The strong exchange degeneracy<sup>4</sup> assumption predicts  $-45^\circ$  for the regeneration phase of the nucleon target.

Since the regeneration from nucleons is a much weaker process than elastic scattering, i.e.,

$$\left. \frac{d\sigma_{K_L p \rightarrow K_S p}}{dt} / \frac{d\sigma_{K_L p \rightarrow K_L p}}{dt} \right|_{t=0} < 0.05$$

for hydrogen target in the momentum region of our interest, it may be reasonably assumed that  $K_L^0 \rightarrow K_S^0$  occurs at most once as the  $K_L^0$  (and  $K_S^0$ ) repeatedly scatter elastically on traversing the nucleus. Then the coherent nuclear regeneration amplitude from nucleus  $A$  is given by<sup>1,5</sup>

$$f_{K_L^0 A \rightarrow K_S^0 A}(q) = \frac{ik}{2\pi} \sum_{j=1}^A \int d^2b d^3r_1 \cdots d^3r_A |\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)|^2 e^{i\vec{q} \cdot \vec{b}} \\ \times \prod_{z_i < z_j} [1 - \Gamma_{K_L}(\vec{b} - \vec{S}_i)] \Gamma_{\text{Reg}}(\vec{b} - \vec{S}_j) \prod_{z_h > z_l} [1 - \Gamma_{K_S}(\vec{b} - \vec{S}_h)]. \quad (2)$$

where  $\psi$  is the target wave function,  $\vec{b}$  is the impact parameter of the incident particle, and  $\vec{S}_i$  is the impact parameter of the  $i$ th nucleon (the transverse part of  $\vec{r}_i$ ). It has been assumed that the nucleus is left in the initial state; by explicit calculation we have verified that the cross section for nuclear excitation is small in the forward direction. The  $\Gamma$ 's are profile functions which are defined in terms of the regeneration and elastic scattering amplitudes of the nucleon by

$$\Gamma_{\text{Reg}}(\vec{b}) = \frac{1}{2\pi ik} \int f_{K_L N \rightarrow K_S N}(\vec{q}) e^{-i\vec{q} \cdot \vec{b}} d^2q,$$

$$\Gamma_{K_{L,S}}(\vec{b}) = \frac{1}{2\pi ik} \int f_{K_{L,S} N \rightarrow K_{L,S} N}(\vec{q}) e^{-i\vec{q} \cdot \vec{b}} d^2q.$$

Since the  $K^0 N \rightarrow \bar{K}^0 N$  transition is much weaker than

the elastic  $K^0(\bar{K}^0)N \rightarrow K^0(\bar{K}^0)N$  reaction, we can safely approximate

$$f_{K_L N} \equiv f_{K_L N \rightarrow K_L N} \\ = f_{K_S N \rightarrow K_S N} \\ = \frac{1}{2} (f_{K^0 N \rightarrow K^0 N} + f_{\bar{K}^0 N \rightarrow \bar{K}^0 N}).$$

Therefore,

$$\Gamma_{K_L}(\vec{b}) = \Gamma_{K_S}(\vec{b}).$$

We approximate the nucleus wave function by neglecting correlations and using a product of single-nucleon density functions,

$$|\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)|^2 = \prod_{i=1}^A \rho(\vec{r}_i). \quad (3)$$

This assumption simplifies Eq. (2) to the form

$$\begin{aligned}
f_{K_L^+ A \rightarrow K_S^+ A}(\vec{q}) = & \left[ \left( \frac{N_p}{A} \right) f_{K_L^+ p \rightarrow K_S^+ p}(0) \right. \\
& \left. + \left( 1 - \frac{N_p}{A} \right) f_{K_L^+ n \rightarrow K_S^+ n}(0) \right] \\
& \times \int e^{i \vec{q} \cdot \vec{b}} T(\vec{b}) e^{-(1-i\alpha)\sigma T(\vec{b})/2} d^2 b,
\end{aligned} \quad (4)$$

assuming that the nuclear density  $\rho(\vec{S}, z)$  varies slowly compared with  $\Gamma(\vec{b} - \vec{S})$  as a function of  $S$ .  $N_p$  is the number of protons in the nucleus, where

$$\begin{aligned}
\sigma &= \left( \frac{N_p}{A} \right) \sigma_{K_L^+ p}^T + \left( 1 - \frac{N_p}{A} \right) \sigma_{K_L^+ n}^T, \\
\alpha &= \frac{(N_p/A) \operatorname{Re} f_{K_L^+ p}(0) + (1 - N_p/A) \operatorname{Re} f_{K_L^+ n}(0)}{(N_p/A) \operatorname{Im} f_{K_L^+ p}(0) + (1 - N_p/A) \operatorname{Im} f_{K_L^+ n}(0)},
\end{aligned}$$

and

$$T(\vec{b}) = A \int \rho(\vec{r}) dz.$$

The coherent regeneration cross section is then

$$\frac{d\sigma_A}{dt} = \left( \frac{d\sigma_N}{dt} \right)_0 \left| \int J_0(qb) T(b) e^{-(1-i\alpha)\sigma T(b)/2} b db \right|^2, \quad (5)$$

where we have defined an average nucleon regeneration forward cross section by

$$\left( \frac{d\sigma_N}{dt} \right)_0 = \left| \left( \frac{N_p}{A} \right) f_{K_L^+ p \rightarrow K_S^+ p}(0) + \left( 1 - \frac{N_p}{A} \right) f_{K_L^+ n \rightarrow K_S^+ n}(0) \right|^2. \quad (6)$$

In the numerical calculation we use a Woods-Saxon nuclear density for both proton and neutron:

$$\rho(b, z) = \rho_0 \left[ 1 + \exp\left(\frac{r-c}{a}\right) \right]^{-1}.$$

We use  $c = 1.20 A^{1/3}$  fm and  $a = 0.6$  fm. These nuclear parameters are larger<sup>6</sup> than those from electron scattering experiments due to the finite-range, strong interaction of  $K$  mesons.

The calculation requires the real parts of the nucleon regeneration amplitudes, which have not been well investigated experimentally in the energy range of interest to us. An alternative would be to use dispersion relations, but this would require fairly accurate high-energy total cross sections, which are also not available. We use instead the strong exchange-degeneracy hypothesis,<sup>4</sup> which gives, using the optical theorem,

$$\alpha = \frac{\frac{1}{2} [ - (N_p/A) C_p P_L^{-\beta_p} - (1 - N_p/A) C_n P_L^{-\beta_n} ]}{(N_p/A) \sigma_{K_L^+ p}^T + (1 - N_p/A) \sigma_{K_L^+ n}^T},$$

where we have parameterized  $KN$  total cross sec-

tions as constant and the  $\bar{K}N$  as

$$\sigma_{\bar{K}^0 N}^T = \sigma_{\bar{K}^0 N}^0 + C_N P_L^{-\beta_N},$$

and where  $P_L$  is in units of GeV/c.

The numerical values used are

$$\sigma_{K^0 p}^T = 17.6 \text{ mb},$$

$$\sigma_{K^0 n}^T = 17.7 \text{ mb},$$

$$\sigma_{\bar{K}^0 p}^T = (18.2 + 7.2 P_L^{-0.52}) \text{ mb},$$

and

$$\sigma_{\bar{K}^0 n}^T = (20.2 + 19.5 P_L^{-0.92}) \text{ mb},$$

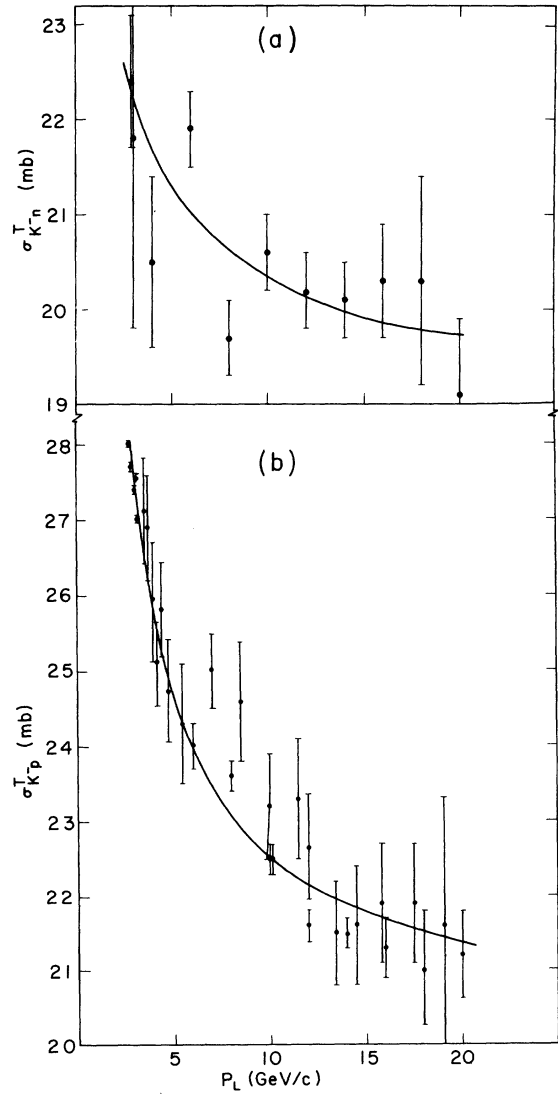


FIG. 1.  $K^-p$  and  $K^-n$  total cross sections (data points from Ref. 7), together with the best fit of the forms  $\sigma_{KN} = \sigma_N^0 + C_N P_L^{-\beta_N}$ . All data points between 2.5 GeV/c and 20.0 GeV/c are used to obtain the parameters, but only representative data points are shown here.

extracted from  $K^\pm$  scattering experiments<sup>7</sup> using charge symmetry. The fits involved in the latter two cross sections are shown in Fig. 1.

We first assumed the neutron and proton to have the same regeneration amplitude; then

$$\left(\frac{d\sigma_N}{dt}\right)_0 = \left(\frac{d\sigma_H}{dt}\right)_0 = 3.17 P_L^{-1.28},$$

which is shown in Fig. 2. Finally, we obtained the results shown (as dashed lines) in Fig. 3(a), which may be compared with the best fits to experiment of the form  $P_L^{-nA}$  (solid line). The agreement is, on the whole, satisfactory, although there is a tendency for the results of the calculation to fall too fast with  $P_L$ . This potential discrepancy may be an indication of a different energy dependence of the proton and neutron regeneration cross sections. The regeneration phases of nuclei  $\phi_A$  are plotted in Fig. 3(b).<sup>8</sup> Again the agreement is satisfactory, the difference between the nuclear regeneration phases and that of hydrogen is larger for heavier nuclei and goes to zero with increasing momentum. Note that the fact that the nuclear regeneration phases are nearly equal to that of hydrogen strongly justifies the assumption that proton and neutron regeneration phases are approximately equal. In addition, there is one experimental point in this momentum region for a carbon target<sup>9</sup> (which has no neutron excess) in which this calculation would disagree with the forward differential cross-section measurement by a factor of about 2, but would agree with the regeneration-phase measurement. Further experimental studies would be valuable in clarifying this situation.

The regeneration effect in Cu has also been successfully fitted using an optical model.<sup>10</sup> The difference between the optical model and this one-step coherent production model can be summarized in two integrands:

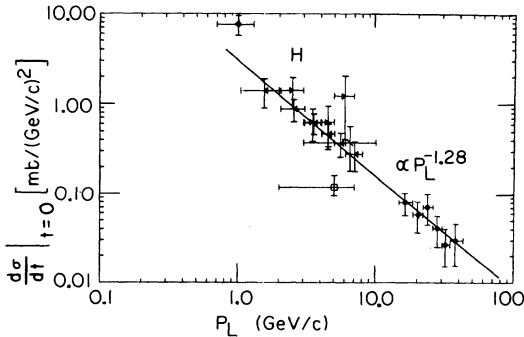


FIG. 2. Hydrogen regeneration cross section extracted from Ref. 3.

$$I^{\text{coh}} = i[\bar{P}(b) - P(b)] e^{i[P(b) + \bar{P}(b)]/2}, \quad (7)$$

$$I^{\text{op}} = e^{i\bar{P}(b)} - e^{iP(b)}, \quad (8)$$

where

$$P(b) = \frac{4\pi}{k} \left[ \left( \frac{N_p}{A} \right) f_{K_0 p}(0) + \left( 1 - \frac{N_p}{A} \right) f_{K_0 n}(0) \right] T(b),$$

and the  $I$ 's are normalized as

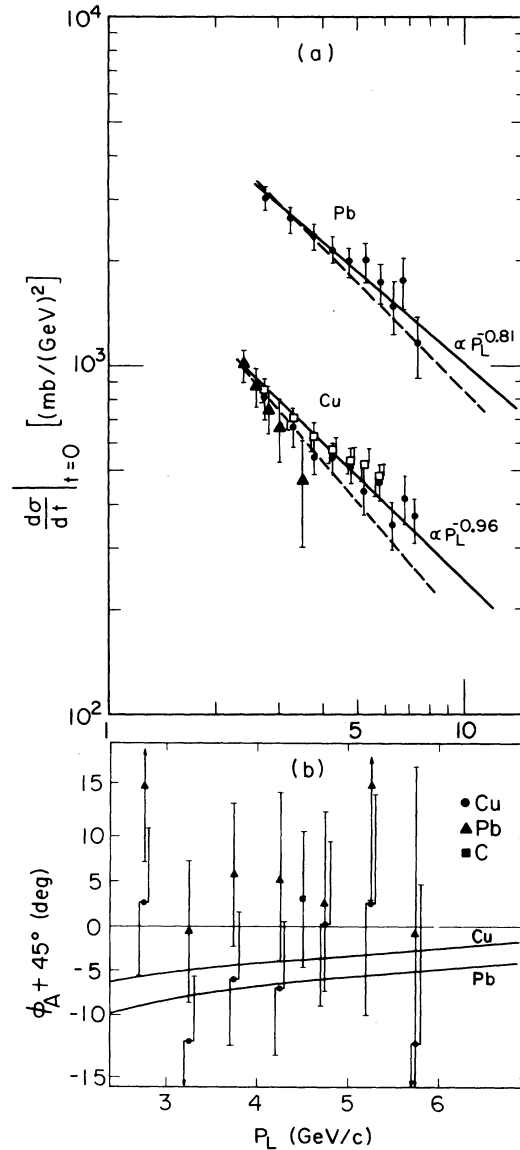


FIG. 3. (a) Nuclear regeneration cross sections. The experimental points are from Ref. 2; the solid line is the best fit to experiment of the form  $P_L^{-nA}$ , and the broken line is the result of the calculation. (b) Calculation of the nuclear regeneration phases. The points are from Ref. 2; the values of  $\phi_A - \phi_H$  were obtained using  $\phi_\pm = 42.0^\circ \pm 3^\circ$  (Ref. 8).

$$f_{\kappa_L p \rightarrow \kappa_S p}(q) = \frac{ik}{4\pi} \int I(b) J_0(qb) b db. \quad (9)$$

Equations (7) and (8) are easily verified to be equivalent if the difference  $\Delta P(b) = \bar{P}(b) - P(b)$  is smaller than both 1 and  $P(b)$  [and  $\bar{P}(b)$ ]. This is not always true for heavy nuclei at  $b \approx 0$  due to large  $T(b)$ . However, the extra  $b$  in the integrand lessens the difference. The detailed quantitative discussion of the difference is given in Ref. 6. The advantage of the present treatment is the clean separation of hadronic effects and nuclear effects shown in

Eq. (5). The first factor (*nucleon effects*) gives most of the energy dependence, while the second (*nuclear effects*) controls the angular distribution. In the optical model this factorization does not occur.

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## Deck-model calculation of $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4,44}^*$ †

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A Reggeized Deck model is used to calculate  $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4,44}^*$ , using a Monte Carlo approach where events are generated from the model and then analyzed as if they were experimental data. A partial-wave fit is made to the results of the calculation. The effect of absorption by the nuclear matter of the outgoing dipion system is estimated, and a brief effort is made to calculate  $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C$ , where the final state of C is not observed.

### I. INTRODUCTION

An experimental effort to elucidate  $3\pi$  production mechanisms by studying the reaction  $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C^*$ , where  $C^*$  is the  $I=0$ ,  $J^P=2^+$  excited state of the nucleus at 4.44 MeV, is underway at the University of Illinois.<sup>1</sup> Since calculations

using a Reggeized Deck model<sup>2,3</sup> have recently attained good agreement<sup>4</sup> with experiment in  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$ , it is natural to ask what part is played in  $3\pi$  production on carbon by the Deck effect. Accordingly, the computational techniques developed in Ref. 4 have been modified to calculate  $3\pi C^*$ , using a Monte Carlo approach. The effect